JEE Main June 2022

Question Paper With Text Solution

26 June | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JUNE 2022 | 26TH JUNE SHIFT-2

SECTION – A

Question ID : 181

Function

1. Let $f : R \to R$ be defined as $f(x) = x - 1$ and $g : R - \{1, -1\} \to R$ be defined as $g(x) = \frac{x^2}{x^2 - 1}$. Then the function fog is :

(1) one-one but not onto (2) onto but not one-one

(3) both one-one and onto (4) neither one-one nor onto

माना $f : R \to R$ तथा $g : R - \{1, -1\} \to R$, $f(x) = x - 1$ तथा $g(x) = \frac{x^2}{x^2 - 1}$ द्वारा परिभाषित हैं। तो फलन fog :

(1) एकैकी है परन्तु आच्छादक नहीं है (2) आच्छादक है परन्तु एकैकी नहीं है

(3) एकैकी तथा आच्छादक दोनों है (4) न तो एकैकी है न ही आच्छादक है

Ans. Official Answer NTA (4)

Sol. $f(x) = x - 1$ $\quad g : R - \{1, -1\} \to R$

$f : R \to R$ $\quad g(x) = \frac{x^2}{x^2 - 1}$

$$f(g(x)) = \frac{x^2 - 1}{x^2 - 1} = \frac{x^2 - x^2 + 1}{x^2 - 1}$$

$$f(g(x)) = \frac{1}{x^2 - 1} \qquad fog : R - \{1, -1\} \to R$$

Neither one–one nor onto

Question ID : 182

Determinant

2. If the system of equations $\alpha x + y + z = 5$, $x + 2y + 3z = 4$, $x + 3y + 5z = \beta$ has infinitely many solutions, then the ordered pair (α, β) is equal to :

यदि समीकरण निकाय $\alpha x + y + z = 5$, $x + 2y + 3z = 4$, $x + 3y + 5z = \beta$ के अनंत हल हैं, क्रमित युग्म (α, β) बराबर है :

(1) (1, –3) (2) (–1, 3) (3) (1, 3) (4) (–1, –3)

Ans. Official Answer NTA (3)

Sol. $\alpha x + y + \cancel{z} = 5$

$x+2y+3\cancel{z}=4$

$x+3y+5\cancel{z}=\beta$

$$D=\begin{vmatrix}\alpha & 1 & 1\\ 1 & 2 & 3\\ 1 & 3 & 5\end{vmatrix}=0$$

$\alpha(1)-1(2)+1(1)=0$

$\alpha-2+1=0$

$\alpha-1=0$

$\alpha=1$

$D_x=0$

$$\begin{vmatrix}5 & 1 & 1\\ 4 & 2 & 3\\ \beta & 3 & 5\end{vmatrix}=0$$

$5(1)-1(20-3\beta)+1(12-2\beta)=0$

$5-20+3\beta+12-2\beta=0$

$-3+\beta=0\Rightarrow\beta=3$ `

$(\alpha,\beta)=(1,3)$

Question ID : 183

Sequence & progression

3. If $A=\sum_{n=1}^{\infty}\frac{1}{\left(3+(-1)^n\right)^n}$ and $B=\sum_{n=1}^{\infty}\frac{(-1)^n}{\left(3+(-1)^n\right)^n}$, then $\frac{A}{B}$ is equal to :

यदि $A=\sum_{n=1}^{\infty}\frac{1}{\left(3+(-1)^n\right)^n}$ तथा $B=\sum_{n=1}^{\infty}\frac{(-1)^n}{\left(3+(-1)^n\right)^n}$ हैं, तो $\frac{A}{B}$ बराबर है :

(1) $\frac{11}{9}$ (2) 1 (3) $-\frac{11}{9}$ (4) $-\frac{11}{3}$

Ans. Official Answer NTA (3)

Sol. $A=\sum_{n=1}^{\infty}\frac{1}{\left(3+(-1)^n\right)^n}$ $B=\sum_{n=1}^{\infty}\frac{(-1)^n}{\left(3+(-1)^n\right)^n}$

$A=\frac{1}{2}+\frac{1}{4^2}+\frac{1}{2^3}+\frac{1}{4^4}+.......$

$$A=\left(\frac{1}{2}+\frac{1}{2^3}+\frac{1}{2^5}+.......\right)+\left(\frac{1}{4^2}+\frac{1}{4^4}+........\right)$$

$$A=\frac{\frac{1}{2}}{1-\frac{1}{2^2}}+\frac{\frac{1}{4^2}}{1-\frac{1}{4^2}}$$

$$A=\frac{\frac{1}{2}}{\frac{3}{4}}+\frac{\frac{1}{16}}{\frac{15}{16}}$$

$$A=\frac{2}{3}+\frac{1}{15}$$

$$A=\frac{10+1}{15}=\frac{11}{15}$$

$$B=\sum_{n=1}^{\infty}\frac{(-1)^n}{\left(3+(-1)^n\right)^n}$$

$$B=\frac{-1}{2}+\frac{1}{4^2}-\frac{1}{2^3}+\frac{1}{4^4}+.........$$

$$B=-\left(\frac{1}{2}+\frac{1}{2^3}+\frac{1}{2^5}........\right)+\left(\frac{1}{4^2}+\frac{1}{4^4}+......\right)$$

$$B=-\left(\frac{\frac{1}{2}}{1-\frac{1}{2^2}}\right)+\left(\frac{\frac{1}{4^2}}{1-\frac{1}{4^2}}\right)$$

$$B=-\left(\frac{\frac{1}{2}}{\frac{3}{4}}\right)+\frac{\frac{1}{16}}{\frac{15}{16}}$$

$$B=\frac{-2}{3}+\frac{1}{15}\qquad=\frac{-10+1}{15}=\frac{-9}{15}$$

$$\frac{A}{B}=\frac{\frac{11}{15}}{\frac{-9}{15}} \qquad =\frac{-11}{9}$$

Question ID : 184

Limit

4. $\lim\limits_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}$ is equal to :

$\lim\limits_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}$ बराबर है :

(1) $\frac{1}{3}$ (2) $\frac{1}{4}$ (3) $\frac{1}{6}$ (4) $\frac{1}{12}$

Ans. Official Answer NTA (3)

Sol. $\lim\limits_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}$

$$\lim_{x\to 0}\frac{-2\sin\left(\frac{x+\sin x}{2}\right)\sin\left(\frac{\sin x-x}{2}\right)}{x^4}$$

$$=-2\lim_{x\to 0}\frac{\sin\left(\frac{\sin x+x}{2}\right)}{\left(\frac{\sin x+x}{2}\right)}\cdot\frac{\sin\left(\frac{\sin x-x}{2}\right)}{\left(\frac{\sin x+x}{2}\right)}\frac{\left(\sin^2 x-x^2\right)}{4x^2}$$

$$=\frac{-2}{4}\lim_{x\to 0}\left(\frac{\sin^2 x-x^2}{x^4}\right);\left(\frac{0}{0}\text{form}\right)\text{L'Hospitla rule}$$

$$=\frac{-1}{2}\lim_{x\to 0}\left(\frac{\sin 2x-2x}{4x^3}\right);\left(\frac{0}{0}\text{form}\right)\text{L'Hospitla rule}$$

$$=\frac{-1}{2}\lim_{x\to 0}\left(\frac{2\cos 2x-2}{12x^2}\right)$$

$$=\frac{-1}{24}\lim_{x\to 0}\left(\frac{2-4\sin^2 x-2}{x^2}\right)$$

$$=\frac{4}{24}\lim_{x\to 0}\left(\frac{\sin^2 x}{x^2}\right)$$

$$=\frac{1}{6}$$

Question ID : 185

Continuity & Differentiability

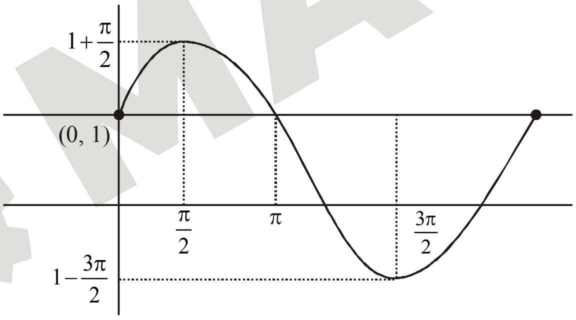
5. Let $f(x) = \min\{1, 1 + x\sin x\}, 0 \le x \le 2\pi$. If m is the number of points, where f is not differentiable and n is the number of points, where f is not continuous, then the ordered pair (m, n) is equal to :

माना $f(x) = \min\{1, 1 + x\sin x\}, 0 \le x \le 2\pi$ है। यदि उन बिन्दुओं की संख्या, जहाँ f अवकलनीय नहीं है, m है तथा उन बिन्दुओं की संख्या, जहाँ f संतत नहीं है, n है, तो क्रमित युग्म (m, n) बराबर है :

(1) (2, 0) (2) (1, 0) (3) (1, 1) (4) (2, 1)

Ans. Official Answer NTA (2)

Sol. $f(x) = \min\{1, 1 + x\sin x\}\ 0 \le x \le 2\pi$



$$f(x)=\begin{cases}1 & 0\le x<\pi\\ 1+x\sin x & \pi\le x\le 2\pi\end{cases}$$

$f(0^+) = 1$ $f(0) = 1$

$f(\pi^-) = 1$ $f(\pi^+) = 1$ $f(\pi) = 1$

$f(2\pi^-) = 1$ $f(2\pi) = 1$

$n = 0$

$$f'(x)=\begin{cases}0 & 0<x<\pi\\ x\cos x+\sin x & \pi<x<2\pi\end{cases}$$

$f'(\pi^-)=0$ $\qquad$ $f'(\pi^+)=\pi(-1)=-\pi$

m = 1

(m, n) = (1, 0)

Question ID : 186

Tangent and normal

6. Consider a cuboid of sides 2x, 4x and 5x and a closed hemisphere of radius r. If the sum of their surface areas is a constant k, then the ratio x : r, for which the sum of their volumes is maximum, is :

भुजाओं 2x, 4x तथा 5x के एक घनाभ तथा त्रिज्या r के एक बंद गोलार्ध का विचार कीजिए। यदि उनके पृष्ठीय क्षेत्रफलों का योग एक अचर k है, तो उनके आयतन का योग अधिकतम होने के लिए अनुपात x : r है :

(1) 2 : 5 (2) 19 : 45 (3) 3 : 8 (4) 19 : 15

Ans. Official Answer NTA (2)

Sol. T.S.A of cuboid = $2\left(8x^2+20x^2+10x^2\right)=76x^2$

T.S.A of closed Hemis pheve = $3\pi r^2$

$$76x^2+3\pi r^2=k$$

$$152x\frac{dx}{dr}+6\pi r=0$$

$$\frac{dx}{dr}=-\frac{6\pi r}{152x}$$

Volume = $40x^3+\frac{2}{3}\pi r^3$

$$\frac{dv}{dr}=120x^2\frac{dx}{dr}+2\pi r^2=0$$

$$120x^2\times\left(\frac{-6\pi r}{152x}\right)+2\pi r^2=0$$

$$120x \times \left(+ \frac{6}{152} \right) = +2r$$

$$\frac{x}{r} = \frac{2 \times 152}{6 \times 120}$$

$$\frac{x}{r} = \frac{19}{3 \times 15} = \frac{19}{45}$$

Question ID : 187

Area Under Curve

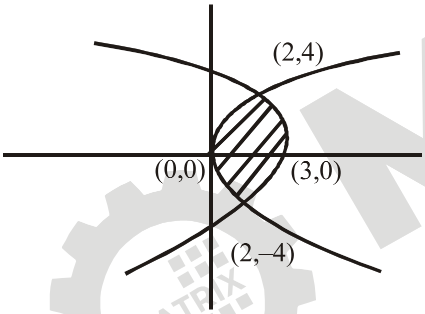
7. The area of the region bounded by $y^2 = 8x$ and $y^2 = 16(3 - x)$ is equal to :

वक्रों $y^2 = 8x$ तथा $y^2 = 16(3 - x)$ द्वारा घिरे क्षेत्र का क्षेत्रफल है :

(1) $\frac{32}{3}$ (2) $\frac{40}{3}$ (3) 16 (4) 19

Ans. Official Answer NTA (3)

Sol.



$$8x = 16(3 - x)$$

$$x = 6 - 2x$$

$$3x = 6 \Rightarrow x = 2$$

$$y^2 = 16 \Rightarrow y = \pm 4$$

$x = y^2/8$,

$$x = \frac{48 - y^2}{16}$$

$$\text{Area} = 2\int_0^4 \left(\left(\frac{48 - y^2}{16} \right) - \left(\frac{y^2}{8} \right) \right) dy = 16$$

Question ID : 188

Indefinite Integration

8. If $\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}\,dx = g(x)+c,\ g(1)=0$, then $g\left(\frac{1}{2}\right)$ is equal to :

यदि $\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}\,dx = g(x)+c,\ g(1)=0$ हैं, तो $g\left(\frac{1}{2}\right)$ बराबर है :

(1) $\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)+\frac{\pi}{3}$

(2) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)+\frac{\pi}{3}$

(3) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)-\frac{\pi}{3}$

(4) $\frac{1}{2}\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)-\frac{\pi}{6}$

Ans. Official Answer NTA (1)

Sol. $\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx = g(x)+c$ $\qquad$ g(1) = 0

x = cos θ

dx = – sin 0 dθ

$\int \frac{1}{\cos\theta}\sqrt{\frac{1-\cos\theta}{1+\sin\theta}}$ $\qquad$ x – sinθ dx

$-\int \frac{\sin\theta}{\cos\theta}\sqrt{\frac{\sin^2\left(\frac{\theta}{2}\right)}{\cos^2\left(\frac{\theta}{2}\right)}}\,d\theta$ $\qquad$ $= -\int \frac{\sin\theta}{\cos\theta}\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\,d\theta$

$-\int \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\times\sin\frac{\theta}{2}}{\cos\theta\cos\frac{\theta}{2}}\,d\theta$

$-\int \frac{2\sin^2\left(\frac{\theta}{2}\right)}{\cos\theta}\,d\theta = -\int\frac{1-\cos\theta}{\cos\theta}\,d\theta$

$-\int \sec\theta d\theta + \int d\theta$

$\theta - \int \sec\theta\ d\theta + c$

θ – ln(sec θ + tan θ) + c

$\cos^{-1}n - ln(\sec(\cos^{-1}x) + \tan(\cos^{-1}x)) + c$

$g(x) = \cos^{-1}x - ln(\sec(\cos^{-1}x) + \tan^{-1}(\cos^{-1} 1))$

g(1) = 0 – ln(1 + tan θ) = 0

$$(g)\frac{1}{2}=\frac{\pi}{3}-ln\left(\sec\frac{\pi}{3}+\tan\frac{\pi}{3}\right)$$

$$(g)\frac{1}{2}=\frac{\pi}{3}-ln\left(2+\sqrt{3}\right)$$

$$(g)\frac{1}{2}=\frac{\pi}{3}+ln\left(\frac{1}{2+\sqrt{3}}\right)=\frac{\pi}{3}+ln\left(2-\sqrt{3}\right)$$

Question ID : 189

Differential Equation

9. If y = (x) is the solution of the differential equation $x\frac{dy}{dx}+2y=xe^x, y(1)=0$ then the local maximum value of function $z(x)=x^2y(x)-e^x, x\in R$ is :

यदि अवकल समीकरण $x\frac{dy}{dx}+2y=xe^x, y(1)=0$ का हल $y=y(x)$ है, तो फलन $z(x)=x^2y(x)-e^x, x\in R$ का स्थानीय अधिकतम मान है :

(1) 1 – e (2) 0 (3) 1/2 (4) $\frac{4}{e}-e$

Ans. Official Answer NTA (4)

Sol. $x\frac{dy}{dx}+zy=xe^x$

$$\frac{dy}{dx}+\left(\frac{2}{x}\right)y=e^x$$

$$\text{I. F} = e^{\int\frac{2dx}{x}}=e^{2\ell nx}e^{\ell nx^2}=x^2$$

$$yx^2=\int e^4\times x^2dx$$

$$yx^2=x^2\times e^x-\int 2x\times e^x dy$$

$$yx^2=x^2e^x-2\left[xe^x-\int e^x dx\right]$$

$$x^2y=x^2e^x-2xe^x+2e^x+c$$

$$1\times 0=1\times e-\frac{2}{e}+\frac{2}{e}+c$$

$$c=-e$$

$$x^2y(x)=x^2e^x-2xe^x+2e^x-e$$

$$\cancel{z}(x)=x^2e^x-2xe^x+2e^x-e-e^x$$

$\not{z}\left(x\right)=x^2e^x-2xe^x+e^x-e$

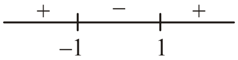
$\not{z}'\left(x\right)=x^2e^x+2xe^x-2\left(xe^x+e^x\right)+e^x$

$\not{z}'\left(x\right)=x^2e^x+2xe^x-2xe^x-2e^x+e^x$

$\not{z}'\left(x\right)=x^2e^x-e^x=0$

$\Rightarrow e^x(x^2-1)=0$

$\Rightarrow x=\pm1$



$\not{z}\left(-1\right)=1\times e^{-1}-2\left(-1\right)\times e^{-1}+e^{-1}-e$

$=\frac{1}{e}+\frac{2}{e}+\frac{1}{e}-e$

$=\frac{4}{e}-e$

Question ID : 1810

Differential Equation

10. If the solution of the differential equation $\frac{dy}{dx}+e^x\left(x^2-2\right)y=\left(x^2-2x\right)\left(x^2-2\right)e^{2x}$ satisfies y(0) = 0, then the value of y(2) is :

यदि अवकल समीकरण $\frac{dy}{dx}+e^x\left(x^2-2\right)y=\left(x^2-2x\right)\left(x^2-2\right)e^{2x}$ का हल y(0) = 0 को संतुष्ट करता है, तो y(2) का मान है :

(1) – 1 (2) 1 (3) 0 (4) e

Ans. Official Answer NTA (3)

Sol. $\frac{dy}{dx}+e^x\left(x^2-2\right)y=\left(x^2-2x\right)\left(x^2-2\right)e^{2x}$

IF = $e^{\int e^x\left(x^2-2\right)dx}$

Solution:

$y.e^{e^x\left(x^2-2x\right)}=\int e^{e^x\left(x^2-2x\right)}\left(x^2-2x\right)\left(x^2-2\right)e^{2x}dx$

$t=e^x\left(x^2-2x\right)$

$dt=e^x\left(x^2-2\right)dx$

$ye^t=\int e^t t.dt$

$$ye^{t} = te^{t} - e^{t} + c$$

$$ye^{e^{x}\left(x^{2}-2x\right)} = e^{e^{x}\left(x^{2}-2x\right)}\left(e^{x}\left(x^{2}-2x\right)-1\right)+c$$

y (0) = 0

$0 = e^{0}(e^{0}0 - 1) + c$

c = 1

$$ye^{e^{x}\left(x^{2}-2x\right)} = e^{e^{x}\left(x^{2}-2x\right)}\left(e^{x}\left(x^{2}-2x\right)-1\right)+1$$

y(2) : ye

$$ye^{e^{2}(0)} = e^{e^{2}0}\left(e^{2}0-1\right)+1$$

y = 0

Question ID : 1811

Ellipse

11. If m is the slope of a common tangent to the curves $\frac{x^{2}}{16}+\frac{y^{2}}{9}=1$ and $x^2 + y^2 = 12$, then $12m^2$ is equal to:

यदि वक्रों $\frac{x^{2}}{16}+\frac{y^{2}}{9}=1$ तथा $x^2 + y^2 = 12$ की एक उभ्यनिष्ठ स्पर्श रेखा की प्रवणता m है, तो $12m^2$ बराबर है :

(1) 6 (2) 9 (3) 10 (4) 12

Ans. Official Answer NTA (2)

Sol. $\frac{x^{2}}{16}+\frac{y^{2}}{9}=1$ $x^2 + y^2 = 12$

$$y = mx \pm \sqrt{a^{2}m^{2}+b^{2}}$$

$$y = mx \pm \sqrt{16m^{2}+9}$$

Apply COT

$$mn - 9 \pm \sqrt{16m^{2}+9} = 0$$

$$\left|\frac{\pm\sqrt{16m^{2}+9}}{\sqrt{1+m^{2}}}\right| = 2\sqrt{3}$$

$16m^2 + 9 = 12(1 + m^2)$

$16m^2 + 9 = 12 + 12\ m^2$

$4m^2 = 3$

$$m = \pm\frac{\sqrt{3}}{2}$$

$$m^2 = \frac{3}{4}$$

$12m^2 = 12\left(\frac{3}{4}\right) = 9$

Question ID : 1812

Ellipse

12. The locus of the mid point of the line segment joining the point (4, 3) and the points on the ellipse $x^2 + 2y^2 = 4$ is an ellipse with ecentricity :

दीर्घवृत्त $x^2 + 2y^2 = 4$ के बिन्दुओं को बिन्दु (4, 3) से मिलाने वाले रेखाखण्ड के मध्यबिन्दु का बिन्दुपथ एक दीर्घवृत्त है, जिसकी उत्केन्द्रता है :

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{2\sqrt{2}}$ (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{2}$

Ans. Official Answer NTA (3)

Sol. $\frac{x^2}{4}+\frac{y^2}{2}=1$

$a = 2$ $b=\sqrt{2}$

$p(\theta)=\left(2\cos\theta, \sqrt{2}\sin\theta\right)$ A(4, 3)

$$S(h,k)=\left(\frac{2\cos\theta+4}{2}, \frac{\sqrt{2}\sin\theta+3}{2}\right)$$

$\frac{2h-4}{2}=\cos\theta$ ——(1)

$\frac{2k-3}{\sqrt{2}}=\sin\theta$ ——(2)

$(1)^2 + (2)^2$

$$1=\left(\frac{2h-4}{2}\right)^2+\left(\frac{2k-3}{\sqrt{2}}\right)^2$$

$$1=(h-2)^2+\frac{(2k-3)^2}{2}$$

$$\frac{(x-2)^2}{1}+\frac{(2y-3)^2}{2}=1$$

$e^2 = 1 - \frac{b^2}{a^2}$

$e^2 = 1 - \frac{1}{2}$

$e^2 = \frac{1}{2}$ $\Rightarrow e = \frac{1}{\sqrt{2}}$

Question ID : 1813

Hyperbola

13. The normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{9} = 1$ at the point $\left(8, 3\sqrt{3}\right)$ on it passes through the point :

अतिपरवलय $\frac{x^2}{a^2} - \frac{y^2}{9} = 1$ के बिन्दु $\left(8, 3\sqrt{3}\right)$ पर अभिलंब किस बिन्दु से होकर जाता है?

(1) $\left(15, -2\sqrt{3}\right)$ (2) $\left(9, 2\sqrt{3}\right)$ (3) $\left(-1, 9\sqrt{3}\right)$ (4) $\left(-1, 6\sqrt{3}\right)$

Ans. Official Answer NTA (3)

Sol. $\frac{x^2}{a^2} - \frac{y^2}{9} = 1$ $\left(8, 3\sqrt{3}\right)$

$\frac{64}{a^2} - 3 = 1$

$\frac{64}{a^2} = 4$ $\Rightarrow \frac{16}{a^2} = 1$

$\frac{x^2}{16} - \frac{y^2}{9} = 1$ $a^2 = 16$

$\frac{16x}{8} + \frac{9y}{3\sqrt{3}} = 25$

$2y + \sqrt{3}y = 25$

$\left(-1, 9\sqrt{3}\right)$ $\Rightarrow -2 + 27 = 25$

Question ID : 1814

3D Geometry

14. If the plane 2x + y – 5z = 0 is rotated about its line of intersection with the plane 3x – y + 4z – 7 = 0 by an angle of $\frac{\pi}{2}$, then the plane after the rotation passes throught the point :

समतल 2x + y – 5z = 0 को, इसकी समतल 3x – y + 4z – 7 = 0 से प्रतिच्छेदन रेखा के सापेक्ष $\frac{\pi}{2}$ कोण तक घुमाने पर प्राप्त समतल किस बिन्दु से होकर जाता है :

(1) (2, –2, 0) (2) (–2, 2, 0) (3) (1, 0, 2) (4) (–1, 0, –2)

Ans. Official Answer NTA (3)

Sol. $(2x+y-5z)+\lambda(3x-y+4z-7)=0$

Rotated by $\frac{\pi}{2}$

$(2+3\lambda)x+(1-\lambda)y+(-5+4\lambda)z-7\lambda=0$

& 2x + y –5z = 0

$\Rightarrow 2(2+3\lambda)+1-(1-\lambda)-5(-5+4\lambda)-7\lambda=0$

$30=15\lambda$

$\lambda=2$

Required plane : 8x –y + 32 –14 = 0

check options :

Question ID : 1815

Vectors

15. If the lines $\vec{r}=(\hat{i}-\hat{j}+\hat{k})+\lambda(3\hat{j}-\hat{k})$ and $\vec{r}=(\alpha\,\hat{i}-\hat{j})+\mu(2\hat{i}-3\hat{k})$ are coplanar, then the distance of the plane containing these two lines from the point (α, 0,0) is :

यदि रेखाएँ $\vec{r}=(\hat{i}-\hat{j}+\hat{k})+\lambda(3\hat{j}-\hat{k})$ तथा $\vec{r}=(\alpha\,\hat{i}-\hat{j})+\mu(2\hat{i}-3\hat{k})$ सह–तलीय हैं, तो इन रेखाओं से होकर जाने वाले समतल की बिन्दु (α, 0,0) से दूरी है :

(1) $\frac{2}{9}$ (2) $\frac{2}{11}$ (3) $\frac{4}{11}$ (4) 2

Ans. Official Answer NTA (2)

Sol. $\vec{r}=(1,3\lambda-1,1-\lambda)$

$\vec{r}=(\alpha+2\mu,-1,-3\mu)$

$$\begin{bmatrix} \alpha-1 & 0 & -1 \\ 0 & 3 & -1 \\ 2 & 0 & -3 \end{bmatrix} = 0$$

$$\Rightarrow \alpha = \frac{5}{3}$$

$$\vec{n} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & -1 \\ 2 & 0 & -3 \end{bmatrix} = \hat{i}(-9) - \hat{j}(2) + \hat{k}(-6)$$

$$= -9\hat{i} - 2\hat{j} - 6\hat{k}$$

equation of plane

$9(x-1) + 2(y+1) + 6(z-1) = 0$

$9x + 2y + 6z = 13$

Perpendicular distance from (5/3, 0, 0)

$$= \left| \frac{9\frac{5}{3} + 0 + 0 - 13}{\sqrt{81+4+36}} \right| = \left| \frac{2}{\sqrt{121}} \right| = \frac{2}{11}$$

Question ID : 1816

Vectors

16. Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}, \vec{b} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$ be three given vectors. Let $\vec{v}$ be a vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ is $\frac{2}{\sqrt{3}}$. If $\vec{v}.\hat{j} = 7$, then $\vec{v}.(\hat{i} + \hat{k})$ is equal to :

माना तीन सदिश $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}, \vec{b} = 2\hat{i} - 3\hat{j} + \hat{k}$ तथा $\vec{c} = \hat{i} - \hat{j} + \hat{k}$ है। माना $\vec{a}$ तथा $\vec{b}$ के समतल में एक सदिश $\vec{v}$ का $\vec{c}$ पर प्रक्षेप $\frac{2}{\sqrt{3}}$ है। यदि $\vec{v}.\hat{j} = 7$ है, तो $\vec{v}.(\hat{i} + \hat{k})$ बराबर है :

(1) 6 (2) 7 (3) 8 (4) 9

Ans. Official Answer NTA (4)

Sol. $\vec{v} = x\vec{a} + y\vec{b}$

$\vec{v} = x\left(\hat{i}+\hat{j}+2\hat{k}\right)+y\left(2\hat{i}-3\hat{j}+\hat{k}\right)$

$\dfrac{\vec{v}.\vec{c}}{|\vec{c}|} = 2\sqrt{3}$

$x + 2y - (x - 3y) + 2x + y = 2$
$2x + 6y = 2$
$x + 3y = 1 \quad$ ——(1)

$\vec{v}.\hat{j} = 7$
$x - 3y = 7 \quad$ ——(2)
$2y = 8 \quad \Rightarrow x = 4$
$3y = -3 \quad \Rightarrow y = -1$

$\vec{v} = 2\hat{i}+7\hat{j}+7\hat{k}$

$\vec{v}.\left(\hat{i}+k\right)$

$2 + 7 = 9$

Question ID : 1817

Statistics

17. The mean and standard deviation of 50 observations are 15 and 2 respectively. It was found that one incorrect observation was taken such that the sum of correct and incorrect observations is 70. If the correct mean is 16, then the correct variance is equal to :

50 प्रेक्षणों के माध्य तथा मानक विचलन क्रमशः 15 तथा 2 हैं। यह पाया गया कि एक गलत प्रेक्षण लिया गया था तथा सही और गलत प्रेक्षणों का योग 70 है। यदि सही माध्य 16 है, तो सही प्रसरण बराबर है :

(1) 10 (2) 36 (3) 43 (4) 60

Ans. Official Answer NTA (3)

Sol. Given $\overline{x} = 15$

$\sigma = 2 \Rightarrow \sigma^2 = 4$

$x_1 + x_2 + ... + x_{50} = 15 \times 50 = 750$

$4 = \frac{x_1^2 + x_2^2 + ... + x_{50}^2}{50} - 225$

$x_1^2 + x_2^2 + ... + x_{50}^2 = 50 \times 229$

Let a be the correct observation and b is the incorrect observation

a + b = 70

$16 = \frac{750 - b + a}{50}$

a – b = 50

= a = 60 b = 10

correct variance $\frac{50 \times 229 + (60)^2 - (10)^2}{50} - 256$

= 43

Let a be the correct observation and b is the incorrect observation

a + b = 70

Question ID : 1818

Trigonometric Ratio and Identities

18. 16 sin (20º) sin (40º) sin (80º) is equal to :

16 sin (20º) sin (40º) sin (80º) बराबर है :

(1) $\sqrt{3}$ (2) $2\sqrt{3}$ (3) 3 (4) $4\sqrt{3}$

Ans. Official Answer NTA (2)

Sol. 16 sin20° sin40° sin80°

8 sin20°(2 sin80° sin40°)

8 sin20°(cos 40° – cos 120°)

$8\sin{}^\circ\left(\cos 40^\circ + \frac{1}{2}\right)$

8 cos40° sin20° + 4 sin20°

4(2 cos40° sin20° + sin20°)

$$4(\sin 60°) = 4\times\frac{\sqrt{3}}{2} = 2\sqrt{3}$$

Question ID : 1819

ITF

19. If the inverse trignometric functions take principal values, then $\cos^{-1}\left(\frac{3}{10}\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right)+\frac{2}{5}\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)\right)$ is equal to :

यदि प्रतिलोम त्रिकोणमितीय फलन मुख्यमान लेते हैं, तो $\cos^{-1}\left(\frac{3}{10}\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right)+\frac{2}{5}\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)\right)$ बराबर है :

(1) 0 (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{6}$

Ans. Official Answer NTA (3)

Sol. $\cos^{-1}\left(\frac{3}{10}\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right)+\frac{2}{5}\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)\right) = 0$

$\Rightarrow \tan\theta = \frac{4}{3}$

$\cos^{-1}\left(\frac{3}{10}\cos\theta+\frac{2}{5}\sin\theta\right)$

$\tan\theta = \frac{4}{3} = \frac{P}{B}$

$H = \sqrt{25} = 25$

$\cos\theta = \frac{3}{5}$ $\sin\theta = \frac{4}{5}$

$\cos^{-1}\left(\frac{3}{10}\times\frac{3}{5}+\frac{2}{5}\times\frac{4}{5}\right) = \cos^{-1}\left(\frac{9}{50}+\frac{8}{25}\right)$

$\cos^{-1}\left(\frac{25}{50}\right) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Question ID : 1820

Mathematical Reasoning

20. Let $r \in \{p, q, \sim p, \sim q\}$ be such that the logical statement $r \vee (\sim p) \Rightarrow (p \wedge q) \vee r$ is a tautology. Then r is

equal :

माना $r \in \{p, q, \sim p, \sim q\}$ के लिए तर्क संगत कथन $r \vee (\sim p) \Rightarrow (p \wedge q) \vee r$ एक पुनरुक्ति है। तो r बराबर है :

(1) p (2) q (3) ~ p (4) ~ q

Ans. Official Answer NTA (3)

Sol. $\therefore \sim p \vee \sim p = \sim p$

and $(p \vee q) \vee \sim p = p$

$\therefore \sim p \Rightarrow p = \text{tautology}$

SECTION – B

Question ID : 1821

Methods of Differentiation

21. Let $f : R \to R$ safisfy $f(x+y) = 2^x f(y) + 4^y f(x), \forall x, y \in R$. If f(2) = 3 then $14.\dfrac{f'(4)}{f'(2)}$ is equal to ___.

माना $f : R \to R$, $f(x+y) = 2^x f(y) + 4^y f(x), \forall x, y \in R$ को संतुष्ट करता है। यदि f(2) = 3 है, तो $14.\dfrac{f'(4)}{f'(2)}$ बराबर है ___.

Ans. Official Answer NTA (248)

Sol. $f(x + y) = 2^x f(y) + 4^y f(x)$

$f'(x+y) \times 1 = 2^x \ell n2 f(y) + 4^y f'(x)$

$x = y = 2$

$f'(4) = 4\ell n2 f(2) + 16 f'(2)$

$f'(4) - 16 f'(2) = 12\ell n2$ ——(1)

$f'(x+y) \times 1 = 2^x f'(y) + f(x) 4^y \ell n4$

$x = y = 2$

$f'(4) = 4 f'(2) + 48\ell n4$

$f'(4) - 4 f'(2) = 48\ell n4$ ——(2)

$12 f'(2) = 48\ell n4 - 12\ell n2$

$12 f'(2) = 96\ell n2 - 12\ell n2$

$f'(2) = 7\ell n2$

$f'(4) - 16 \times 7\ell n2 = 12\ell n2$

$$f'(4)=124\ell n2$$

$$\frac{f'(4)}{f'(4)}=124\times 2=248$$

Question ID : 1822

Quadratic Equation

22. Let p and q be two real numbers such that $p + q = 3$ and $p^4 + q^4 = 369$. Then $\left(\frac{1}{p}+\frac{1}{q}\right)^{-2}$ is equal to ____.

माना दो वास्तविक संख्याओं p तथा q के लिए $p + q = 3$ तथा $p^4 + q^4 = 369$ हैं। तो $\left(\frac{1}{p}+\frac{1}{q}\right)^{-2}$ बराबर है ______

Ans. Official Answer NTA (4)

Sol. $p + q = 3$ $\qquad$ $p^4 + q^4 = 369$

$$\left(\frac{1}{p}+\frac{1}{q}\right)^{-2}=\left(\frac{p+q}{pq}\right)^{-2}=\left(\frac{pq}{p+q}\right)^{2}$$

$$=\frac{(pq)^2}{9}$$

$$(p)^2+\left(q^2\right)^2=\left(p^2+q^2\right)^2-2(pq)^2$$

$$369=\left((p+q)^2-(2pq)\right)^2-2(pq)^2$$

$$369=\left(9-(2pq)\right)^2-2(pq)^2$$

$pq = t$

$$369=(9-2t)^2-2t^2$$

$$369=81+4t^2-36t-2t^2$$

$2t^2 - 36t - 288 = 0$

$t^2 - 18t - 144 = 0$

$t^2 - 24t + 6t - 144 = 0$

$t(t - 24) + 6(t - 24) = 0$

$t = 24$ $\quad$ or $\quad$ $t = -6$

$$\left(\frac{1}{p}+\frac{1}{q}\right)^{-2}=\frac{(pq)^2}{9}=\frac{36}{9}=4$$

or $\frac{576}{9}=64$

Question ID : 1823

Complex Number

23. If $z^2 + z + 1 = 0$, $z \in C$, then $\left|\sum_{n=1}^{15}\left(z^n + (-1)^n \frac{1}{z^n}\right)^2\right|$ is equal to _______.

यदि $z^2 + z + 1 = 0$, $z \in C$ हैं, तो $\left|\sum_{n=1}^{15}\left(z^n + (-1)^n \frac{1}{z^n}\right)^2\right|$ बराबर है ________

Ans. Official Answer NTA (2)

Sol. If $z^2 + z + 1 = 0$, $z \in c$
$z = w, w^2$

$$\left|\sum_{n=1}^{15}\left(z^n + (-1)^n \frac{1}{z^n}\right)^2\right|$$

$$\left|\sum_{n=1}^{15}\omega^{2n} + \sum_{n=1}^{15}\frac{1}{\omega^{2n}} + 2(-1)\right|$$

$$\left|\frac{\omega^2\left(\omega^{30}-1\right)}{\left(\omega^2-1\right)} + \frac{\frac{1}{\omega^2}\left(\frac{1}{\omega^{30}}-1\right)}{\left(\frac{1}{\omega^2}-1\right)} - 2\right|$$

$$\left|\frac{\omega^2\left(\omega^{30}-1\right)}{\left(\omega^2-1\right)} + \frac{\omega^{30}-1}{\omega^2\left(\omega^2-1\right)} - 2\right|$$

$$\left|\frac{\omega^2(1-1)}{\left(\omega^2-1\right)} + \frac{1-1}{\omega^2\left(\omega^2-1\right)} - 2\right|$$

$= |0 + 0 - 2| = 2$

Question ID : 1824

Matrices

24. Let $X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, Y = \alpha I + \beta X + \gamma X^2$ and $Z = \alpha^2 I - \alpha\beta X + \left(\beta^2 - \alpha\gamma\right)X^2, \alpha, \beta, \gamma \in R$.

If $Y^{-1} = \begin{bmatrix} 1/5 & -2/5 & 1/5 \\ 0 & 1/5 & -2/5 \\ 0 & 0 & 1/5 \end{bmatrix}$, then $(\alpha - \beta + \gamma)^2$ is equal to ________.

माना $X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, Y = \alpha I + \beta X + \gamma X^2$ तथा $Z = \alpha^2 I - \alpha\beta X + (\beta^2 - \alpha\gamma)X^2, \alpha, \beta, \gamma \in R$ हैं। यदि

$Y^{-1} = \begin{bmatrix} 1/5 & -2/5 & 1/5 \\ 0 & 1/5 & -2/5 \\ 0 & 0 & 1/5 \end{bmatrix}$, है, तो $(\alpha - \beta + \gamma)^2$ बराबर है __________.

Ans. Official Answer NTA (100)

Sol. $X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$y = \alpha I + \beta x + \gamma x^2$

$$X^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Y^2 = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{bmatrix} + \begin{bmatrix} 0 & \beta & 0 \\ 0 & 0 & \beta \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \gamma \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \alpha & \beta \\ 0 & 0 & \alpha \end{bmatrix}$$

$\therefore\ Y.\,Y^{-1} = I$

$$\begin{bmatrix} \alpha & \beta & \gamma \\ 0 & 0 & \beta \\ 0 & 0 & \alpha \end{bmatrix} . \begin{bmatrix} 1/5 & -2/5 & 1/5 \\ 0 & 1/5 & -2/5 \\ 0 & 0 & 1/5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\frac{\alpha}{5}=1 \Rightarrow \alpha=5$$

$$\frac{-2\alpha}{5}+\frac{\beta}{5}=0$$

$$\frac{\beta}{5}=2 \Rightarrow \beta=10$$

$$\frac{\alpha}{5}-\frac{2\beta}{5}+\frac{\gamma}{5}=0$$

$$\gamma=15$$

$$(\alpha-\beta+\gamma)^2=(5-10+15)^2=10^2=100$$

Question ID : 1825

P & C

25. The total number of 3-digit numbers, whose greatest common divisor with 36 is 2, is _______.

तीन अंकों की संख्याओं, जिनका 36 के साथ महत्तम सार्व भाजक 2 है, की कुल संख्या है _______

Ans. Official Answer NTA (150)

Sol. H.C. F. (n, 36) = 2

n : even no. but not multiple for 4 & 3

3 digit no. = 900

3 digit even no. = 450

$n(4 \cup 3) = n(4) + n(3) - n(4 \cap 3)$

No. divisible by 4 = 225

No. divisible by 3 = 150

No. divisible by 12 = 75

n = 450 – 225 – 150 + 75

= 150

Question ID : 1826

Binomial Theorem

26. If $\left({}^{40}C_0\right)+\left({}^{41}C_1\right)+\left({}^{42}C_2\right)+.....+\left({}^{60}C_{20}\right)=\frac{m}{n}\,{}^{60}C_{20}$ m and n are coprime, then m + n is equal to ____.

यदि $\left({}^{40}C_0\right)+\left({}^{41}C_1\right)+\left({}^{42}C_2\right)+.....+\left({}^{60}C_{20}\right)=\frac{m}{n}\,{}^{60}C_{20}$ है m, n असहभाज्य हैं, तो m + n बराबर है ________

Ans. Official Answer NTA (102)

Sol. $${}^{40}C_0+{}^{41}C_1+........+{}^{60}C_{20}=\left(\frac{m}{n}\right){}^{60}C_{20}$$

$${}^{41}C_0+{}^{41}C_1+{}^{42}C_2+........+{}^{60}C_{20}$$

$${}^{42}C_1+{}^{42}C_2+........+{}^{60}C_{20}={}^{60}C_{20}$$

$$\frac{m}{n}\times\frac{\lfloor 60}{\lfloor 20\lfloor 40}=\frac{\lfloor 61}{\lfloor 20\lfloor 41}$$

$$\frac{m}{n}\times\frac{61}{41}$$

m + n = 102

Question ID : 1827

Sequence & progression

27. If a_1 (> 0), a_2, a_3, a_4, a_5 are in G.P., $a_2 + a_4 = 2a_3 + 1$ and $3a_2 + a_3 = 2a_4$, then $a_2 + a_4 + 2a_5$ is equal to ___.

यदि a_1 (> 0), a_2, a_3, a_4, a_5 एक G.P. में हैं, $a_2 + a_4 = 2a_3 + 1$ तथा $3a_2 + a_3 = 2a_4$ हैं, तो $a_2 + a_4 + 2a_5$ बराबर है _____

Ans. Official Answer NTA (40)

Sol. $a_2 + a_4 = 2a_3 + 1$

$ar + ar^3 = 2ar^2 + 1$ ....(1)

$3a_2 + a_3 = 2a_4$

$3ar + ar^2 = 2ar^3$

$3r + r^2 = 2r^3$

$r(3 + r) = 2r^3$

$2r^2 - r - 3 = 0$

$$r=\frac{3}{2}, r=-1$$

when $r=\frac{3}{2}$

$$\frac{3a}{2}+\frac{27a}{8}=2a\frac{9}{4}+1$$

$a = \frac{r}{3}$

for r = – 1, a < 0

$\therefore\ a_2 + a_4 + 2a_5 = ar + ar^3 + 2ar^4$

$= a\,[r + r^3 + 2r^4\,]$

$= 40$

Question ID : 1828

Definite Integration

28. The integral $\frac{24}{\pi}\int_0^{\sqrt{2}} \frac{\left(2-x^2\right)dx}{\left(2+x^2\right)\sqrt{4+x^4}}$ is equal to _____.

समाकलन $\frac{24}{\pi}\int_0^{\sqrt{2}} \frac{\left(2-x^2\right)dx}{\left(2+x^2\right)\sqrt{4+x^4}}$ बराबर है _______

Ans. Official Answer NTA (3)

Sol. $$= \frac{24}{\pi}\int_0^{\sqrt{2}} \frac{x^2\left(\frac{2}{x^2}-1\right)dx}{x\left(\frac{2}{x}+x\right)x\sqrt{\frac{4}{x^2}+x^2}}$$

$$= \frac{24}{\pi}\int_0^{\sqrt{2}} \frac{\left(\frac{2}{x^2}-1\right)dx}{\left(\frac{2}{x}+x\right)\sqrt{\left(\frac{2}{x}+x\right)^2-4}}$$

Let $t = \frac{2}{x} + x$

$-\,dt = \left(\frac{2}{x^2}-1\right)dx$

$$= -\frac{24}{\pi}\int \frac{dt}{t\sqrt{t^2-4}}$$

$$= -\frac{24}{\pi} \times \frac{1}{2}\left[\sec^{-1}\left(\frac{t}{2}\right)\right]$$

$$= -\frac{12}{\pi}\left[\sec^{-1}\left(\frac{x}{2}+\frac{1}{x}\right)\right]_0^{\sqrt{2}}$$

$$= -\frac{12}{\pi}\left[\sec^{-1}\sqrt{2} - \sec^{-1}\infty\right]$$

$$= -\frac{12}{\pi}\left[\frac{\pi}{4}-\frac{\pi}{2}\right]$$

$$= 3$$

Question ID : 1829

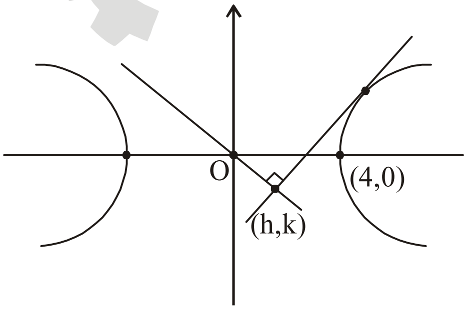
Hyperbola

29. Let a line L_1 be tangent to the hyperbola $\frac{x^2}{16}-\frac{y^2}{4}=1$ and let L_2 be the line passing through the origin and perpendicular to L_1. If the locus of the point of intersection of L_1 and L_2 is $(x^2 + y^2)^2 = \alpha x^2 + \beta y^2$, then $\alpha + \beta$ is equal to ___.

माना L_1 अतिपरवलय $\frac{x^2}{16}-\frac{y^2}{4}=1$ की एक स्पर्श रेखा है तथा रेखा L_2 मूलबिन्दु से होकर जाती है और L_1 के लम्बवत है। यदि L_1 तथा L_2 के प्रतिच्छेदन बिन्दु का बिन्दुपथ $(x^2 + y^2)^2 = \alpha x^2 + \beta y^2$ है, तो $\alpha + \beta$ बराबर है ______

Ans. Official Answer NTA (12)

Sol.



tangent :

$$\frac{x\sec\theta}{4}-\frac{y\tan\theta}{2}=1$$

$$m_1 = \frac{\sec\theta}{4}\times\frac{2}{\tan\theta}=\frac{\sec\theta}{2\tan\theta}$$

$$m_2 = \frac{k}{h}$$

$$\because \quad m_1 \times m_2 = -1$$

$$\frac{k}{h} \times \frac{\sec\theta}{2\tan\theta} = -1$$

$$\frac{k}{2h\sin\theta} = -1$$

$$\sin\theta = -\frac{k}{2h}$$

$$\cos\theta = \frac{\sqrt{4h^2 - k^2}}{2h}$$

$$\frac{h}{4\cos\theta} - \frac{k\sin\theta}{2\cos\theta} = 1$$

$$\frac{h.2h}{4\sqrt{4h^2 - k^2}} + \frac{k \times \frac{k}{2h}}{2 \times \frac{\sqrt{4h^2 - k^2}}{2h}} = 1$$

$$\frac{h^2}{2\sqrt{4h^2 - k^2}} + \frac{k^2}{2\sqrt{4h^2 - k^2}} = 1$$

$$\left(h^2 + k^2\right) = 2\sqrt{4h^2 - k^2}$$

$$\left(x^2 + y^2\right) = 2\sqrt{4x^2 - y^2}$$

$$\left(x^2 + y^2\right)^2 = 4\left(4x^2 - y^2\right)$$

$\alpha = 16, \beta = -4$

$\alpha + \beta = 16 - 4 = 12$

Question ID : 1830

Probability

30. If the probability that a randomly chosen 6-digit number formed by using digits 1 and 8 only is a multiple of 21 is p, then 96 p is equal to ________.

यदि एक यादृच्छया चुनी गई 6 अंकों की संख्या, जो केवल अंको 1 तथा 8 के प्रयोग से बनाई गई है, के 21 का गुणज होने की प्रायिकता p है, तो 96 p बराबर है ________

Ans. Official Answer NTA (33)

Sol. Total cases for randomly 6-digit number formed by using digit 1 and 8

$2^6 = 64$

divisible by 21 when divided by 3

3 favourable cases

1. All 1's _________(1)
2. All 8's _________(1)