

JEE Main July 2022
Question Paper With Text Solution
27 July | Shift-1

MATHEMATICS



JEE Main & Advanced | XI-XII Foundation | VI-X Pre-Foundation

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JEE MAIN JULY 2022 | 27TH JULY SHIFT-1**SECTION - A**

Question ID : 1169401

Set & Relations

1. Let R_1 and R_2 be two relations defined on R by $a R_1 b \Leftrightarrow ab \geq 0$ and $a R_2 b \Leftrightarrow a \geq b$. Then :
माना R पर दो संबंध R_1 तथा R_2 , $a R_1 b \Leftrightarrow ab \geq 0$ तथा $a R_2 b \Leftrightarrow a \geq b$ द्वारा परिभाषित हैं। तो :

(1) R_1 is an equivalence relation but not R_2

R_1 एक तुल्यता संबंध है परन्तु R_2 नहीं है

(2) R_2 is an equivalence relation but not R_1

R_2 एक तुल्यता संबंध है परन्तु R_1 नहीं है

(3) Both R_1 and R_2 are equivalence relations

R_1 तथा R_2 दोनों तुल्यता संबंध हैं

(4) Neither R_1 nor R_2 is an equivalence relation

न तो R_1 न ही R_2 एक तुल्यता संबंध है

Ans. Official Answer NTA (4)

Sol. $R_1 = \{xy \geq 0, x, y \in R\}$

For reflexive $x \times x \geq 0$ which is true.

For symmetric

If $xy \geq 0 \Rightarrow yx \geq 0$

If $x = 2, y = 0$ and $z = -2$

Then $x.y \geq 0$ & $y.z \geq 0$ but $x.z \geq 0$ is not true

\Rightarrow not transitive relation.

$\Rightarrow R_1$ is not equivalence

R_2 if $a \geq b$ it does not implies $b \geq a$

$\Rightarrow R_2$ is not equivalence relation

$\Rightarrow D$

Question ID : 1169402

Function

2. Let $f, g : N - \{1\} \rightarrow N$ be functions defined by $f(a) = \alpha$, where α is the maximum of the powers of those primes p such that p^α divides a , and $g(a) = a + 1$, for all $a \in N - \{1\}$. Then, the function $f + g$ is :

माना $f, g : N - \{1\} \rightarrow N$, $f(a) = \alpha$, जहाँ उन अभाज्य संख्याओं p , जिनके लिए p^α , a को विभाजित करता है, की



घातों में α अधिकतम है तथा $g(a) = a + 1$ सभी $a \in \mathbb{N} - \{1\}$ के लिए द्वारा परिभाषित हैं। तो फलन $f + g$:

(1) one-one but not onto

एकैकी है परन्तु आच्छादक नहीं है

(2) onto one-one and onto

आच्छादक है परन्तु एकैकी नहीं है

(3) both one-one and onto

एकैकी तथा आच्छादक दोनों है

(4) neither one-one nor onto

न तो एकैकी है न ही आच्छादक

Ans. Official Answer NTA (4)

Sol. $f: \mathbb{N} - \{1\} \rightarrow \mathbb{N}$ $f(a) = \alpha$

Where α is max of powers of prime P such that p^α divides a. Also $g(a) = a + 1$

$$\therefore f(2) = 1 \quad g(2) = 3$$

$$f(3) = 1 \quad g(3) = 4$$

$$f(4) = 2 \quad g(4) = 5$$

$$f(5) = 1 \quad g(5) = 6$$

$$\Rightarrow f(2) + g(2) = 4$$

$$(f(3) + g(3)) = 5$$

$$f(4) + g(4) = 7$$

$$f(5) + g(5) = 7$$

\therefore Many one $f(x) + g(x)$ does not contain 1

\Rightarrow into function

\therefore Ans. (D) [neither one-one nor onto]

Question ID : 1169403

Complex Number

3. Let the minimum value v_0 of $v = |z|^2 + |z-3|^2 + |z-6i|^2$, $z \in \mathbb{C}$ is attained at $z = z_0$. Then $|2z_0^2 - \bar{z}_0^3 + 3|^2 + v_0^2$ is equal to :

माना v_0 of $v = |z|^2 + |z-3|^2 + |z-6i|^2$, $z \in \mathbb{C}$ का न्यूनतम मान $z = z_0$ पर प्राप्त होता है। तो $|2z_0^2 - \bar{z}_0^3 + 3|^2 + v_0^2$ बराबर है :

(1) 1000

(2) 1024

(3) 1105

(4) 1196

Ans. Official Answer NTA (1)

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Sol. $z_0 = \left(\frac{0+3+0}{3}, \frac{0+6+0}{3} \right) = (1, 2)$

$$v_0 = |1+2i|^2 + |1+2i-3|^2 + |1+2i+6i|^2 = 30$$

Then $\left| 2z_0^2 - \bar{z}_0 + 3 \right|^2 + v_0^2$

$$= \left| 2(1+2i)^2 - (1-2i) + 3 \right|^2 + 900$$

$$= \left| 2(1-4+4i) - (1-4-4i) + 3 \right|^2 + 900$$

$$= |8+6i|^2 + 900 = 100 + 900 = 1000$$

Question ID : 1169404

Matrices

4. Let $A = \begin{pmatrix} 1 & 2 \\ -2 & -5 \end{pmatrix}$. Let $\alpha, \beta \in \mathbb{R}$ be such that $\alpha A^2 + \beta A = 2I$. Then $\alpha + \beta$ is equal to :

माना $A = \begin{pmatrix} 1 & 2 \\ -2 & -5 \end{pmatrix}$ है तथा माना $\alpha, \beta \in \mathbb{R}$ इस प्रकार हैं कि $\alpha A^2 + \beta A = 2I$ है। तो $\alpha + \beta$ का मान है :

(1) -10

(2) -6

(3) 6

(4) 10

Ans. Official Answer NTA (4)

Sol. Characteristic equation of matrix A

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 2 \\ -2 & -5-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^2 + 4\lambda = 1$$

$$\Rightarrow A^2 + 4A = I$$

$$\Rightarrow 2A^2 + 8A = 2I$$

$$\Rightarrow 2A^2 + 8A = 2I \quad \dots (1)$$

$$\text{Given that } \alpha A^2 + \beta A = 2I \quad \dots (2)$$

Comparing equation (1) & (2) we get

$$\alpha = 2, \quad \beta = 8$$

$$\therefore \alpha + \beta = 10$$

Question ID : 1169405

**Binomial Theorem**

5. The remainder when $(2021)^{2022} + (2022)^{2021}$ is divided by 7 is :

$(2021)^{2022} + (2022)^{2021}$ को 7 से विभाजित करने पर शेषफल है :

- (1) 0 (2) 1 (3) 2 (4) 6

Ans. Official Answer NTA(1)

Sol. $(2021)^{2022} + (2022)^{2021}$
 $= (2023 - 2)^{2022} + (2023 - 1)^{2021}$
 $= 7n_1 + 2^{2022} + 7n_2 - 1$
 $= 7(n_1 + n_2) + 8^{674} - 1$
 $= 7(n_1 + n_2) + (7 - 1)^{674} - 1$
 $= 7(n_1 + n_2) + 7n_3 + 1 - 1$
 $= 7(n_1 + n_2 + n_3)$

∴ Given number is divisible by 7 hence remainder is zero

Question ID : 1169406

Sequence & progression

6. Suppose $a_1, a_2, \dots, a_n, \dots$ be an arithmetic progression of natural numbers. If the ratio of the sum of first five terms to the sum of first nine terms of the progression is $5 : 17$ and $110 < a_{15} < 120$, then the sum of the first terms of the progression is equal to :

माना $a_1, a_2, \dots, a_n, \dots$ वास्तविक संख्याओं की एक समांतर श्रेणी है। यदि इस श्रेणी के प्रथम पाँच पदों का योग का, नौ पदों के योग से अनुपात $5 : 17$ है तथा $110 < a_{15} < 120$ है, तो इस श्रेणी के प्रथम दस पदों का योग है :

- (1) 290 (2) 380 (3) 460 (4) 510

Ans. Official Answer NTA(2)

Sol. $\frac{S_5}{S_9} = \frac{5}{17} \Rightarrow \frac{\frac{5}{2}(2a + 4d)}{\frac{9}{2}(2a + 8d)} = \frac{5}{17}$

$\Rightarrow d = 4a$

$a_{15} = a + 14d = 57a$

Now, $110 < a_{15} < 120$

$\Rightarrow 110 < 57a < 120$

$\Rightarrow a = 2 \therefore d = 8$

$S_{10} = \frac{10}{2}(2 \times 2 + 9 \times 8) = 380$



Question ID : 1169407

Definite Integration

7. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined as $f(x) = a \sin\left(\frac{\pi[x]}{2}\right) + [2-x]$, $a \in \mathbb{R}$, where $[t]$ is the greatest integer

less than or equal to t if $\lim_{x \rightarrow -1} f(x)$ exists, then the value of $\int_0^4 f(x) dx$ is equal to :

माना एक फलन $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = a \sin\left(\frac{\pi[x]}{2}\right) + [2-x]$, $a \in \mathbb{R}$, द्वारा परिभाषित है, जहाँ $[t]$ महत्तम पूर्णांक \leq

t है। यदि $\lim_{x \rightarrow -1} f(x)$ का अस्तित्व है, तो $\int_0^4 f(x) dx$ का मान बराबर है :

- (1) -1 (2) -2 (3) 1 (4) 2

Ans. Official Answer NTA (2)

Sol. $\lim_{x \rightarrow -1^+} a \sin\left(\frac{\pi[x]}{2}\right) + [2-x] = -a + 2$

$\lim_{x \rightarrow -1^-} a \sin\left(\frac{\pi[x]}{2}\right) + [2-x] = 0 + 3 = 3$

$\lim_{x \rightarrow -1^-} f(x)$ exist when $a = -1$

Now,

$$\begin{aligned} \int_0^4 f(x) dx &= \int_0^1 f(x) dx + \int_1^2 f(x) dx + \int_2^3 f(x) dx + \int_3^4 f(x) dx \\ &= \int_0^1 (0+1) dx + \int_1^2 (-1+0) dx + \int_2^3 (0-1) dx + \int_3^4 (1-2) dx \\ &= 1 - 1 - 1 - 1 = -2 \end{aligned}$$

Question ID : 1169408

Definite Integration

8. Let $I = \int_{\pi/4}^{\pi/3} \left(\frac{8 \sin x - \sin 2x}{x}\right) dx$. Then :

माना $I = \int_{\pi/4}^{\pi/3} \left(\frac{8 \sin x - \sin 2x}{x}\right) dx$ है, तो :

- (1) $\frac{\pi}{2} < I < \frac{3\pi}{4}$ (2) $\frac{\pi}{4} < I < \frac{5\pi}{12}$ (3) $\frac{5\pi}{12} < I < \frac{\sqrt{2}}{3}\pi$ (4) $\frac{3\pi}{4} < I < \pi$

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Ans. Official Answer NTA (3)

Sol. Consider

$$f(x) = 8\sin x - \sin 2x$$

$$f'(x) = 8\cos x - 2\cos 2x$$

$$f''(x) = -8\sin x + 4\sin 2x$$

$$= -8\sin x (1 - \cos x)$$

$$\therefore f'(x) < 0 \quad x \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$$

$\therefore f'(x)$ is \downarrow function

$$f'\left(\frac{\pi}{3}\right) < f'(x) < f'\left(\frac{\pi}{4}\right)$$

$$5 < f'(x) < \frac{8}{\sqrt{2}}$$

$$5 < f'(x) < 4\sqrt{2}$$

$$5x < f(x) < 4\sqrt{2}x$$

$$5 < \frac{f(x)}{x} < 4\sqrt{2}$$

$$\int_{\pi/4}^{\pi/3} 5 < \int_{\pi/4}^{\pi/3} \frac{f(x)}{x} < \int_{\pi/4}^{\pi/3} 4\sqrt{2}$$

$$\int_{\pi/4}^{\pi/3} 5 < \int_{\pi/4}^{\pi/3} \frac{8\sin x - \sin 2x}{x} < \int_{\pi/4}^{\pi/3} 4\sqrt{2}$$

$$\frac{5\pi}{12} < I < \frac{\sqrt{2}\pi}{3}$$

Question ID : 1169409

Area Under Curve

9. The area of the smaller region enclosed by the curves $y^2 = 8x + 4$ and $x^2 + y^2 + 4\sqrt{3}x - 4 = 0$ is equal to :

वक्रों $y^2 = 8x + 4$ तथा $x^2 + y^2 + 4\sqrt{3}x - 4 = 0$ से घिरे छोटे क्षेत्र का क्षेत्रफल है :

(1) $\frac{1}{3}(2 - 12\sqrt{3} + 8\pi)$

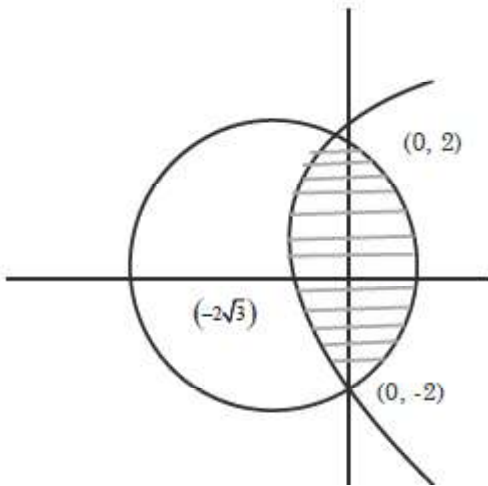
(2) $\frac{1}{3}(2 - 12\sqrt{3} + 6\pi)$

(3) $\frac{1}{3}(4 - 12\sqrt{3} + 8\pi)$

(4) $\frac{1}{3}(4 - 12\sqrt{3} + 6\pi)$



Ans. Official Answer NTA (3)



Sol.

$$x^2 + y^2 + 4\sqrt{3}x - 4 = 0$$

$$y^2 = 8x + 4$$

Point of intersections are $(0, 2)$ & $(0, -2)$

Both are symmetric about x-axis

$$\text{Area} = 2 \int_0^2 \left(\sqrt{16 - y^2} - 2\sqrt{3} \right) - \left(\frac{y^2 - 4}{8} \right) dy$$

$$\text{On solving Area} = \frac{1}{3} [8\pi + 4 - 12\sqrt{3}]$$

Question ID : 11694010

Differential Equation

10. Let $y = y_1(x)$ and $y = y_2(x)$ be two distinct solutions of the differential equation $\frac{dy}{dx} = x + y$, with $y_1(0) = 0$ and $y_2(0) = 1$ respectively. Then, the number of points of intersection of $y = y_1(x)$ and $y = y_2(x)$ is :

माना अवकल सीमकरण $\frac{dy}{dx} = x + y$ के $y_1(0) = 0$ तथा $y_2(0) = 1$ के लिए दो भिन्न हल क्रमशः $y = y_1(x)$ तथा

$y = y_2(x)$ के प्रतिच्छेदन बिन्दुओं की संख्या है :

(1) 0

(2) 1

(3) 2

(4) 3

Ans. Official Answer NTA (1)

Sol. $\frac{dy}{dx} = x + y \Rightarrow \frac{dy}{dx} - y = x$

$$I f = e^{-x}$$



$$\therefore \text{solution is } ye^{-x} = \int xe^{-x} dx$$

$$\Rightarrow ye^{-x} = -xe^{-x} - e^{-x} + c$$

$$\Rightarrow y = -x - 1 + ce^x$$

$$y_1(0) = 0 \Rightarrow c = 1$$

$$\therefore y_1 = -x - 1 + e^x \quad \dots\dots(1)$$

$$y_2(0) = 1 \Rightarrow c = 2$$

$$\therefore y_2 = -x - 1 + 2e^x \quad \dots\dots(2)$$

$$\text{Now } y_2 - y_1 = e^x > 0 \therefore y_2 \neq y_1$$

\therefore Number of points of intersection of y_1 & y_2 is zero.

Question ID : 11694011

Parabola

11. Let $P(a, b)$ be a point on the parabola $y^2 = 8x$ such that the tangent at P passes through the centre of the circle $x^2 + y^2 - 10x - 14y + 65 = 0$. Let A be the product of all possible values of a and B be the product of all possible values of b . Then the value of $A + B$ is equal to :

माना परवलय $y^2 = 8x$ के बिन्दु $P(a, b)$ पर स्पर्श रेखा, वृत्त $x^2 + y^2 - 10x - 14y + 65 = 0$ के केन्द्र से होकर जाती है। माना a के सभी संभव मानों का गुणनफल A है तथा b के सभी संभव मानों का गुणनफल B है। तो $A + B$ का मान है :

- (1) 0 (2) 25 (3) 40 (4) 65

Ans. Official Answer NTA (4)

Sol. $P(a, b)$ is point on $y^2 = 8x$, such that tangent at P pass through centre of $x^2 + y^2 - 10x - 14y + 65 = 0$ i.e. (5, 7)

Tangent at $P(at^2, 2at)$ is $ty = x + at^2$

$A = 2$ & it pass through (5, 7)

$$7t = 5 + 2t^2$$

$$\Rightarrow t = 1, t = \frac{5}{2}$$

$$\therefore P(at^2, 2at) \Rightarrow (2, 4) \text{ when } t = 1$$

$$\& \left(\frac{25}{2}, 10 \right) \text{ when } t = \frac{5}{2}$$

$$\therefore A = 2 \times \frac{25}{2} = 25$$



$$B = 4 \times 10 = 40 \quad \therefore A + B = 65$$

Question ID : 11694012

Vectors

12. Let $\vec{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}$ and $\vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$ be two vectors, such that $\vec{a} \times \vec{b} = -\hat{i} + 9\hat{j} + 12\hat{k}$. Then the projection of $\vec{b} - 2\vec{a}$ on $\vec{b} + \vec{a}$ is equal to :

माना दो सदिशों $\vec{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}$ तथा $\vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$ के लिए $\vec{a} \times \vec{b} = -\hat{i} + 9\hat{j} + 12\hat{k}$ है, तो $\vec{b} - 2\vec{a}$ का $\vec{b} + \vec{a}$ पर प्रक्षेप बराबर है :

(1) 2

(2) $\frac{39}{5}$

(3) 9

(4) $\frac{46}{5}$

Ans. Official Answer NTA (4)

Sol. Let $\vec{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}$, $\vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$

$$\vec{a} \times \vec{b} = -\hat{i} + 9\hat{j} + 12\hat{k}$$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 1 & \beta \\ 3 & -5 & 4 \end{vmatrix}$$

$$\Rightarrow (4 + 5\beta)\hat{i} + (3\beta - 4\alpha)\hat{j} + (-5\alpha - 3)\hat{k}$$

$$= -\hat{i} + 9\hat{j} + 12\hat{k}$$

$$\therefore 4 + 5\beta = -1, 3\beta - 4\alpha = 9, -5\alpha - 3 = 12$$

$$\beta = -1, \alpha = -3$$

$$\therefore \vec{a} = -3\hat{i} + \hat{j} - \hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$$

$$\therefore \vec{a} + \vec{b} = -4\hat{j} + 3\hat{k}$$

$$|\vec{a}|^2 = 11, |\vec{b}|^2 = 50$$

$$\vec{a} \cdot \vec{b} = -9 + (-5) - 4 = -18$$

\therefore Projection of $(2\vec{b} - 2\vec{a})$ on $\vec{a} + \vec{b}$ is

$$\frac{(\vec{b} - 2\vec{a}) \cdot (\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|}$$



$$= \frac{|\vec{b}|^2 - 2|\vec{a}|^2 - (\vec{a} \cdot \vec{b})}{|\vec{a} + \vec{b}|} = \frac{50 - 22 - (-18)}{5} = \frac{46}{5}$$

Ans. $\left(\frac{46}{5}\right)$

Question ID : 11694013

Vectors

13. Let $\vec{a} = 2\hat{i} - \hat{j} + 5\hat{k}$ and $\vec{b} = \alpha\hat{i} + \beta\hat{j} + 2\hat{k}$. If $((\vec{a} \times \vec{b}) \times \hat{i}) \cdot \hat{k} = \frac{23}{2}$, then $|\vec{b} \times 2\hat{j}|$ is equal to :

माना $\vec{a} = 2\hat{i} - \hat{j} + 5\hat{k}$ तथा $\vec{b} = \alpha\hat{i} + \beta\hat{j} + 2\hat{k}$ हैं। यदि $((\vec{a} \times \vec{b}) \times \hat{i}) \cdot \hat{k} = \frac{23}{2}$ है, तो $|\vec{b} \times 2\hat{j}|$ बराबर है:

(1) 4

(2) 5

(3) $\sqrt{21}$

(4) $\sqrt{17}$

Ans. Official Answer NTA (2)

Sol. $\vec{a} = 2\hat{i} - \hat{j} + 5\hat{k}$, $\vec{b} = \alpha\hat{i} + \beta\hat{j} + 2\hat{k}$

$$((\vec{a} \times \vec{b}) \times \hat{i}) \cdot \hat{k} = \frac{23}{2}, \text{ then } |\vec{b} \times 2\hat{j}| \text{ is}$$

$$((\vec{a} \cdot \hat{i})\vec{b} - (\vec{b} \cdot \hat{i})\vec{a}) \cdot \hat{k} = \frac{23}{2}$$

$$(\vec{a} \cdot \hat{i})(\vec{b} \cdot \hat{i}) - (\vec{b} \cdot \hat{i})(\vec{a} \cdot \hat{k}) = \frac{23}{2}$$

$$2 \times 2 - \alpha \times 5 = \frac{23}{2} \Rightarrow 5\alpha = 4 - \frac{23}{2} \Rightarrow \alpha = \frac{-3}{2}$$

$$\vec{b} \times 2\hat{j} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & \beta & 2 \\ 0 & 2 & 0 \end{vmatrix} = -4\hat{i} + 2\alpha\hat{k}$$

$$\therefore |\vec{b} \times 2\hat{j}| = \sqrt{16 + 4\alpha^2} = \sqrt{16 + 4 \times \frac{9}{4}} = 5$$

Question ID : 11694014

Probability

14. Let S be the sample space of all five digit numbers. It p is the probability that a randomly selected number from

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S, is multiple of 7 but not divisible by 5, then $9p$ is equal to :

माना पाँच अंकों की सभी संख्याओं की प्रतिदर्श समष्टि S में से यह एक यादृच्छया चुनी गई संख्या 7 का गुणज होने तथा 5 से विभाज्य न होने की प्रायिकता p है, तो $9p$ बराबर है :

- (1) 1.0146 (2) 1.2085 (3) 1.0285 (4) 1.1521

Ans. Official Answer NTA(3)

Sol. $n(S) = \text{all 5 digit nos} = 9 \times 10^4$

A : no is multiple of 7 but not divisible by 5

Smallest 5 digit divisible by 7 is 10003

Largest 5 digit divisible by 7 is 99995

$$\therefore 99995 = 10003 + (n - 1) 7 \quad n = 12857$$

Numbers divisible by 35

$$99995 = 10010 + (P-1) 35 \quad P = 2572$$

\therefore Numbers divisible by 7 but not by 35 are

$$12857 - 2572 = 10285$$

$$\therefore P = \frac{10285}{90000} \quad \therefore 9P = 1.0285$$

Ans. (C) [1.0285]

Question ID : 11694015

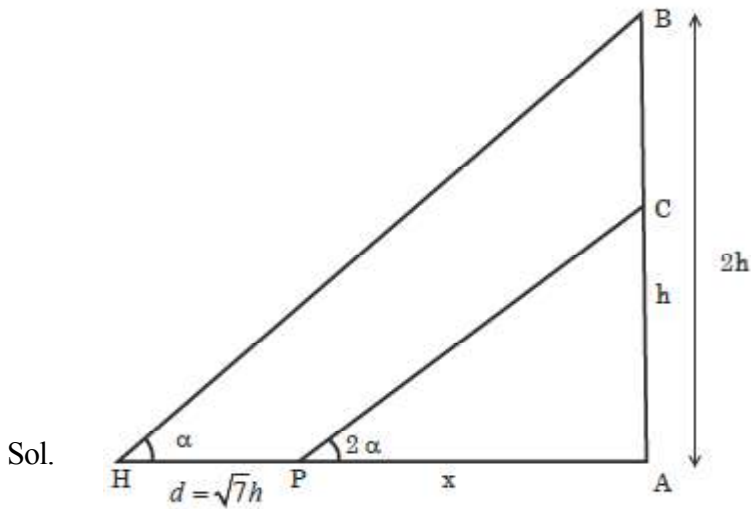
Heights & Distances

15. Let a vertical tower AB of height $2h$ stand on a horizontal ground. Let from a point P on the ground a man can see upto height h of the tower with an angle of elevation 2α . When from P, he moves a distance d in the direction of \overline{AP} , he can see the top B of the tower with an angle of elevation α . If $d = \sqrt{7} h$, then $\tan \alpha$ is equal to :

माना $2h$ ऊँचाई का एक ऊर्ध्वाधर टावर AB एक क्षैतिज धरातल पर खड़ा है। धरातल के एक बिन्दु P से एक पुरुष टावर को h ऊँचाई तक उन्नयन कोण 2α के साथ देख सकता है। जब वह P से \overline{AP} की दिशा में d दूरी तक जाता है, तो वह टावर का शिखर B को उन्नयन कोण α के साथ देख सकता है। यदि $d = \sqrt{7} h$ तो $\tan \alpha$ बराबर है :

- (1) $\sqrt{5} - 2$ (2) $\sqrt{3} - 1$ (3) $\sqrt{7} - 2$ (4) $\sqrt{7} - \sqrt{3}$

Ans. Official Answer NTA(3)



$$\tan 2\alpha = \frac{h}{x}$$

$$\text{and } \tan \alpha = \frac{2h}{x + \sqrt{7}h}$$

$$\tan \alpha = \frac{2h}{h \cot 2\alpha + \sqrt{7}h}$$

$$\tan \alpha = \frac{2}{\frac{(1 - \tan^2 \alpha)}{2 \tan \alpha} + \sqrt{7}}$$

Put $\tan \alpha = t$ & simplify

$$\Rightarrow \tan \alpha = \sqrt{7} - 2$$

Question ID : 11694016

Mathematical Reasoning

16. $(p \wedge r) \Leftrightarrow (p \wedge (\sim q))$ is equivalent to $(\sim p)$ when r is :

$(p \wedge r) \Leftrightarrow (p \wedge (\sim q))$, $(\sim p)$ के तुल्य है, जब r है :

- (1) p (2) $\sim q$ (3) q (4) $\sim q$

Ans. Official Answer NTA (3)

Sol. Given $(p \wedge r) \Leftrightarrow (p \wedge (\sim q)) \equiv (\sim p)$

Taking $r = q$



p	q	$\sim p$	$\sim q$	$p \wedge q$	$p \wedge \sim q$	$(p \wedge r) \Leftrightarrow (p \wedge (\sim q))$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	F	T
F	F	T	T	F	F	T

So, clear $(p \wedge r) \Leftrightarrow (p \wedge (\sim q)) \equiv (\sim p)$

Question ID : 11694017

3D Geometry

17. If the plane P passes through the intersection of two mutually perpendicular planes $2x + ky - 5z = 1$ and $3kx - ky + z = 5$, $k < 3$ and intercepts a unit length on positive x-axis, then the intercept made by the plane P on the y-axis is :

यदि दो परस्पर लंब समतलों $2x + ky - 5z = 1$ तथा $3kx - ky + z = 5$, $k < 3$ के प्रतिच्छेदन से होकर जाने वाले समतल P धनात्मक x-अक्ष पर अंतःखण्ड लंबाई एक है, तो P का y-अक्ष पर अंतः खण्ड है :

- (1) $\frac{1}{11}$ (2) $\frac{5}{11}$ (3) 6 (4) 7

Ans. Official Answer NTA (4)

Sol. Two given planes mutually perpendicular

$$2(3k) + k(-k) + (-5)1 = 0$$

$$k = 1, 5$$

$$\text{but } k < 3 \quad \text{So } k = 1$$

Plane passing through these planes is

$$2x + y - 5z - 1 + \lambda(3x - y + z - 5) = 0$$

$$\frac{x}{\frac{5\lambda+1}{2+3\lambda}} + \frac{y}{\frac{5\lambda+1}{1-\lambda}} + \frac{z}{\frac{5\lambda+1}{\lambda-5}} = 1$$

$$\text{Given } \frac{5\lambda+1}{2+3\lambda} = 1 \Rightarrow \lambda = \frac{1}{2}$$

$$\text{So intercept on y-axis is } = \frac{5\lambda+1}{1-\lambda} = 7$$

Question ID : 11694018

Straight Line

18. Let A(1, 1), B(-4, 3), C(-2, -5) be vertices of a triangle ABC, P be a point on side BC, and Δ_1 and Δ_2 be the

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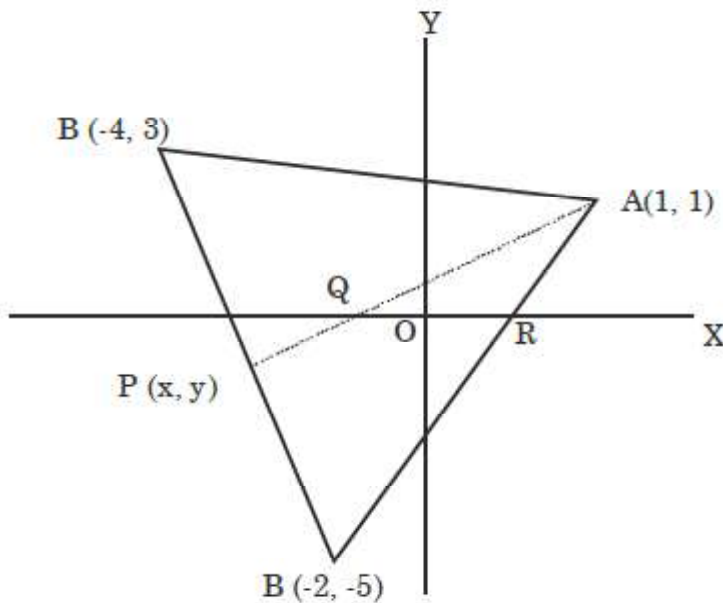
areas of triangle APB and ABC, respectively. If $\Delta_1 : \Delta_2 = 4 : 7$ then the area enclosed by the lines AP, AC and the x-axis is :

माना एक त्रिभुज ABC के शीर्ष $A(1, 1), B(-4, 3), C(-2, -5)$ हैं, भुजा BC पर एक बिन्दु P है तथा त्रिभुजों APB तथा ABC के क्षेत्रफल क्रमशः Δ_1 तथा Δ_2 हैं। यदि $\Delta_1 : \Delta_2 = 4 : 7$ हैं, तो रेखाओं AP, AC और x-अक्ष से घिरे क्षेत्र का क्षेत्रफल है :

- (1) $\frac{1}{4}$ (2) $\frac{4}{3}$ (3) $\frac{1}{2}$ (4) 1

Ans. Official Answer NTA (3)

Sol.



$$\text{Given } \Delta_1 = \frac{1}{2} \begin{vmatrix} x & y & 1 \\ 1 & 1 & 1 \\ -4 & 3 & 1 \end{vmatrix}$$

$$\& \Delta_2 = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ -4 & 3 & 1 \\ -2 & -5 & 1 \end{vmatrix}$$

$$\text{Given } \frac{\Delta_1}{\Delta_2} = \frac{4}{7} \Rightarrow \frac{-2x - 5y + 7}{36} = \frac{4}{7}$$

$$\Rightarrow 14x + 35y = -95 \quad \dots(1)$$

$$\text{Equation of BC is } 4x + y = -13 \quad \dots(2)$$

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Solve equation (1) & (2)

Point P $\left(\frac{-20}{7}, \frac{-11}{7}\right)$

Here point Q $\left(\frac{-1}{2}, 0\right)$ & R $\left(\frac{1}{2}, 0\right)$

So Area of triangle AQR = $\frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

Question ID : 11694019

Circle

19. If the circle $x^2 + y^2 - 2gx + 6y - 19c = 0$, $g, c \in \mathbb{R}$ passes through the point $(6, 1)$ and its centre lies on the line $x - 2cy = 8$, then the length of intercept made by the circle on x-axis is :

यदि वृत्त $x^2 + y^2 - 2gx + 6y - 19c = 0$, $g, c \in \mathbb{R}$ बिन्दु $(6, 1)$ से होकर जाता है तथा इसका केन्द्र रेखा $x - 2cy = 8$ पर है, तो वृत्त द्वारा x-अक्ष पर बनाए गए अंतःखंड की लंबाई :

- (1) $\sqrt{11}$ (2) 4 (3) 3 (4) $2\sqrt{23}$

Ans. Official Answer NTA (4)

Sol. Given circle $x^2 + y^2 - 2gx + 6y - 19c = 0$ Passes through $(6, 1)$

$12g + 19c = 43 \quad \dots(1)$

Centre $(g, -3)$ lies on given line

$So, g + 6c = 8 \quad \dots(2)$

Solve equation (1) & (2)

$c = 1 \text{ \& } g = 2$

equation of circle $x^2 + y^2 - 4x + 6y - 19 = 0$

Length of intercept on x-axis

$= 2\sqrt{g^2 - c} = 2\sqrt{23}$

Question ID : 11694020

Monotonicity20. Let a function $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as :माना एक फलन $f: \mathbb{R} \rightarrow \mathbb{R}$



$$f(x) = \begin{cases} \int_0^x (5 - |t-3|) dt & , x > 4 \\ x^2 + bx & , x \leq 4 \end{cases}$$

where be $b \in \mathbb{R}$. If f is continuous at $x=4$, then which of the following statements is NOT true ?

जहाँ $b \in \mathbb{R}$ है, द्वारा परिभाषित है। यदि $x=4$ पर f संतत है, तो निम्न में से कौनसा कथन सत्य नहीं है ?

(1) f is not differentiable at $x=4$

$x=4$ पर f अवकलनीय नहीं है

(2) $f'(3) + f'(5) = \frac{35}{4}$

$f'(3) + f'(5) = \frac{35}{4}$ है

(3) f is increasing in $\left(-\infty, \frac{1}{8}\right) \cup (8, \infty)$

$\left(-\infty, \frac{1}{8}\right) \cup (8, \infty)$ में f वर्धमान है

(4) f has a local minima at $x = \frac{1}{8}$

$x = \frac{1}{8}$, f का एक स्थानीय निम्नतम बिन्दु है

Ans. Official Answer NTA (3)

Sol. Given $f(x) = \begin{cases} \int_0^x (5 - |t-3|) dt, & x > 4 \\ x^2 + bx & , x \leq 4 \end{cases}$

$f(x)$ is continuous at $x=4$

So $\lim_{x \rightarrow 4^-} f(x) = \lim_{x \rightarrow 4^+} f(x) = f(4)$

So $16 + 4b = \int_0^3 (2-t) dt + \int_3^4 (8-t) dt$

$\Rightarrow 16 + 4b = 15$

So $b = \frac{-1}{4}$

At $x=4$



$$\text{LHD} = 2x + b = \frac{31}{4}$$

$$\text{RHD} = 5 - |x - 3| = 4$$

LHD \neq RHD

Option (A) is true

$$\text{and } f'(3) + f'(5) = \frac{23}{4} + 3 = \frac{35}{4}$$

Option (B) is true

$$\therefore f(x) = x^2 - \frac{x}{4} \text{ at } x \leq 4$$

$$f'(x) = 2x - \frac{1}{4}$$

This function is not increasing.

In the interval in $x \in \left(-\infty, \frac{1}{8}\right)$

Option (C) is NOT TRUE.

This function $f(x)$ is also local minima at $x = \frac{1}{8}$

SECTION - B

Question ID : 11694021

ITF

21. For $k \in \mathbb{R}$, let the solutions of the equation $\cos(\sin^{-1}(x \cot(\tan^{-1}(\cos(\sin^{-1} x)))))) = k$, $0 < |x| < \frac{1}{\sqrt{2}}$ be α and β , where the inverse trigonometric functions take only principal values. If the solutions of the equation $x^2 - bx - 5 = 0$ are $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ and $\frac{\alpha}{\beta}$, then $\frac{b}{k^2}$ is equal to _____.

माना $k \in \mathbb{R}$ के लिए समीकरण $\cos(\sin^{-1}(x \cot(\tan^{-1}(\cos(\sin^{-1} x)))))) = k$, $0 < |x| < \frac{1}{\sqrt{2}}$ के हल α तथा β हैं, जहाँ प्रतिलोम त्रिकोणमितीय फलन केवल मुख्य मान लेते हैं। यदि समीकरण $x^2 - bx - 5 = 0$ के हल $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ तथा $\frac{\alpha}{\beta}$ है, तो $\frac{b}{k^2}$ बराबर है _____.

Ans. Official Answer NTA (12)

Sol. $\cos(\sin^{-1} x) = \cos(\cos^{-1} \sqrt{1-x^2}) = \sqrt{1-x^2}$

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$$\cot(\tan^{-1} 1 - x^2) = \cot \cot^{-1} \left(\sqrt{\frac{1}{\sqrt{1-x^2}}} \right) = \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \cos \left(\sin^{-1} \left(\frac{x}{1-x^2} \right) \right) = \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}} = k$$

$$\Rightarrow 1-2x^2 = k^2(1-x^2)$$

$$\Rightarrow (k^2-2)x^2 = k^2-1$$

$$x^2 = \frac{k^2-1}{k^2-2}$$

$$\alpha = \sqrt{\frac{k^2-1}{k^2-2}} \Rightarrow \alpha = \frac{k^2-1}{k^2-2}$$

$$\beta = \sqrt{\frac{k^2-1}{k^2-2}} \Rightarrow \beta^2 = \frac{k^2-1}{k^2-2}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = 2 \left(\frac{k^2-2}{k^2-1} \right) \& \frac{\alpha}{\beta} = -1$$

$$\text{Sum of roots} = \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{\alpha}{\beta} = b$$

$$\Rightarrow \frac{2(k^2-2)}{k^2-1} - 1 = b \dots (1)$$

$$\text{Products of roots} = \left(\frac{1}{\alpha^2} + \frac{1}{\beta^2} \right) \frac{\alpha}{\beta} = -5$$

$$\Rightarrow \frac{2(k^2-2)}{k^2-1} (-1) = -5$$

$$\Rightarrow 2k^2 - 4 = 5k^2 - 5$$

$$\Rightarrow 3k^2 = 1 \Rightarrow k^2 = \frac{1}{3} \dots \text{Put in (1)}$$

$$\Rightarrow b = \frac{2(k^2-2)}{k^2-1} - 1 = 5 - 1 = 4$$



$$\frac{b}{k^2} = \frac{4}{\frac{1}{3}} = 12$$

Question ID : 11694022

Statistics

22. The mean and variance of 10 observations were calculated as 15 and 15 respectively by a student who took by mistake 25 instead of 15 for one observation. Then, the correct standard deviation is _____.

एक विद्यार्थी द्वारा 10 प्रेक्षणों के माध्य तथा प्रसरण क्रमशः 15 तथा 15 निकाले गए। विद्यार्थी ने एक परीक्षण 15 को गलती से 25 लिया। तो सही मानक विचलन है _____.

Ans. Official Answer NTA (2)

Sol. $n = 10, \bar{x} = \frac{\sum x_i}{10} = 15$

$$6^2 = \frac{\sum x_i^2}{10} - (\bar{x})^2 = 15$$

$$\Rightarrow \sum_{i=1}^{10} x_i = 150$$

$$\Rightarrow \sum_{i=1}^9 x_i + 25 = 150$$

$$\Rightarrow \sum_{i=1}^9 x_i = 125$$

$$\Rightarrow \sum_{i=1}^9 x_i + 15 = 140$$

$$\text{Actual mean} = \frac{140}{10} = 14 = \bar{x}_{\text{new}}$$

$$\sum_{i=1}^9 \frac{x_i^2 + 25^2 - 15^2}{10} = 15$$

$$\Rightarrow \sum_{i=1}^9 x_i^2 + 625 = 2400$$

$$\sum_{i=1}^9 x_i^2 = 1775$$

$$\sum_{i=1}^9 x_i^2 + 15^2 = 2000 = (\sum x_i^2)_{\text{actual}}$$

$$6_{\text{actual}}^2 = \frac{(\sum x_i^2)_{\text{actual}} - (\bar{x}_{\text{new}})^2}{10}$$

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$$= \frac{2000}{10} - 14^2$$

$$= 200 - 196 = 4$$

$$(\text{S.D})_{\text{actual}} = 6 = 2$$

Question ID : 11694023

3D Geometry

23. Let the line $\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z-3}{-4}$ intersect the plane containing the lines $\frac{x-4}{1} = \frac{y+1}{-2} = \frac{z}{1}$ and

$4ax - y + 5z - 7a = 0 = 2x - 5y - z - 3$, $a \in \mathbb{R}$ at the point $P(\alpha, \beta, \gamma)$. Then the value of $\alpha + \beta + \gamma$ equals _____.

माना रेखा $\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z-3}{-4}$ एक समतल, जिसमें रेखाएँ $\frac{x-4}{1} = \frac{y+1}{-2} = \frac{z}{1}$ तथा

$4ax - y + 5z - 7a = 0 = 2x - 5y - z - 3$, $a \in \mathbb{R}$ स्थित हैं, तो बिन्दु $P(\alpha, \beta, \gamma)$ पर काटती है। तो $\alpha + \beta + \gamma$ का मान है _____.

Ans. Official Answer NTA (12)

Sol. Equation of plane

$$4ax - y + 5z - 7a + \lambda (2x - 5y - z - 3) = 0$$

this satisfy $(4, -1, 0)$

$$16a + 1 - 7a + \lambda (8 + 5 - 3) = 0$$

$$9a + 1 + 10\lambda = 0 \quad \dots(1)$$

Normal vector of the plane A is $(4a + 2\lambda, -1 - 5\lambda, 5 - \lambda)$ vector along the line which contained the plane A is $i - 2j + k$

$$\therefore 4a + 2\lambda + 2 + 10\lambda + 5 - \lambda = 0$$

$$11\lambda + 4a + 7 = 0 \quad \dots(2)$$

Solve (1) and (2) to get $a = 1, \lambda = -1$

Now equation of plane

$$x + 2y + 3z - 2 = 0$$

Let the point in the line $\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z-3}{-4} = t$

is $(7t + 3, -t + 2, -4t + 3)$ satisfy the equation of plane A

$$7t + 3 - 2t + 4 + 9 - 12t - 2 = 0$$

$$t = 2$$

$$\text{So } \alpha + \beta + \gamma = 2t + 8 = 12$$

Question ID : 11694024

Hyperbola**MATRIX JEE ACADEMY**

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24. An ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through the vertices of the hyperbola $H: \frac{x^2}{49} - \frac{y^2}{64} = -1$. Let the major and minor axes of the ellipse E coincide with the transverse and conjugate axes of the hyperbola H . Let the product of the eccentricities of E and H be $\frac{1}{2}$. If l is the length of the latus rectum of the ellipse E , then the value of $113l$ is equal to _____.

एक दीर्घवृत्त $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, अतिपरवलय $H: \frac{x^2}{49} - \frac{y^2}{64} = -1$ के शीर्षों से होकर जाता है। माना दीर्घवृत्त E के दीर्घ तथा लघु अक्ष क्रमशः अतिपरवलय H के अनुप्रस्थ तथा संयुग्मी अक्ष के सम्पाती हैं। माना E तथा H की उत्केन्द्रताओं का गुणनफल $\frac{1}{2}$ है। यदि दीर्घवृत्त E की नाभिलंब जीवा की लंबाई l है, तो $113l$ का मान है _____.

Ans. Official Answer NTA (1552)

Sol. Hyp: $\frac{y^2}{64} - \frac{x^2}{49} = 1$

An ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through the vertices of the hyperbola $H: \frac{x^2}{49} - \frac{y^2}{64} = -1$.

So $b^2 = 64$

$$e_H = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{1 + \frac{49}{64}}$$

Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$e_E = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{a^2}{64}}$$

$$b = 8, \sqrt{\frac{1-a^2}{64}} \times \frac{\sqrt{113}}{8} = \frac{1}{2} \Rightarrow \sqrt{64-a^2} \times \sqrt{113} = 32$$

$$(64 - a^2) = \frac{32^2}{113}$$

$$\Rightarrow a^2 = 64 - \frac{32^2}{113}$$

$$l = \frac{2a^2}{b} = \frac{2}{8} \left(64 - \frac{32^2}{113} \right) = \frac{1552}{113}$$

$$113l = 1552$$

Question ID : 11694025

**Differential Equation**

25. Let $y = y(x)$ be the solution curve of the differential equation

$$\sin(2x^2) \log_e(\tan x^2) dy + \left(4xy - 4\sqrt{2}x \sin\left(x^2 - \frac{\pi}{4}\right)\right) dx = 0, 0 < x < \sqrt{\frac{\pi}{2}}, \text{ which passes through the point}$$

$$\left(\sqrt{\frac{\pi}{6}}, 1\right). \text{ then } \left|y\left(\sqrt{\frac{\pi}{3}}\right)\right| \text{ is equal to } \underline{\hspace{2cm}}.$$

माना अवकल समीकरण $\sin(2x^2) \log_e(\tan x^2) dy + \left(4xy - 4\sqrt{2}x \sin\left(x^2 - \frac{\pi}{4}\right)\right) dx = 0, 0 < x < \sqrt{\frac{\pi}{2}}$ का हल

वक्र $y = y(x)$, बिन्दु $\left(\sqrt{\frac{\pi}{6}}, 1\right)$ से होकर जाता है। तो $\left|y\left(\sqrt{\frac{\pi}{3}}\right)\right|$ बराबर है $\underline{\hspace{2cm}}$.

Ans. Official Answer NTA (1)

Sol. $\sin(2x^2) \ln(\tan x^2) dy + \left(4xy - \sqrt{2}x \sin\left(x^2 - \frac{\pi}{4}\right)\right) dx = 0$

$$\ln(\tan x^2) dy + \frac{4xy dx}{\sin(2x^2)} - \frac{4\sqrt{2}x \sin\left(x^2 - \frac{\pi}{4}\right)}{\sin(2x^2)} dx = 0$$

$$d(y \ln(\tan x^2)) - 4\sqrt{2}x \frac{(\sin x^2 - \cos x^2)}{\sqrt{2} - 2 \sin x^2 \cos x^2} dx = 0$$

$$d(y \ln(\tan x^2)) - \frac{4x(\sin x^2 - \cos x^2)}{(\sin x^2 + \cos^2) - 1} dx = 0$$

$$\Rightarrow \int d(y \ln(\tan x^2)) + 2 \int \frac{dt}{t^2 - 1} = \int 0$$

$$\Rightarrow y \ln(\tan x^2) + 2 \cdot \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| = c$$

$$y \ln(\tan x^2) + \ln \left(\frac{\sin x^2 + \cos x^2 - 1}{\sin x^2 + \cos x^2 + 1} \right) = c$$

$$\text{Put } y = 1 \text{ and } x = \sqrt{\frac{\pi}{6}}$$



$$1 \ln\left(\frac{1}{\sqrt{3}}\right) + \ln\left(\frac{\frac{1}{2} + \frac{\sqrt{3}}{2} - 1}{\frac{1}{2} + \frac{\sqrt{3}}{2} + 1}\right) = c$$

$$\text{Now } x = \sqrt{\frac{\pi}{3}} \Rightarrow y(\ln \sqrt{3}) + \ln\left(\frac{\frac{1}{2} + \frac{\sqrt{3}}{2} - 1}{\frac{1}{2} + \frac{\sqrt{3}}{2} + 1}\right) = \ln\left(\frac{1}{\sqrt{3}}\right) + \ln\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$$

$$y(\ln \sqrt{3}) = \ln\left(\frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow y = -1$$

$$|y| = 1$$

Question ID : 11694026

Tangent and normal

26. Let M and N be the number of points on the $y^5 - 9xy + 2x = 0$, where the tangents to the curve are parallel to x-axis and y-axis, respectively. Then the value of M + N equals _____.

माना वक्र $y^5 - 9xy + 2x = 0$ पर उन बिन्दुओं, जिन पर स्पर्श रेखा x-अक्ष y-अक्ष के समान्तर हैं, कि संख्या क्रमशः M तथा N है। तो M + N बराबर है _____.

Ans. Official Answer NTA (2)

Sol. $y^5 - 9xy + 2x = 0$

$$5y^4 \frac{dy}{x} - 9x \frac{dy}{dx} - 9y + 2 = 0$$

$$\frac{dy}{dx}(5y^4 - 9x) = 9y - 2$$

$$\frac{dy}{dx} = \frac{9y - 2}{5y^4 - 9x} = 0 \text{ (for horizontal tangent)}$$

$$y = \frac{2}{9} \Rightarrow \text{Which does not satisfy the original equation} \Rightarrow M = 0.$$

Now $5y^4 - 9x = 0$ (for vertical tangent)

$$5y^4(9y - 2) - 9y \cdot 5 = 0$$

$$y^4[45y - 10 - 9y] = 0$$

$$y = 0 \text{ (Or) } 36y = 10$$



$$y = \frac{5}{18}$$

$$y = 0 \Rightarrow x = 0 \text{ \& } y = \frac{5}{18} \Rightarrow x =$$

$$(0, 0) \quad \left(x, \frac{5}{18}\right)$$

$$N = 2$$

$$M + N = 0 + 2 = 2$$

Question ID : 11694027

Sequence & progression

27. Let $f(x) = 2x^2 - x - 1$ and $S = \{n \in \mathbb{Z} : |f(n)| \leq 800\}$. Then, the value of $\sum_{n \in S} f(n)$ is equal to _____.

माना $f(x) = 2x^2 - x - 1$ तथा $S = \{n \in \mathbb{Z} : |f(n)| \leq 800\}$ हैं। तो $\sum_{n \in S} f(n)$ का मान है _____.

Ans. Official Answer NTA (10620)

Sol. $f(x) = 2x^2 - x - 1$

$$|f(x)| \leq 800$$

$$2n^2 - n - 801 \leq 0$$

$$n^2 - \frac{1}{2}n - \frac{801}{2} \leq 0$$

$$\left(n - \frac{1}{4}\right)^2 - \frac{801 - \frac{1}{16}}{2} \leq 0$$

$$\left(n - \frac{1}{4}\right)^2 - \frac{6409}{16} \leq 0$$

$$\left(n - \frac{1}{4} - \frac{\sqrt{6409}}{4}\right) \left(n - \frac{1}{4} + \frac{\sqrt{6409}}{16}\right) \leq 0$$

$$\frac{1 - \sqrt{6409}}{4} \leq n \leq \frac{1 + \sqrt{6409}}{4}$$

$$n = \{-19, -18, -17, \dots, 0, 1, 2, \dots, 20\}$$

$$\sum_{n \in S} f(x) = \sum (2x^2 - x - 1)$$

$$= 2 [19^2 + 18^2 + \dots + 1^2 + 1^2 + 2^2 + \dots + 19^2 + 20^2]$$

$$= 4 [1^2 + 2^2 + \dots + 19^2] + 2 [20^2] - 20 - 40$$

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$$\begin{aligned}
 &= \frac{4 \times 19 \times 20 \times (2 \times 19 + 1)}{6} + 2 \times 400 - 60 \\
 &= \frac{4 \times 19 \times 20 \times 39}{6} + 800 - 60 - 9880 + 800 - 60 \\
 &= 10620
 \end{aligned}$$

Question ID : 11694028

Matrices

28. Let S be the set containing all 3×3 matrices with entries from $\{-1, 0, 1\}$. The total number of matrices $A \in S$ such that the sum of all the diagonal elements of $A^T A$ is 6 is _____.

माना 3×3 के सभी आव्यूहों जिनके अवयव $\{-1, 0, 1\}$ में से हैं, का समुच्चय S है। आव्यूह $A \in S$ जिसके लिए $A^T A$ के विकर्ण के सभी अवयवों का योग 6 है, की कुल संख्या है _____.

Ans. Official Answer NTA (5376)

Sol. $\text{Tr}(AA^T) = 6$

$$AA^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\begin{aligned}
 \text{Now given } a^2 + d^2 + g^2 + b^2 + e^2 + h^2 + c^2 + f^2 + i^2 &= 6 \\
 &= {}^9C_3 \times 2^6 \\
 &= 5376
 \end{aligned}$$

Question ID : 11694029

Ellipse

29. If the length of the latus rectum of the ellipse $x^2 + 4y^2 + 2x + 8y - \lambda = 0$ is 4, and l is the length of its major axis, then $\lambda + l$ is equal to _____.

यदि दीर्घवृत्त $x^2 + 4y^2 + 2x + 8y - \lambda = 0$ की नाभिलंब जीवा की लम्बाई 4 है तथा इसके दीर्घअक्ष की लंबाई l है, तो $\lambda + l$ बराबर है _____.

Ans. Official Answer NTA (75)

Sol. $\lambda + l = 75$

$$x^2 + 4y^2 + 2x + 8y - \lambda = 0$$

$$\frac{(x+1)^2}{\lambda+5} + \frac{(y+1)^2}{\frac{\lambda+5}{4}} = 1$$

$$\therefore \frac{2b^2}{a} = 4$$



$$\Rightarrow \lambda = 59$$

$$\lambda \neq -5$$

$$l = 2a = 2\sqrt{\lambda + 5} = 2\sqrt{65} = 16$$

$$\Rightarrow \lambda + l = 59 + 16 = 75$$

Question ID : 11694030

Complex Number

30. Let $S = \{z \in \mathbb{C} : z^2 + \bar{z} = 0\}$. Then $\sum_{z \in S} (\text{Re}(z) + \text{Im}(z))$ is equal to _____.

माना $S = \{z \in \mathbb{C} : z^2 + \bar{z} = 0\}$ है, तो $\sum_{z \in S} (\text{Re}(z) + \text{Im}(z))$ बराबर है _____.

Ans. Official Answer NTA (0)

Sol. $S = \{z \in \mathbb{C} : z^2 + \bar{z} = 0\}$

$$\text{Let } z = x + iy$$

$$z^2 = x^2 - y^2 + 2ixy$$

$$\bar{z} = x - iy$$

$$z^2 + \bar{z} = x^2 - y^2 + x + i(2xy - y) = 0$$

$$\Rightarrow x^2 + x - y^2 = 0 \text{ \& } 2xy - y = 0$$

$$y = 0 \text{ or } x = \frac{1}{2}$$

$$\text{If } y = 0; x = 0, -1$$

$$\text{If } x = \frac{1}{2}; y = \frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$$

$$\sum_{z \in S} (\text{Re}(z) + \text{Im}(z)) = \left(0 - 1 + \frac{1}{2} + \frac{1}{2}\right) + 0 + 0 \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}$$