JEE Main August 2021

Question Paper With Text Solution

27 August. | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911
Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in

JEE MAIN AUGUST 2021 | 27TH AUGUST SHIFT-1

SECTION – A

1. If $U_n = \left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)^2 \dots \left(1+\frac{n^2}{n^2}\right)^n$, then $\lim_{n\to\infty} (U_n)^{\frac{-4}{n^2}}$ is equal to :

यदि $U_n = \left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)^2 \dots \left(1+\frac{n^2}{n^2}\right)^n$ है, तो $\lim_{n\to\infty} (U_n)^{\frac{-4}{n^2}}$ बराबर है :

(1) $\frac{16}{e^2}$ (2) $\frac{4}{e}$ (3) $\frac{4}{e^2}$ (4) $\frac{e^2}{16}$

Question ID : 86435120597
Option 1 ID : 86435168381
Option 2 ID : 86435168379
Option 3 ID : 86435168380
Option 4 ID : 86435168382

Ans. Official Answer NTA (4)

Sol. $$y = \lim_{n\to\infty}\left\{\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)^2 ----\left(1+\frac{n^2}{n^2}\right)^n\right\}^{\frac{-4}{n^2}}$$

$$y = \lim_{n\to\infty}\left(\prod_{r=1}^{n}\left(1+\frac{r^2}{n^2}\right)^r\right)^{\frac{-4}{n^2}}$$

$$\ln(y) = \lim_{n\to\infty}\sum_{r=1}^{n} \frac{-4r}{n^2}\ln\left(1+\frac{r^2}{n^2}\right)$$

$$\ln(y) = \int_0^1 -4x\ln\left(1+x^2\right)dx$$

Let $1 + x^2 = t$
$2x\, dx = dt$

$$\ln y = \int_1^2 -2\ln t\, dt = -2[t\ln t - t]_1^2$$

$\ln y = 2 - \ln 16$
$y = e^{2-\ln 16}$

$$y = \frac{e^2}{16}$$

2. If $\left(\sin^{-1}x\right)^2-\left(\cos^{-1}x\right)^2=a; 0<x<1, a\neq 0$, then the value of $2x^2-1$ is :

यदि $\left(\sin^{-1}x\right)^2-\left(\cos^{-1}x\right)^2=a; 0<x<1, a\neq 0$ है, तो $2x^2-1$ का मान है :

(1) $\sin\left(\frac{4a}{\pi}\right)$ (2) $\cos\left(\frac{2a}{\pi}\right)$ (3) $\cos\left(\frac{4a}{\pi}\right)$ (4) $\sin\left(\frac{2a}{\pi}\right)$

Question ID : 86435120605
Option 1 ID : 86435168413
Option 2 ID : 86435168411
Option 3 ID : 86435168414
Option 4 ID : 86435168412

Ans. Official Answer NTA (4)

Sol. $(\sin^{-1}x+\cos^{-1}x)(\sin^{-1}x-\cos^{-1}x)=a$

$$\Rightarrow \frac{\pi}{2}\left(\frac{\pi}{2}-2\cos^{-1}x\right)=a$$

$$\Rightarrow 2\cos^{-1}x=\frac{\pi}{2}-\frac{2a}{\pi}$$

$$\Rightarrow \cos^{-1}(2x^2-1)=\frac{\pi}{2}-\frac{2a}{\pi}$$

$$\Rightarrow 2x^2-1=\cos\left(\frac{\pi}{2}-\frac{2a}{\pi}\right)$$

$$\Rightarrow 2x^2-1=\sin\left(\frac{2a}{\pi}\right)$$

3 . If α, β are the distinct roots of $x^2+bx+c=0$, then $\lim_{x\to\beta}\frac{e^{2(x^2+bx+c)}-1-2(x^2+bx+c)}{(x-\beta)^2}$ is equal to :

यदि α, β समीकरण $x^2+bx+c=0$ के दो भिन्न मूल हैं, तो $\lim_{x\to\beta}\frac{e^{2(x^2+bx+c)}-1-2(x^2+bx+c)}{(x-\beta)^2}$ बराबर है :

(1) $2(b^2+4c)$ (2) b^2-4c (3) b^2+4c (4) $2(b^2-4c)$

Question ID : 86435120594
Option 1 ID : 86435168369
Option 2 ID : 86435168367
Option 3 ID : 86435168368
Option 4 ID : 86435168370

Ans. Official Answer NTA (4)

Sol. $$\lim_{x\to\beta}\frac{\left\{1+2(x^2+bx+c)+\frac{\{2(x^2+bx+c)\}^2}{2!}+......\right\}-1-2(x^2+bx+c)}{(x-\beta)^2}$$

$$\lim_{x\to\beta}\frac{2\left(x^2+bx+c\right)^2}{(x-\beta)^2}=\lim_{x\to\beta}\frac{2\left(x-\alpha\right)^2\left(x-\beta\right)^2}{\left(x-\beta\right)^2}$$

$$\Rightarrow 2\left(\beta-\alpha\right)^2=2\left(\left(\alpha+\beta\right)^2-4\alpha\beta\right)$$

$$=2\left(b^2-4c\right)$$

4. If $x^2+9y^2-4x+3=0, x, y \in R$, then x and y respectively lie in the intervals :

यदि $x^2+9y^2-4x+3=0, x, y \in R$ है, तो x तथा y क्रमशः निम्न में से किस अंतराल में है?

(1) $\left[-\frac{1}{3},\frac{1}{3}\right]$ and $\left[-\frac{1}{3},\frac{1}{3}\right]$

(2) [1, 3] and $\left[-\frac{1}{3},\frac{1}{3}\right]$

(3) $\left[-\frac{1}{3},\frac{1}{3}\right]$ and [1, 3]

(4) [1, 3] and [1, 3]

Question ID : 86435120590
Option 1 ID : 86435168354
Option 2 ID : 86435168352
Option 3 ID : 86435168353
Option 4 ID : 86435168351

Ans. Official Answer NTA (2)

Sol. $x^2+9y^2-4x+3=0$

$$\frac{\left(x-2\right)^2}{1}+\frac{y^2}{1/9}=1$$

$x-2 \in [-1, 1]$ & $y \in \left[-\frac{1}{3},\frac{1}{3}\right]$

$x \in [1, 3]$

5. Let A be a fixed point (0, 6) and B be a moving point (2t, 0). Let M be the mid-point of AB and the perpendicular bisector of AB meets the y-axis at C. The locus of the mid-point P of MC is :

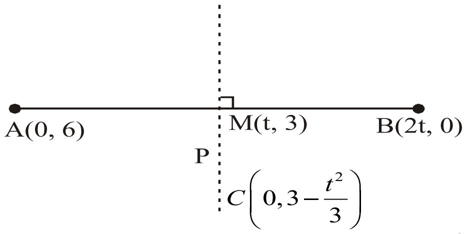
माना A एक स्थिर बिन्दु (0, 6) है तथा B एक चर बिन्दु (2t, 0) है। माना AB का मध्य बिन्दु है तथा AB का लंबद्विभाजक y-अक्ष को बिन्दु C पर मिलता है तो MC के मध्य बिन्दु P का बिन्दुपथ है :

(1) $3x^2 - 2y - 6 = 0$ (2) $2x^2 - 3y + 9 = 0$ (3) $3x^2 + 2y - 6 = 0$ (4) $2x^2 + 3y - 9 = 0$

Question ID : 86435120596
Option 1 ID : 86435168377
Option 2 ID : 86435168375
Option 3 ID : 86435168378
Option 4 ID : 86435168376

Ans. Official Answer NTA (4)

Sol.



$$m_{AB} = \frac{-3}{t}$$

$$m_{MC} = \frac{t}{3}$$

Eq. of MC $\Rightarrow (y-3) = \frac{t}{3}(x-t)$

$$x = 0, y = 3 - \frac{t^2}{3} \quad \Rightarrow C\left(0, 3-\frac{t^2}{3}\right)$$

$$P = \left(\frac{t}{2}, 3-\frac{t^2}{6}\right) \equiv (h, k)$$

$$2h = t \Rightarrow 4h^2 = t^2$$

$$3 - \frac{t^2}{6} = k \Rightarrow 3 - \frac{4h^2}{6} = k$$

$$\Rightarrow 2h^2 + 3k - 9 = 0$$

$$\Rightarrow 2x^2 + 3y - 9 = 0$$

6. When a certain biased die is rolled, a particular face occurs with probability $\frac{1}{6}-x$ and its opposite face occurs with probability $\frac{1}{6}+x$. All other faces occur with probability $\frac{1}{6}$.

Note that opposite faces sum to 7 in any die. If $0 < x < \frac{1}{6}$, and the probability of obtaining total sum = 7, when such a die is rolled twice, is $\frac{13}{96}$, then the value of x is :

जब एक अभिनत पासा फेंका जाता है, तो एक विशेष फलक के प्राप्त होने की प्रायिकता $\frac{1}{6}-x$ है तथा इसकी सम्मुख फलक के प्राप्त होने की प्रायिकता $\frac{1}{6}+x$ है। शेष सभी फलकों के प्राप्त होने की प्रायिकता $\frac{1}{6}$ है।

गौर कीजिए कि किसी भी पासे के सम्मुख फलकों का योग 7 होता है। यदि $0 < x < \frac{1}{6}$, है तथा ऐसे दो पासे बाद फेंकने पर कुल योग 7 प्राप्त करने की प्रायिकता $\frac{13}{96}$ है, तो x का मान है :

(1) $\frac{1}{9}$ (2) $\frac{1}{16}$ (3) $\frac{1}{12}$ (4) $\frac{1}{8}$

Question ID : 86435120604
Option 1 ID : 86435168407
Option 2 ID : 86435168408
Option 3 ID : 86435168409
Option 4 ID : 86435168410

Ans. Official Answer NTA (4)

Sol. Let $P(1)=\frac{1}{6}-x$

then $P(6)=\frac{1}{6}+x, P(2)=P(3)=P(4)=P(5)=\frac{1}{6}$

P (sum = 7) = P {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

$$P(\text{sum}=7)=\left(\frac{1}{6}-x\right)\cdot\left(\frac{1}{6}+x\right)+\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}+\left(\frac{1}{6}+x\right)\left(\frac{1}{6}-x\right)=\frac{13}{96}$$

$x=\frac{1}{8}$

7. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} = 2(y + 2\sin x - 5)x - 2\cos x$ such that y(0) = 7. Then $y(\pi)$ is equal to :

माना अवकल समीकरण $\frac{dy}{dx} = 2(y + 2\sin x - 5)x - 2\cos x$ का हल y = y(x) है, जिसके लिए y(0) = 7 है तो $y(\pi)$ बराबर है :

(1) $7e^{\pi^2} + 5$ (2) $2e^{\pi^2} + 5$ (3) $3e^{\pi^2} + 5$ (4) $e^{\pi^2} + 5$

Question ID : 86435120598
Option 1 ID : 86435168385
Option 2 ID : 86435168384
Option 3 ID : 86435168386
Option 4 ID : 86435168383

Ans. Official Answer NTA (2)

Sol. $\frac{dy}{dx} - 2xy = 2x(2\sin x - 5) - 2\cos x$

I.F. $= e^{\int -2x dx} = e^{-x^2}$

$y \cdot e^{-x^2} = \int e^{-x^2}\left\{2x\left(2\sin x - 5\right) - 2\cos x\right\} dt$

$y \cdot e^{-x^2} = \int e^{-x^2}\left\{(-2x)(-2\sin x) + (-2\cos x)\right\} dx + 5\int e^{-x^2}(-2x)\, dx$

$y \cdot e^{-x^2} = e^{-x^2}\left(-2\sin x\right) + 5e^{-x^2} + C$

$y = 5 - 2\sin x + C \cdot e^{x^2}$

$y(0) = 7 \Rightarrow C = 2$

$y = 5 - 2\sin x + 2 \cdot e^{x^2}$

$y(\pi) = 5 + 2e^{\pi^2}$

8. $\int_6^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e(x^2 - 44x + 484)} dx$ is equal to :

$\int_6^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e(x^2 - 44x + 484)} dx$ के बराबर है :

(1) 8 (2) 6 (3) 10 (4) 5

Question ID : 86435120606
Option 1 ID : 86435168416
Option 2 ID : 86435168417

Option 3 ID : 86435168415

Option 4 ID : 86435168418

Ans. Official Answer NTA (4)

Sol. $I=\int_{6}^{16}\frac{\log_e x^2dx}{\log_e x^2+\log_e(22-x)^2}$........(i)

Use $\int_{a}^{b} f(\mathrm{x})\,\mathrm{dx}=\int_{a}^{b} f(\mathrm{a+b-x})\,\mathrm{dx}$

$I=\int_{6}^{16}\frac{\log_e(22-\mathrm{x})^2}{\log_e(22-\mathrm{x})^2+\log_e x^2}$.......(ii)

$2I=\int_{6}^{16} dx=10$

I = 5

9. The statement $(\mathrm{p}\wedge(\mathrm{p}\rightarrow q)\wedge(\mathrm{q}\rightarrow \mathrm{r}))\rightarrow \mathrm{r}$ is :

(1) equivalent to $q\rightarrow\sim r$ (2) a fallacy

(3) equivalent to $p\rightarrow\sim r$ (4) a tautology

कथन $(\mathrm{p}\wedge(\mathrm{p}\rightarrow q)\wedge(\mathrm{q}\rightarrow \mathrm{r}))\rightarrow \mathrm{r}$:

(1) $q\rightarrow\sim r$ के तुल्य है (2) एक पुनरूक्ति है

(3) $p\rightarrow\sim r$ के तुल्य है (4) एक विरोधोक्ति है

Question ID : 86435120608
Option 1 ID : 86435168426
Option 2 ID : 86435168424
Option 3 ID : 86435168425
Option 4 ID : 86435168423

Ans. Official Answer NTA (4)

Sol. $(\mathrm{p}\wedge(\mathrm{p}\rightarrow \mathrm{q})\wedge(\mathrm{q}\rightarrow \mathrm{r}))\rightarrow \mathrm{r}$

$\Rightarrow(\mathrm{p}\wedge(\sim p\vee \mathrm{q})\wedge(\sim \mathrm{q}\vee \mathrm{r}))\rightarrow \mathrm{r}$

$\Rightarrow(\{(\mathrm{p}\wedge\sim \mathrm{p})\vee(\mathrm{p}\wedge \mathrm{q})\}\wedge(\sim \mathrm{q}\vee \mathrm{r}))\rightarrow \mathrm{r}$

$\Rightarrow\left(f\vee(\mathrm{p}\wedge \mathrm{q})\right)\wedge(\sim \mathrm{q}\vee \mathrm{r})\rightarrow \mathrm{r}$

$\Rightarrow(\mathrm{p}\wedge \mathrm{q})\wedge(\sim \mathrm{q}\vee r)\rightarrow \mathrm{r}$

$\Rightarrow p\wedge\left(q\wedge\left(\sim q\vee r\right)\right)\rightarrow r$

$\Rightarrow p\wedge\left(\left(q\wedge\sim q\right)\vee\left(q\wedge r\right)\right)\rightarrow r$

$\Rightarrow p\wedge\left(q\wedge r\right)\rightarrow r$

$\Rightarrow \sim (p \wedge q \wedge r) \vee r$

$\Rightarrow \sim p \vee \sim q \vee \sim r \vee r$

$\Rightarrow \sim p \vee \sim q \vee t$

$\Rightarrow t$

10. A wire of length 20 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a regular hexagon. Then the length of the side (in meters) of the hexagon, so that the combined area of the square and the hexagon is minimum, is :

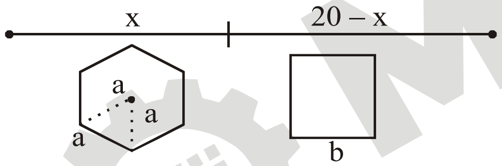
20 m लम्बाई की एक तार को दो भागों में काटा जाना है। एक भाग से एक वर्ग बनाना है तथा दूसरे भाग से एक सम षड्भुज बनाना है। तो दोनों वर्ग तथा षड्भुज के कुल क्षेत्रफल के न्यूनतम होने के लिए षड्भुज की भुजा की लम्बाई (मीटर में) है :

(1) $\frac{10}{2+3\sqrt{3}}$ (2) $\frac{10}{3+2\sqrt{3}}$ (3) $\frac{5}{3+\sqrt{3}}$ (4) $\frac{5}{2+\sqrt{3}}$

Question ID : 86435120595
Option 1 ID : 86435168374
Option 2 ID : 86435168373
Option 3 ID : 86435168372
Option 4 ID : 86435168371

Ans. Official Answer NTA (2)

Sol.



6a = x 4b = 20 – x

a = x/6 b = 5 – x/4

$$A = 6\cdot\left(\frac{\sqrt{3}}{4}a^2\right) + b^2 = \frac{3\sqrt{3}}{2}\cdot\frac{x^2}{36} + \left(5-\frac{x}{4}\right)^2$$

$$\frac{dA}{dx} = \frac{\sqrt{3}x}{12} + 2\left(5-\frac{x}{4}\right)\left(\frac{-1}{4}\right) = 0$$

$$\Rightarrow x = \frac{60}{3+2\sqrt{3}}$$

$$a = \frac{x}{6} = \frac{10}{3+2\sqrt{3}}$$

11. Let us consider a curve, y = f(x) passing through the point (–2, 2) and the slope of the tangent to the curve at any point (x, f(x)) is given by $f(x) + xf'(x) = x^2$. Then :

माना एक वक्र y = f(x) बिन्दु (–2, 2) से होकर जाता है तथा वक्र के किसी बिन्दु (x, f(x)) पर स्पर्शरेखा की प्रवणता $f(x) + xf'(x) = x^2$ द्वारा दी गई है।

(1) $x^3 - 3xf(x) - 4 = 0$
(2) $x^3 + xf(x) + 12 = 0$
(3) $x^2 + 2xf(x) + 4 = 0$
(4) $x^2 + 2xf(x) - 12 = 0$

Question ID : 86435120600
Option 1 ID : 86435168391
Option 2 ID : 86435168393
Option 3 ID : 86435168394
Option 4 ID : 86435168392

Ans. Official Answer NTA (1)

Sol. $f(x) + x f'(x) = x^2$

Let y = f(x)

$$y + x\frac{dy}{dx} = x^2$$

$$\frac{dy}{dx} + \frac{y}{x} = x$$

If = $e^{\int \frac{dx}{x}} = x$

$$y \cdot x = \int x^2 dx = \frac{x^3}{3} + C$$

Passes through (–2, 2)

$$\Rightarrow C = \frac{-4}{3}$$

$$yx = \frac{x^3}{3} - \frac{4}{3}$$

$$\Rightarrow x^3 - x\ f(x) - 4 = 0$$

12. If $S=\left\{z\in\mathbb{C}:\dfrac{z-i}{z+2i}\in\mathbb{R}\right\}$, then :

(1) S contains exactly two elements
(2) S is a circle in the complex plane
(3) S is a straight line in the complex plane
(4) S contains only one element

यदि $S=\left\{z\in\mathbb{C}:\dfrac{z-i}{z+2i}\in\mathbb{R}\right\}$, है, तो :

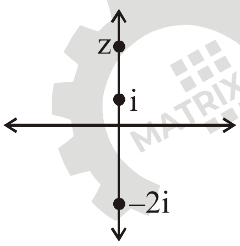
(1) S में ठीक दो अवयव है
(2) S सम्मिश्र समतल में, एक वृत्त है
(3) S सम्मिश्र समतल में, एक सरल है
(4) S में केवल एक अवयव है

Question ID : 86435120591
Option 1 ID : 86435168356
Option 2 ID : 86435168358
Option 3 ID : 86435168357
Option 4 ID : 86435168355

Ans. Official Answer NTA (3)

Sol. $\dfrac{z-i}{z-(-2i)}\in R$

$\arg\left(\dfrac{z-i}{z-(-2\mathrm{i})}\right)=0$ or π



z lies on the imaginary axis.

13. If the matrix $A=\begin{pmatrix}0 & 2\\ K & -1\end{pmatrix}$ satisfies $A(A^3+3I)=2I$, then the value of K is :

यदि आव्यूह $A=\begin{pmatrix}0 & 2\\ K & -1\end{pmatrix}$ समीकरण $A(A^3+3I)=2I$ को संतुष्ट करता है, तो K का मान है :

(1) $-\dfrac{1}{2}$
(2) 1
(3) – 1
(4) $\dfrac{1}{2}$

Question ID : 86435120592
Option 1 ID : 86435168359
Option 2 ID : 86435168362
Option 3 ID : 86435168360
Option 4 ID : 86435168361

Ans. Official Answer NTA (4)

Sol. $A(A^3 + 3I) = 2I$

$A^4 + 3A - 2I = 0$

$$A = \begin{pmatrix} 0 & 2 \\ k & -1 \end{pmatrix}$$

tr (A) = –1
|A| = –2K
Characteristic equation
$A^2 + A - 2KI = 0$
$A^2 = 2KI - A$
$A^4 = 4K^2I - 4KA + A^2$
$A^4 = 4K^2I - 4KA + (2KI - A)$
$A^4 = (4K^2 + 2K)I - (4K + 1)A$
$A^4 + 3A - 2I = 0$
$(4K^2 + 2K)I - (4K + 1)A + 3A - 2I = 0$
$(4K - 2)A = (4K^2 + 2K - 2)I$
$(2K - 1)A = (2K - 1)(K + 1)I$

$$K = \frac{1}{2}$$

14. If 0 < x < 1, then $\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + ...$, is equal to :

यदि 0 < x < 1 है, तो $\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + ...$, बराबर है :

(1) $x\left(\frac{1+x}{1-x}\right) + \log_e(1-x)$

(2) $x\left(\frac{1-x}{1+x}\right) + \log_e(1-x)$

(3) $\left(\frac{1+x}{1-x}\right) + \log_e(1-x)$

(4) $\left(\frac{1-x}{1+x}\right) + \log_e(1-x)$

Question ID : 86435120607
Option 1 ID : 86435168419
Option 2 ID : 86435168421
Option 3 ID : 86435168420
Option 4 ID : 86435168422

Ans. Official Answer NTA (1)

Sol. $S = \frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + ...\infty$

$$S = \sum_{r=1}^{\infty} \frac{(2r+1)}{(r+1)} x^{r+1} = \sum_{r=1}^{\infty} \frac{\{2(r+1) - 1\}}{(r+1)}, x^{r+1}$$

$$S=\sum_{r=1}^{\infty} 2\cdot x^{r+1}-\sum_{r=1}^{\infty} \frac{x^{r+1}}{r+1}$$

$$S=2(x^2+x^3+x^4+....\infty)-(\frac{x^2}{2}+\frac{x^3}{3}+....\infty)$$

$$S=\frac{2x^2}{1-x}-(-\ln(1-x)-x)=\frac{x(1+x)}{1-x}+\ln(1+x)$$

15. $\sum_{k=0}^{20} ({}^{20}C_k)^2$ is equal to :

$\sum_{k=0}^{20} ({}^{20}C_k)^2$ के बराबर है :

(1) ${}^{40}C_{20}$ (2) ${}^{41}C_{20}$ (3) ${}^{40}C_{21}$ (4) ${}^{40}C_{19}$

Question ID : 86435120609
Option 1 ID : 86435168429
Option 2 ID : 86435168427
Option 3 ID : 86435168430
Option 4 ID : 86435168428

Ans. Official Answer NTA (1)

Sol. $\sum_{k=0}^{20} {}^{20}C_k.{}^{20}C_k=\sum_{k=0}^{20} {}^{20}C_k.{}^{20}C_{20-k}$

$\Rightarrow {}^{20}C_0\,{}^{20}C_{20}+{}^{20}C_1\,{}^{20}C_{19}+..........+{}^{20}C_{20}\,{}^{20}C_0$

$\Rightarrow {}^{40}C_{20}$

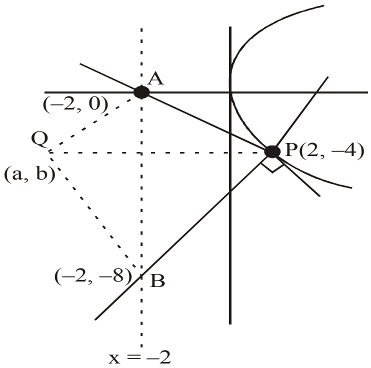
16. A tangent and a normal are drawn at the point P(2, –4) on the parabola $y^2 = 8x$, which meet the directrix of the parabola at the points A and B respectively. If Q(a, b) is a point such that AQBP is a square, then 2a + b is equal to :

परवलय $y^2 = 8x$ के बिन्दु P(2, –4) पर एक स्पर्श रेखा तथा एक अभिलम्ब खींचे गए हैं, जो परवलय की नियता को क्रमशः बिन्दुओं A तथा B पर मिलते है। यदि Q(a, b) एक ऐसा बिन्दु है, जिसके लिए AQBP एक वर्ग है, तो 2a + b बराबर है :

(1) – 20 (2) – 16 (3) – 12 (4) – 18

Question ID : 86435120599
Option 1 ID : 86435168390
Option 2 ID : 86435168388
Option 3 ID : 86435168387
Option 4 ID : 86435168389

Ans. Official Answer NTA (2)

Sol. 

$y^2 = 8x$

Tangent at $(2, -4)$

$T = 0:\ \ y(-4) = 8\left(\frac{x+2}{2}\right)$

$x + y + 2 = 0$

at $x = -2, y = 0$

$A(-2, 0)$

$N = 0 \quad x - y = 6$

at $x = -2, y = -8$

$B(-2, -8)$

Mid point of AB and PQ must be same

$a + 2 = -2 - 2 \Rightarrow a = -6$

$b - 4 = 0 - 8 \Rightarrow b = -4$

$2a + b = -16$

17. Equation of a plane at a distance $\sqrt{\frac{2}{21}}$ from the origin, which contains the line of intersection of the planes x – y – z – 1 = 0 and 2x + y – 3z + 4 = 0, is :

मूलबिन्दु से $\sqrt{\frac{2}{21}}$ की दूरी पर एक समतल, जिसमें समतलों $x - y - z - 1 = 0$ तथा $2x + y - 3z + 4 = 0$ की प्रतिच्छेदन रेखा स्थित है, का समीकरण है :

(1) $3x - 4z + 3 = 0$
(2) $-x + 2y + 2z - 3 = 0$
(3) $4x - y - 5z + 2 = 0$
(4) $3x - y - 5z + 2 = 0$

Question ID : 86435120601
Option 1 ID : 86435168395
Option 2 ID : 86435168396

Option 3 ID : 86435168398

Option 4 ID : 86435168397

Ans. Official Answer NTA (3)

Sol. $P_1 : x - y - z - 1 = 0$

$P_2 : 2x - y - 3z + 4 = 0$

$P : P_1 + \lambda P_2 = 0$

$(x - y - z - 1) + \lambda(2x + y - 3z + 4) = 0$

$\Rightarrow (1 + 2\lambda)x + (-1 + \lambda)y + (-1 - 3\lambda)z - 1 + 4\lambda = 0$

Distance from $(0, 0) = \sqrt{\frac{2}{21}}$

$$\left|\frac{0 + 0 + 0 - 1 + 4\lambda}{\sqrt{(1 + 2\lambda)^2 + (-1 + \lambda)^2 + (-1 - 3\lambda)^2}}\right| = \sqrt{\frac{2}{21}}$$

$\Rightarrow 308\lambda^2 - 184\lambda + 15 = 0$

$(2\lambda - 1)(154\lambda - 15) = 0$

$\lambda = \frac{1}{2}, \frac{15}{154}$

for $\lambda = \frac{1}{2}$

$P : (x - y - z - 1) + \frac{1}{2}(2x + y - 3z + 4) = 0$

$P : 4x - y - 5z + 2 = 0$

18. Let $\frac{\sin A}{\sin B} = \frac{\sin(A - C)}{\sin(C - B)}$, where A, B, C are angles of a triangle ABC. If the lengths of the sides opposite these angles are a, b, c respectively, then :

(1) a^2, b^2, c^2 are in A.P.

(2) c^2, a^2, b^2 are in A.P.

(3) $b^2 - a^2 = a^2 + c^2$

(4) b^2, c^2, a^2 are in A.P.

माना $\frac{\sin A}{\sin B} = \frac{\sin(A - C)}{\sin(C - B)}$, है, जहां A, B, C त्रिभुज ABC के कोण हैं। यदि इन कोणों के सम्मुख भुजाओं की लम्बाईयाँ क्रमशः a, b, c है__________, तो :

(1) a^2, b^2, c^2 A.P. में है

(2) c^2, a^2, b^2 A.P. में है

(3) $b^2 - a^2 = a^2 + c^2$

(4) b^2, c^2, a^2 A.P. में है

Question ID : 86435120603
Option 1 ID : 86435168403
Option 2 ID : 86435168405
Option 3 ID : 86435168406
Option 4 ID : 86435168404

Ans. Official Answer NTA (4)

Sol. $\frac{\sin A}{\sin B} = \frac{\sin(A-C)}{\sin(C-B)}$

$\Rightarrow \frac{\sin(B+C)}{\sin(A+C)} = \frac{\sin(A-C)}{\sin(C-B)}$

$\Rightarrow \sin(C+B).\sin(C-B) = \sin(A+C).\sin(A-C)$

$\Rightarrow \sin^2 C - \sin^2 B = \sin^2 A - \sin^2 C$

$\Rightarrow 2\sin^2 C = \sin^2 A + \sin^2 B$

$\Rightarrow 2c^2 = a^2 + b^2$

b^2, c^2, a^2 are in A.P.

19. The distance of the point (1, –2, 3) from the plane x – y + z = 5 measured parallel to a line, whose direction ratios are 2, 3, –6 is :

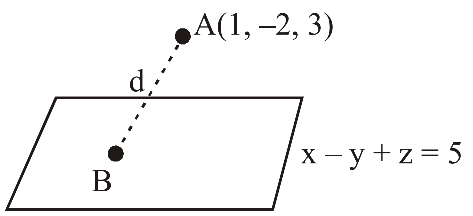
बिन्दु (1, –2, 3) की, एक रेखा जिसके दिक् अनुपात 2, 3, –6 हैं, के समान्तर समतल $x - y + z = 5$ से दूरी है :

(1) 3 (2) 2 (3) 5 (4) 1

Question ID : 86435120602
Option 1 ID : 86435168400
Option 2 ID : 86435168399
Option 3 ID : 86435168402
Option 4 ID : 86435168401

Ans. Official Answer NTA (4)

Sol.



DR's of AB = 2, 3, – 6

Equation of AB $= \frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda$

$B(1+2\lambda, -2+3\lambda, 3-6\lambda)$

Lies on given plane

$1+2\lambda+2-3\lambda+3-6\lambda=5$

$\lambda=\frac{1}{7}$

$\overrightarrow{AB}=2\lambda i+3\lambda j-6\lambda k$

$d=|\overrightarrow{AB}|=\lambda.7=\frac{1}{7}.7=1$

20. If for x, y $\in$ **R**, x > 0, $y=\log_{10}x+\log_{10}x^{1/3}+\log_{10}x^{1/9}+....$upto ∞ terms and $\frac{2+4+6+...+2y}{3+6+9+...+3y}=\frac{4}{\log_{10}x}$, then the ordered pair (x, y) is equal to :

यदि x, y $\in$ **R**, x > 0 के लिए $y=\log_{10}x+\log_{10}x^{1/3}+\log_{10}x^{1/9}+....$अनंत पदों तक तथा $\frac{2+4+6+...+2y}{3+6+9+...+3y}=\frac{4}{\log_{10}x}$, है, तो क्रमित युग्म (x, y) बराबर है :

(1) $(10^6, 6)$ (2) $(10^4, 6)$ (3) $(10^2, 3)$ (4) $(10^6, 9)$

Question ID : 86435120593
Option 1 ID : 86435168363
Option 2 ID : 86435168365
Option 3 ID : 86435168364
Option 4 ID : 86435168366

Ans. Official Answer NTA (4)

Sol. $y=\log_{10}x+\log_{10}x^{1/3}+\log_{10}x^{1/9}+.........\infty$

$y=\left(1+\frac{1}{3}+\frac{1}{9}+......\infty\right)\log_{10}x$

$y=\frac{3}{2}\log_{10}x$

$\frac{2+4+6+.......+2y}{3+6+9+.......+3y}=\frac{4}{\log_{10}x}$

$\Rightarrow\frac{2}{3}=\frac{4}{\log_{10}x}\Rightarrow\log_{10}x=6$

$x=10^6$

$y=\frac{3}{2}\log_{10}10^6$

$y=9$

$(x,y)=(10^6,9)$

SECTION - B

1. If the system of linear equations

$2x + y - z = 3$

$x - y - z = \alpha$

$3x + 3y + \beta z = 3$

has infinitely many solution, then $\alpha + \beta - \alpha\beta$ is equal to_________.

यदि रैखिक समीकरण निकाय

$2x + y - z = 3$

$x - y - z = \alpha$

$3x + 3y + \beta z = 3$

के अनन्त हल हैं, तो $\alpha + \beta - \alpha\beta$ बराबर है_________ I

Question ID : 86435120611

Ans. Official Answer NTA (5)

Sol. $2x + y - z = 3$

$x - y - z = \alpha$

$3x + 3y + \beta z = 3$

Eliminate y

$3x - 2z = 3 + \alpha$

$6x + (\beta - 3)z = 3\alpha + 3$

for infinite solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{3}{6} = \frac{-2}{\beta - 3} = \frac{3 + \alpha}{3\alpha + 3}$$

$\Rightarrow \alpha = 3,\ \beta = -1$

$\alpha + \beta - \alpha\beta = 5$

2. The number of distinct real roots of the equation $3x^4 + 4x^3 - 12x^2 + 4 = 0$ is__________.

समीकरण $3x^4 + 4x^3 - 12x^2 + 4 = 0$ के भिन्न वास्तविक मूलों की संख्या है_________ ।

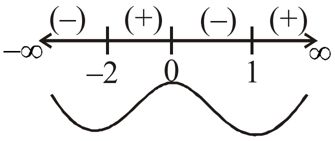
Question ID : 86435120614

Ans. Official Answer NTA (4)

Sol. $f(x) = 3x^4 + 4x^3 - 12x^2 + 4$

$f'(x) = 12x(x-1)(x+2)$

$f'(x) =$

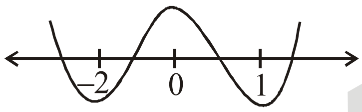


$f(-2) = -28(-)\text{ve}$

$f(0) = 4(+)\text{ve}$

$f(1) = -1(-)\text{ve}$

$f(x) =$



No. of roots = 4

3. Let the equation $x^2 + y^2 + px + (1-p)y + 5 = 0$ represent circles of varying radius $r \in (0, 5]$. Then the number of elements in the set $S = \{q : q = p^2 \text{ and } q \text{ is an integer}\}$ is________.

माना समीकरण $x^2 + y^2 + px + (1-p)y + 5 = 0$ उन वृत्तों को दर्शाती है, जिनकी चर त्रिज्या $r \in (0, 5]$ है तो समुच्चय $S = \{q : q = p^2$ तथा q एक पूर्णांक है$\}$ में अवयवों की संख्या है________ ।

Question ID : 86435120615

Ans. Official Answer NTA (61)

Sol. $x^2 + y^2 + px + (1-p)y + 5 = 0$

$$r = \sqrt{\frac{p^2}{4} + \frac{(p-1)^2}{4} - 5}$$

$$r = \sqrt{\frac{2p^2 - 2p - 19}{4}} \in (0,5]$$

$0 < 2p^2 - 2p - 19 \le 100$

$2p^2 - 2p - 19 > 0 \ \& \ 2p^2 - 2p - 119 \le 0$

$$p \in \left[\frac{1-\sqrt{239}}{2}, \frac{1-\sqrt{39}}{2}\right) \cup \left(\frac{1+\sqrt{39}}{2}, \frac{1+\sqrt{239}}{2}\right]$$

$\Rightarrow p^2 \in (6.88, 67.73)$

$\Rightarrow q = \{7, 8, ..., 67\}$

Total = 61

4. Let $\vec{a} = \hat{i} + 5\hat{j} + a\hat{k}, \vec{b} = \hat{i} + 3\hat{j} + \beta\hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{j} - 3\hat{k}$ be three vectors such that, $\left|\vec{b} \times \vec{c}\right| = 5\sqrt{3}$ and $\vec{a}$ is perpendicular to $\vec{b}$. Then the greatest amongst the values of $\left|\vec{a}\right|^2$ is _______.

माना तीन सदिशों $\vec{a} = \hat{i} + 5\hat{j} + a\hat{k}, \vec{b} = \hat{i} + 3\hat{j} + \beta\hat{k}$ तथा $\vec{c} = -\hat{i} + 2\hat{j} - 3\hat{k}$ के लिए $\left|\vec{b} \times \vec{c}\right| = 5\sqrt{3}$ है तथा सदिश $\vec{a}$ सदिश $\vec{b}$ के लम्बवत् है तो $\left|\vec{a}\right|^2$ के मानों में अधिकतम मान है _______।

Question ID : 86435120618

Ans. Official Answer NTA (90)

Sol. $\vec{a} = i + 5j + \alpha k$

$\vec{b} = i + 3j + \beta k$

$\vec{c} = -i + 2j - 3k$

$|\vec{b} \times \vec{c}|^2 = 75$

$\Rightarrow b^2c^2 - (\vec{b}.\vec{c})^2 = 75$

$\Rightarrow (10 + \beta^2).14 - (5 - 3\beta)^2 = 75$

$\Rightarrow \beta = -2, -4$

$\vec{a}.\vec{b} = 0$

$\Rightarrow 1 + 15 + \alpha\beta = 0$

$\Rightarrow \alpha = 8, 4$

$|\vec{a}|^2 = 1 + 25 + \alpha^2$

$|\vec{a}|^2_{max} = 1 + 25 + 64$

$= 90$

5. Let n be an odd natural number such that the variance of 1, 2, 3, 4...., n is 14. Then n is equal to______.

माना n एक विषम प्राकृतिक संख्या है जिसके लिए 1, 2, 3, 4...., n का प्रसरण 14 है तो n बराबर है______।

Question ID : 86435120617

Ans. Official Answer NTA (13)

Sol. $\sigma^2 = \left(\frac{\Sigma n^2}{n}\right) - \left(\frac{\Sigma n}{n}\right)^2 = 14$

$$\Rightarrow \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4} = 14$$

$$\Rightarrow n^2 = 169 \Rightarrow n = 13$$

6. If $y^{1/4} + y^{-1/4} = 2x$, and $(x^2-1)\frac{d^2y}{dx^2} + \alpha x\frac{dy}{dx} + \beta y = 0$, then $|\alpha - \beta|$ is equal to_______.

यदि $y^{1/4} + y^{-1/4} = 2x$ तथा $(x^2-1)\frac{d^2y}{dx^2} + \alpha x\frac{dy}{dx} + \beta y = 0$, है, तो $|\alpha - \beta|$ बराबर है_______ ।

Question ID : 86435120613

Ans. Official Answer NTA (17)

Sol. $y^{\frac{1}{4}} + y^{-\frac{1}{4}} = 2x$

$$\left(\frac{1}{4}.y^{-\frac{3}{4}} - \frac{1}{4}y^{-\frac{5}{4}}\right)\frac{dy}{dx} = 2$$

$$\frac{1}{4y}\left(y^{\frac{1}{4}} - y^{-\frac{1}{4}}\right)\frac{dy}{dx} = 2$$

$$\sqrt{\left(y^{\frac{1}{4}} + y^{-\frac{1}{4}}\right)^2 - 4}\,\frac{dy}{dx} = 8y$$

$$\Rightarrow \sqrt{4x^2-4}\,\frac{dy}{dx} = 8y$$

$$\Rightarrow \sqrt{x^2-1}\,\frac{dy}{dx} = 4y$$

$$\Rightarrow \sqrt{x^2-1}\,\frac{d^2y}{dx^2} + \frac{1}{2\sqrt{x^2-1}}2x\frac{dy}{dx} = 4\frac{dy}{dx} = 4\frac{4y}{\sqrt{x^2-1}}$$

$$\Rightarrow (x^2-1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 16y = 0$$

$$\alpha = 1, \beta = -16$$

$$|\alpha - \beta| = 17$$

7. If $A = \{x \in R : |x-2| > 1\}$, $B = \left\{x \in R : \sqrt{x^2-3} > 1\right\}$, $C = \left\{x \in R : |x-4| \geq 2\right\}$ and Z is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^c \cap Z$ is________.

यदि $A = \{x \in R : |x-2| > 1\}$, $B = \left\{x \in R : \sqrt{x^2-3} > 1\right\}$, $C = \left\{x \in R : |x-4| \geq 2\right\}$ है तथा सभी पूर्णांको का समुच्चय

Z है, तो समुच्चय $(A\cap B\cap C)^c \cap Z$ के उपसमुच्चयों की संख्या है________।

Question ID : 86435120610

Ans. Official Answer NTA (256)

Sol. $A : x-2<-1$ or $x-2>1$

$x<1$ or $x>3$

$x\in(-\infty,1)\cup(3,\infty)$

$B : \sqrt{x^2-3}>1 \quad \Rightarrow x^2-3>1$

$\Rightarrow x^2>4$

$\Rightarrow x\in(-\infty,-2)\cup(2,\infty)$

$C :\ x-4\le -2$ or $x-4\ge 2$

$\Rightarrow x\le 2$ or $x\ge 6$

$\Rightarrow x\in(-\infty,2]\cup[6,\infty)$

$A\cap B\cap C=(-\infty,-2)\cup[6,\infty)$

$(A\cap B\cap C)^C=[-2,6)$

$(A\cap B\cap C)^C\cap Z=\{-2,-1,0,1,2,3,4,5\}$

Number of subsets = $2^8 = 256$

8.. A number is called a palindrome if it reads the same backward as well as forward. For example 285582 is a six digit palindrome. The number of six digit palindromes, which are divisible by 55, is________.

एक संख्या को विलोमपद (palindrome) कहते हैं यदि इसे आगे तथा पीछे से पढ़ने पर एक ही संख्या मिलती है। उदाहरण के लिए 285582 छः अंको का एक विलोमपद है। छः अंको के विलोमपदों जो 55 से विभाजित होते है, की संख्या है________।

Question ID : 86435120612

Ans. Official Answer NTA (100)

Sol. 5xyyz5

Number of ways = $1\times 10\times 10 = 100$

9. If $\int \frac{dx}{(x^2+x+1)^2} = a\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+b\left(\frac{2x+1}{x^2+x+1}\right)+C, x>0$ where C is the constant of integration, then the value of $9(\sqrt{3}a+b)$ is equal to_______.

यदि $\int \frac{dx}{(x^2+x+1)^2} = a\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+b\left(\frac{2x+1}{x^2+x+1}\right)+C, x>0$ है, जहाँ C समाकलन अचर है, तो $9(\sqrt{3}a+b)$ का मान बराबर है _______।

Question ID : 86435120619

Ans. Official Answer NTA (15)

Sol. $I = \int \frac{dx}{\left(\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2\right)^2}$

Let $x + \frac{1}{2} = \frac{\sqrt{3}}{2}\tan\theta$

$dx = \frac{\sqrt{3}}{2}\sec^2\theta\, d\theta$

$I = \frac{\sqrt{3}}{2}\int \frac{\sec^2\theta d\theta}{\frac{9}{16}\sec^4\theta}$

$I = \frac{8}{3\sqrt{3}}\int\left(\frac{1+\cos 2\theta}{2}\right)d\theta$

$I = \frac{4}{3\sqrt{3}}\left[\theta + \frac{\sin 2\theta}{2}\right] + C$

$I = \frac{4}{3\sqrt{3}}\left[\theta + \frac{\tan\theta}{1+\tan^2\theta}\right] + C$

$I = \frac{4}{3\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + \frac{1}{3}\left(\frac{2x+1}{x^2+x+1}\right) + C$

$a = \frac{4}{3\sqrt{3}}$

$b = \frac{1}{3}$

$9(\sqrt{3}\,a + b) = 15$

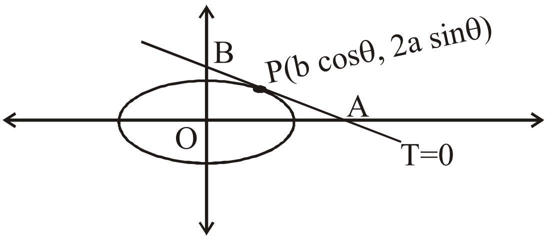
10. If the minimum area of the triangle formed by a tangent to the ellipse $\frac{x^2}{b^2} + \frac{y^2}{4a^2} = 1$ and the co-ordinate axis is kab, then k is equal to_______.

यदि दीर्घवृत्त $\frac{x^2}{b^2} + \frac{y^2}{4a^2} = 1$ की एक स्पर्श रेखा तथा निर्देशांक अक्षों द्वारा बने त्रिभुज का न्यूनतम क्षेत्रफल kab है, तो k बराबर है_______ ।

Question ID : 86435120616

Ans. Official Answer NTA (2)

Sol.



$E : \frac{x^2}{b^2} + \frac{y^2}{4a^2} = 1$

$\underline{T=0}\ \frac{x\cos\theta}{b} + \frac{y\sin\theta}{2a} = 1$

$A\left(\frac{b}{\cos\theta}, 0\right), B\left(0, \frac{2a}{\sin\theta}\right)$

$\therefore$ Area $\Delta OAB = \frac{1}{2}.\frac{b}{\cos\theta}\frac{2a}{\sin\theta} = \frac{2ab}{\sin 2\theta}$

$A_{Min} = 2ab = Kab$

$K = 2$