JEE Main July 2022

Question Paper With Text Solution

26 July | Shift-2

MATHEMATICS

JEE MAIN JULY 2022 | 26TH JULY SHIFT-2

SECTION – A

Question ID : 144961

Quadratic Equation

1. The minimum value of the sum of the square of the roots of $x^2 + (3 - a)x + 1 = 2a$ is :

समीकरण $x^2 + (3 - a)x + 1 = 2a$ के मूलों के वर्गों के योगफल का निम्नतम मान है :

(1) 4 (2) 5 (3) 6 (4) 8

Ans. Official Answer NTA (3)

Sol. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

let $f(a) = (3 - a)^2 - 2(1 - 2a)$

$f(a) = a^2 - 2a + 7$

$f(a) = (a - 1)^2 + 6$

$f(a))_{min.} = 6$

Question ID : 144962

Complex number

2. If $z = x + iy$ satisfies $|z| - 2 = 0$ and $|z - i| - |z + 5i| = 0$, then :

यदि $z = x + iy$ समीकरणों $|z| - 2 = 0$ तथा $|z - i| - |z + 5i| = 0$ को संतुष्ट करता है, तो :

(1) $x + 2y - 4 = 0$ (2) $x^2 + y - 4 = 0$

(3) $x + 2y + 4 = 0$ (4) $x^2 - y + 3 = 0$

Ans. Official Answer NTA (3)

Sol. $|z-i|-|z+5i|=0$

$\Rightarrow |x + (y - 1)i| = |x + (y + 5)i|$

$x^2 + (y - 1)^2 = x^2 + (y + 5)^2$

$(y - 1)2 - (y + 5)^2 = 0$

$(2y + 4)(-6) = 0$

$y = -2$

$\therefore x^2 + (-2)^2 = 4$

$x = 0$

$Z \equiv (0, -2)$, check options

Question ID : 144963

Matrices

3. Let $A=\begin{bmatrix}1\\1\\1\end{bmatrix}$ and $B=\begin{bmatrix}9^2 & -10^2 & 11^2\\12^2 & 13^2 & -14^2\\-15^2 & 16^2 & 17^2\end{bmatrix}$, then the value of the A′BA is :

माना $A=\begin{bmatrix}1\\1\\1\end{bmatrix}$ तथा $B=\begin{bmatrix}9^2 & -10^2 & 11^2\\12^2 & 13^2 & -14^2\\-15^2 & 16^2 & 17^2\end{bmatrix}$ हैं, तो A′BA का मान है :

(1) 1224 (2) 1042 (3) 540 (4) 539

Ans. Official Answer NTA (4)

Sol. $A'BA=[1\ 1\ 1]\begin{bmatrix}9^2 & -10^2 & 11^2\\12^2 & 13^2 & -14^2\\-15^2 & 16^2 & 17^2\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}$

$=[9^2+12^2-15^2-10^2+13^2+16^2\ 11^2-14^2+17^2]\begin{bmatrix}1\\1\\1\end{bmatrix}$

$=[9^2+12^2-15^2-10^2+13^2+16^2+11^2-14^2+17^2]$

= [539

Question ID : 144964

Binomial Theorem

4. $\sum_{\substack{i,j=0\\i\neq j}}^{n} {}^nC_i\ {}^nCj$ is equal to :

$\sum_{\substack{i,j=0\\i\neq j}}^{n} {}^nC_i\ {}^nCj$ बराबर है :

(1) $2^{2n}-{}^{2n}C_n$

(2) $2^{2n-1}-{}^{2n-1}C_{n-1}$

(3) $2^{2n}-\frac{1}{2}\,{}^{2n}C_n$

(4) $2^{n-1}+{}^{2n-1}C_n$

Ans. Official Answer NTA (2)

Sol. $\sum_{\substack{i,j=0\\i\rightleftharpoons j}}^{n} {}^nC_i\ \ {}^nC_j$

$= \sum_{i=0}^{n} {}^nC_i \cdot \sum_{j=0}^{n} {}^nC_j - \sum_{i=j=0}^{n} \left({}^nC_i\right)^2$

$= (2^n)(2^n) - {}^{2n}C_n$

$= 2^{2n} - {}^{2n}C_n$

Question ID : 144965

Tangent and normal

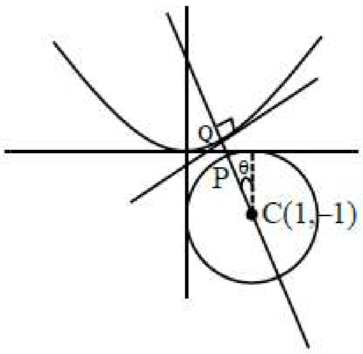
5. Let P and Q be any points on the curves $(x-1)^2 + (y+1)^2 = 1$ and $y = x^2$, respectively. The distance between P and Q is minimum for some value of the abscissa of P in the interval :

माना वक्रों $(x-1)^2 + (y+1)^2 = 1$ तथा $y = x^2$ पर क्रमशः P तथा Q कोई भी बिन्दु हैं। यदि P तथा Q के बीच दूरी निम्नतम है, तो P के भुज का मान किस अंतराल में है :

(1) $\left(0, \frac{1}{4}\right)$ (2) $\left(\frac{1}{2}, \frac{3}{4}\right)$ (3) $\left(\frac{1}{4}, \frac{1}{2}\right)$ (4) $\left(\frac{3}{4}, 1\right)$

Ans. Official Answer NTA (3)

Sol.



$Q = (t, t^2)$

$m_{CQ} = m_{normal}$

$\frac{t^2+1}{t-1} = -\frac{1}{2t}$

Let $f(t) = 2t^3 + 3t - 1$

$f\left(\frac{1}{4}\right) f\left(\frac{1}{3}\right) < 0 \Rightarrow t \in \left(\frac{1}{4}, \frac{1}{3}\right)$

$P \equiv (1 + \cos(90+\theta), -1 + \sin(90+\theta))$

$P = (1 - \sin\theta, -1 + \cos\theta)$

$m_{normal} = m_{CP} \Rightarrow -\frac{1}{2t} = \frac{\cos\theta}{-\sin\theta} \Rightarrow \tan\theta = 2t$

$x = 1 - \sin\theta = 1 - \frac{2t}{\sqrt{1+4t^2}} = g(t)\,(\text{let})$

$\Rightarrow g'(t) < 0$

$g(t) \downarrow$ function

$t \in \left(\frac{1}{4}, \frac{1}{3}\right)$

$\Rightarrow g(t) \in (0.44, 0.485) \in \left(\frac{1}{4}, \frac{1}{2}\right)$

Question ID : 144966

Monotonocity

6. If the maximum value of a, for which the function $f_a(x) = \tan^{-1} 2x - 3ax + 7$ is non-decreasing in $\left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$, is $\bar{a}$, then $f_{\bar{a}}\left(\frac{\pi}{8}\right)$ is equal to :

यदि a का अधिकतम मान, जिसके लिए फलन $f_a(x) = \tan^{-1} 2x - 3ax + 7$, अंतराल $\left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$ में ह्रासमान नहीं है, $\bar{a}$ है तो $f_{\bar{a}}\left(\frac{\pi}{8}\right)$ बराबर है :

(1) $8 - \frac{9\pi}{4(9+\pi^2)}$

(2) $8 - \frac{4\pi}{9(4+\pi^2)}$

(3) $8\left(\frac{1+\pi^2}{9+\pi^2}\right)$

(4) $8 - \frac{\pi}{4}$

Ans. Official Answer NTA (1)

Sol. $f(x) = \tan^{-1} 2x - 3ax + 7$

$f'_a(x) = \frac{2}{1+4x^2} - 3a \geq 0$

$a \leq \left(\frac{2}{3(1+4x^2)}\right)_{\min.}$ at $x = \pm\frac{\pi}{6}$

$a_{\max} = \bar{a} = \frac{6}{9+\pi^2}$

$f_{\bar{a}}\left(\frac{\pi}{8}\right) = \tan^{-1}\frac{\pi}{4} - 3\frac{6}{9+\pi^2 8} + 7 = \tan^{-1}\frac{\pi}{4} - \frac{9\pi}{4(\pi^2+9)} + 7$

Question ID : 144967

Limit

7. Let $\beta = \lim_{x\to 0}\frac{\alpha x-(e^{3x}-1)}{\alpha x(e^{3x}-1)}$ for some $\alpha \in R$. Then the value of $\alpha+\beta$ is :

माना किसी $\alpha \in R$ के लिए $\beta = \lim_{x\to 0}\frac{\alpha x-(e^{3x}-1)}{\alpha x(e^{3x}-1)}$ है। तो $\alpha+\beta$ का मान है :

(1) $\frac{14}{5}$ (2) $\frac{3}{2}$ (3) $\frac{5}{2}$ (4) $\frac{7}{2}$

Ans. Official Answer NTA (3)

Sol. $\beta = \lim_{x\to 0}\frac{\alpha x-(e^{3x}-1)}{\alpha x(e^{3x}-1)}$

$$\beta = \lim_{x\to 0}\frac{1+\alpha x-\left[1+3x+\frac{9x^2}{2!}+....\right]}{(\alpha x)\frac{(e^{3x}-1)}{3x}3x}$$

$$\beta = \lim_{x\to 0}\frac{(\alpha x-3x)-\frac{9x^2}{2!}}{3\alpha x^2}$$

For existence of limit $\alpha-3=0$

$\alpha = 3$

Limit $\beta = \frac{-3}{2\alpha}$

$\beta = -\frac{1}{2}$

Now,

$\alpha+\beta = \frac{5}{2}$

Question ID : 144968

Methods of Differentiation

8. The value of $\log_e 2\frac{d}{dx}\left(\log_{\cos x} \operatorname{cosec} x\right)$ at $x=\frac{\pi}{4}$ is :

$x=\frac{\pi}{4}$ पर $\log_e 2\frac{d}{dx}\left(\log_{\cos x} \operatorname{cosec} x\right)$ का मान है:

(1) $-2\sqrt{2}$ (2) $2\sqrt{2}$ (3) –4 (4) 4

Ans. Official Answer NTA (4)

Sol. $\log_e 2\frac{d}{dx}\left(\log_{\cos x} \operatorname{cosec} x\right)$

Let,

$y=\log_{\cos x} \operatorname{cosec} x$

$$y=-\frac{\ln(\sin x)}{\ln(\cos x)}$$

$$\frac{dy}{dx}=-\frac{\left[\cot x.\ln(\cos x)+\tan x.\ln(\sin x)\right]}{\left(\ln(\cos x)\right)^2}$$

$$\left.\frac{dy}{dx}\right)_{x=\frac{\pi}{4}}=\frac{4}{\ln 2}$$

Now,

$$\Rightarrow \log_e 2.\frac{4}{\ln 2}=4$$

Question ID : 144969

Definite Integration

9. $\int_0^{20\pi}\left(|\sin x|+|\cos x|\right)^2 dx$ is equal to :

$\int_0^{20\pi}\left(|\sin x|+|\cos x|\right)^2 dx$ बराबर है :

(1) 10(π + 4) (2) 10(π + 2) (3) 20(π – 2) (4) 20(π + 2)

Ans. Official Answer NTA (4)

Sol. $I=\int_0^{20\pi}\left(|\sin x|+|\cos x|\right)^2 dx$; (Jack property)

$$I=40\int_0^{\pi/2}\left(\sin x+\cos x\right)^2 dx$$

$I = 40\int_0^{\pi/2}(1+\sin 2x)dx$

$I = 20[\pi + 2]$

Question ID : 1449610

Differential Equation

10. Let the solution curve y = f(x) of the differential equation $\frac{dy}{dx}+\frac{xy}{x^2-1}=\frac{x^4+2x}{\sqrt{1-x^2}}$, $x \in (-1, 1)$ pass through the origin. Then $\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx$ is equal to :

माना अवकल समीकरण $\frac{dy}{dx}+\frac{xy}{x^2-1}=\frac{x^4+2x}{\sqrt{1-x^2}}$, $x \in (-1, 1)$ का हल वक्र y = f(x) मूल बिन्दु से होकर जाता है।

तो $\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx$ बराबर है :

(1) $\frac{\pi}{3}-\frac{1}{4}$ (2) $\frac{\pi}{3}-\frac{\sqrt{3}}{4}$ (3) $\frac{\pi}{6}-\frac{\sqrt{3}}{4}$ (4) $\frac{\pi}{6}-\frac{\sqrt{3}}{2}$

Ans. Official Answer NTA (2)

Sol. $\frac{dy}{dx}+\frac{xy}{x^2-1}=\frac{x^4+2x}{\sqrt{1-x^2}}$

$I.F = e^{\int \frac{x}{x^2-1}dx}$

$I.F = \sqrt{1-x^2}$

Solution of D.E.

$y.\sqrt{1-x^2} = \int \frac{x^4+2x}{\sqrt{1-x^2}}\sqrt{1-x^2}dx$

$y.\sqrt{1-x^2} = \int (x^4+2x)dx$

$y.\sqrt{1-x^2} = \frac{x^5}{5}+x^2+C$

At x = 0, y = 0, get C = 0

$$y = \frac{x^5}{5\sqrt{1-x^2}} + \frac{x^2}{\sqrt{1-x^2}}$$

Now,

$$\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx = \int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \frac{x^5}{5\sqrt{1-x^2}}dx + \int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \frac{x^2}{\sqrt{1-x^2}}dx$$

$$\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx = 0 + 2\int_{0}^{\frac{\sqrt{3}}{2}} \frac{x^2}{\sqrt{1-x^2}}dx$$

$$\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx = \frac{\pi}{3} - \frac{\sqrt{3}}{4}$$

Question ID : 1449611

Ellipse

11. The acute angle between the pair of tangents drawn to the ellipse $2x^2 + 3y^2 = 5$ from the point (1, 3) is:

बिन्दु (1, 3) से दीर्घवृत्त $2x^2 + 3y^2 = 5$ पर डाली गई दो स्पर्श रेखाओं के बीच न्यून कोण हैं :

(1) $\tan^{-1}\left(\frac{16}{7\sqrt{5}}\right)$ (2) $\tan^{-1}\left(\frac{24}{7\sqrt{5}}\right)$ (3) $\tan^{-1}\left(\frac{32}{7\sqrt{5}}\right)$ (4) $\tan^{-1}\left(\frac{3+8\sqrt{5}}{35}\right)$

Ans. Official Answer NTA (2)

Sol. Equation of tangent to the ellipse $2x2 + 3y^2 = 5$ is

$$y = mx \pm \sqrt{\frac{5}{2}m^2 + \frac{5}{3}}$$

It pass through (1, 3)

$$3 = m \pm \sqrt{\frac{5}{2}m^2 + \frac{5}{3}}$$

$$3m^2 + 12m - \frac{44}{3} = 0$$

Let θ be the angle between the tangents

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$\tan\theta = \left|\frac{3\sqrt{320}}{-35}\right|$

$\theta = \tan^{-1}\left(\frac{24}{7\sqrt{5}}\right)$

Question ID : 1449612

Parabola

12. The equation of a common tangent to the parabolas $y = x^2$ and $y = -(x-2)^2$ is :

परवलयों $y = x^2$ तथा $y = -(x-2)^2$ की एक उभयनिष्ठ स्पर्श रेखा का समीकरण है :

(1) $y = 4(x-2)$ (2) $y = 4(x-1)$ (3) $y = 4(x+1)$ (4) $y = 4(x+2)$

Ans. Official Answer NTA (2)

Sol. Equation of tangent of $y = x^2$ be

$tx = y + at^2$ ...(1)

$y = tx - \frac{t^2}{4}$

Solve with $y = -(x-2)^2$

$tx = \frac{t^2}{4} = -(x-2)^2$

$x^2 + x(t-4) - \frac{t^2}{4} + 4 = 0$

$D = 0$

$(t-4)^2 - 4.\left(4 - \frac{t^2}{4}\right) = 0$

$t^2 - 4t = 0$

$t = 0$ or $t = 4$

From eq. (1), required common tangent is

$y = 4(x-1)$

Question ID : 1449613

Circle

13. Let the abscissae of the two points P and Q on a circle be the roots of $x^2 - 4x - 6 = 0$ and the ordinates of P and Q be the roots of $y^2 + 2y - 7 = 0$. If PQ is a diameter of the circle $x^2 + y^2 + 2ax + 2by + c = 0$, then the value of $(a + b - c)$ is :

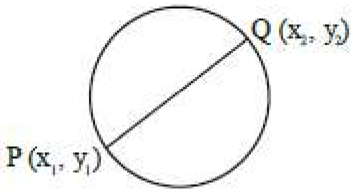
माना $x^2 - 4x - 6 = 0$ के मूल एक वृत्त पर दो बिन्दुओं P तथा Q के भुज हैं तथा $y^2 + 2y - 7 = 0$ के मूल P तथा Q

के भुज हैं तथा $x^2 + y^2 + 2ax + 2by + c = 0$ का एक व्यास है, तो $(a + b - c)$ का मान है :

(1) 12 (2) 13 (3) 14 (4) 16

Ans. Official Answer NTA (1)

Sol.



Equation of circle diameter form

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

(where x_1, x_2 are the roots of $x^2 - 4x - 6 = 0$ and y_1, y_2 are the roots of $y^2 + 2y - 7 = 0$)

$x^2 + y^2 - 4x + 2y - 13 = 0$

Now,

Compare it with the given equation, we get

$a = -2, b = 1, c = -13$

Now

$a + b - c = 12$

Question ID : 1449614

Hyperbola

14. If the line $x - 1 = 0$ is a directrix of the hyperbola $kx^2 - y^2 = 6$, then the hyperbola passes through the point:

यदि रेखा $x - 1 = 0$, अतिपरवलय $kx^2 - y^2 = 6$ की एक नियता है, तो यह अतिपरवलय किस बिन्दु से होकर जाता है:

(1) $\left(-2\sqrt{5}, 6\right)$ (2) $\left(-\sqrt{5}, 3\right)$ (3) $\left(\sqrt{5}, -2\right)$ (4) $\left(2\sqrt{5}, 3\sqrt{6}\right)$

Ans. Official Answer NTA (3)

Sol. $\dfrac{x^2}{6/k} - \dfrac{y^2}{6} = 1$ ......(1)

$e^2 = 1 + \dfrac{6}{6/k}$

$e = \sqrt{1 + k}$

$a = \sqrt{\frac{6}{k}}$

Equation of directrix $x = \frac{a}{e} \Rightarrow x = \sqrt{\frac{6}{k(k+1)}}$

$\frac{6}{k(k+1)} = 1$

$k = 2$

From eq. (1) ,we get $2x^2 - y^2 = 6$

Check options

Question ID : 1449615

Vectors

15. A vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}, \hat{i}+\hat{j}$ and the plane determined by the vectors $\hat{i}-\hat{j}, \hat{i}+\hat{k}$. The obtuse angle between $\vec{a}$ and the vector $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$ is :

सदिशों $\hat{i}, \hat{i}+\hat{j}$ द्वारा प्राप्त समतल तथा सदिशों $\hat{i}-\hat{j}, \hat{i}+\hat{k}$ द्वारा प्राप्त समतल की प्रतिच्छेदन रेखा के समांतर एक सदिश $\vec{a}$ है। $\vec{a}$ तथा सदिश $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$ के बीच अधिक कोण है :

(1) $\frac{3\pi}{4}$ (2) $\frac{2\pi}{3}$ (3) $\frac{4\pi}{5}$ (4) $\frac{5\pi}{6}$

Ans. Official Answer NTA (1)

Sol. $\vec{n}_1 = \hat{i} \times (\hat{i} + \hat{j}) = \hat{i}$

$\vec{n}_2 = (\hat{i} + \hat{j}) \times (\hat{i} - \hat{j})$

$= \hat{i} + \hat{j} - \hat{k}$

Line of intersection along $\vec{n}_1 \times \vec{n}_2$

$= \hat{k} \times (\hat{i} + \hat{j} - \hat{k}) = -\hat{i} + \hat{j}$

D.R of $\vec{a} = -\hat{i} + \hat{j}$

D.E of $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$

$\vec{a}.\vec{b} = -3$ and $(\vec{a} \wedge \vec{b}) = \theta$

$\cos\theta = \frac{-3}{\sqrt{2} \times 3}$

$\theta = \frac{3\pi}{4}$

Question ID : 1449616

ITF

16. If $0 < x < \frac{1}{\sqrt{2}}$ and $\frac{\sin^{-1} x}{\alpha} = \frac{\cos^{-1} x}{\beta}$, then a value of $\sin\left(\frac{2\pi\alpha}{\alpha+\beta}\right)$ is :

यदि $0 < x < \frac{1}{\sqrt{2}}$ तथा $\frac{\sin^{-1} x}{\alpha} = \frac{\cos^{-1} x}{\beta}$ हैं, तो $\sin\left(\frac{2\pi\alpha}{\alpha+\beta}\right)$ का एक समान है :

(1) $4\sqrt{(1-x^2)}(1-2x^2)$ (2) $4x\sqrt{(1-x^2)}(1-2x^2)$

(3) $2x\sqrt{(1-x^2)}(1-4x^2)$ (4) $4\sqrt{(1-x^2)}(1-4x^2)$

Ans. Official Answer NTA (2)

Sol. $\frac{\sin^{-1} x}{\alpha} = \frac{\cos^{-1} x}{\beta} = k$

$\sin^{-1} x = k\alpha$

$\cos^{-1} x = k\beta$

$k = \frac{\pi}{2(\alpha+\beta)}$ .....(i)

$\sin\left(\frac{2\pi\alpha}{\alpha+\beta}\right) = \sin\left(4\sin^{-1} x\right)$

$= 2\sin\left(2\sin^{-1} x\right)\cos\left(2\sin^{-1} x\right)$

$= 4x\sqrt{1-x^2}\left(1-2x^2\right)$

Question ID : 1449617

Mathematical Reasoning

17. Negation of the Boolean expression $p \Leftrightarrow (q \Rightarrow p)$ is :

बूलीय व्यंजक $p \Leftrightarrow (q \Rightarrow p)$ का निषेधन है :

(1) $(\sim p) \wedge q$ (2) $p \wedge (\sim q)$ (3) $(\sim p) \vee (\sim q)$ (4) $(\sim p) \wedge (\sim q)$

Ans. Official Answer NTA (4)

Sol. $\sim\left(p \leftrightarrow (q \rightarrow p)\right)$

$\sim(p \leftrightarrow q) = (p \wedge \sim q) \vee (q \wedge \sim p)$

$\sim\left(p \leftrightarrow (q \rightarrow p)\right) = \left(p \wedge \sim (q \rightarrow p)\right) \vee \left((q \rightarrow p) \wedge \sim p\right)$

$\left(p \wedge \sim (q \rightarrow p)\right) = p \wedge (q \wedge \sim p) = (p \wedge \sim p) \wedge q = c$

$(q \to p) \wedge \sim p = (\sim q \vee p) \wedge \sim p = \sim p \wedge (\sim q \vee p)$

$= (\sim p \wedge \sim q) \vee (\sim p \wedge p) = \sim p \wedge \sim q$

$\sim (p \leftrightarrow (q \to p)) = c \vee (\sim p \wedge \sim q) = \sim p \wedge \sim q$

Question ID : 1449618

Probability

18. Let X be a binomially distributed random variable with mean 4 and variance $\frac{4}{3}$. Then, $54\,P(X \le 2)$ is equal to :

एक यादृच्छिक चर X, जिसका बंटन द्विपद है, का माध्य 4 तथा प्रसरण $\frac{4}{3}$ हैं। तो $54\,P(X \le 2)$ बराबर है:

(1) $\frac{73}{27}$ (2) $\frac{146}{27}$ (3) $\frac{146}{81}$ (4) $\frac{126}{81}$

Ans. Official Answer NTA (2)

Sol. np = 4

npq = 4/3

n = 6, p = 2/3, q = 1/3

54(P(X = 2) + P (X = 1) + P(X = 0))

$$54\left({}^6C_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^4 + {}^6C_1\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^5 + {}^6C_0\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^6 \right)$$

$$= \frac{146}{27}$$

Question ID : 1449619

Indefinite Integration

19. The integral $\int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x - \sin x)}{\left(1+\frac{2}{\sqrt{3}}\sin 2x\right)}dx$ is equal to :

समाकलन $\int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x - \sin x)}{\left(1+\frac{2}{\sqrt{3}}\sin 2x\right)}dx$ बराबर है :

(1) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}\right|+C$

(2) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{3}\right)}\right|+C$

(3) $\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}\right|+C$

(4) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}-\frac{\pi}{6}\right)}\right|+C$

Ans. Official Answer NTA (1)

Sol. $$I=\int\frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(1+\frac{2}{\sqrt{3}}\sin 2x\right)}dx$$

$$\frac{\sqrt{3}}{2}=\int\frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(\frac{\sqrt{3}}{2}+\sin 2x\right)}dx$$

$$\int\frac{\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)(\cos x-\sin x)}{\sin 60^\circ+\sin 2x}dx$$

$$\int\frac{\left(\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\sin x\right)}{2\sin\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{6}\right)}dx$$

$$\int\frac{\left(\cos\left(x-\frac{\pi}{6}\right)-\sin\left(x+\frac{\pi}{6}\right)\right)}{2\sin\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{6}\right)}dx$$

$$\frac{1}{2}\left(\int\frac{dx}{\sin\left(x+\frac{\pi}{6}\right)}-\int\frac{dx}{\cos\left(x-\frac{\pi}{6}\right)}\right)$$

$$\frac{1}{2}\ln\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}\right|$$

Question ID : 1449620

Area Under Curve

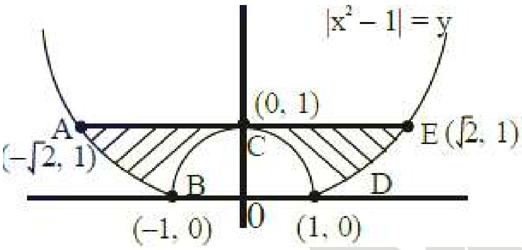
20. The area bounded by the curves $y = |x^2 - 1|$ and $y = 1$ is :

वक्रों $y = |x^2 - 1|$ तथा $y = 1$ द्वारा घिरे क्षेत्र का क्षेत्रफल है :

(1) $\frac{2}{3}\left(\sqrt{2}+1\right)$ (2) $\frac{4}{3}\left(\sqrt{2}-1\right)$ (3) $2\left(\sqrt{2}-1\right)$ (4) $\frac{8}{3}\left(\sqrt{2}-1\right)$

Ans. Official Answer NTA (4)

Sol. $y = |x^2 - 1|$



Arera = ABCDEA

$$= 2\left(\int_0^1\left(1-\left(1-x^2\right)\right)dx + \int_1^{\sqrt{2}}\left(1-\left(x^2-1\right)\right)dx\right)$$

$$= \frac{8}{3}\left(\sqrt{2}-1\right)$$

SECTION – B

Question ID : 1449621

Set & Relations

21. Let A = {1, 2, 3, 4, 5, 6, 7} and B = {3, 6, 7, 9}. Then the number of elements in the set $\{C \subseteq A : C \cap B \neq \phi\}$ is _________.

माना A = {1, 2, 3, 4, 5, 6, 7} तथा B = {3, 6, 7, 9} हैं। तो समुच्चय $\{C \subseteq A : C \cap B \neq \phi\}$ में अवयवों की संख्या है _________.

Ans. Official Answer NTA (112)

Sol. A = {1,2,3,4,5,6,7} and

B = {3,6,7,9}

Total subset of A = $2^7 = 128$

$C \cap B = \phi$ when set C contains the element 1, 2, 4, 5

$\therefore\ S = \{C \subseteq A;\ C \cap B \neq \phi\}$

= Total . $(C \cap B = \phi)$

$= 128 - 2^4 = 112$

Question ID : 1449622

Vectors

22. The largest value of a, for which the perpendicular distance of the plane containing the lines $\vec{r} = (\hat{i}+\hat{j}) + \lambda(\hat{i}+a\hat{j}-\hat{k})$ and $\vec{r} = (\hat{i}+\hat{j}) + \mu(-\hat{i}+\hat{j}-a\hat{k})$ from the point (2, 1, 4) is $\sqrt{3}$, is ________.

a का अधिकतम मान, जिसके लिए रेखाओं $\vec{r} = (\hat{i}+\hat{j}) + \lambda(\hat{i}+a\hat{j}-\hat{k})$ तथा $\vec{r} = (\hat{i}+\hat{j}) + \mu(-\hat{i}+\hat{j}-a\hat{k})$ को अंतर्विष्ट करने वाले समतल की बिन्दु (2, 1, 4) से लंबवत् दूरी $\sqrt{3}$ है, है________।

Ans. Official Answer NTA (2)

Sol. $\vec{r} = (\hat{i}+\hat{j}) + \lambda(\hat{i}+a\hat{j}-\hat{k})$

$\vec{r} = (\hat{i}+\hat{j}) + \mu(-\hat{i}+\hat{j}-a\hat{k})$

D.R's of plane containing these lines is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & a & -1 \\ -1 & 1 & -a \end{vmatrix} = \hat{i}(1-a^2) - \hat{j}(-a-1) + \hat{k}(1+a)$$

$\vec{n} = (1-a)\hat{i} + \hat{j} + \hat{k}$

One point in plane : (1, 1, 0)

$\therefore$ equation of plane is

$(1 - a)(x - 1) + (y - 1) + (z - 0) = 0$

$(1 - a)x + y + z + a - 2 = 0$

$$\therefore D = \frac{|(1-a)2+1+4+a-2|}{\sqrt{(1-a)^2+1+1}}$$

$\Rightarrow |5-a| = \sqrt{3}.\sqrt{a^2-2a+3}$

$\Rightarrow a^2 + 2a - 8 = 0$

$\Rightarrow a = 2, -4$

$\therefore$ largest value of a = 2

Question ID : 1449623

P & C

23. Number are to be formed between 1000 and 3000, which are divisible by 4, using the digits 1, 2, 3, 4, 5 and 6 without repetition of digits. Then the total number of such numbers is _________.

अकों 1, 2, 3, 4, 5 तथा 6 के प्रयोग से बिना पुनरावृत्ति के 1000 तथा 3000 के बीच 4 से विभाज्य संख्याऐं बनाई जानी हैं। इस प्रकार की संख्याओं की कुल संख्या है _____।

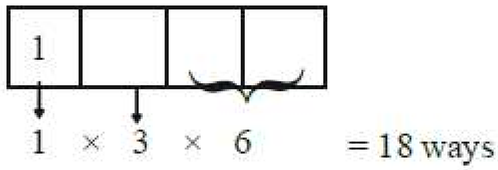
Ans. Official Answer NTA (30)

Sol. Here 1st digit is 1 or 2 only

Case-I

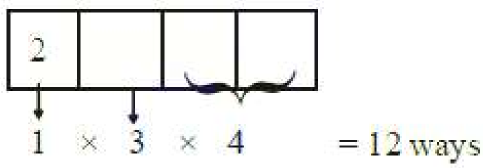
If first digit is 1

Then last two digits can be 24, 32, 36, 52, 56, 64



Case – II

If first digit is 2 then last two digit can be 16, 36, 56, 64



Total ways = 12 + 18 = 30 ways

Question ID : 1449624

Sequence & progression

24. If $\sum_{k=1}^{10}\frac{k}{k^4+k^2+1}=\frac{m}{n}$, where m and n are co-prime, then m + n is equal to _________.

यदि $\sum_{k=1}^{10}\frac{k}{k^4+k^2+1}=\frac{m}{n}$ है, जहाँ m तथा n असहभाज्य हैं, तो m + n बराबर है ________.

Ans. Official Answer NTA (166)

Sol. $\sum_{k=1}^{10}\frac{k}{k^4+k^2+1}=\frac{m}{n}$

$$\Rightarrow \frac{1}{2}\sum_{k=1}^{10}\frac{(k^2+k+1)-(k^2-k+1)}{(k^2+k+1)(k^2-k+1)}$$

$$\Rightarrow \frac{1}{2}\left(\sum_{k=1}^{10}\left(\frac{1}{\left(k^2-k+1\right)}-\frac{1}{k^2+k+1}\right)\right)$$

$$\Rightarrow \frac{55}{111}=\frac{m}{n}$$

$m + n = 166$

Question ID : 1449625

Trigonometric Equation

25. If the sum of solutions of the system of equations $2\sin^2\theta - \cos 2\theta = 0$ and $2\cos^2\theta + 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is $k\pi$, then k is equal to _________.

यदि समीकरण निकाय $2\sin^2\theta - \cos 2\theta = 0$ तथा $2\cos^2\theta + 3\sin\theta = 0$ के अंतराल $[0, 2\pi]$ में हलों का योगफल $k\pi$ है, तो k बराबर है _______।

Ans. Official Answer NTA (3)

Sol. $2\sin^2\theta - \cos 2\theta = 0$

$2\sin^2\theta - (1 - 2\sin^2\theta) = 0$

$$\Rightarrow \sin^2\theta = \left(\frac{1}{2}\right)^2$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$2\cos^2\theta + 3\sin\theta = 0$

$\Rightarrow 2\sin^2\theta - 3\sin\theta - 2 = 0$

$$\therefore \sin\theta = -\frac{1}{2}$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

So, the common solution is

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

K = 3

Question ID : 1449626

Statistics

26. The mean and standard deviation of 40 observations are 30 and 5 respectively. It was noticed that two of these observations 12 and 10 were wrongly recorded. If σ is the standard deviation of the data after omitting the two wrong observations from the data, then $38\sigma^2$ is equal to _________.

40 प्रेक्षणों का माध्य तथा मानक विचलन क्रमशः 30 तथा 5 हैं। यह पाया गया कि इनमें से दो प्रेक्षण 12 तथा 10 गलती से लिखे गए। यदि गलती से लिखे दो प्रक्षेणों को हटाने के पश्चात् शेष आंकड़ों का मान विचलन σ है, तो $38\sigma^2$ बराबर है_________।

Ans. Official Answer NTA (238)

Sol. Wrong mean = $\mu_1 = 30$

Wrong S.D = $\sigma_1 = 5$

$\frac{\Sigma x_i}{40} = 30$

$\Rightarrow \Sigma x_i = 1200$

$\sigma_1^2 = 25$

$\Rightarrow \frac{\Sigma x_i^2}{40} - 30^2 = 25$

$\Rightarrow \Sigma x_i^2 = 925 \times 40 = 37000$

New sum $= \Sigma x_i' = 1200 - 10 - 12 = 1178$

New mean $= \mu_i' = \frac{1178}{38} = 31$

New $\Sigma x_i^2 = 37000 - (10)^2 - (12)^2 = 36756$

New S.D, $\sigma_1' = \sqrt{\frac{36756}{38} - (31)^2} = \sigma$

$36756 - (31)^2 \times 38 = 38\sigma^2$

$\Rightarrow 38\sigma^2 = 238$

Question ID : 1449627

3D Geometry

27. The plane passing through the line L : $lx - y + 3(1 - l)z = 1$, $x + 2y - z = 2$ and perpendicular to the plane $3x + 2y + z = 6$ is $3x - 8y + 7z = 4$. If θ is the acute angle between the line L and the y-axis, then $415 \cos^2\theta$ is equal to _________.

रेखा L : $lx - y + 3(1 - l)z = 1$, $x + 2y - z = 2$ से होकर जाने वाले तथा समतल $3x + 2y + z = 6$ के लम्बवत् समतल $3x - 8y + 7z = 4$ है। यदि रेखा L तथा y-अक्ष के बीच न्यून कोण θ है, तो $415 \cos^2\theta$ बराबर है _______।

Ans. Official Answer NTA (125)

Sol. $\vec{n}_1 = \ell\hat{i} - \hat{j} + 3(1-\ell)\hat{k}$

$\vec{n}_2 = \hat{i} + 2\hat{j} - \hat{k}$

Direction ratio of line = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \ell & -1 & 3(1-\ell) \\ 1 & 2 & -1 \end{vmatrix}$

$=(6\ell-5)\hat{i}+(3-2\ell)\hat{j}+(2\ell+1)\hat{k}$

$3x-8y+7z=4$ will contain the line

$(6\ell-5)\hat{i}+(3-2\ell)\hat{j}+(2\ell+1)\hat{k}$

Normal of $3x-8y+7z=4$ will be perpendicular to the line

$=3(6\ell-5)+(3-2\ell)(-8)+7(2\ell+1)=0$

$\Rightarrow \ell=\frac{2}{3}$

$\therefore$ direction ratio of line $\left(-1,\frac{5}{3},\frac{7}{3}\right)$

Angle with y axis

$\cos\theta=\frac{5/3}{\sqrt{1+\frac{25}{9}+\frac{49}{9}}}$

$\cos\theta=\frac{5}{\sqrt{83}}$

$\therefore 415\cos^2\theta=\frac{25}{83}\times 415=125$

Question ID : 1449628

Differential Equation

28. Suppose $y=y(x)$ be the solution curve to the differential equation $\frac{dy}{dx}-y=2-e^{-x}$ such that $\lim\limits_{x\to\infty} y(x)$ is finite. If a and b are respectively the x- and y- intercepts of the tangent to the curve at $x=0$, then the value of $a-4b$ is equal to _________.

माना अवकल समीकरण $\frac{dy}{dx}-y=2-e^{-x}$ के हल वक्र $\lim\limits_{x\to\infty} y(x)$ परिमित है। यदि वक्र की $x=0$ पर स्पर्श रेखा के $x-$ तथा $y-$ अंतः खण्ड क्रमशः a तथा b हैं, तो $a-4b$ का मान बराबर है _________।

Ans. Official Answer NTA (3)

Sol. $\frac{dy}{dx}-y=2-e^{-x}$

I.F. $=e^{-\int dx}=e^{-x}$

$\therefore$ solution of D.E.

$y.e^{-x} = \int \left(2e^{-x} - e^{-2x}\right)dx$

$\Rightarrow y = -2 + \frac{e^{-x}}{2} + C.e^{x}$

$\because \lim_{x\to\infty} y$ is finite

$\because \lim_{x\to\infty}\left(-2 + \frac{e^{-x}}{2} + C.e^{x}\right) \to$ finite

This is possible only when $C = 0$

$\therefore y = y(x) = -2 + \frac{e^{-x}}{2}$

$\frac{dy}{dx} = -\frac{1}{2}e^{-x}$

$\left.\frac{dy}{dx}\right|_{x=0} = -\frac{1}{2} = m, y(0) = -2 + \frac{1}{2} = \frac{-3}{2}$

$\therefore$ equation of tangent

$y + \frac{3}{2} = -\frac{1}{2}(x - 0)$

$\Rightarrow x + 2y = -3$

$a = -3, b = \frac{-3}{2}$

$a - 4b = -3 + 6 = 3$

Question ID : 1449629

Sequence & progression

29. Different A.P.'s are constructed with the first term 100, the last term 199, and integral common differences. The sum of the common differences of all such A.P.'s having at least 3 terms and at most 33 terms is _________.

भिन्न A.P. बनाई गई हैं, जिनके प्रथम पद 100, अंतिम पद 199 तथा सार्व अंतर पूर्णांक हैं। इस प्रकार की सभी A.P. जिनमें कम से कम 3 पद तथा अधिक से अधिक 33 पद हैं, के सार्व अंतरों का योगफल हैं _____।।

Ans. Official Answer NTA (53)

Sol. 1st term = 100 = a

Last term = 199 = ℓ

If 3 term

a, a + d, a + 2d

$a_n = \ell = a + (n - 1)d$

$d_i = \frac{\ell - a}{n - 1}$

$n \to$ number of terms

$n = 3,\ d_1 = \frac{199-100}{2}$

$= \frac{99}{2} \notin I$

$n = 4,\ d_2 = \frac{99}{3} = 33 \in I$

$n = 10,\ d_3 = \frac{99}{9} = 11 \in I$

$n = 12,\ d_4 = \frac{99}{11} = 9 \in I$

$\therefore\ \Sigma d_1 = 33 + 11 + 9 = 53$

Question ID : 1449630

Matrices

30. The number of matrices $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where a, b, c, d $\in \{-1, 0, 1, 2, 3, ...10\}$, such that $A = A^{-1}$, is _________.

अव्यहों $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, जहाँ a, b, c, d $\in \{-1, 0, 1, 2, 3, ...10\}$ हैं $A = A^{-1}$ है, की संख्या है _______ ।

Ans. Official Answer NTA (50)

Sol. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Given $A = A^{-1}$

$\therefore\ A^2 = A \cdot A^{-1} = I$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore\ a^2 + bc = 1$ .....(1)

$ab + bd = 0$ .....(2)

$ac + cd = 0$ .....(3)

$bc + d^2 = 1$ .....(4)

(1) – (4) gives

$a^2 - d^2 = 0$

$\Rightarrow (a + d) = 0$ or $a - d = 0$

Case – I

$a + d = 0 \Rightarrow (a, d) = (-1, 1), (0, 0), (1, -1)$

(a) (a, d) = (.1, 1)
$\therefore$ from equation (1)
$1 + bc = 1 \Rightarrow bc = 0$
b = 0 C = 12 possibilities
c = 0 b = 12 possibilities
but (0, 0) is repeated
$\therefore 2 \times 12 = 24$
24 – 1 (repeated) = 23 pairs
(b) $(a, d) = (1, -1) \Rightarrow bc = 0 \rightarrow$ 23 pairs
(c) $(a, d) = (0, 0) \Rightarrow bc = 1$
$\Rightarrow (b, c) = (1, 1)$ & $(-1, -1)$, 2 pairs

Case – II

a = d
from (2) and (3)
$a \neq 0$ then $b = c = 0$
$a^2 = 1$
$a = \pm 1 = d$
$(a, d) = (1, 1), (-1, -1) \rightarrow$ 2 pairs
$\therefore$ Total = 23 + 23 + 2 + 2
= 50 pairs