JEE Main July 2022

Question Paper With Text Solution

26 July | Shift-1

MATHEMATICS

JEE MAIN JULY 2022 | 26^{TH} JULY SHIFT-1

SECTION – A

Question ID : 100101

Limit

1. Let $f : R \to R$ be a continuous function such that $f(3x) - f(x) = x$. If $f(8) = 7$, then $f(14)$ is equal to :

माना $f : R \to R$ एक संतत फलन है जिसके लिए $f(3x) - f(x) = x$ है। यदि $f(8) = 7$ है, तो $f(14)$ बराबर है :

(1) 4 (2) 10 (3) 11 (4) 16

Ans. Official Answer NTA (2)

Sol. $f(x) - f(x/3) = x/3$

$f(x/3) - f(x/3^2) = x/3^2$

.... on adding

$$f(x) - \lim_{n\to\infty}\left(\frac{x}{3^n}\right) = x\left(\frac{1}{3} + \frac{1}{3^2}....\infty\right)$$

$$f(x) - f(0) = \frac{x}{2}$$

$f(8) = 7$; $f(0) = 3$

$f(x) = x/2 + 3$

$f(14) = 10$

Question ID : 100102

Complex Number

2. Let O be the origin and A be the point $z_1 = 1 + 2i$. If B is the point z_2, $Re(z_2) < 0$, such that OAB is a right angled isosceles triangle with OB as hypotenuse, then which of the following is NOT true?

माना O मूल बिन्दु है तथा $z_1 = 1 + 2i$ है। यदि B, बिन्दु z_2, $Re(z_2) < 0$, है तथा OAB एक समद्विबाहु समकोण त्रिभुज है, जिसका कर्ण OB है, तो निम्न में से कौनसा सत्य नहीं है ?

(1) $\arg z^2 = \pi - \tan^{-1} 3$

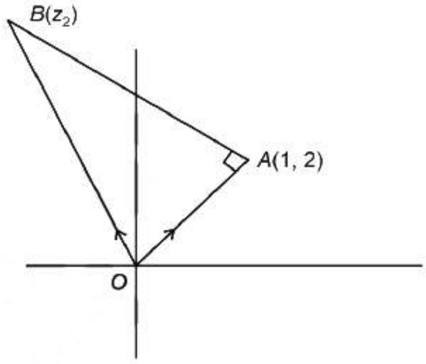
(2) $\arg (z_1 - 2z_2) = -\tan^{-1}\frac{4}{3}$

(3) $|z_2| = \sqrt{10}$

(4) $|2z_1 - z_2| = 5$

Ans. Official Answer NTA (4)

Sol.



$$\frac{z_2 - 0}{(1+2i)-0} = \frac{|OB|}{|OA|} e^{\frac{i\pi}{4}}$$

$$\Rightarrow \frac{z_2}{1+2i} = \sqrt{2} e^{\frac{i\pi}{4}}$$

Or $z_2 = (1 + 2i)(1 + i)$

$= -1 + 3i$

$\arg z_2 = \pi - \tan^{-1} 3$

$|z_2| = \sqrt{10}$

$z_1 - 2z_2 = (1 + 2i) + 2 - 6i = 3 - 4i$

$\arg(z_1 - 2z_2) = -\tan^{-1}\frac{4}{3}$

$|2z_1 - z_2| = |2 + 4i + 1 - 3i| = |3 + i|$

$= \sqrt{10}$

Question ID : 100103

Determinant

3. If the system of linear equations.

यदि रैखिक समीकरण निकाय

$8x + y + 4z = -2$

$x + y + z = 0$

$\lambda x - 3y = \mu$

has infinitely many solutions, then the distance of the point $\left(\lambda, \mu, -\frac{1}{2}\right)$ from the plane $8x + y + 4z + 2 = 0$ is:

के अनंत हल हैं, तो समतल $8x + y + 4z + 2 = 0$ से बिन्दु $\left(\lambda, \mu, -\frac{1}{2}\right)$ की दूरी है :

(1) $3\sqrt{5}$ (2) 4 (3) $\frac{26}{9}$ (4) $\frac{10}{3}$

Ans. Official Answer NTA (4)

Sol. $D = \begin{vmatrix} 8 & 1 & 4 \\ 1 & 1 & 1 \\ \lambda & -3 & 0 \end{vmatrix} = 0 \Rightarrow \lambda = 4$

Also $D_1 = D_2 = D_3 = 0$
So $\mu = -2$

Point $\left(4, -2, -\frac{1}{2}\right)$

Distance from plane $= \frac{10}{3}$

Question ID : 100104

Matrices

4. Let A be a 2×2 matrix with $\det(A) = -1$ and $\det((A + I)(\text{Adj}(A) + I)) = 4$. Then the sum of the diagonal elements of A can be :

माना A एक 2×2 का आव्यूह है जिसके लिए $\det(A) = -1$ तथा $\det((A + I)(\text{Adj}(A) + I)) = 4$ हैं। तो A के विकर्ण के अवयवों का योग हो सकता है :

(1) –1 (2) 2 (3) 1 (4) $-\sqrt{2}$

Ans. Official Answer NTA (2)

Sol. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $ad - bc = -1$

$|A + I|\,|\text{adj } A + I| = 4$
$\Rightarrow ad - bc + a + d + 1 = 2 \text{ or } -2$
$a + d = 2 \text{ or } -2$

Question ID : 100105

Area Under Curve

5. The odd natural number a, such that the area of the region bounded by $y = 1$, $y = 3$, $x = 0$, $x = y^a$ is $\frac{364}{3}$, is equal to :

विषम पूर्णांक संख्या a, जिसके लिए $y = 1, y = 3, x = 0, x = y^a$ से घिरे क्षेत्र का क्षेत्रफल $\frac{364}{3}$ है, बराबर है:

(1) 3 (2) 5 (3) 7 (4) 9

Ans. Official Answer NTA (2)

Sol. a is a odd natural number and

$$\left|\int_1^3 y^a dy\right| = \frac{364}{3}$$

$$\Rightarrow \left|\frac{1}{a+1}\left(y^{a+1}\right)_1^3\right| = \frac{364}{3}$$

$$\Rightarrow \frac{3^{a+1}-1}{a+1} = \pm\frac{364}{3}$$

Solving with (–) sign,

$$\frac{3^{a+1}-1}{a+1} = \frac{-364}{3}\text{, No a exist}$$

$\therefore$ (a = 5)

Question ID : 100106

Sequence & progression

6. Consider two G.Ps. $2, 2^2, 2^3, \ldots$ and $4, 4^2, 4^3, \ldots$ of 60 and n terms respectively. If the geometric mean of all the $60 + n$ terms is $(2)^{\frac{225}{8}}$, then $\sum_{k=1}^{n} k(n-k)$ is equal to :

60 तथा n पदों की दो G.P. क्रमशः $2, 2^2, 2^3, \ldots$ तथा $4, 4^2, 4^3, \ldots$ हैं। यदि सभी $60 + n$ पदों गुणोत्तर माध्य $(2)^{\frac{225}{8}}$ है, तो $\sum_{k=1}^{n} k(n-k)$ बराबर है :

(1) 560 (2) 1540 (3) 1330 (4) 2600

Ans. Official Answer NTA (3)

Sol. $$\left(\left(2^1 2^2 \ldots 2^{60}\right)\left(4^1 \cdot 4^2 \ldots\ldots 4^n\right)\right)^{\frac{1}{60+n}} = 2^{\frac{225}{8}}$$

$$\left(2^{30\times61} 4^{\frac{n(n+1)}{2}}\right)\frac{1}{60+n} = 2^{\frac{225}{8}}$$

$$2^{1830+n^2+n} = 2^{\frac{(225)(60+n)}{8}}$$

$$= 8n^2 - 217n + 1140 = 0$$

$$n = 20, \frac{57}{8}$$

$$\sum_{k=1}^{n} nk - k^2 = \frac{n^2(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}$$

$= 1330$

Question ID : 100107

Continuity & Differentiability

7. If the function $f(x) = \begin{cases} \dfrac{\log_e(1-x+x^2)+\log_e(1+x+x^2)}{\sec x - \cos x} & , \quad x \in \left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right) - \{0\} \\ k & , \quad x = 0 \end{cases}$ is continuous at $x = 0$, then k is equal to :

यदि फलन $f(x) = \begin{cases} \dfrac{\log_e(1-x+x^2)+\log_e(1+x+x^2)}{\sec x - \cos x} & , \quad x \in \left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right) - \{0\} \\ k & , \quad x = 0 \end{cases}$

$x = 0$, पर संतत है, तो k बराबर है :

(1) 1 (2) –1 (3) e (4) 0

Ans. Official Answer NTA (1)

Sol. $\lim\limits_{x\to 0} \dfrac{\left(\ln\left(1+x^2+x^4\right)\right)\cos x}{1-\cos^2 x}$

$$\lim_{x\to 0} \frac{\left(\dfrac{\ln\left(1+x^2+x^4\right)}{x^2+x^4}\right)x^2\left(1+x^2\right)\cos x}{\left(\dfrac{\sin^2 x}{x^2}\right)x^2} = 1$$

$\therefore k = 1$

Question ID : 100108

Continuity & Differentiability

8. If $f(x)=\begin{cases} x+a & , \quad x\le 0 \\ |x-4| & , \quad x>0 \end{cases}$ and $g(x)=\begin{cases} x+1 & , \quad x<0 \\ (x-4)^2+b & , \quad x\ge 0 \end{cases}$ are continuous on R, then (gof) (2) + (fog) (–2) is equal to :

यदि $f(x)=\begin{cases} x+a & , \quad x\le 0 \\ |x-4| & , \quad x>0 \end{cases}$ तथा $g(x)=\begin{cases} x+1 & , \quad x<0 \\ (x-4)^2+b & , \quad x\ge 0 \end{cases}$ R पर संतत तो (gof) (2) + (fog) (–2) बराबर है :

(1) –10 (2) 10 (3) 8 (4) –8

Ans. Official Answer NTA (4)

Sol. $f(x)=\begin{cases} x+a, & x\le 0 \\ |x-4|, & x>0 \end{cases}$ and $g(x)=\begin{cases} x+1, & x<0 \\ (x-4)^2+b, & x\ge 0 \end{cases}$

$\because$ f(x) and g(x) are continuous on R

$\therefore$ a = 4 and b = 1 – 16 = –15

then (gof)(2) + (fog) (–2)

= g(2) + f(–1)

= –11 + 3 = – 8

Question ID : 100109

Maxima & Minima

9. Let $f(x)=\begin{cases} x^3-x^2+10x-7, & x\le 1 \\ -2x+\log_2(b^2-4), & x>1 \end{cases}$. Then the set of all values of b, for which f(x) has maximum value at x = 1, is :

माना $f(x)=\begin{cases} x^3-x^2+10x-7, & x\le 1 \\ -2x+\log_2(b^2-4), & x>1 \end{cases}$ है। तो b के सभी मानों, जिनके लिए f(x) का अधिकतम मान x = 1 पर है, का समुच्चय है :

(1) (–6, –2) (2) (2, 6) (3) [–6, –2) ∪ (2, 6] (4) $\left[-\sqrt{6},-2\right)\cup\left(2,\sqrt{6}\right]$

Ans. Official Answer NTA (3)

Sol. f(1) = 3

For x < 1, $f'(x)=3x^2-2x+10>0$

$\Rightarrow$ f(x) is increasing

For x >1 , f '(x)< 0

$\Rightarrow$ function is decreasing.

$\lim_{x \to 1^+} f(x) = -2 + \log_2(b^2 - 4)$

For maximum value at x= 1

$3 \geq -2 + \log_2(b^2 - 4)$

$32 \geq b^2 - 4 > 0$

$b \in [-6, -2] \cup (2, 6]$

Question ID : 100110

Definite Integration

10. If $a = \lim_{n \to \infty} \sum_{k=1}^{n} \frac{2n}{n^2 + k^2}$ and $f(x) = \sqrt{\frac{1 - \cos x}{1 + \cos x}}$, $x \in (0, 1)$, then :

यदि $a = \lim_{n \to \infty} \sum_{k=1}^{n} \frac{2n}{n^2 + k^2}$ तथा $f(x) = \sqrt{\frac{1 - \cos x}{1 + \cos x}}$, $x \in (0, 1)$ हैं, तो :

(1) $2\sqrt{2} f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$

(2) $f\left(\frac{a}{2}\right) f'\left(\frac{a}{2}\right) = \sqrt{2}$

(3) $\sqrt{2} f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$

(4) $f\left(\frac{a}{2}\right) = \sqrt{2} f'\left(\frac{a}{2}\right)$

Ans. Official Answer NTA (3)

Sol. $a = \frac{1}{n} \sum_{k=1}^{n} \frac{2}{1 + \left(\frac{k}{n}\right)^2} = \int_0^1 \frac{2}{1 + x^2} dx = \frac{\pi}{2}$

$f(x) = \tan\left(\frac{x}{2}\right); x \in (0, 1)$

$f\left(\frac{\pi}{4}\right) = \sqrt{2} - 1$

$f'\left(\frac{\pi}{4}\right) = \frac{1}{2} \sec^2\left(\frac{\pi}{8}\right) = \frac{\sqrt{2}}{\sqrt{2} + 1}$

$f'\left(\frac{\pi}{4}\right) = \sqrt{2} f\left(\frac{\pi}{4}\right)$

Question ID : 100111

Differential Equation

11. If $\frac{dy}{dx}+2y\tan x=\sin x, 0<x<\frac{\pi}{2}$ and $y\left(\frac{\pi}{3}\right)=0$, then the maximum value of $y(x)$ is :

यदि $\frac{dy}{dx}+2y\tan x=\sin x, 0<x<\frac{\pi}{2}$, $y\left(\frac{\pi}{3}\right)=0$ हैं, तो $y(x)$ का अधिकतम मान है :

(1) $\frac{1}{8}$ (2) $\frac{3}{4}$ (3) $\frac{1}{4}$ (4) $\frac{3}{8}$

Ans. Official Answer NTA (1)

Sol. $\frac{dy}{dx}+2y\tan x=\sin x$

which is a first order linear differential equation.

Integrating factor (I.F.) $=e^{\int 2\tan x dx}$

$=e^{2\ln|\sec x|}=\sec^2 x$

Solution of differential equation can be written as

$y.\sec^2 x=\int \sin x.\sec^2 x dx=\int \sec x.\tan x dx$

$y\sec^2 x=\sec x+C$

$y\left(\frac{\pi}{3}\right)=0, 0=\sec\frac{\pi}{3}+C \Rightarrow \quad C=-2$

$y=\frac{\sec x-2}{\sec^2 x}=\cos x-2\cos^2 x$

$=\frac{1}{8}-2\left(\cos x-\frac{1}{4}\right)^2$

$y_{max}=\frac{1}{8}$

Question ID : 100112

Straight Line

12. A point P moves so that the sum of squares of its distances from the points (1, 2) and (–2, 1) is 14. Let f(x, y) = 0 be the locus of P, which intersects the x-axis at the points A, B and the y-axis at the points C, D. Then the area of the quadrilateral ACBD is equal to :

एक बिन्दु P इस प्रकार चलायमान है कि इसकी बिन्दुओं (1, 2) तथा (–2, 1) से दूरियों के वर्गो का योग 14 है। माना P का बिन्दुपथ f(x, y) = 0 है, जो x-अक्ष को बिन्दुओं A, B पर तथा y-अक्ष को बिन्दुओं C, D पर काटता है। तो चतुर्भुज ACBD का क्षेत्रफल बराबर है :

(1) $\frac{9}{2}$ (2) $\frac{3\sqrt{17}}{2}$ (3) $\frac{3\sqrt{17}}{4}$ (4) 9

Ans. Official Answer NTA (2)

Sol. $(x-1)^2+(y-2)^2+(x+2)^2+(y-1)^2=14$

$\Rightarrow x^2+y^2+x-3y-2=0$

Put $x = 0$

$\Rightarrow y^2-3y-2=0$

$\Rightarrow y=\frac{3\pm\sqrt{17}}{2}$

Put $y = 0$

$\Rightarrow x^2+x-2=0$

$(x+2)(x-1)=0$

$\therefore A(-2,0), B(1,0), C\left(0,\frac{3+\sqrt{17}}{2}\right), D\left(0,\frac{3-\sqrt{17}}{2}\right)$

Area $=\frac{1}{2}\cdot 3\cdot\sqrt{17}=\frac{3\sqrt{17}}{2}$

Question ID : 100113

Hyperbola

13. Let the tangent drawn to the parabola $y^2 = 24x$ at the point (α, β) is perpendicular to the line $2x + 2y = 5$. Then the normal to the hyperbola $\frac{x^2}{\alpha^2}-\frac{y^2}{\beta^2}=1$ at the point $(\alpha + 4, \beta + 4)$ does NOT pass through the point :

माना परवलय $y^2 = 24x$ के बिन्दु (α, β) पर स्पर्शरेखा, $2x + 2y = 5$ के लंबवत है। तो अतिपरवलय $\frac{x^2}{\alpha^2}-\frac{y^2}{\beta^2}=1$ के बिन्दु $(\alpha + 4, \beta + 4)$ पर अभिलंब किस बिन्दु से होकर नहीं जाता है ?

(1) (25, 10) (2) (20, 12) (3) (30, 8) (4) (15, 13)

Ans. Official Answer NTA (4)

Sol. Tangent at (α, β) has slope 1

$\beta^2 = 24\alpha$

Equation of tangent $y\beta=12(x+\alpha), \frac{12}{\beta}=1$

$\Rightarrow \alpha = 6, \beta = 12$

$\therefore (\alpha + 4, \beta + 4) = (10,16)$

Normal at (10, 16) to $\frac{x^2}{36}-\frac{y^2}{144}=1$ is

$2x + 5y = 100$

Question ID : 100114

3D Geometry

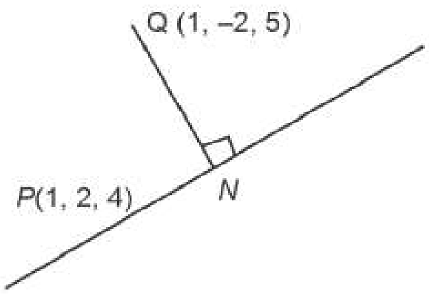
14. The length of the perpendicular from the point (1, –2, 5) on the line passing through (1, 2, 4) and parallel to the line $x + y - z = 0 = x - 2y + 3z - 5$ is :

बिन्दु (1, 2, 4) से होकर जाने वाली तथा रेखा $x + y - z = 0 = x - 2y + 3z - 5$ के समांतर रेखा की, बिन्दु (1, – 2, 5) से लंब की लंबाई है :

(1) $\sqrt{\frac{21}{2}}$ (2) $\sqrt{\frac{9}{2}}$ (3) $\sqrt{\frac{73}{2}}$ (4) 1

Ans. Official Answer NTA (1)

Sol.



The line $x + y - z = 0 = x - 2y + 3z - 5$ is parallel to the vector

$$\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & -1\\ 1 & -2 & 3\end{vmatrix}=(1,4,-3)$$

Equation of line though P(1, 2, 3) and parallel to $\vec{b}$

$$\frac{x-1}{1}=\frac{y-2}{-4}=\frac{z-4}{-3}$$

Let $N\equiv(\lambda+1,-4\lambda+2,-3\lambda+4)$

$\overrightarrow{QN}=(\lambda,-4\lambda+4,-3\lambda-1)$

$\overrightarrow{QN}$ is perpendicular to $\vec{b}$

$\Rightarrow(\lambda,-4\lambda+4,-3\lambda-1).(1,4,-3)=0$

$\Rightarrow \lambda = \frac{1}{2}$

Hence $\overrightarrow{QN} = \left(\frac{1}{2}, 2, \frac{-5}{2}\right)$ and $\left|\overrightarrow{QN}\right| = \sqrt{\frac{21}{2}}$

Question ID : 100115

Vectors

15. Let $\vec{a} = \alpha\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}$, $\alpha > 0$. If the projection of $\vec{a} \times \vec{b}$ on the vector $-\hat{i} + 2\hat{j} - 2\hat{k}$ is 30, then α is equal to :

माना $\vec{a} = \alpha\hat{i} + \hat{j} - \hat{k}$ तथा $\vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}$, $\alpha > 0$ यदि $\vec{a} \times \vec{b}$ का सदिश $-\hat{i} + 2\hat{j} - 2\hat{k}$ पर प्रक्षेप 30 है, तो α बराबर है :

(1) $\frac{15}{2}$ (2) 8 (3) $\frac{13}{2}$ (4) 7

Ans. Official Answer NTA (4)

Sol. $\vec{a} \times \vec{b} = (1-\alpha)\hat{i} + \left(\alpha^2 - 2\right)\hat{j} + (\alpha - 2)\hat{k}$

Projection of $\vec{a} \times \vec{b}$ on $-\hat{i} + 2\hat{j} - 2\hat{k}$

$= \frac{(\vec{a} \times \vec{b}) \cdot (-\hat{i} + 2\hat{j} - 2\hat{k})}{3} = 30$

$\Rightarrow 2\alpha^2 - \alpha - 91 = 0$

$\Rightarrow \alpha = 7, -\frac{13}{2}$

Question ID : 100116

Statistics

16. The mean and variance of a binomial distribution are α and $\frac{\alpha}{3}$ respectively. If $P(X = 1) = \frac{4}{243}$, then $P(X = 4$ or $5)$ is equal to :

एक द्विपद बंटन के माध्य तथा प्रसरण क्रमशः α तथा $\frac{\alpha}{3}$ हैं। यदि $P(X = 1) = \frac{4}{243}$ है, तो $P(X = 4$ या $5)$ बराबर है :

(1) $\frac{5}{9}$ (2) $\frac{64}{81}$ (3) $\frac{16}{27}$ (4) $\frac{145}{243}$

Ans. Official Answer NTA (3)

Sol. $np = \alpha$ .....(1)

$npq = \alpha/3$ .....(2)

From (1) & (2)

$q = 1/3$ & $p = 2/3$

$^{n}C_1 q^{n-1} p^1 = \frac{4}{243}$

$\frac{n}{3^n} = \frac{2}{243}$

$n = 6$

$P\,(4 \text{ or } 5) = {}^6c_4 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^2 + {}^6C_5 \left(\frac{2}{3}\right)^5 . \left(\frac{1}{3}\right)^0$

$= \frac{16}{27}$

Question ID : 100117

Probability

17. Let E_1, E_2, E_3 be three mutually exclusive events such that $P(E_1) = \frac{2+3p}{6}$, $P(E_2) = \frac{2-p}{8}$ and $P(E_3) = \frac{1-p}{2}$. If the maximum and minimum values of p are p_1 and p_2, then $(p_1 + p_2)$ is equal to :

माना E_1, E_2, E_3 तीन परस्पर अपवर्जी घटनाएँ है तथा तथा $P(E_1) = \frac{2+3p}{6}$, $P(E_2) = \frac{2-p}{8}$ तथा $P(E_3) = \frac{1-p}{2}$ हैं। यदि p के अधिकतम तथा निम्नतम मान p_1 तथा p_2 हैं, तो $(p_1 + p_2)$ बराबर है :

(1) $\frac{2}{3}$ (2) $\frac{5}{3}$ (3) $\frac{5}{4}$ (4) 1

Ans. Official Answer NTA (2)

Sol. $0 \le \frac{2+3P}{6} \le 1 \Rightarrow P \in \left[-\frac{2}{3}, \frac{4}{3}\right]$

$0 \le \frac{2-3P}{8} \le 1 \Rightarrow P \in [-6, 2]$

$0 \le \frac{1-P}{2} \le 1 \Rightarrow P \in [-1, 1]$

$0 < P(E_1) + P(E_2) + P(E_3) \le 1$

$0 < \frac{13}{12} - \frac{P}{8} \le 1$

$P \in \left[\frac{2}{3}, \frac{26}{3}\right]$

Taking intersection of all

$P \in \left[\frac{2}{3}, 1\right)$

$P_1 + P_2 = \frac{5}{3}$

Question ID : 100118

Trigonometric Equation

18. Let $S = \{\theta \in [0, 2\pi] : 8^{2\sin^2\theta} + 8^{2\cos^2\theta} = 16\}$. Then $n(S) + \sum_{\theta \in S}\left(\sec\left(\frac{\pi}{4} + 2\theta\right)\operatorname{cosec}\left(\frac{\pi}{4} + 2\theta\right)\right)$ is equal to :

माना $S = \{\theta \in [0, 2\pi] : 8^{2\sin^2\theta} + 8^{2\cos^2\theta} = 16\}$ है। तो $n(S) + \sum_{\theta \in S}\left(\sec\left(\frac{\pi}{4} + 2\theta\right)\operatorname{cosec}\left(\frac{\pi}{4} + 2\theta\right)\right)$ बराबर है :

(1) 0 (2) – 2 (3) – 4 (4) 12

Ans. Official Answer NTA (3)

Sol. $8^{2\sin^2\theta} + 8^{2-2\sin^2\theta} = 16$

$y + \frac{64}{y} = 16$

$\Rightarrow y = 8$

$\Rightarrow \sin^2\theta = 1/2$

$n(S) + \sum_{\theta \in S}\frac{1}{\cos(\pi/4 + 2\theta)\sin(\pi/4 + 2\theta)}$

$= 4 + (-2) \times 4 = -4$

Question ID : 100119

ITF

19. $\tan\left(2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{\sqrt{5}}{2} + 2\tan^{-1}\frac{1}{8}\right)$ is equal to :

$\tan\left(2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{\sqrt{5}}{2} + 2\tan^{-1}\frac{1}{8}\right)$ बराबर है :

(1) 1 (2) 2 (3) $\frac{1}{4}$ (4) $\frac{5}{4}$

Ans. Official Answer NTA (2)

Sol. $\tan\left(2\left(\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}\right)+\tan^{-1}\left(\frac{1}{2}\right)\right)$

$= \tan\left[2\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{2}\right)\right]$

$= 2$

Question ID : 100120

Mathematical Reasoning

20. The statement $(\sim(p \Leftrightarrow \sim q)) \wedge q$ is :

(1) a tautology (2) a contradiction

(3) equivalent ot $(p \Rightarrow q) \wedge q$ (4) equivalent to $(p \Rightarrow q) \wedge p$

कथन $(\sim(p \Leftrightarrow \sim q)) \wedge q$ है :

(1) एक पुनरुक्ति है (2) एक विरोधोक्ति है

(3) $(p \Rightarrow q) \wedge q$ के तुल्य है (4) $(p \Rightarrow q) \wedge p$ के तुल्य है

Ans. Official Answer NTA (4)

Sol. $\sim(p \Leftrightarrow \sim q) \wedge q$

$= (p \Leftrightarrow q) \wedge q$

p	q	$p \leftrightarrow q$	$(p \leftrightarrow q) \wedge q$	$(p \rightarrow q)$	$(p \rightarrow q) \wedge q$	$(p \rightarrow q) \wedge p$
T	T	T	T	T	T	T
T	F	F	F	F	F	F
F	T	F	F	T	T	F
F	F	T	F	T	F	F

SECTION – B

Question ID : 100121

Quadratic Equation

21. If for some p, q, r $\in$ R, not all have same sign, one of the roots of the equation $(p^2 + q^2)x^2 - 2q(p + r)x + q^2 + r^2 = 0$ is also a root of the equation $x^2 + 2x - 8 = 0$, then $\frac{q^2+r^2}{p^2}$ is equal to _________.

यदि p, q, r $\in$ R, सभी धनात्मक या सभी ऋणात्मक नहीं है, के लिए समीकरण $(p^2 + q^2)x^2 - 2q(p + r)x + q^2 + r^2 = 0$ का एक मूल समीकरण $x^2 + 2x - 8 = 0$ का भी एक मूल है, तो $\frac{q^2+r^2}{p^2}$ बराबर है ________।

Ans. Official Answer NTA (272)

Sol. $(px-q)^2 + (qx-r)^2 = 0$

$\Rightarrow x = \frac{q}{p} = \frac{r}{q} = -4$

$\Rightarrow \frac{q^2+r^2}{p^2} = 272$

Question ID : 100122

P & C

22. The number of 5-digit natural numbers, such that the product of their digits is 36, is _________.

5-अंकों की प्राकृतिक संख्याओं, जिनके अंकों का गुणनफल है 36 है, की संख्या है _______।

Ans. Official Answer NTA (180)

Sol. $3 \times \frac{5!}{2!2!} + \frac{5!}{3! \times 2!} + \frac{5!}{2!} + \frac{5!}{3!} = 180$

Question ID : 100123

Sequence & progression

23. The series of positive multiples of 3 is divided into sets " {3}, {6, 9, 12}, {15, 18, 21, 24, 27},..... Then the sum of the elements in the 11th set is equal to _________.

3 के धनात्मक गुणजों की श्रेणी को समुच्चयों {3}, {6, 9, 12}, {15, 18, 21, 24, 27},.....में विभाजित किया गया है। तो 11 वें समुच्चय में अवयवों का योग बराबर है _________।

Ans. Official Answer NTA (6993)

Sol. $S_{11} = 3[101+102 + ... +121]$

$= \frac{3}{2}(222) \times 21 = 6993$

Question ID : 100124

Monotonocity

24. The number of distinct real roots of the equation $x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0$ is _________.

$x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0$ के भिन्न वास्तविक मूलों की संख्या है _________।

Ans. Official Answer NTA (3)

Sol. $x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0$

$\Rightarrow (x - 1)^2 (x + 1)(x^5 + 3x - 1) = 0$

Let $f(x) = x^5 + 3x - 1$

$f'(x) > 0 \ \forall \ x \in R$

Hence 3 real distinct roots.

Question ID : 100125

Binomial Theorem

25. If the coefficients of x and x^2 in the expansion of $(1 + x)^p (1 - x)^q$, $p, q \le 15$, are -3 and -5 respectively, then the coefficient of x^3 is equal to _________.

$(1 + x)^p (1 - x)^q$, $p, q \le 15$, के प्रसार में x तथा x^2 के गुणांक क्रमशः -3 तथा -5 हैं, तो x^3 का गुणांक बराबर है ________।

Ans. Official Answer NTA (23)

Sol. Since coefficient of x is -3

$\Rightarrow {}^pC_1 - {}^qC_1 = -3$

$\Rightarrow p - q = -3$ ....(1)

Comparing coefficients of x^2

$-{}^pC_1 {}^qC_1 + {}^pC_2 + {}^qC_2 = -5$

$-pq + \frac{p(p-1)}{2} + \frac{q(q-1)}{2} = -5$ ....(2)

Solving (1) and (2)

$p = 8, q = 11$

Coefficient of x^3 is

$-{}^qC_3 + {}^pC_3 + {}^pC_1 {}^qC_2 - {}^pC_2 {}^qC_1$

$= -{}^{11}C_3 + {}^8C_3 + {}^8C_1 {}^{11}C_2 - {}^8C_2 {}^{11}C_1$

$= 23$

Question ID : 100126

Definite Integration

26. If $n(2n+1)\int_0^1 (1-x^n)^{2n} dx = 1177\int_0^1 (1-x^n)^{2n+1} dx$, then $n \in N$ is equal to _________.

यदि $n(2n+1)\int_0^1 (1-x^n)^{2n} dx = 1177\int_0^1 (1-x^n)^{2n+1} dx$ है, तो $n \in N$ बराबर है________।

Ans. Official Answer NTA (24)

Sol. Let $I_1 = \int_0^1 (1-x^n)^{2n} dx, I_2 = \int_0^1 (1-x^n)^{2n+1} dx$

$I_2 = \int_0^1 (1-x^n)^{2n+1} .1dx$

$= (1-x^n)^{2n+1} .x|_0^1 - \int_0^1 (2n+1)(1-x^n)^{2n}(-nx^{n-1})xdx$

$I_2 = -n(2n+1)\{I_2 + I_1\}$

$(2n^2+n+1)I_2 = n(2n+1)I_1$

$\frac{I_1}{I_2} = \frac{2n^2+n+1}{n(2n+1)} = \frac{1177}{n(2n+1)}$

$\Rightarrow 2n^2 + n - 1176 = 0 \Rightarrow n = 24$

Question ID : 100127

Differential Equation

27. Let a curve y = y(x) pass through the point (3, 3) and the area of the region under this curve, above the x-axis and between the abscissae 3 and x(> 3) be $\left(\frac{y}{X}\right)^3$. If this curve also passes through the point $(\alpha, 6\sqrt{10})$ in the first quadrant, then α is equal to _________.

माना एक वक्र y = y(x) बिन्दु (3, 3) से होकर जाता है तथा इस वक्र के नीचे, x-अक्ष के ऊपर तथा भुजों 3 और x(> 3) के बीच के क्षेत्र का क्षेत्रफल $\left(\frac{y}{X}\right)^3$ है। यदि वह वक्र प्रथम चतुर्थांश में बिन्दु $(\alpha, 6\sqrt{10})$ से भी होकर जाता है, तो α बराबर है ________ ।

Ans. Official Answer NTA (6)

Sol. $x^4 = 3yx.y' - 3y^2$

$\Rightarrow 3xy\frac{dy}{dx} = 3y^2 + x^4$

Put $y^2 = t, y\frac{dy}{dx} = \frac{1}{2}\frac{dt}{dx}$

$\frac{dt}{dx} - \frac{2}{x}t = \frac{2}{3}x^3$

$\therefore \frac{t}{x^2} = \frac{x^2}{3} + C$

$\Rightarrow \frac{y^2}{x^2} = \frac{x^2}{3} - 2$

Put (3, 3) , C = –2

$\therefore \frac{y^2}{x^2} = \frac{x^2}{3} - 2$

$3y^2 = x^4 - 6x^2$

$x^4 - 6x^2 = 1080$

$\therefore x = 6$

Question ID : 100128

Straight Line

28. The equations of the sides AB, BC and CA of a triangle ABC are $2x + y = 0$, $x + py = 15a$ and $x - y = 3$ respectively. If its orthocentre is (2, a), $-\frac{1}{2} < a < 2$, then p is equal to _________.

एक त्रिभुज ABC की भुजाओं AB, BC तथा CA के समीकरण क्रमशः $2x + y = 0$, $x + py = 15a$ तथा $x - y = 3$ हैं। इसका लंब केन्द्र (2, a), $-\frac{1}{2} < a < 2$ है, तो p बराबर है_______।

Ans. Official Answer NTA (3)

Sol. Coordinates of $A(1,-2), B\left(\frac{15a}{1-2p}, \frac{-30a}{1-2p}\right)$ and orthocentre H(2, a)

Slope of AH = p

$a + 2 = p$ .....(1)

Slope of BH = –1

$31a - 2ab = 15a + 4p - 2$ .....(2)

From (1) and (2)

$a = 1$ & $p = 3$

Question ID : 100129

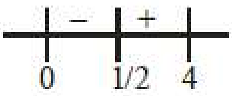
Monotonocity

29. Let the function $f(x) = 2x^2 - \log_e x$, $x > 0$, be decreasing in (0, a) and increasing in (a, 4). A tangent to the parabola $y^2 = 4ax$ at a point P on it passes through the point (8a, 8a – 1) but does not pass through the point $\left(-\frac{1}{a}, 0\right)$. If the equation of the normal at P is $\frac{x}{\alpha} + \frac{y}{\beta} = 1$, then $\alpha + \beta$ is equal to _________.

फलन $f(x) = 2x^2 - \log_e x, x > 0$, अंतराल (0, a) में ह्रासमान है तथा (a, 4) में वर्धमान है। $y^2 = 4ax$ के एक बिन्दु P पर स्पर्श रेखा, बिन्दु (8a, 8a – 1) से होकर जाती है, परन्तु बिन्दु $\left(-\frac{1}{a}, 0\right)$ से होकर नहीं जाती है। यदि बिन्दु P अभिलंब का समीकरण $\frac{x}{\alpha} + \frac{y}{\beta} = 1$ है, तो $\alpha + \beta$ बराबर है।

Ans. Official Answer NTA (45)

Sol.



$f'(x) = 4x - \frac{1}{x}$

$a = \frac{1}{2}$

Let $P(x_1, y_1)$ be any point on $y^2 = 4ax$

$\frac{1}{y_1} = \frac{3 - y_1}{4 - x_1} \Rightarrow y_1^2 - 6y_1 + 8 = 0$

$y_1 = 2, 4$

$\Rightarrow$ P(8, 4) as P(2, 2) rejected

Equation of normal at P.

$y - 4 = -4(x - 8)$

$\frac{x}{9} + \frac{y}{36} = 1$

$\alpha = 9, \beta = 36$

$\alpha + \beta = 45$

Question ID : 100130

3D Geometry

30. Let Q and R be two points on the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z-1}{2}$ at a distance $\sqrt{26}$ from the point P(4, 2, 7). Then the square of the area of the triangle PQR is _________.

माना रेखा $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z-1}{2}$ पर बिन्दु P (4, 2, 7) से $\sqrt{26}$ दूरी पर दो बिन्दु Q तथा R है। तो त्रिभुज PQR के क्षेत्रफल का वर्ग बराबर है ________।

Ans. Official Answer NTA (153)

Sol. Let $(2\lambda - 1, 3\lambda - 2, 2\lambda + 1)$ be any point on the line

$(2\lambda - 5)^2 + (3\lambda - 4)^2 + (2\lambda - 6)^2 = 26$

$\lambda = 1, 3$

Q (1, 1, 3) ; R (5, 7, 7) ; P (4, 2, 7)

Area of triangle $PQR = \frac{1}{2}\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right|$

$= \sqrt{153}$