JEE Main July 2022

Question Paper With Text Solution

27 July | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JULY 2022 | 27TH JULY SHIFT-2

SECTION – A

Question ID : 100401

Function

1. The domain of the function $f(x)=\sin^{-1}\left[2x^2-3\right]+\log_2\left(\log_{\frac{1}{2}}\left(x^2-5x+5\right)\right)$, where [t] is the greatest integer function, is :

फलन $f(x)=\sin^{-1}\left[2x^2-3\right]+\log_2\left(\log_{\frac{1}{2}}\left(x^2-5x+5\right)\right)$, जहाँ [t] महत्तम पूर्णांक फलन है, का प्रांत है

(1) $\left(-\sqrt{\frac{5}{2}},\frac{5-\sqrt{5}}{2}\right)$

(2) $\left(\frac{5-\sqrt{5}}{2},\frac{5+\sqrt{5}}{2}\right)$

(3) $\left(1,\frac{5-\sqrt{5}}{2}\right)$

(4) $\left[1,\frac{5+\sqrt{3}}{2}\right)$

Ans. Official Answer NTA (3)

Sol. $f(x)=\sin^{-1}[2x^2-3]+\log_2\left(\log_{\frac{1}{2}}\left(x^2-5x+5\right)\right)$

$P_1:-1\le[2x^2-2]<1$

$\Rightarrow -1\le 2x^2-3<2$

$\Rightarrow 2<2x^2<5$

$\Rightarrow 1<x^2<\frac{5}{2}$

$\Rightarrow P_1: x\in\left(-\sqrt{\frac{5}{2}},-1\right)\cup\left(1,\sqrt{\frac{5}{2}}\right)$

$P_2: x^2-5x+5>0$

$\Rightarrow\left(x-\left(\frac{5-\sqrt{5}}{2}\right)\right)\left(x-\left(\frac{5-\sqrt{5}}{2}\right)\right)>0$

$P_3:\log_{\frac{1}{2}}\left(x^2-5x+5\right)>0$

$\Rightarrow x^2-5x-5<1$

$\Rightarrow x^2-5x+4<0$

$\Rightarrow P_3 : x \in (1,4)$

So, $P_1 \cap P_2 \cap P_3 = \left(1, \frac{5-\sqrt{5}}{2}\right)$

Question ID : 100402

Complex number

2. Let S be the set of all (α, β), $\pi < \alpha, \beta < 2\pi$, for which the complex number $\frac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$ is purely imaginary and $\frac{1 + i\cos\beta}{1 - 2i\cos\beta}$ is purely real. Let $Z_{\alpha\beta} = \sin 2\alpha + i\cos 2\beta$, $(\alpha, \beta) \in S$. Then $\sum_{(\alpha,\beta)\in S}\left(iZ_{\alpha\beta} + \frac{1}{i\bar{Z}_{\alpha\beta}}\right)$ is equal to :

माना सभी (α, β), $\pi < \alpha, \beta < 2\pi$, जिनके लिए सम्मिश्र संख्या $\frac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$ विशुद्ध काल्पनिक तथा $\frac{1 + i\cos\beta}{1 - 2i\cos\beta}$ विशुद्ध वास्तविक है, का समुच्चय S है। माना $Z_{\alpha\beta} = \sin 2\alpha + i\cos 2\beta$, $(\alpha, \beta) \in S$ हैं। तो $\sum_{(\alpha,\beta)\in S}\left(iZ_{\alpha\beta} + \frac{1}{i\bar{Z}_{\alpha\beta}}\right)$ बराबर है :

(1) 3 (2) $3i$ (3) 1 (4) $2 - i$

Ans. Official Answer NTA (3)

Sol. $\pi < \alpha, \beta < 2\pi$

$\frac{1 - i\sin\alpha}{1 + i(2\sin\alpha)} =$ Purely imaginary

$\Rightarrow \frac{(1 - i\sin\alpha)(1 - i(2\sin\alpha))}{1 + 4\sin^2\alpha} =$ Purely imaginary

$\Rightarrow \frac{1 - 2\sin^2\alpha}{1 + 4\sin^2\alpha} = 0$

$\Rightarrow \sin^2\alpha = \frac{1}{2}$

$\Rightarrow \alpha = \left\{\frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

& $\frac{1 + i\cos\beta}{1 + i(-2\cos\beta)} =$ Purely real

$\Rightarrow \frac{(1+i\cos\beta)(1+2i\cos\beta)}{1+4\cos^2\beta} = \text{Purely real}$

$\Rightarrow 3\cos\beta = 0$

$\Rightarrow \beta = \frac{3\pi}{2}$

$\Rightarrow z_{\alpha\beta} = \sin\frac{5\pi}{2} + i\cos 3\pi = 1 - i$

or

$z_{\alpha\beta} = \sin\frac{7\pi}{2} + i\cos 3\pi = -1 - i$

Required value $= \left[i(1-i) + \frac{1}{i(1+i)}\right]\left[i(-1-i) + \frac{1}{i(-1+i)}\right]$

$= i(-2i) + \frac{1}{i}\frac{2i}{i(-2)} \Rightarrow 2 - 1 = 1$

Question ID : 100403

Quadratic Equation

3. If α, β are the roots of the equation $x^2 - \left(5 + 3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_5 3}}\right)x + 3\left(3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log_5 3)^{\frac{2}{3}}} - 1\right) = 0$, then the equation, whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$, is :

यदि समीकरण $x^2 - \left(5 + 3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_5 3}}\right)x + 3\left(3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log_5 3)^{\frac{2}{3}}} - 1\right) = 0$ के मूल α, β हैं, तो वह समीकरण, जिसके मूल $\alpha + \frac{1}{\beta}$ तथा $\beta + \frac{1}{\alpha}$ हैं, है :

(1) $3x^2 - 20x - 12 = 0$ (2) $3x^2 - 10x - 4 = 0$

(3) $3x^2 - 10x + 2 = 0$ (4) $3x^2 - 20x + 16 = 0$

Ans. Official Answer NTA (2)

Sol. Bonus because ‘x’ is missing the correct will be,

$x^2 - \left(5 + 3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_5 3}}\right)x + 3\left(3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log_5 3)^{\frac{2}{3}}} - 1\right) = 0$

$3^{\sqrt{\log_3 5}} = 3^{\sqrt{\log_3 5}.\sqrt{\log_3 5}.\sqrt{\log_5 3}} = 3^{\log_3 5.\sqrt{\log_5 3}}$

$$= \left(3^{\log_3 5}\right)^{\sqrt{\log_5 3}} = 5^{\sqrt{\log_5 3}}$$

$$3^{\sqrt[3]{\log_3 5}} = 3^{\log_3 5 . \sqrt[3]{(\log_5 3)^2}} = \left(3^{\log_3 5}\right)^{(\log_3 5)^{2/3}}$$

$$= 5^{(\log_5 3)^{2/3}}$$

So, equation is $x_2 - 5x - 3 = 0$ and roots are α & β $\{\alpha + \beta = 5; \alpha\beta = -3\}$

New roots are $\alpha + \frac{1}{\beta} \& \beta + \frac{1}{\alpha}$

i.e., $\frac{\alpha\beta+1}{\beta} \& \frac{\alpha\beta+1}{\alpha}$ i.e., $\frac{-2}{\beta} \& \frac{-2}{\alpha}$

Let $\frac{-2}{\alpha} = t \Rightarrow \alpha = \frac{-2}{t}$

As $\alpha^2 - 5\alpha - 3 = 0$

$\Rightarrow \frac{4}{t^2} + \frac{10}{t} - 3 = 0$

$\Rightarrow 4 + 10t - 3t^2 = 0$

$\Rightarrow 3t^2 - 10t - 4 = 0$

i.e., $3x^2 - 10x - 4 = 0$

Question ID : 100404

Matrices

4. Let $A = \begin{pmatrix} 4 & -2 \\ \alpha & \beta \end{pmatrix}$. If $A^2 + \gamma A + 18I = O$, then det (A) is equal to :

माना $A = \begin{pmatrix} 4 & -2 \\ \alpha & \beta \end{pmatrix}$ है। यदि $A^2 + \gamma A + 18I = O$ है, तो det (A) बराबर है :

(1) –18 (2) 18 (3) –50 (4) 50

Ans. Official Answer NTA (2)

Sol. The characteristic equation for A is A $|A - \lambda I| = 0$

$\Rightarrow \begin{vmatrix} 4-\lambda & -2 \\ \alpha & \beta-\lambda \end{vmatrix} = 0$

$\Rightarrow (4-\lambda)(\beta-\lambda) + 2\alpha = 0$

$\Rightarrow \lambda^2 - (\beta+4)\lambda + 4\beta + 2\alpha = 0$

Put $\lambda = A$

$A^2-(\beta-4)A+(4\beta+2\alpha)I=0$

On comparison

$-9(\beta+4)=\gamma$ & $4\beta+2\alpha=18$

and $|A|=4\beta+2\alpha=18$

Question ID : 100405

Continuity & Differentiability

5. If for $p\neq q\neq 0$, the function $f(x)=\dfrac{\sqrt[7]{p(729+x)}-3}{\sqrt[3]{729+qx}-9}$ is continuous at $x=0$, then :

यदि $p\neq q\neq 0$ के लिए, फलन $f(x)=\dfrac{\sqrt[7]{p(729+x)}-3}{\sqrt[3]{729+qx}-9}$, $x=0$ पर संतत है, तो :

(1) $7pq\, f(0)-1=0$
(2) $63q\, f(0)-p^2=0$
(3) $21q\, f(0)-p^2=0$
(4) $7pq\, f(0)-9=0$

Ans. Official Answer NTA (2)

Sol. $f(0)=\lim_{x\to 0} f(x)$

Limit should be $\frac{0}{0}$ form

So, $\sqrt[7]{p.729}-3=0 \Rightarrow p.3^6=3^7 \Rightarrow p=3$

Now, $f(0)=\lim_{x\to 0}\dfrac{\sqrt[7]{3(3^6+x)}-3}{\sqrt[3]{3^6+qx}-9}$

$$=\lim_{x\to 0}\frac{3\left[\left(1+\frac{x}{3^6}\right)^{1/7}-1\right]}{9\left[\left(1+\frac{qx}{3^6}\right)^{1/3}-1\right]}=\frac{3}{9}\times\frac{\frac{1}{7.3^6}}{\frac{q}{3.3^6}}$$

$\Rightarrow f(0)=\frac{1}{3}\times\frac{3}{7q}=\frac{1}{7q}$

$\Rightarrow 7qf(0)-1=0$

$\Rightarrow 7.p^2.qf(0)-p^2=0$ (for option)

$\Rightarrow 63qf(0)-p^2=0$

Question ID : 100406

Definite Integration

6. Let f(x) = 2 + |x| – |x – 1| + |x + 1|, x ∈ R. Consider

(S1) : $f'\left(-\frac{3}{2}\right)+f'\left(-\frac{1}{2}\right)+f'\left(\frac{1}{2}\right)+f'\left(\frac{3}{2}\right)=2$

(S2) : $\int_{-2}^{2} f(x)\,dx = 12$

Then :

(1) Both (S1) and (S2) are correct (2) Both (S1) and (S2) are wrong

(3) Only (S1) is correct(4) Only (S2) is correct

माना f(x) = 2 + |x| – |x – 1| + |x + 1|, x ∈ R है। तो

(S1) : $f'\left(-\frac{3}{2}\right)+f'\left(-\frac{1}{2}\right)+f'\left(\frac{1}{2}\right)+f'\left(\frac{3}{2}\right)=2$

(S2) : $\int_{-2}^{2} f(x)\,dx = 12$

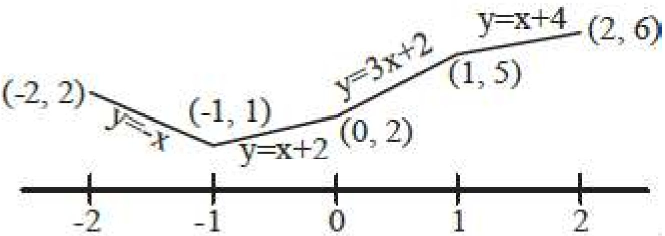
में :

(1) (S1) तथा (S2) दोनों सही हैं (2) (S1) तथा (S2) दोनों गलत हैं

(3) केवल (S1) सही है (4) केवल (S2) सही है

Ans. Official Answer NTA (4)

Sol.



(S1) : $f'\left(-\frac{3}{2}\right)+f'\left(-\frac{1}{2}\right)+f'\left(\frac{1}{2}\right)+f'\left(\frac{3}{2}\right)=4$

(S2) : $\int_{-2}^{2} f(x)\,dx = 12$

∴ (D)

Question ID : 100407

Sequence & progression

7. Let the sum of an infinite G.P., whose first term is a and the common ratio is r, be 5. Let the sum of its first five terms be $\frac{98}{25}$. Then the sum of the first 21 terms of an AP, whose first term is 10ar, n^{th} term is a_n and the common difference is $10ar^2$, is equal to :

माना एक अपरिमित G.P., जिसका पहला पद a है तथा सार्व अनुपात r है, का योग 5 है। माना इसके प्रथम पाँच पदों का योग $\frac{98}{25}$ है। तो उस AP, जिसका पहला पद 10ar है, n^{th} पद a_n है तथा सार्व अंतर $10ar^2$ है, के प्रथम 21 पदों का योग है :

(1) $21\ a_{11}$ (2) $22\ a_{11}$ (3) $15\ a_{16}$ (4) $14\ a_{16}$

Ans. Official Answer NTA (1)

Sol. $S_{21} = \frac{21}{2}[20ar + 20.10ar^2]$

$= 21\ [10\ ar + 100\ ar^2]$

$= 21.\ a_{11}$

Question ID : 100408

Area Under Curve

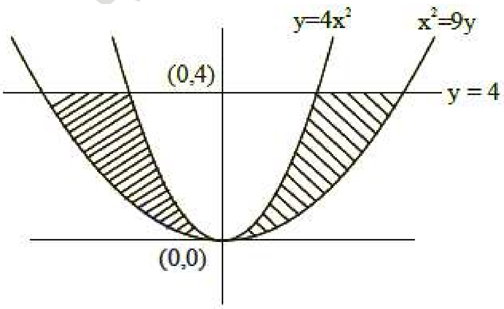
8. The area of the region enclosed by $y \le 4x^2$, $x^2 \le 9y$ and $y \le 4$, is equal to :

$y \le 4x^2$, $x^2 \le 9y$ तथा $y \le 4$ घिरे क्षेत्र का क्षेत्रफल है :

(1) $\frac{40}{3}$ (2) $\frac{56}{3}$ (3) $\frac{112}{3}$ (4) $\frac{80}{3}$

Ans. Official Answer NTA (4)

Sol.



$$\Delta = 2.\int_0^4 \left(3\sqrt{y} - \frac{\sqrt{y}}{2}\right) dy$$

$$= 2.\int_0^4 \frac{5}{2}\sqrt{y}\ dy = \frac{80}{3}$$

Question ID : 100409

Definite Integration

9. $\int_0^2\left(\left|2x^2-3x\right|+\left[x-\frac{1}{2}\right]\right)dx$, where [t] is the greatest integer function, is equal to :

$\int_0^2\left(\left|2x^2-3x\right|+\left[x-\frac{1}{2}\right]\right)dx$, जहाँ [t] महत्तम पूर्णांक फलन है, बराबर है :

(1) $\frac{7}{6}$ (2) $\frac{19}{12}$ (3) $\frac{31}{12}$ (4) $\frac{3}{2}$

Ans. Official Answer NTA (2)

Sol. $\int_0^2\left|2x^2-3x\right|dx$

$$=\int_0^{\frac{3}{2}}\left(3x-2x^2\right)dx+\int_{\frac{3}{2}}^{2}\left(2x^2-3x\right)dx=\frac{19}{12.}$$

$$\int_0^2\left[x-\frac{1}{2}\right]dx=\int_{\frac{-1}{2}}^{\frac{3}{2}}[t]dt$$

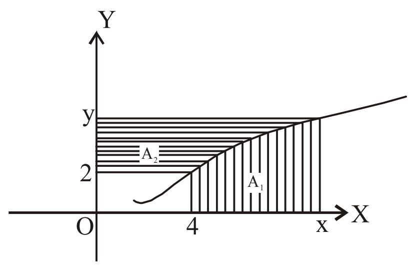
$$=\int_{\frac{1}{2}}^{0}\left(-1\right)dt+\int_0^1 0.dt\int_1^{\frac{3}{2}}1.dt=0.$$

Question ID : 100410

Area Under Curve

10. Consider a curve y = y(x) in the first quadrant as shown in the figure. Let the area A_1 is twice the area A_2. Then the normal to the curve perpendicular to the line 2x – 12y = 15 does NOT pass through the point:

चित्र में दिखाए अनुसार y = y(x) प्रथम चतुर्थांश में एक वक्र है। माना क्षेत्रफल A_1, क्षेत्रफल A_2 का दो गुना है। तो वक्र पर, रेखा 2x – 12y = 15 के लंबवत, अभिलंब किस बिन्दु से होकर नहीं जाता है :



(1) (6, 21)

(2) (8, 9)

(3) (10, –4)

(4) (12, –15)

Ans. Official Answer NTA (3)

Sol. Given that $A_1 = 2A_2$

from the graph $A_1 + A_2 = xy - 8$

$$\Rightarrow \frac{3}{2}A_1 = xy - 8$$

$$\Rightarrow A_1 = \frac{2}{3}xy - \frac{16}{3}$$

$$\Rightarrow \int_4^x f(x)\,dx = \frac{2}{3}xy - \frac{16}{3}$$

$$\Rightarrow f(x) = \frac{2}{3}\left(x\frac{dy}{dx} + y\right)$$

$$\Rightarrow 2\int\frac{dy}{y} = \int\frac{dx}{x}$$

$\Rightarrow 2\ell ny = \ell nx + \ell nc$

$\Rightarrow y^2 = cx$

As $f(4) = 2 \Rightarrow c = 1$

so $y^2 = x$

slope of normal = –6

$$y = -(x) - \frac{1}{2}(-6) - \frac{1}{4}(-6)^3$$

$\Rightarrow y = -6x + 3 + 54$

$\Rightarrow y + 6x = 57$

Now check options and (C) will not satisfy.

Question ID : 100411

Straight Line

11. The equation of the sides AB, BC and CA of a triangle ABC are $2x + y = 0$, $x + py = 39$ and $x - y = 3$ respectively and P(2, 3) is its circumcentre. Then which of the following is NOT true?

(1) $(AC)^2 = 9p$ (2) $(AC)^2 + p^2 = 136$

(3) $32 < \text{area }(\Delta ABC) < 36$ (4) $34 < \text{area }(\Delta ABC) < 38$

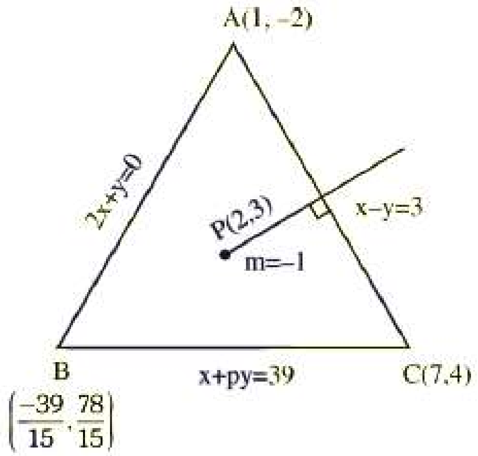
त्रिभुज ABC की भुजाओं AB, BC तथा CA के समीकरण क्रमशः $2x + y = 0$, $x + py = 39$ तथा $x - y = 3$ हैं तथा इसका परिकेन्द्र P(2, 3) है। तो निम्न में से कौनसा सत्य नहीं है ?

(1) $(AC)^2 = 9p$ (2) $(AC)^2 + p^2 = 136$

(3) 32 < क्षेत्रफल (ΔABC) < 36 (4) 34 < क्षेत्रफल (ΔABC) < 38

Ans. Official Answer NTA (4)

Sol.



Perpendicular bisector of AB

$x + y = 5$

Take image of A

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{-2(-6)}{2} = 6$$

(7, 4)

7 + 4ap = 39

p = 8

solving x + 8y = 39 and y = –2x

$$x = \frac{-39}{15} \qquad y = \frac{78}{15}$$

$AC^2 = 72 = 9p$

$AC^2 + p^2 = 72 + 64 = 136$

$$\Delta ABC = \frac{1}{2}\begin{vmatrix} 1 & -2 & 1 \\ 7 & 4 & 1 \\ \frac{-39}{15} & \frac{78}{15} & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[4 - \frac{78}{15} + 2\left(7 + \frac{39}{15}\right) + 7\left(\frac{78}{15}\right) + \frac{4 \times 39}{15}\right]$$

$$= \frac{1}{2}\left[18 + 18 \times \frac{13}{5}\right]$$

$= 9\left[\frac{18}{5}\right] = \frac{162}{5} = 32.4$

Question ID : 100412

Circle

12. A circle C_1 passes through the origion O and has diameter 4 on the positive x-axis. The line y = 2x gives a chord OA of circle C_1. Let C_2 be the circle with OA as a diameter. If the tangent to C_2 at the point A meets the x-axis at P and y-axis at Q, then QA : AP is equal to :

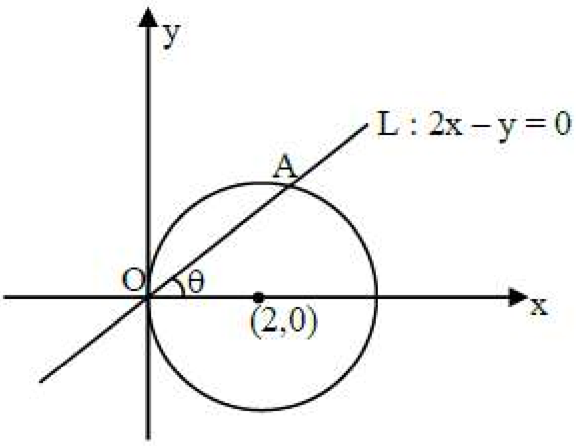
एक वृत्त C_1 मूल बिन्दु O से होकर जाता है तथा धनात्मक x-अक्ष पर इसका व्यास 4 है। रेखा y = 2x से वृत्त C_1 की जीवा OA बनती है। माना C_2 वह वृत्त है, जिसका एक व्यास OA है। यदि बिन्दु A पर C_2 की स्पर्श रेखा x-अक्ष पर P तथा y-अक्ष पर Q पर मिलती है, तो QA : AP बराबर है :

(1) 1 : 4 (2) 1 : 5 (3) 2 : 5 (4) 1 : 3

Ans. Official Answer NTA (1)

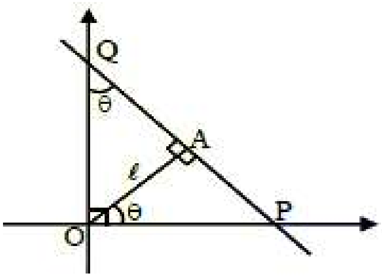
Sol. $C_1 : x^2 + y^2 - 4x = 0$

$\tan\theta = 2$



C_2 is a circle with OA as diameter.

So, tangent at A on C_2 is perpendicular to OR



Let $OA = \ell$

$\therefore \frac{QA}{AP} = \frac{\ell \cot\theta}{\ell \tan\theta}$

$= \frac{1}{\tan^2\theta} = \frac{1}{4}$

Question ID : 100413

Parabola

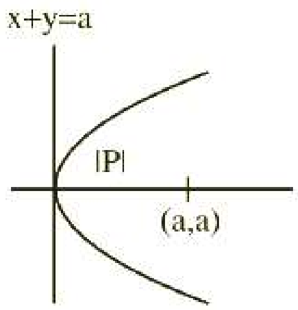
13. If the length of the latus rectum of a parabola, whose focus is (a, a) and the tangent at its vertex is $x + y = a$, is 16, then |a| is equal to :

यदि एक परवलय, जिसकी नाभि (a,a) है तथा जिसके शीर्ष पर स्पर्श रेखा का समीकरण $x + y = a$ है, की नाभिलंब जीवा की लंबाई 16 है, तो |a| बराबर है :

(1) $2\sqrt{2}$ (2) $2\sqrt{3}$ (3) $4\sqrt{2}$ (4) 4

Ans. Official Answer NTA (3)

Sol.



$|P| = \left|\frac{a}{\sqrt{2}}\right| = \frac{16}{4} = 4$

$|a| = 4\sqrt{2}$

Question ID : 100414

3D Geometry

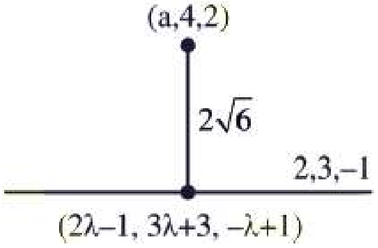
14. If the length of the perpendicular drawn from the point P(a, 4, 2), a > 0 on the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}$ is $2\sqrt{6}$ units and $Q(\alpha_1, \alpha_2, \alpha_3)$ is the image of the point P in this line, then $a + \sum_{i=1}^{3} \alpha_1$ is equal to :

यदि बिन्दु P(a, 4, 2), a > 0, से रेखा $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}$ पर डाले गए लंब की लंबाई $2\sqrt{6}$ इकाई है तथा इस रेखा में बिन्दु P का प्रतिबिम्ब $Q(\alpha_1, \alpha_2, \alpha_3)$ है, तो $a + \sum_{i=1}^{3} \alpha_1$ बराबर है :

(1) 7 (2) 8 (3) 12 (4) 14

Ans. Official Answer NTA (2)

Sol.



$$(2\lambda-1-a)2+(3\lambda-1)3+(-\lambda-1)(-1)=0$$

$$\Rightarrow 4\lambda-2-2a+9\lambda-3+\lambda+1=0$$

$$\Rightarrow 14\lambda-4-2a=0$$

$$\Rightarrow 7\lambda-2-a=0$$

and,

$$(2\lambda-1-a)^2+(3\lambda-1)^2+(\lambda+1)^2=24$$

$$\Rightarrow (5\lambda-1)^2+(3\lambda-1)^2+(\lambda+1)^2=24$$

$$\Rightarrow 35\lambda^2-14\lambda-21=0$$

$$\Rightarrow (\lambda-1)(35\lambda+21)=0$$

For, $\lambda=1$ $\Rightarrow a=5$

Let $(\alpha_1,\alpha_2,\alpha_3)$ be reflection of point P

$\alpha_1+5=2$ $\alpha_2+4=12$ $\alpha_3+2=0$

$\alpha_1=-3$ $\alpha_2=8$ $\alpha_3=-2$

$a+\alpha_1+\alpha_2+\alpha_3=8$

Question ID : 100415

3D Geometry

15. If the line of intersection of the planes $ax+by=3$ and $ax+by+cz=0$, $a>0$ makes an angle 30° with the plane $y-z+2=0$, then the direction cosines of the line are :

यदि समतलों $ax+by=3$ तथा $ax+by+cz=0$, $a>0$ की प्रतिच्छेदन रेखा L, समतल $y-z+2=0$ से 30º का कोण बनाती है, तो रेखा L की दिक् कोज्या हैं :

(1) $\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0$ (2) $\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}},0$ (3) $\frac{1}{\sqrt{5}},-\frac{2}{\sqrt{5}},0$ (4) $\frac{1}{2},-\frac{\sqrt{3}}{2},0$

Ans. Official Answer NTA (2)

Sol. $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & 0 \\ a & b & c \end{vmatrix}$

$= bc\hat{i} - ac\hat{j}$

Direction ratios of line are **b, –a, 0**

Direction ratios of normal of the plane are **0, 1, – 1**

$\cos 60^o = \left|\frac{-a}{\left|\sqrt{2}\right|\sqrt{b^2+a^2}}\right| = \frac{1}{2}$

$\Rightarrow \left|\frac{a}{\sqrt{a^2+b^2}}\right| = \frac{1}{\sqrt{2}}$

$\Rightarrow b = \pm a$

So, D.R.'s can be $(\pm a, -a, 0)$

$\therefore$ D.C.'s can be $\pm\left(\frac{\pm 1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$

Question ID : 100416

Probability

16. Let X have a binomial distribution B(n, p) such that the sum and the product of the mean and variance of X are 24 and 128 respectively. If $P(X > n - 3) = \frac{k}{2^n}$, then k is equal to :

माना X का द्विपद बंटन B(n, p) है। X के माध्य और प्रसरण का योग तथा गुणनफल क्रमशः 24 तथा 128 है। यदि $P(X > n - 3) = \frac{k}{2^n}$ है, तो k बराबर है :

(1) 528 (2) 529 (3) 629 (4) 630

Ans. Official Answer NTA (2)

Sol. Let α = Mean & β = Variance $(\alpha > \beta)$

So, $\alpha + \beta = 24$, $\alpha\beta = 128$

$\Rightarrow \alpha = 16$ & $\beta = 8$

$\Rightarrow np = 16$ $npq = 8 \Rightarrow q = \frac{1}{2}$

$\therefore p = \frac{1}{2}, n = 32$

$p(x > n-3) = \frac{1}{2^n}\left({}^nC_{n-2} + {}^nC_{n-1} + {}^nC_n\right)$

$\therefore k = {}^{32}C_{30} + {}^{32}C_{31} + {}^{32}C_{32} = \frac{32 \times 31}{2} + 32 + 1$

$= 496 + 33 = 529$

Question ID : 100417

Probability

17. A six faced die is biased such that

$3 \times P(\text{a prime number}) = 6 \times P(\text{a composite number}) = 2 \times P(1)$.

Let X be a random variable that counts the number of times one gets a perfect square on some throws of this die. If the die is thrown twice, then the mean of X is :

छः फलकों का एक पासा इस प्रकार अभिनत है कि

3 × P(एक अभाज्य संख्या) = 6 × P(एक भाज्य संख्या) = 2 × P(1).

माना X एक यादृच्छिक चर है, जो यह दर्शाता है कि इस पासे को कुछ बार फेंकने पर कितनी बार पूर्ण वर्ग प्राप्त होता है। यदि इस पासे को दो बार फेंका जाता है, तो X का माध्य है :

(1) $\frac{3}{11}$ (2) $\frac{5}{11}$ (3) $\frac{7}{11}$ (4) $\frac{8}{11}$

Ans. Official Answer NTA (4)

Sol. Let $\frac{P(\text{a prime number})}{2} = \frac{P(\text{a composite})}{1} = \frac{P(1)}{3} = k$

So, P(a prime number) = 2k,
P(a composite number) = k,
& P(1) = 3k
& 3 × 2k + 2 × k + 3k = 1

$\Rightarrow k = \frac{1}{11}$

$P(\text{success}) = P(1 \text{ or } 4) = 3k + k = \frac{4}{11}$

Number of trials, n = 2

$\therefore \text{mean} = np = 2 \times \frac{4}{11} = \frac{8}{11}$

Question ID : 100418

Heights & Distances

18. The angle of elevation of the top P of a vertical tower PQ of height 10 from a point A on the horizontal ground is 45°. Let R be a point on AQ and from a point B, vertically above R, the angle of elevation of P is 60°. If $\angle BAQ = 30°$, AB = d and the area of the trapezium PQRB is α, then the ordered pair (d, α) is :

10 ऊँचाई के एक ऊर्ध्वाधर टावर के शिखर P का क्षैतिज धरातल पर एक बिन्दु A से उन्नयन कोण 45° है। माना AQ पर एक बिन्दु R है तथा के ऊर्ध्वाधर ऊपर एक बिन्दु B से P का उन्नयन कोण 60° है। यदि $\angle BAQ = 30°$, AB = d तथा समलंब PQRB का क्षेत्रफल α है, तो क्रमित युग्म (d, α) है :

(1) $\left(10\left(\sqrt{3}-1\right), 25\right)$ (2) $\left(10\left(\sqrt{3}-1\right), \frac{25}{2}\right)$

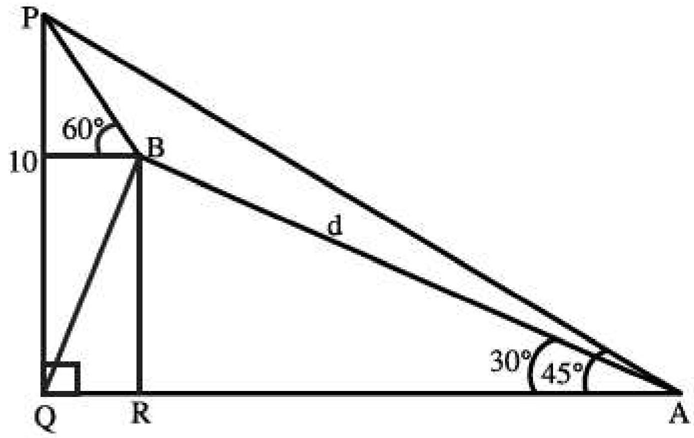
(3) $\left(10\left(\sqrt{3}+1\right), 25\right)$ (4) $\left(10\left(\sqrt{3}+1\right), \frac{25}{2}\right)$

Ans. Official Answer NTA (1)

Sol. $QA = 10 \qquad RA = d\cos 30^\circ = \frac{\sqrt{3}d}{2}$

$QR = 10 - \frac{\sqrt{3}d}{2}$

$BR = d\sin 30^\circ = \frac{d}{2}$



$$\tan 60^\circ = \frac{PQ - BR}{QR} = \frac{10 - \frac{d}{2}}{10 - \frac{\sqrt{3}d}{2}}$$

$$\Rightarrow \sqrt{3} = \frac{20 - d}{20 - \sqrt{3}d}$$

$$\Rightarrow 20\sqrt{3} - 3d = 20 - d$$

$\Rightarrow 2d = 20\left(\sqrt{3}-1\right)$

$\Rightarrow d = 10\left(\sqrt{3}-1\right)$

$ar\left(PQRB\right) = \alpha = \frac{1}{2}\left(PQ+BR\right).QR$

$= \frac{1}{2}\left(10+\frac{d}{2}\right).\left(10-\frac{\sqrt{3}d}{2}\right)$

$= \frac{1}{2}\left(10+5\sqrt{3}-5\right)\left(10-15+5\sqrt{3}\right)$

$= \frac{1}{2}\left(5\sqrt{3}+5\right)\left(5\sqrt{3}-5\right) = \frac{1}{2}\left(75-25\right) = 25$

Question ID : 100419

Trigonometric Equation

19. Let $S = \left\{\theta \in \left(0,\frac{\pi}{2}\right) : \sum_{m=1}^{9} \sec\left(\theta+(m-1)\frac{\pi}{6}\right)\sec\left(\theta+\frac{m\pi}{6}\right) = -\frac{8}{\sqrt{3}}\right\}$. Then :

माना $S = \left\{\theta \in \left(0,\frac{\pi}{2}\right) : \sum_{m=1}^{9} \sec\left(\theta+(m-1)\frac{\pi}{6}\right)\sec\left(\theta+\frac{m\pi}{6}\right) = -\frac{8}{\sqrt{3}}\right\}$ है। तो :

(1) $S = \left\{\frac{\pi}{12}\right\}$

(2) $S = \left\{\frac{2\pi}{3}\right\}$

(3) $\sum_{\theta\in S} \theta = \frac{\pi}{2}$

(4) $\sum_{\theta\in S} \theta = \frac{3\pi}{4}$

Ans. Official Answer NTA (3)

Sol. Let $\alpha = \theta + (m-1)\frac{\pi}{6}$

& $\beta = \theta + m\frac{\pi}{6}$

So, $\beta - \alpha = \frac{\pi}{6}$

Here, $\sum_{m=1}^{9} \sec\alpha.\sec\beta = \sum_{m=1}^{9} \frac{1}{\cos\alpha.\cos\beta}$

$= 2\sum_{m=1}^{9} \frac{\sin(\beta-\alpha)}{\cos\alpha.\cos\beta} = 2\sum_{m=1}^{9}\left(\tan\beta - \tan\alpha\right)$

$$=2\sum_{m=1}^{9}\left(\tan\left(\theta+m\frac{\pi}{6}\right)-\tan\left(\theta+(m-1)\frac{\pi}{6}\right)\right)$$

$$=2\left(\tan\left(\theta+\frac{9\pi}{6}\right)-\tan\theta\right)=2(-\cot\theta-\tan\theta)=-\frac{8}{\sqrt{3}}$$

(Given)

$$\therefore \tan\theta+\cot\theta=\frac{4}{\sqrt{3}}$$

$$\Rightarrow \tan\theta=\frac{1}{\sqrt{3}} \text{ or } \sqrt{3}$$

So, $S=\left\{\frac{\pi}{6},\frac{\pi}{3}\right\}$

$$\sum_{\theta\in S}\theta=\frac{\pi}{6}+\frac{\pi}{3}=\frac{\pi}{2}$$

Question ID : 100420

Mathematical Reasoning

20. If the truth value of the statement $(P\wedge(\sim R))\rightarrow((\sim R)\wedge Q)$ is F, then truth value of which of the following is F?

यदि कथन $(P\wedge(\sim R))\rightarrow((\sim R)\wedge Q)$ का सत्य मान F है, तो निम्न में से किस का सत्य मान है F है ?

(1) $P\vee Q\rightarrow \sim R$ (2) $R\vee Q\rightarrow \sim P$

(3) $\sim(P\vee Q)\rightarrow \sim R$ (4) $\sim(R\vee Q)\rightarrow \sim P$

Ans. Official Answer NTA (4)

Sol. $X\Rightarrow Y$ is a false

when X is true and Y is false

So, $P\rightarrow T$, $Q\rightarrow F$, $R\rightarrow F$

(A) $P\vee Q\rightarrow \sim R$ is T

(B) $R\vee Q\rightarrow \sim P$ is T

(C) $\sim(P\vee Q)\rightarrow \sim R$ is T

(D) $\sim(R\vee Q)\rightarrow \sim P$ is F

SECTION – B

Question ID : 100421

Matrices

21. Consider a matrix $A=\begin{bmatrix}\alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\\ \beta+\gamma & \gamma+\alpha & \alpha+\beta\end{bmatrix}$, where α, β, γ are three distinct natural numbers. If

$\frac{\det\left(\text{adj}\left(\text{adj}\left(\text{adj}\left(\text{adj } A\right)\right)\right)\right)}{(\alpha-\beta)^{16}(\beta-\gamma)^{16}(\gamma-\alpha)^{16}}=2^{32}\times 3^{16}$, then the number of such 3-tuples (α, β, γ) is _________.

आव्यूह $A=\begin{bmatrix}\alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\\ \beta+\gamma & \gamma+\alpha & \alpha+\beta\end{bmatrix}$ है, जहाँ α, β, γ तीन भिन्न पूर्णांक संख्याऐं हैं। यदि

$\frac{\det\left(\text{adj}\left(\text{adj}\left(\text{adj}\left(\text{adj } A\right)\right)\right)\right)}{(\alpha-\beta)^{16}(\beta-\gamma)^{16}(\gamma-\alpha)^{16}}=2^{32}\times 3^{16}$ है, तो इस प्रकार के त्रिकों (α, β, γ) की संख्या है _________.

Ans. Official Answer NTA (42)

Sol. $A=\begin{bmatrix}\alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\\ \beta+\gamma & \gamma+\alpha & \alpha+\beta\end{bmatrix}$

$R_3 \to R_3 + R_1$

$\Rightarrow |A| = |\alpha+\beta+\gamma|\begin{vmatrix}\alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\\ 1 & 1 & 1\end{vmatrix}$

$\Rightarrow |A| = (\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$

$\because\ |\text{adj}A| = |A|^{n-1}$

$|\text{adj}(\text{adj}A)| = |A|^{(n-1)^2}$

$|\text{adj}(\text{adj}(\text{adj}(\text{adj}A)))| = |A|^{(n-1)^4} = |A|^{2^4} = |A|^{16}$

$\therefore (\alpha+\beta+\gamma)^{16} = 2^{32}.3^{16}$

$\Rightarrow (\alpha+\beta+\gamma)^{16} = (2^2.3)^{16} = (12)^{16}$

$\Rightarrow \alpha+\beta+\gamma = 12$

$\because \alpha,\beta,\gamma \in N$

$(\alpha-1)+(\beta-1)+(\gamma-1)=9$

number all tuples $(\alpha, \beta, \gamma) = {}^{11}C_2 = 55$

1 case for $\alpha = \beta = \gamma$

& 12 case when any two of these are equal

So, No. of distinct tuples (α, β, γ)

$= 55 - 13 = 42$

Question ID : 100422

Function

22. The number of functions f, from the set $A = \{x \in N : x^2 - 10x + 9 \le 0\}$ to the set $B = \{n^2 : n \in N\}$ such that $f(x) \le (x - 3)^2 + 1$, for every $x \in A$, is _________.

समुच्चय $A = \{x \in N : x^2 - 10x + 9 \le 0\}$ से समुच्चय $B = \{n^2 : n \in N\}$ ऐसे फलनों, जिनके लिए $f(x) \le (x - 3)^2 + 1$, $x \in A$ है, की संख्या है _________.

Ans. Official Answer NTA (1440)

Sol. $(x^2 - 10x + 9) \le T\ 0 \Rightarrow (x - 1)(x - 9) \le 0$

$\Rightarrow x \in [1, 9] \Rightarrow A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$f(x) \le (x - 3)^2 + 1$

$x = 1 : f(1) \le 5 \Rightarrow 1^2, 2^2$

$x = 2 : f(2) \le 2 \Rightarrow 1^2$

$x = 3 : f(3) \le 1 \Rightarrow 1^2$

$x = 4 : f(4) \le 2 \Rightarrow 1^2$

$x = 5 : f(5) \le 5 \Rightarrow 1^2, 2^2$

$x = 6 : f(6) \le 10 \Rightarrow 1^2, 2^2, 3^2$

$x = 7 : f(7) \le 17 \Rightarrow 1^2, 2^2, 3^2, 4^2$

$x = 8 : f(8) \le 26 \Rightarrow 1^2, 2^2, 3^2, 4^2, 5^2$

$x = 9 : f(9) \le 37 \Rightarrow 1^2, 2^2, 3^2, 4^2, 5^2, 6^2$

Total number of such function

$= 2(6!) = 2(720) = 1440$

Question ID : 100423

Binomial Theorem

23. Let for the 9th term in the binomial expansion of $(3 + 6x)^n$, in the increasing powers of 6x, to be the greatest for $x = \frac{3}{2}$, the least value of n is n_0. If k is the ratio of the coefficient of x^6 to the coefficient of x^3, then $k + n_0$ is equal to : _________.

माना 6x की बढ़ती घात में $(3+6x)^n$ के द्विपद प्रसार में $x=\frac{3}{2}$ पर 9वें पद का मान अधिकतम होने के लिए, n का fuEur e eku n_0 है। यदि x^6 के गुणांक का x^3 के गुणांक से अनुपात k हैं, तो $k+n_0$ बराबर है ________.

Ans. Official Answer NTA (24)

Sol. $(3 + 6x)^n = {}^nC_0 3^n + {}^nC_1 3^{n-1}(6x)^1 + \ldots$

$T_{r+1}\ {}^nC_r 3^{n-r}.(6x)r = {}^nC_r 3^{n-r}.6_r.\ x_r$

$= {}^nC_r 3^{n-r}.3^r.2^r.\left(\frac{3}{2}\right)^r = {}^nC_r 3^n.3^r$ [for $x=\frac{3}{2}$]

T_9 is greatest of $x=\frac{3}{2}$

So, $T_9 > T_{10}$ and $T_9 > T_8$
(concept of numerically greatest term)

Here, $\frac{T_9}{T_{10}} > 1$ and $\frac{T_9}{T_8} > 1$

$\Rightarrow \frac{{}^nC_8 3^n.3^8}{{}^nC_9 3^n.3^9} > 1$ and $\frac{{}^nC_8 3^n.3^8}{{}^nC_7 3^n.3^7} > 1$

and $\frac{{}^nC_8}{{}^nC_7} > \frac{1}{3}$

and $\frac{n-7}{8} > \frac{1}{3}$

$\Rightarrow \frac{29}{3} < n < 11 \Rightarrow n = 10 = n_0$

So, in $(3 + 6x)^n$ for $n = n_0 = 10$
i.e., in $(3 + 6x)^{10}$, here $T_{r+1} = {}^{10}C_r 3^{10-r}6^r x^r$
$T_7 = {}^{10}C_6 3^4.6^6.x^6 = 210.3^{10}.2^6x^6$
$T_4 = {}^{10}C_3 3^7 6^3 x^3 = 120.3^{10}.2^3x^3$
Ratio of coefficient of x6 and coefficient of $x^3 = k$

$\therefore k = \frac{210.3.{}^{10}2^6}{120.3^{10}.2^3} = \frac{7}{4}\times 2^3 = 14$

So, $k + n_0 = 14 + 1 = 24$

Question ID : 100424

Sequence & progression

24. $\frac{2^3-1^3}{1\times 7}+\frac{4^3-3^3+2^2-1^3}{2\times 11}+\frac{6^3-5^3+4^3-3^3+2^3-1^3}{3\times 15}+\ldots..+\frac{30^3-29^3+28^3-27^3+\ldots.+2^3-1^3}{15\times 63}$ is equal to ________.

$$\frac{2^3-1^3}{1\times 7}+\frac{4^3-3^3+2^2-1^3}{2\times 11}+\frac{6^3-5^3+4^3-3^3+2^3-1^3}{3\times 15}+.....+\frac{30^3-29^3+28^3-27^3+....+2^3-1^3}{15\times 63}$$

बराबर है______।

Ans. Official Answer NTA (120)

Sol. $$T_n=\frac{2\sum_{r=1}^{n}(2r)^3-\left(\sum_{r=1}^{2n}r^3\right)}{n(4n+3)}$$

$$\Rightarrow T_n=n$$

So, $\sum_{n=1}^{15}T_n=120$

Question ID : 100425

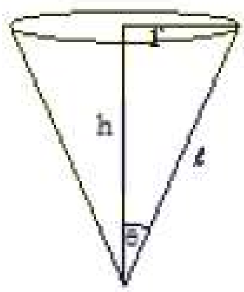
Tangent and normal

25. A water tank has the shape of a right circular cone with axis vertical and vertex downwards. Its semi-vertical angle is $\tan^{-1}\frac{3}{4}$. Water is poured in it at a constant rate of 6 cubic meter per hour. The rate (in square meter per hour), at which the wet curved surface area of the tank is increasing, when the depth of water in the tank is 4 meters, is _________.

एक पानी की टंकी लंब वृत्तीय शंकु, जिसका अक्ष ऊर्ध्वाधर तथा शीर्ष नीचे की ओर है, के आकार की है। इसका अध ा शीर्ष कोण $\tan^{-1}\frac{3}{4}$ है। इसमें 6 घन मीटर प्रति घंटे की दर से पानी डाला जाता है। जब टंकी में पानी की गहराई 4 मीटर है, उस समय टंकी के गीले वक्र पृष्ठीय क्षेत्रफल के बढ़ने की दर (वर्ग मीटर प्रति घंटे में) है ________।

Ans. Official Answer NTA (5)

Sol.



$$\tan\theta=\frac{3}{4}=\frac{r}{h}$$

$$\frac{dV}{dt}=6$$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi h^3 \tan^2\theta = \frac{9\pi}{48}h^3 = \frac{3\pi}{16}h^3$

$\Rightarrow \frac{dV}{dt} = \frac{3\pi}{16}.3h^2.\frac{dh}{dt} = 6 \Rightarrow \left(\frac{dh}{dt}\right)_{h=4} = \frac{2}{3\pi} m/hr$

Now, $S = \pi r\ell = \frac{15}{16}\pi h^2$

$\Rightarrow \frac{dS}{dt} = \frac{15\pi}{16}.2h\frac{dh}{dt}$

$\Rightarrow \left(\frac{dS}{dt}\right)_{h=4} = 5m^3/hr$

Question ID : 100426

Methods of Differentiation

26. For the curve $C : (x^2 + y^2 - 3) + (x^2 - y^2 - 1)^5 = 0$, the value of $3y' - y^3y''$, at the point (α, α), $\alpha > 0$, on C, is equal to _________.

वक्र $C : (x^2 + y^2 - 3) + (x^2 - y^2 - 1)^5 = 0$ के बिन्दु (α, α), $\alpha > 0$ पर $3y' - y^3y''$ का मान बराबर है ______ ।

Ans. Official Answer NTA (16)

Sol. (α, α) lies on

$C : x^2 + y^2 - 3 + x^2 - y^2 - 1^5 = 0$

Put $(\alpha,\alpha), 2\alpha^2 - 3 + -1^5 = 0$

$\Rightarrow \alpha = \sqrt{2}$

Now, differentiate C

$2x + 2y.y' + 5(x^2 - y^2 - 1)^4(2x - 2yy') =$ ....(1)

At $(\sqrt{2}, \sqrt{2})$

$\sqrt{2} + \sqrt{2}y' + 5(-1)^4(\sqrt{2} - \sqrt{2}y') = 0$

$\Rightarrow y' = \frac{3}{2}$ .....(2)

Diff. (1) w.r.t. x

Again, Diff. (1) w.r.t. x

$1 + (y')^2 + yy'' + 20(x^2 - y^2 - 1)^3(x - yy')^2.2$

$+5(x^2 - y^2 - 1)^4(1 - (y')^2 - yy'') = 0$

At $\left(\sqrt{2},\sqrt{2}\right)$ and $y'=\frac{3}{2}$

We have,

$$\left(1+\frac{9}{4}\right)+\sqrt{2}y''-40\left(\sqrt{2}-\sqrt{2}.\frac{3}{2}\right)^2$$

$$+5(1)\left(1-\frac{9}{4}-\sqrt{2}y''\right)=0$$

$$\Rightarrow 4\sqrt{2}y''=-23$$

$$\therefore 3y'-y^3y''=\frac{9}{2}+\frac{23}{2}=16$$

Question ID : 100427

Definite Integration

27. Let f(x) = min{[x – 1], [x – 2], ..., [x – 10]} where [t] denotes the greatest integer ≤ t. Then $\int_0^{10} f(x)dx+\int_0^{10}\left(f(x)\right)^2dx+\int_0^{10}\left|f(x)\right|dx$ is equal to : ________.

माना f(x) = min{[x – 1], [x – 2], ..., [x – 10]} है, जहाँ [t] महत्तम पूर्णांक ≤ t है। तो $\int_0^{10} f(x)dx+\int_0^{10}\left(f(x)\right)^2dx+\int_0^{10}\left|f(x)\right|dx$ बराबर है ________।

Ans. Official Answer NTA (385)

Sol. f(x) = [x] – 10

$$\int_0^{10} f(x).dx=-10-9-8-.....-1$$

$$=-\frac{10.11}{2}=-55$$

$$\int_0^{10}\left(f(x)\right)^2dx=10^2+9^2+8^2+....+1^2$$

$$=\frac{10.11.21}{6}=385$$

$$\int_0^{10}\left|f(x)\right|=10+9+8+....+1$$

$$=\frac{10.11}{2}=55$$

= – 55 +385 + 55 = 385

Question ID : 100428

Definite Integration

28. Let f be differentiable function satisfying $f(x)=\frac{2}{\sqrt{3}}\int_0^{\sqrt{3}} f\left(\frac{\lambda^2 x}{3}\right)d\lambda$, $x>0$ and $f(1)=\sqrt{3}$. If $y=f(x)$ passes through the point $(\alpha, 6)$ then α is equal to _________.

माना f एक अवकलनीय फलन है, $f(x)=\frac{2}{\sqrt{3}}\int_0^{\sqrt{3}} f\left(\frac{\lambda^2 x}{3}\right)d\lambda$, $x>0$ तथा $f(1)=\sqrt{3}$ हैं। यदि $y=f(x)$ बिन्दु $(\alpha, 6)$ से होकर जाता है, तो α बराबर है _________।

Ans. Official Answer NTA (12)

Sol. Let, $\frac{\lambda^2 x}{3}=t$

$\Rightarrow \frac{2\lambda x}{3}d\lambda = dt$

$\Rightarrow d\lambda = \frac{3}{2}.\frac{1\sqrt{x}}{x.\sqrt{3}\sqrt{t}}dt$

$\Rightarrow d\lambda = \frac{\sqrt{3}}{2}.\frac{1}{\sqrt{x}}.\frac{dt}{\sqrt{t}}$

So, $f(x)=\frac{1}{\sqrt{x}}\int_0^x \frac{f(t)}{\sqrt{t}}dt$

$\Rightarrow \sqrt{x}.f'(x)+\frac{f(x)}{2\sqrt{x}}=\frac{f(x)}{\sqrt{x}}$

$\Rightarrow \sqrt{x}.f'(x)=\frac{f(x)}{2\sqrt{x}}$

$\Rightarrow \frac{dy}{y}=\frac{dx}{2x}$

$\Rightarrow \ln y=\frac{1}{2}\ln x + c \Rightarrow f(x)=\sqrt{x}$

$\Rightarrow y=\sqrt{3x} \quad \{\text{as } f(1)=\sqrt{3}\}$

So, $f(x)=\sqrt{3x}$

Now, $f(\alpha)=6 \Rightarrow 36=3\alpha$

$\Rightarrow \alpha = 12$

Question ID : 100429

Hyperbola

29. A common tangent T to the curve $C_1 \frac{x^2}{4} + \frac{y^2}{9} = 1$ and $C_2 : \frac{x^2}{42} - \frac{y^2}{143} = 1$ does not pass through the fourth quadrant. If T touches C_1 at (x_1, y_1) and C_2 at (x_2, y_2), then $|2x_1 + x_2|$ is equal to _________.

वक्रों $C_1 \frac{x^2}{4} + \frac{y^2}{9} = 1$ तथा $C_2 : \frac{x^2}{42} - \frac{y^2}{143} = 1$ की एक उभयनिष्ठ स्पर्श रेखा T चतुर्थ चतुर्थांश से होकर नहीं जाती है। यदि T, वक्र C_1 को (x_1, y_1) पर तथा वक्र C_2 को (x_2, y_2) को स्पर्श करती है, तो $|2x_1 + x_2|$ बराबर है _____।

Ans. Official Answer NTA (20)

Sol. Let common tangents are

$T_1 : y = mx \pm \sqrt{4m^2 + 9}$

& $T_2 : y = mx \pm \sqrt{42m^2 - 13}$

So, $4m^2 + 9 = 42m^2 - 143$

$\Rightarrow 38m^2 = 152$

$\Rightarrow m = \pm 2$

& $c = \pm 5$

For given tangent not pass through 4^{th} quadrant

$T : y = 2x + 5$

Now, comparing with $\frac{xx_1}{4} + \frac{yy_1}{9} = 1$

We get, $\frac{x_1}{8} = \frac{1}{5} \Rightarrow x_1 = -\frac{8}{5}$

$\frac{xx_2}{42} - \frac{yy_2}{143} = 1$

2x /0 y = –5 we have

$x_2 = -\frac{84}{5}$

So, $|2x_1 + x_2| = \left|\frac{-100}{5}\right| = 20$

Question ID : 100430

Vectors

30. Let $\vec{a},\vec{b},\vec{c}$ be three non-coplanar vectors such that $\vec{a}\times\vec{b}=4\vec{c}, \vec{b}\times\vec{c}=9\vec{a}$ and $\vec{c}\times\vec{a}=\alpha\vec{b}$, $\alpha > 0$. If $|\vec{a}|+|\vec{b}|+|\vec{c}|=36$, then α is equal to _________.

माना तीन सदिश $\vec{a},\vec{b},\vec{c}$ सहतलीय नहीं है तथा $\vec{a}\times\vec{b}=4\vec{c}, \vec{b}\times\vec{c}=9\vec{a}, \vec{c}\times\vec{a}=\alpha\vec{b}$, $\alpha > 0$ हैं। यदि $|\vec{a}|+|\vec{b}|+|\vec{c}|=36$ है, तो α बराबर है ________।

Ans. Official Answer NTA (36)

Sol. $\vec{a}+\vec{b}=4\vec{c} \Rightarrow \vec{a}.\vec{c}=0=\vec{b}.\vec{c}$

$\vec{b}\times\vec{c}=9\vec{a} \Rightarrow \vec{a}.\vec{b}=0=\vec{a}.\vec{c}$

$\therefore \vec{a},\vec{b},\vec{c}$ are mutually $\perp$ set of vectors.

$\Rightarrow |\vec{a}||\vec{b}|=4|\vec{c}|, |\vec{b}||\vec{c}|=9|\vec{a}|$ & $|\vec{c}||\vec{a}|=\alpha|\vec{b}|$

$\Rightarrow \frac{|\vec{a}|}{|\vec{c}|}=\frac{4}{9}\frac{|\vec{c}|}{|\vec{a}|}$

$\Rightarrow \frac{|\vec{c}|}{|\vec{a}|}=\frac{3}{2}$

$\therefore$ If $|a|=\lambda, |c|=\frac{3\lambda}{2}$ & $|b|=6$

Now $|a|+|b|+|c|=\frac{1}{36}$

$\Rightarrow \frac{5}{2}\lambda+6=\frac{1}{36}, \lambda=\frac{-43}{18}=|a|$

which gives negative value of λ or $|a|$ which is NOT possible & hence data seems to be wrong.

But if $|\vec{a}|+|\vec{b}|+|\vec{c}|=36$

$\frac{5}{2}\lambda+6=36$

$\lambda=12$

$\alpha=\frac{|\vec{c}||\vec{a}|}{|\vec{b}|}=\frac{3\times12}{2}\times\frac{12}{6}$

$\alpha=36$