JEE Main August 2021

Question Paper With Text Solution

27 August. | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN AUGUST 2021 | 27TH AUGUST SHIFT-2

SECTION – A

1. Two poles, AB of length a metres and CD of length a+b ($b \neq a$) metres are erected at the same horizontal level with bases at B and D. If BD = x and $\tan\lfloor ACB = \frac{1}{2}$, then:

दो पोल, a मीटर लंबा AB तथा a+b ($b \neq a$) मीटर लंबा CD एक क्षैतिज धरातल पर खड़े है। इनके आधार B तथा D हैं। यदि BD = x तथा $\tan\lfloor ACB = \frac{1}{2}$ हैं, तो :

(1) $x^2 + 2(a+2b)x+a(a+b) =0$

(2) $x^2 - 2ax+b(a+b)=0$

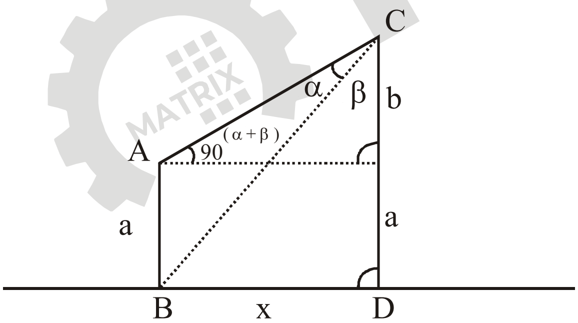
(3) $x^2 + 2(a+2b)x-b(a+b) =0$

(4) $x^2 - 2ax+a(a+b)=0$

Question ID : 86435120699
Option 1 ID : 86435168700
Option 2 ID : 86435168699
Option 3 ID : 86435168697
Option 4 ID : 86435168698

Ans. Official Answer NTA (2)

Sol.



$$\tan(90-(\alpha+\beta)) = \frac{b}{x}$$

$$\cot(\alpha+\beta) = \frac{b}{x}$$

$$\frac{\cot\alpha\cot\beta-1}{\cot\beta+\cot\alpha} = \frac{b}{x}$$

Also

$$\tan\beta = \frac{x}{a+b}$$

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$$\frac{\frac{2\times(a+b)}{x}-1}{2+\frac{a+b}{x}}=\frac{b}{x}$$

$$\frac{2(a+b)-x}{(a+b)+2x}=\frac{b}{x}$$

$2x\,(a+b)-x^2=(a+b)\,b+2bx$

$x^2-2ax+b(a+b)=0$

2. The value of the integral $\int_0^1 \frac{\sqrt{x}dx}{(1+x)(1+3x)(3+x)}$ is

समाकलन $\int_0^1 \frac{\sqrt{x}dx}{(1+x)(1+3x)(3+x)}$ का मान है :

(1) $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{2}\right)$

(2) $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{2}\right)$

(3) $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{6}\right)$

(4) $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{6}\right)$

Question Type : MCQ
Question ID : 86435120689
Option 1 ID : 86435168657
Option 2 ID : 86435168660
Option 3 ID : 86435168659
Option 4 ID : 86435168658Ans.

Official Answer NTA (1)

Sol. $\int_0^1 \frac{\sqrt{x}dx}{(1+x)(1+3x)(3+x)}$ $\qquad x=t^2$

$I=\int_0^1 \frac{2t^2dt}{(1+t^2)(1+3t^2)(3+t^2)}$ $\qquad dx=2tdt$

$$I=\int_0^1 \frac{(3t^2+1)-(t^2+1)dt}{(t^2+1)(3t^2+1)(t^2+3)}$$

$$I=\int_0^1 \frac{dt}{(1+t^2)(t^2+3)}-\int_0^1 \frac{dt}{(3t^2+1)(t^2+3)}$$

$$I=\frac{1}{2}\int_0^1 \frac{(t^2+3)-(t^2+1)}{(1+t^2)(t^2+3)}dt+\frac{1}{8}\int_0^1 \frac{(1+3t^2)-3(3+t^2)}{(3t^2+1)(t^2+3)}dt$$

$$I=\frac{1}{2}\int_0^1 \frac{dt}{1+t^2}-\frac{1}{2}\int_0^1 \frac{dt}{t^2+3}+\frac{1}{8}\int_0^1 \frac{dt}{d^2+3}-\frac{3}{8}\int_0^1 \frac{dt}{3t^2+1}$$

$$I=\frac{1}{2}\int_0^1 \frac{dt}{1+t^2}-\frac{3}{8}\int_0^1 \frac{dt}{t^2+3}-\frac{3}{8}\int_0^1 \frac{dt}{1+3t^2}$$

$$I=\frac{1}{2}\tan^{-1}\Big]_0^1-\frac{3}{8\sqrt{3}}\tan^{-1}\left(\frac{t}{\sqrt{3}}\right)\Big]_0^1-\frac{3}{8\sqrt{3}}\tan^{-1}(\sqrt{3}t)\Big]_0^1$$

$$I=\frac{1}{2}\left(\frac{\pi}{2}-0\right)-\frac{3}{8\sqrt{3}}\left(\frac{\pi}{6}\right)-\frac{3}{8\sqrt{3}}\left(\frac{\pi}{3}\right)$$

$$I=\frac{\pi}{8}-\frac{\sqrt{3}}{16}\pi$$

$$I=\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{2}\right)$$

3. Let [λ] be the greatest integer less than or equal to λ The set of all values of λ for which the system of linear equations $x+y+z=4$, $3x+2y+5z=3$, $9x+4y+(28+[\lambda])z=[\lambda]$ has a solution is:

माना [λ] महत्तम पूर्णांक $\le \lambda$ है। λ के सभी मानों, जिनके लिए रैखिक समीकरण निकाय $x+y+z=4$, $3x+2y+5z=3$, $9x+4y+(28+[\lambda])z=[\lambda]$ का हल है, का समुच्चय है :

(1) $(-\infty,-9)\cup(-9,\infty)$

(2) $[-9,-8]$

(3) $(-\infty,-9)\cup[-8,\infty)$

(4) R

Question Type : MCQ
Question ID : 86435120682
Option 1 ID : 86435168632
Option 2 ID : 86435168629
Option 3 ID : 86435168630
Option 4 ID : 86435168631

Ans. Official Answer NTA (4)

Sol. $D=\begin{vmatrix}1 & 1 & 1\\ 3 & 2 & 5\\ 9 & 4 & 28+[\lambda]\end{vmatrix}=[\lambda]+9$

If $[\lambda]+9\neq 0$ then system will have unique solution.

and if $[\lambda]+9=0$ then $D_1=D_2=D_3=0$ so system will have infinite solution.

4. The equation of the plane passing through the line of intersection of the planes $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=1$ and $\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0$ and parallel to the x-axis is:

समतलों $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=1$ तथा $\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0$ की प्रतिच्छेदन रेखा से होकर जाने वाले तथा x-अक्ष के समान्तर समतल का समीकरण है :

(1) $\vec{r}\left(\hat{j}-3\hat{k}\right)+6=0$

(2) $\vec{r}\left(\hat{j}-3\hat{k}\right)-6=0$

(3) $\vec{r}\left(\hat{i}-3\hat{k}\right)+6=0$

(4) $\vec{r}\left(\hat{i}+3\hat{k}\right)+6=0$

Question Type : MCQ
Question ID : 86435120696
Option 1 ID : 86435168688
Option 2 ID : 86435168687
Option 3 ID : 86435168685
Option 4 ID : 86435168686

Ans. Official Answer NTA (1)

Sol. Given plane in cartesian form

$x+y+z=1$ _______(1)

$2x+3y-z+4=0$ _______(2)

Required plane

$(x + y + z - 1) + \lambda\ (2x + 3y - z\ + 4) = 0$

$(1 + 2\lambda)x + (1 + 3\lambda)y + (1 - \lambda)z + 4\lambda - 1 = 0$

Here, normal is perpendicular to x-axis.

Hence $(1+2\lambda)+(1+3\lambda)(0)+(1-\lambda)(0)=0$

$d = -\frac{1}{2}$

Now required plane

$y - 3z + 6 = 0$

which is form option (1).

5. The angle between teh straight lines, whose directioncosines are given by the equations $2l + 2m - n = 0$ and $mn + nl + lm = 0$, is:

सरल रेखाओं, जिनके दिक्–कोसाइन समीकरणों $2l + 2m - n = 0$ तथा $mn + nl + lm = 0$ द्वारा दिए गए हैं, के बीच का कोण है:

(1) $\frac{\pi}{3}$

(2) $\cos^{-1}\left(\frac{8}{9}\right)$

(3) $\frac{\pi}{2}$

(4) $\pi - \cos^{-1}\left(\frac{4}{9}\right)$

Question Type : MCQ
Question ID : 86435120695
Option 1 ID : 86435168681
Option 2 ID : 86435168682
Option 3 ID : 86435168683
Option 4 ID : 86435168684

Ans. Official Answer NTA (3)

Sol. $n = 2(l + m)$ ______ (1)

$n(m + l) + lm = 0$ ______ (2)

$2(l + m)^2 + lm = 0$

$2l^2 + 5lm + 2m^2 = 0$

$(m + 2l)(2m + l) = 0$

$l=-2m$	$l=-\frac{m}{2}$
Case - I	
$<-2m, m, -2m,>$	$<-\frac{m}{2}, m, m>$
$<-2, 1, -2>$	$<-1, 2, 2>$

Hence, angle between these lines

$$\cos\theta=\frac{2+2-4}{\sqrt{9}\sqrt{9}}$$

$$\cos\theta=0$$

$$\theta=\frac{\pi}{2}$$

6. If $0<x<1$ and $y=\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{3}{4}x^4+\ ...$, then the value of e^{1+y} at $x=\frac{1}{2}$ is

यदि $0<x<1$ तथा $y=\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{3}{4}x^4+\ ...$, हैं, तो $x=\frac{1}{2}$ पर e^{1+y} का मान है :

(1) $\frac{1}{2}e^2$

(2) $\frac{1}{2}\sqrt{e}$

(3) 2e

(4) $2e^2$

Question Type : MCQ
Question ID : 86435120684
Option 1 ID : 86435168640
Option 2 ID : 86435168639
Option 3 ID : 86435168637
Option 4 ID : 86435168638Ans.

Official Answer NTA (1)

Sol. $y = \left(1-\frac{1}{2}\right)x^2 + \left(1-\frac{1}{3}\right)x^3 + \left(1-\frac{1}{4}\right)x^4 + ...\infty$

$y = (x^2 + x^3 + x^4.....\infty) - \left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + ...\infty\right)$

$y = \frac{x^2}{1-x} + x - \left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + ...\infty\right)$

$y = \frac{x}{1-x} + ln(1-x)$

Now at $x = \frac{1}{2}$

$y = 1 - ln2$

Now e^{1+y}

$= e^{1+1-ln2}$

$= e^{2-ln2}$

$= e^2 \times \frac{1}{2}$

$= \frac{e^2}{2}$

7. If the solution curve of the differential equation $(2x - 10y^3)dy + ydx = 0$, passes through the points (0, 1) and (2, β), then β is a root of the equation:

यदि अवकल समीकरण $(2x - 10y^3)dy + ydx = 0$ का हल वक्र, बिन्दुओं (0, 1) तथा (2, β) से होकर जाता है, तो β निम्न में से किस समीकरण का एक मूल है?

(1) $y^5 - 2y - 2 = 0$

(2) $2y^5 - 2y - 1 = 0$

(3) $y^5 - y^2 - 1 = 0$

(4) $2y^5 - y^2 - 2 = 0$

Question Type : MCQ
Question ID : 86435120692
Option 1 ID : 86435168672
Option 2 ID : 86435168669
Option 3 ID : 86435168671
Option 4 ID : 86435168670

Ans. Official Answer NTA (4)

Sol. $y\dfrac{dx}{dy} = -2x + 10y^3$

$\dfrac{dx}{dy} + x\left(\dfrac{2}{y}\right) = 10y^2$

I.F. $= e^{\int \frac{2}{y}dy} = y^2$

= e

Now

$x(IF) = \int Q(IF)dy$

$xy^2 = \int 10y^2(y^2)dy$

$xy^2 = 2y^5 + c$

from (0, 1)

$c = -2$

$xy^2 = 2y^5 - 2$

$xy^5 - xy^2 - 2 = 0$

from (2, B)

$2B^5 - 2B^2 - 2 = 0$

$B^5 - B^2 - 1 = 0$

$y^5 - y^2 - 1 = 0$

8. A differential equation representing the family of parabolas with axis parallel to y-axis and whose length of latus rectum is the distance of the point (2, –3) from the line 3x + 4y = 5, is given by:

ऐसे परवलयों के कुल, जिनके अक्ष y-अक्ष के समान्तर हैं तथा जिनकी नाभिलम्ब जीवा की लम्बाई बिन्दु (2, –3) की रेखा 3x + 4y = 5 से दूरी है, को निरूपित करने वाला एक अवकल समीकरण है :

(1) $11\dfrac{d^2x}{dy^2} = 10$

(2) $11\dfrac{d^2y}{dx^2}=10$

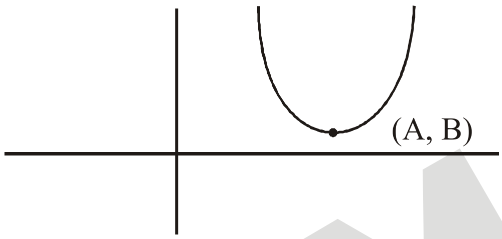
(3) $10\dfrac{d^2x}{dy^2}=11$

(4) $10\dfrac{d^2y}{dx^2}=11$

Question Type : MCQ
Question ID : 86435120691
Option 1 ID : 86435168665
Option 2 ID : 86435168667
Option 3 ID : 86435168668
Option 4 ID : 86435168666

Ans. Official Answer NTA (2)

Sol.



$$4a=\left|\frac{6-12-5}{5}\right|$$

$$4a=\frac{11}{5}$$

$$(x-A)^2=\frac{11}{5}(y-B)$$

$$2(x-A)=\frac{11}{5}\frac{dy}{dx}$$

$$2=\frac{11}{5}\frac{d^2y}{dx^2}$$

$$11\frac{d^2y}{dx^2}=10$$

9. Let $A=\begin{pmatrix}[x+1] & [x+2] & [x+3]\\ [x] & [x+3] & [x+3]\\ [x] & [x+2] & [x+4]\end{pmatrix}$, where [t] denotes the gretest integer less than or equal to t, If det (A) = 192, then the set of values of x is the interval

माना $A=\begin{pmatrix}[x+1] & [x+2] & [x+3]\\ [x] & [x+3] & [x+3]\\ [x] & [x+2] & [x+4]\end{pmatrix}$, जहाँ [t] - महत्तम पूर्णांक $\le$ t को दर्शाता है। यदि det (A) = 192 है, तो x के मानों का समुच्चय निम्न में से कौनसा अंतराल है?

(1) [60, 61]

(2) [62, 63]

(3) [68, 69]

(4) [65, 66]

Question Type : MCQ
Question ID : 86435120683
Option 1 ID : 86435168636
Option 2 ID : 86435168635
Option 3 ID : 86435168633
Option 4 ID : 86435168634

Ans. Official Answer NTA (2)

Sol. $|A| = \begin{vmatrix} [x]+1 & [x]+2 & [x]+3 \\ [x] & [x]+3 & [x]+3 \\ [x] & [x]+2 & [x]+4 \end{vmatrix} = 19^2$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$\begin{vmatrix} [x]+1 & [x]+2 & [x]+3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 19^2$

$[x] = 62$

$x \in [62, 63)$

10. The set of all values of $k > -1$, for which the equation

$(3x^2 + 4x+3)^2 - (k+1)(3x^2 + 4x +3)(3x^2 + 4x+2) + k(3x^2 + 4x +2)^2 = 0$ has real roots, is:

$k > -1$ के सभी मानों, जिनके लिए समीकरण

$(3x^2 + 4x+3)^2 - (k+1)(3x^2 + 4x +3)(3x^2 + 4x+2) + k(3x^2 + 4x +2)^2 = 0$

के वास्तविक मूल हैं, का समुच्चय है :

(1) [2, 3]

(2) $\left(1, \frac{5}{2}\right)$

(3) $\left[\frac{1}{2},\frac{3}{2}\right)-\{1\}$

(4) $\left[-\frac{1}{2},1\right)$

Question ID : 86435120681
Option 1 ID : 86435168626
Option 2 ID : 86435168625
Option 3 ID : 86435168628
Option 4 ID : 86435168627

Ans. Official Answer NTA (2)

Sol. Given equation can be expressed

$$\left(\frac{3x^2+4x+3}{3x^2+4x+2}\right)^2-(k+1)\left(\frac{3x^2+4x+3}{3x^2+4x+2}\right)+k=0$$

$t^2-(k+1)t+k=0$
$t^2-kt-t+k=0$

$t(t-k)-1(t-k)=0$

$(t-1)(t-k)=0$

$t=1$ | $t=k$

$\frac{3x^2+4x+3}{3x^2+4x+2}=k$

$3x^2+4x+3=3x^2k+4xk+2k$

$3x^2(k-1)+4x(k-1)+2k-3=0$

$x\in R$
$D\geq 0$
$16(k-1)^2-k(k-1)(2k-3)\geq 0$

$k\in\left(1,\frac{5}{2}\right]$

11. If two tangents drawn from a point P to the parabola $y^2=16(x-3)$ are at right angles, then the locus of point P is:

यदि एक बिन्दु P से परवलय $y^2 = 16(x-3)$ पर खींची गई दो स्पर्श रेखायें समकोण बनाती हैं,तो बिन्दु P का बिन्दुपथ है :

(1) $x + 4 = 0$

(2) $x + 2 = 0$

(3) $x + 3 = 0$

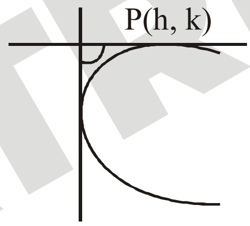
(4) $x + 1 = 0$

Question ID : 86435120694
Option 1 ID : 86435168680
Option 2 ID : 86435168678
Option 3 ID : 86435168679
Option 4 ID : 86435168677

Ans. Official Answer NTA (4)

Sol. Equation of tangent

$$y = m(x-3) + \frac{4}{m}$$



$$k = m(h-3) + \frac{4}{m}$$

$$km = m^2(h-3) + 4$$

$$m^2(h-3) - km + 4 = 0 \quad \langle \; m_1, m_2$$

$$m_1 m_2 = -1$$

$$\frac{4}{h-3} = -1$$

$$x - 3 = 4$$

$$x + 1 = 0$$

12. Each of the persons A and B independently tosses three fair coin. The probability that both of them get the same number of heads is:

दो व्यक्तियों A तथा B में से प्रत्येक तीन न्याय सिक्के उछालता है। दोनों के लिए चित्त की संख्या बराबर आने की प्रायिकता है:

(1) $\frac{5}{16}$

(2) $\frac{1}{8}$

(3) $\frac{5}{8}$

(4) 1

Question ID : 86435120697
Option 1 ID : 86435168691
Option 2 ID : 86435168690
Option 3 ID : 86435168689
Option 4 ID : 86435168692

Ans. Official Answer NTA (1)

Sol. C – I O Head (T T T)

I II

$$\left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^3 = \frac{1}{64}$$

C – II I Head (H T T)

I II

$$\left(\frac{3}{8}\right) \left(\frac{3}{8}\right) = \frac{9}{64}$$

C – III 2 Head (H T T)

$$\left(\frac{3}{8}\right) \left(\frac{3}{8}\right) = \frac{9}{64}$$

C – IV 3 Head (H H H)

$$\left(\frac{1}{8}\right) \left(\frac{1}{8}\right) = \frac{1}{64}$$

$$\text{Total probability} = \frac{1}{64}+\frac{9}{64}+\frac{9}{64}+\frac{1}{64}$$

$$= \frac{20}{64}$$

$$= \frac{5}{16}$$

13. Let Z be the set of all integers,

$$A = \{(x, y) \in Z\times Z : (x-2)^2 + y^2 \le 4\}$$

$$B = \{(x, y) \in Z\times Z : x^2 + y^2 \le 4\}$$

$C = \{(x, y) \in Z \times Z : (x-2)^2 + (y-2)^2 \leq 4$

If the total number of relations from $A \cap B$ to $A \cap C$ is 2^p, then the value of p is:

माना सभी पूर्णांको का समुच्चय Z है,

$A = \{(x, y) \in Z \times Z : (x-2)^2 + 2)^2 + y^2 \leq 4\}$

$B = \{(x, y) \in Z \times Z : x^2 + y^2 \leq 4\}$ तथा

$C = \{(x, y) \in Z \times Z : (x-2)^2 + (y-2)^2 \leq 4$ है।

यदि $A \cap B$ से $A \cap C$ में सम्बन्धों की कुल संख्या 2^p है, तो p का मान है :

(1) 16

(2) 9

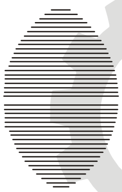
(3) 49

(4) 25

Question ID : 86435120680
Option 1 ID : 86435168622
Option 2 ID : 86435168621
Option 3 ID : 86435168624
Option 4 ID : 86435168623

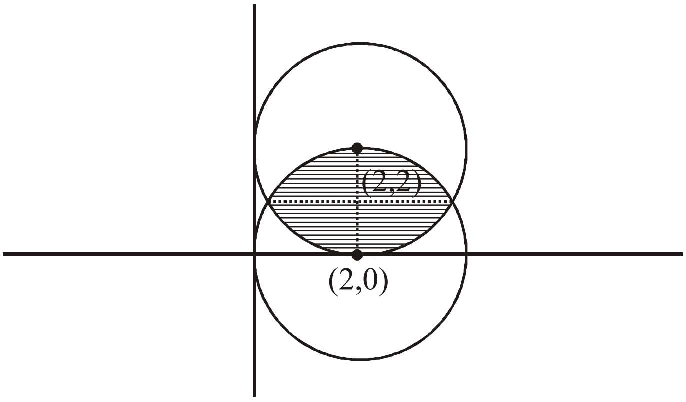
Ans. Official Answer NTA (4)

Sol. $A \cap B$



Integers in $A \cap B = \{(0,0), (1,0), (1,1), (1,-1), (4,0)\} = 5$ points.

$A \cap C$



Integers in $A \cap C = \{(2,0),(2,1),(2,2),(1 \pm \sqrt{3})\} = 5$ points

Relation from $A \cap B$ to $A \cap C = 2^{mn}$

$= 2^{25}$

$P = 25$

14. Let A(a, 0), B(b, 2b + 1) and c(0, b) $b \neq 0$, $|b| \neq 1$, be points such that the area of triangle ABC is 1 sq. unit, then the sum of all possible values of a is:

माना बिन्दुओं A(a, 0), B(b, 2b + 1) तथा c(0, b) $b \neq 0$, $|b| \neq 1$, से बने त्रिभुज ABC का क्षेत्रफल 1 वर्ग इकाई है, तो a के सभी सम्भव मानों का योग है :

(1) $\frac{-2b^2}{b+1}$

(2) $\frac{-2b}{b+1}$

(3) $\frac{2b^2}{b+1}$

(4) $\frac{2b}{b+1}$

Question ID : 86435120693
Option 1 ID : 86435168676
Option 2 ID : 86435168674
Option 3 ID : 86435168675
Option 4 ID : 86435168673

Ans. Official Answer NTA (1)

Sol. $\frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ b & 2b+1 & 1 \\ 0 & b & 1 \end{vmatrix} = \pm 1$

$\begin{vmatrix} a & 0 & 1 \\ b & 2b+1 & 1 \\ 0 & b & 1 \end{vmatrix} = \pm 2$

$a(2b+1-b)+1(b^2) = \pm 2$

$a(b+1)+b^2 = \pm 2$

Case - I

$a = \frac{2-b^2}{b+1}$

Case - II

$a = \frac{-2-b^2}{b+1}$

$\text{Sum} = \frac{-2b^2}{b+1}$

15. If $y(x) = \cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right), x \in \left(\frac{\pi}{2}, \pi\right)$, then $\frac{dy}{dx}$ at $x = \frac{5\pi}{6}$ is

यदि $y(x) = \cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right), x \in \left(\frac{\pi}{2}, \pi\right)$ हैं, तो $\frac{dy}{dx}$ पर $x = \frac{5\pi}{6}$ का मान है :

(1) $\frac{1}{2}$

(2) $-\frac{1}{2}$

(3) -1

(4) 0

Question ID : 86435120686
Option 1 ID : 86435168645
Option 2 ID : 86435168646
Option 3 ID : 86435168648
Option 4 ID : 86435168647

Ans. Official Answer NTA (2)

Sol. $y = \cot^{-1}\left(\dfrac{\left|\sin\frac{x}{2}+\cos\frac{x}{2}\right| + \left|\sin\frac{x}{2}-\cos\frac{x}{2}\right|}{\left|\sin\frac{x}{2}+\cos\frac{x}{2}\right| - \left|\sin\frac{x}{2}-\cos\frac{x}{2}\right|}\right)$

$y = \cot^{-1}\left(\dfrac{2\sin\frac{x}{2}}{2\cos\frac{x}{2}}\right)$

$y = \cot^{-1}\left(\tan\frac{x}{2}\right)$

$y = \cot^{-1}\left(\cot^{-1}\left(\frac{\pi}{2} - x\right)\right)$

$y = \frac{\pi}{2} - x$

$\frac{dy}{dx} = -1$

16. Let M and m respectively be the maximum and minimum values of the function $f(x) = \tan^{-1}(\sin x+\cos x)$ in $\left[0, \frac{\pi}{2}\right]$. Then the value of tan(M–m) is equal to:

माना $\left[0, \frac{\pi}{2}\right]$ में फलन $f(x) = \tan^{-1}(\sin x+\cos x)$ के अधिकतम तथा न्यूनतम मान क्रमशः M तथा m हैं तो tan(M–m) का मान बराबर है :

(1) $3+2\sqrt{2}$

(2) $2-\sqrt{3}$

(3) $3-2\sqrt{2}$

(4) $2+\sqrt{3}$

Question ID : 86435120687
Option 1 ID : 86435168652
Option 2 ID : 86435168650
Option 3 ID : 86435168651
Option 4 ID : 86435168649

Ans. Official Answer NTA (3)

Sol. $y = \tan^{-1}\left(\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)\right)$

$y = \tan^{-1}\left[\sqrt{2}\left[\frac{1}{\sqrt{2}},1\right]\right]$

$y = \tan^{-1}\left[1,\sqrt{2}\right]$

$y = \left[\frac{\pi}{4},\tan^{-1}\sqrt{2}\right]$

Now $m = \frac{\pi}{4}$ $\quad\quad M = \tan^{-1}\sqrt{2} = \lambda \Rightarrow \tan\lambda = \sqrt{2}$

$= \tan(M - m)$

$= \tan\left(\alpha - \frac{\pi}{4}\right)$

$= \frac{\tan\lambda - 1}{\tan\lambda + 1}$

$= \frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$

$= \left(\sqrt{2}-1\right)^2$

$= 2+1-2\sqrt{2}$

$= 3-2\sqrt{2}$

17. A box open from top is made from a rectangular sheet of dimension a×b by cutting squares each of side x from each of the four corners and folding up the flaps. If the volume of the box is maximum, then x is equal to:

a×b (लम्बाई × चौड़ाई) की एक आयताकार चद्दर के प्रत्येक कोने से x भुजा के वर्ग काटकर तथा फलकों को मोड़कर ढ़क्कन रहित एक संदूक बनाया गया है। यदि संदूक का आयतन अधिकतम है, तो x बराबर है :

(1) $\frac{a+b-\sqrt{a^2+b^2+ab}}{6}$

(2) $\frac{a+b-\sqrt{a^2+b^2-ab}}{6}$

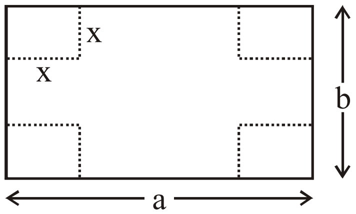
(3) $\frac{a+b-\sqrt{a^2+b^2-ab}}{12}$

(4) $\dfrac{a+b-\sqrt{a^2+b^2-ab}}{6}$

Question ID : 86435120688
Option 1 ID : 86435168655
Option 2 ID : 86435168653
Option 3 ID : 86435168654
Option 4 ID : 86435168656

Ans. Official Answer NTA (2)

Sol.



$$v=(a-2x)(b-2x)x$$

$$v=(4x^2-2x(a+b)+ab)x$$

$$v=4x^3-2x^2(a+b)+abx$$

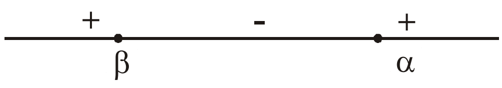
$$\frac{dv}{dx}=12x^2-4x(a+b)+ab=0 \quad \left\langle \begin{matrix}\alpha\\ \beta\end{matrix}\right.$$

$$x=\frac{4(a+b)\pm\sqrt{16(a+b)^2-48ab}}{2\times 12}$$

$$x=\frac{(a+b)\pm\sqrt{a^2+b^2-ab}}{6}$$

Let $\alpha=\dfrac{(a+b)+\sqrt{a^2+b^2-ab}}{6}$

$$\beta=\frac{(a+b)-\sqrt{a^2+b^2-ab}}{6}$$



$$\frac{dv}{dx}=12(x-\alpha)(x-\beta)$$

here maximum value of at $x=\beta$

$$x=\beta=\frac{(a+b)-\sqrt{a^2+b^2-ab}}{6}$$

MATRIX

18. The area of the region bounded by the parabola $(y-2)^2 = (x-1)$, the tangent to it at the point whose ordinate is 3 and the x-axis is:

परवलय $(y-2)^2 = (x-1)$, इसके उस बिन्दु जिसकी कोटि 3 है पर स्पर्श रेखा तथा x-अक्ष द्वारा परिबद्ध क्षेत्र का क्षेत्रफल है :

(1) 6

(2) 4

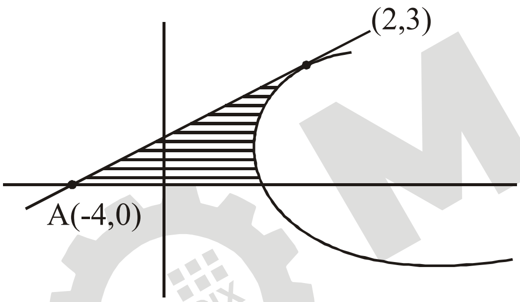
(3) 9

(4) 10

Question ID : 86435120690
Option 1 ID : 86435168661
Option 2 ID : 86435168664
Option 3 ID : 86435168662
Option 4 ID : 86435168663

Ans. Official Answer NTA (3)

Sol. $(y-2)^2 = (x-1)$



slope of tangent

$$2(y-2)\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{2}$$

tangent $y-3 = \frac{1}{2}(x-2)$

$$2y-6 = x-2$$

$$x = 2y-4$$

Required area $= \int_0^3 \left[(y-2)^2 + 1 - (2y-4)\right] dy$

$$=\int_0^3 (y-2)^2-2y+5\ dy$$

$$=\left.\frac{(y-2)^3}{3}-y^2+5y\right]_0^3$$

$$=\frac{1}{3}-9+15+\frac{8}{3}$$

$= 6 + 3$

$= 9$ square unit

19. The boolean expression $(p \wedge q) \Rightarrow ((r\wedge q)\wedge p)$ is equivalent to:

बूलीय व्यंजक $(p \wedge q) \Rightarrow ((r\wedge q)\wedge p)$ निम्न में से किस के तुल्य है :

(1) $(q \wedge r) \Rightarrow (p \wedge q)$

(2) $(p \wedge q) \Rightarrow (r \vee q)$

(3) $(p \wedge r) \Rightarrow (p \wedge q)$

(4) $(p \wedge q) \Rightarrow (r \wedge q)$

Question ID : 86435120698
Option 1 ID : 86435168693
Option 2 ID : 86435168695
Option 3 ID : 86435168696
Option 4 ID : 86435168694

Ans. Official Answer NTA (4)

Sol. $(p \wedge q) \Rightarrow ((r \wedge q) \wedge p)$

$\sim (p \wedge q) \vee ((r \wedge q) \wedge p)$

$\sim (p \wedge q) \vee ((r \wedge p) \wedge (p \wedge q))$

$\Rightarrow [\sim (p \wedge q) \vee (p \wedge q)] \wedge (\sim (p \wedge q) \vee (r \wedge p))$

$\Rightarrow t \wedge [\sim (p \wedge q) \vee (r \wedge p)]$

$\Rightarrow \sim (p \wedge q) \vee (r \wedge p)$

$\Rightarrow (p \wedge q) \Rightarrow (r \wedge p)$

20. If $\lim_{x\to\infty}\left(\sqrt{x^2-x+1}-ax\right)=b$, then the ordered pair (a, b) is:

यदि $\lim_{x\to\infty}\left(\sqrt{x^2-x+1}-ax\right)=b$ है, तो क्रमित युग्म (a, b) है :

(1) $\left(-1,\frac{1}{2}\right)$

(2) $\left(-1,-\frac{1}{2}\right)$

(3) $\left(1,\frac{1}{2}\right)$

(4) $\left(1,-\frac{1}{2}\right)$

Question ID : 86435120685
Option 1 ID : 86435168644
Option 2 ID : 86435168641
Option 3 ID : 86435168643
Option 4 ID : 86435168642

Ans. Official Answer NTA (4)

Sol. $\lim_{x\to\infty} \sqrt{x^2-x+1}-ax=b$

$\lim_{x\to\infty} \frac{x^2-x+1-a^2x^2}{\sqrt{x^2-x+1}+ax}=b$

$\lim_{x\to\infty} \frac{x^2(1-a^2)+1-x}{\sqrt{x^2-x+1}+ax}=b$

If limit exits $a^2=1$

Then $a=\pm 1$

$\lim_{x\to\infty} \frac{1-x}{\sqrt{x^2-x+1}+ax}=b$

$\lim_{x\to\infty} \frac{\frac{1}{x}-1}{\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a}=b$

$\frac{-1}{1+a}=b$ (here $a\neq -1$)

So possible values of b

$-\frac{1}{2} = b$

$\left(1, -\frac{1}{2}\right)$

SECTION - B

1. An online exam is attempted by 50 candidates out of which 20 are boys. The average marks obtained by boys is 12 with a variance 2. The variance of marks obtained by 30 girls is also 2. The average marks of all 50 candidates is 15. If μ is the average marks of girls and σ^2 is the variance of marks of 50 candidates, then $\mu+\sigma^2$ is equal to _______.

50 परीक्षार्थियों द्वारा एक ऑनलाइन परीक्षा दी गई। इन परीक्षार्थियों में 20 लड़के द्वारा प्राप्त किए गए अंको का माध्य 12 तथा प्रसरण 2 हैं। 30 लड़कियों द्वारा प्राप्त अंको का प्रसरण भी 2 है। सभी 50 परीक्षार्थियों के अंको का माध्य 15 है। यदि लड़कियों के अंको का माध्य μ है तथा 50 परीक्षार्थियों के अंको का प्रसरण σ^2 है, तो $\mu+\sigma^2$ बराबर है_______।

Question ID : 86435120709

Ans. Official Answer NTA (25)

Sol. $\sigma_b^2 = 2$ $\quad$ n_1 = No of boys = 20

$\bar{x}_b = 12$ $\quad$ n_2 = No. of girls = 30

$\sigma_g^2 = 2$

$\bar{x}_g =$

$\bar{x} = 15$

$$\text{Now } \bar{x} = \frac{\bar{x}_b n_1 + \bar{x}_g n_2}{n_1 + n_2}$$

$$15 = \frac{12 \times 20 + \bar{x}_g \times 30}{30 + 20}$$

$$750 - 240 = 30\ \bar{x}_g$$

$$\bar{x}_g = 17$$

$$\mu = 17$$

variance of combined group

$$\sigma^2 = \frac{n_1\sigma_b^2 + n_2\sigma_g^2}{n_1 + n_2} + \frac{n_1 n_2}{(n_1 + n_2)^2}(\bar{x}_b - \bar{x}_g)^2$$

$$\sigma^2 = \frac{20\times 2+30\times 2}{20+30}+\frac{20\times 30}{(20+30)^2}(12-17)^2$$

$$\sigma^2 = 8$$

$$\Rightarrow \mu+\sigma^2 = 17+8$$
$= 25$

2. $3\times7^{22}+2\times10^{22}–44$ when divided by 18 leaves the remainder ______.

$3\times7^{22}+2\times10^{22}–44$ को 18 से भाग देने पर शेषफल ______ है।

Question ID : 86435120702

Ans. Official Answer NTA (15)

Sol. $= (6 + 1)^{22} + 2(9 + 1)^{22} - 44$
$= 18I_1 + 3 + 18I_2 + 2 - 44$
$= 18(I_1 + I_2) + 5 - 44$
$= 18I_3 - 39$
$= 18I_3 - 18 \times 3 - 39 + 54$
$= 18(I_3 - 3) + 15$
hence remainder is 15

3. Let S be the sum of all solutions (in radians) of the equation $\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$ in $[0, 4\pi]$. Then $\frac{8S}{\pi}$ is equal to ______.

माना $[0, 4\pi]$ में समीकरण $\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$ के सभी हलों (रेडियन में) का योग S है। तो $\frac{8S}{\pi}$ बराबर है______ ।

Question ID : 86435120708

Ans. Official Answer NTA (56)

Sol. $\sin^4\theta+\cos^4\theta-\sin\theta\cos\theta = 0$

$1-2\sin^2\theta\cos^2\theta-\sin\theta\cos\theta = 0$

$1-\frac{1}{2}\sin^2 2\theta-\frac{1}{2}\sin 2\theta = 0$ $\qquad$ $\sin 2\theta = y$

$2-y^2-y=0$

$y^2-y-2=0$ $\qquad$ If $y=-2$

$(y+2)(y-1)=0$ $\qquad$ $\sin 2\theta = -2$ $\qquad$ not possible

If $y = 1$

$\sin 2\theta = 1$

$2\theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}$

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$

$\text{Sum} = \frac{\pi + 5\pi + 9\pi + 13\pi}{4} = 7\pi$

$\frac{8S}{\pi} = 56$

4. Let z_1 and z_2 be two complex members such that $\arg(z_1 - z_2) = \frac{\pi}{4}$ and z_1, z_2 satisfy the equation $|z-3| = \text{Re}(z)$. Then teh imaginary part of z_1+z_2 is equal to ______.

माना z_1 तथा z_2 दो सम्मिश्र संख्यायें इस प्रकार हैं कि $\arg(z_1 - z_2) = \frac{\pi}{4}$ है तथा z_1, z_2 समीकरण $|z-3| = \text{Re}(z)$ को संतुष्ट करते है तो z_1+z_2 का काल्पनिक भाग है______ ।

Question ID : 86435120700

Ans. Official Answer NTA (6)

Sol. $z_1 = x_1 + iy_1$

$z_2 = x_1 + iy_2$

$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$

$\arg(z_1 - z_2) = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \frac{\pi}{4}$

$y_1 - y_2 = x_1 - x_2$ _______(1)

Now $|z_1 - z_2| = \text{Re}(z_1)$

$(x_1 - 3)^2 + y_1^2 = x_1^2$ _______(2)

$(x_2 - 3)^2 + y_1^2 = x_2^2$ _______(3)

(2) – (3)

$(x_1 - 3)^2 - (x_2 - 3)^2 + y_1^2 - y_2^2 = (x_1^2 - x_2^2)$

$-6x_1 + 6x_2 + (y_1 + y_2)(y_1 - y_2) = 0$

$-6(x_1 - x_1) + (y_1 + y_2)(y_1 - y_2) = 0$

but $x_1 - x_2 = y_1 - y_2$ by (1) them

$-6(y_1 - y_2) + (y_1 + y_2)(y_1 - y_2) = 0$

$y_1 \neq y_2$ then $y_1 + y_2 = 6$

5. The probability distribution of random variable X is given by:

x	1	2	3	4	5
P(x)	K	2K	2K	3K	K

Let $p = p(1 < X < 4 | X < 3)$. If $5p = \lambda k$, then λ is equal to _______.

एक यादृच्छिक चर X का प्रायिकता बंटन

x	1	2	3	4	5
P(x)	K	2K	2K	3K	K

है। माना $p = p(1 < X < 4 | X < 3)$ है। यदि $5p = \lambda k$ है, तो λ बराबर है_______ ।

Question ID : 86435120707

Ans. Official Answer NTA (30)

Sol. Here $P = \frac{2K}{K + 2K}$

$P = \frac{2}{3}$

also $P(x_i) = 1$

K + 2K + 2K + 3K + K = 1

$K = \frac{1}{9}$

$5P = \lambda k$

$5 \times \frac{2}{3} = \lambda \times \frac{1}{9}$

$\lambda = 30$

6. Let S = {1, 2, 3, 4, 5, 6, 9}. Then the number of elements in the set $T = \{A \subseteq S : A \neq \Phi$ and sum of all the elements of A is is not a multiple of 3 } is_______.

माना S ={1, 2, 3, 4, 5, 6, 9} है। तो समुच्चय $T = \{A \subseteq S : A \neq \Phi$ तथा A के सभी अवयवों का योगफल 3 का गुणज नहीं है} में अवयवों की संख्या है_______ ।

Question ID : 86435120701

Ans. Official Answer NTA (80)

Sol. S = {1,2,3,4,5,6,9}

Numbers of 3n type $\rightarrow$ 3,6,9
Numbers of 3n–1 type $\rightarrow$ 2,5
Numbers of 3n–2 type $\rightarrow$ 1,4
Number of subset of S containing one element
which are not divisible by 3 = ${}^2C_1 + {}^2C_1 = 4$
Containing two elements = ${}^3C_1 \times {}^2C_1 + {}^3C_1 \times {}^2C_1 + {}^2C_2 \times {}^2C_2 = 14$
Containing three elements = ${}^3C_2 \times {}^4C_1 + 2({}^2C_2 \times {}^2C_1) + {}^3C_1({}^2C_2 + {}^2C_2) = 22$
Containing four elemnts = ${}^3C_3 \times {}^4C_1 + {}^3C_2({}^2C_2 \times {}^2C_1) + ({}^3C_1{}^2C_1 \times {}^2C_2)2 = 22$
Containing five elemnts = ${}^3C_3({}^2C_2 + {}^2C_2) + ({}^3C_2{}^2C_1 \times {}^2C_2)2 = 2 + 12 = 14$
Containinis six element = 4
Total = 4 + 14 + 22 + 22 + 14 + 4 = 80

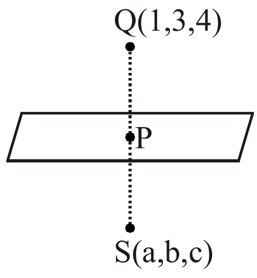
7. Let S be the mirror image of the point Q(1, 3, 4) with respect to the plane 2x–y+z+3=0 and let R (3, 5, λ) be a point of this plane. Then the square of the length of the line segment SR is ______.

माना समतल 2x–y+z+3=0 के सापेक्ष बिन्दु Q(1, 3, 4) का दर्पण प्रतिबिम्ब S है तथा माना इस समतल पर एक बिन्दु R (3, 5, λ) है तो रेखाखण्ड SR की लम्बाई का वर्ग है______ ।

Question ID : 86435120706

Ans. Official Answer NTA (72)

Sol. R (3, 5, r) on the plane
2x – y + z + 3 = 0
then 6 – 5 + r + 3 = 0
r = –4
R(3, 5, –4)
Equation of line QS
$\vec{r} = (1,3,4) + \lambda(2,-1,1)$
$P(1+2\lambda, 3-\lambda, 4+\lambda)$
P is one plane
$2(1+2\lambda) + \lambda - 3 + 4 + \lambda + 3 = 0$



$2+4\lambda+\lambda-3+4+\lambda+3=0$

$6\lambda+6=0$

$\lambda=-1$

$(-1,4,3)$

P is mid-point of QS then $\frac{a+1}{2}=-1$ $\quad$ $a=-3$

$\frac{b+3}{2}=4$ $\quad$ $b=5$

$\frac{c+4}{2}=3$ $\quad$ $c=2$

S(–3, 5, 2)

$RS^2=(\sqrt{36+36})^2=72$

8. Let A $(\sec\theta, 2\tan\theta)$ and $B(\sec\phi, 2\tan\phi)$, where $\theta+\phi=\pi/2$ be two points on the hyperbola $2x^2-y^2=2$. If $(\alpha, \beta)$is the point of the intersection of the normals to the hyperbola at A and B, then $(2\beta)^2$ is equal to _______.

माना अतिपरवलय $2x^2-y^2=2$ पर दो बिन्दु A $(\sec\theta, 2\tan\theta)$ तथा $B(\sec\phi, 2\tan\phi)$, हैं जिनके लिए $\theta+\phi=\pi/2$ है। यदि A तथा B पर अतिपरवलय के अभिलम्बों का प्रतिच्छेदन बिन्दु (α, β) है, तो $(2\beta)^2$ बराबर है_______।

Question ID : 86435120705

Ans. Official Answer NTA (36)

Sol. hyber bola

$2x^2-y^2=2$

$A(\sec\theta.\ 2+\tan\theta)$

$2\sec^2\theta-4\tan^2\theta=2$

$2(1+\tan^2\theta)-4\tan^2\theta=2$

$\tan\theta=0$

$\theta=0$

Similarly $\phi=0$

but given $\theta+\phi=\frac{\pi}{2}$

which is not possible, hence it must be bones.

9. If $\int \frac{2e^x + 3e^{-x}}{4e^x + 7e^{-x}} dx = \frac{1}{14}(ux + v\log_e(4e^x + 7e^{-x})) + C$, where C is as constant of integration, then u + v is equal to ______.

यदि $\int \frac{2e^x + 3e^{-x}}{4e^x + 7e^{-x}} dx = \frac{1}{14}(ux + v\log_e(4e^x + 7e^{-x})) + C$ है, जहाँ C एक समाकलन अचर है, तो u + v बराबर है______ ।

Question ID : 86435120703

Ans. Official Answer NTA (7)

Sol. $I = \int \frac{2e^x + 3e^{-x}}{4e^x + 7e^{-x}} dx$

$2e^x+3e^{-x} = A(4e^x + 7e^{-x}) + B(4e^x - 7e^{-x})$

By comparing the cofficients on both hids

$A + B = \frac{1}{2}$

$A - B = \frac{3}{7}$

$A = \frac{13}{28}$

$B = \frac{1}{28}$

$I = \int \frac{\frac{13}{28}\left(4e^x + 7e^{-x}\right) + \frac{1}{28}\left(4e^x - 7e^{-x}\right)}{4e^x + 7e^{-x}}$

$I = \frac{13}{28}x + \frac{1}{28}\ln\left(4e^x + 7e^{-x}\right) + C$

$I = \frac{1}{14}\left(\frac{13}{2}x + \frac{1}{2}\ln 4e^x + 7e^{-x}\right) + C$

$u = \frac{13}{2}$

$v = \frac{1}{2}$

$u + v = 7$

10. Two circles each of radius 5 units touch each other at the point (1, 2). If the equation of their common tangent is 4x + 3y = 10, and $C_1(\alpha, \beta)$ and $C_2(\gamma,\delta), C_1 \neq C_2$ are their centres, then $|(\alpha+\beta)(\lambda+\delta)|$ is equal to _____.

5 इकाई त्रिज्या के दो वृत्त एक–दूसरे को बिन्दु (1, 2) पर स्पर्श करते है। यदि उनकी उभयनिष्ठ स्पर्श रेखा का समीकरण

$4x + 3y = 10$ है तथा उनके केन्द्र $C_1(\alpha, \beta)$ और $C_2(\gamma,\delta), C_1 \neq C_2$ हैं, तो $|(\alpha+\beta)(\lambda+\delta)|$ बराबर है_____ ।

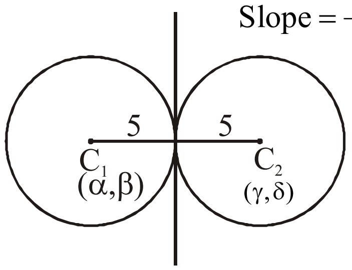
Question ID : 86435120704

Ans. Official Answer NTA (40)

$3x+4y = 10$

$\text{Slope} = \frac{-3}{4}$

Sol.



$\text{Slope of } C_1C_2 = \tan\theta = \frac{4}{3}$

then $\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

to find C_1 & C_2 use parameic from of line

$$\frac{x-1}{\cos\theta} = \frac{y-2}{\sin\theta} = \pm 5$$

$$\frac{x-1}{\frac{3}{5}} = \frac{y-2}{\frac{4}{5}} = \pm 5$$

$x - 1 = \pm 3$ $\quad$ $y - 2 = \pm 4$

$x = 4$ $\quad$ $y = 6$

$= -2$ $\quad$ $y = -2$

$(\alpha,\beta) \Rightarrow (4_1, 6)$

$(\gamma,\delta) \Rightarrow (-2,-2)$

$|(\alpha + B)(r + S)| = |10 \times (-4)| = 40$

MATRIX