JEE Main February 2021

Question Paper With Text Solution

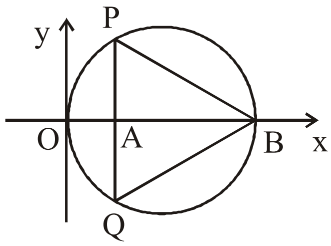
26 Feb.| Shift-1

MATHEMATICS

JEE MAIN FEB 2021 | 26TH FEB SHIFT-1

SECTION – A

1. In the circle given below, let OA = 1 unit, OB = 13 unit and PQ $\perp$ OB. Then, the area of the triangle PQB (in square units) is :



(1) $24\sqrt{3}$ (2) $26\sqrt{3}$ (3) $24\sqrt{2}$ (4) $26\sqrt{2}$

Ans. Official Answer NTA : (1)

Ans. (1)

Sol. $OA = 1,\ \ OB = 13 \Rightarrow AB = 12$

$AP.AQ = AB.AO$

Also AP = AQ

$\Rightarrow AP^2 = 1.12 \Rightarrow AP = \sqrt{12} = 2\sqrt{3} \Rightarrow PQ = 4\sqrt{3}$

Area of $\Delta PQB = \frac{1}{2} PQ.AB = \frac{1}{2} \times 4\sqrt{3} \times 12$

$= 24\sqrt{3}$

2. The sum of infinite series $1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + ...$ is equal to :

(1) $\frac{9}{4}$ (2) $\frac{15}{4}$ (3) $\frac{11}{4}$ (4) $\frac{13}{4}$

Ans. Official Answer NTA : (4)

Ans. (4)

Sol. Sum S = $1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + ...$

Let $S_1 = \frac{2}{3^1} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} .....$ ....(1)

$\Rightarrow \frac{1}{3} S_1 = \frac{2}{3^2} + \frac{7}{3^3} + \frac{12}{3^4} + \frac{17}{3^5} .....$ ....(2)

(1) – (2) $\frac{2}{3} S_1 = \frac{2}{3} + 5\left(\frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} .....\infty\right)$

$$= \frac{2}{3} + 5\left(\frac{\frac{1}{9}}{1-\frac{1}{3}}\right)$$

$\Rightarrow$ $S_1 = \frac{9}{4}$

$S = 1 + S_1 = 1 + \frac{9}{4} = \frac{13}{4}.$

3. If (1, 5, 35), (7, 5, 5), (1, λ, 7) and (2λ, 1, 2) are coplanar, then the sum of all possible values of λ is :

(1) $-\frac{44}{5}$ (2) $-\frac{39}{5}$ (3) $\frac{44}{5}$ (4) $\frac{39}{5}$

Ans. Official Answer NTA : (3)

Ans. (3)

Sol. A (1, 5, 35), B(7, 5, 5), C(1, λ, 7), D(2λ, 1, 2)

are coplanar

$\Rightarrow \overrightarrow{AB}.(\overrightarrow{AC} \times \overrightarrow{AD}) = 0.$

$$\Rightarrow \begin{vmatrix} 6 & 0 & -30 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -33 \end{vmatrix} = 0,$$

$\Rightarrow$ $10\lambda^2 - 88\lambda + 75 = 0$

Sum of values of $\lambda = \frac{44}{5}$

4. The maximum value of the term independent of 't' in the expansion of $\left(tx^{\frac{1}{5}}+\frac{(1-x)^{\frac{1}{10}}}{t}\right)^{10}$ where $x \in (0, 1)$ is :

(1) $\frac{2.10!}{3\sqrt{3}(5!)^2}$ (2) $\frac{2.10!}{3(5!)^2}$ (3) $\frac{10!}{\sqrt{3}(5!)^2}$ (4) $\frac{10!}{\sqrt{3}(5!)^2}$

Ans. Official Answer NTA : (1)

Ans. (1)

Sol. The term independent of t will be middle term :

Middle term $f(x) = {}^{10}C_5 \cdot x \cdot (1-x)^{\frac{1}{2}}$; $x \in (0,1)$.

$$f'(x) = {}^{10}C_5 \cdot \left[(1-x)^{\frac{1}{2}} - \frac{x}{2}(1-x)^{-\frac{1}{2}}\right] = 0$$

$$\Rightarrow 1-x = \frac{x}{2} \Rightarrow \frac{3x}{2} = 1 \Rightarrow x = \frac{2}{3}.$$

Point of maxima.

$$f\left(\frac{2}{3}\right) = {}^{10}C_5 \times \frac{2}{3}\left(1-\frac{2}{3}\right)^{\frac{1}{2}}$$

$$= \frac{2}{3\sqrt{3}} \times \frac{10!}{5!5!}$$

5. The number of seven digit integers with sum of the digits equal to 10 formed by using the digits 1, 2 and 3 only is :

(1) 42 (2) 77 (3) 82 (4) 35

Ans. Official Answer NTA : (2)

Ans. (2)

Sol. The following possibilities are there :

1 1 1 1 1 2 3 or 1 1 1 1 2 2 2

Number of permutations of these $= \frac{7!}{5!} + \frac{7!}{4!3!}$

$= 77.$

6. In an increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of 4^{th}, 6^{th} and 8^{th} terms is equal to :

(1) 35 (2) 30 (3) 32 (4) 26

Ans. Official Answer NTA : (1)

Ans. (1)

Sol. Let first term = a and common ratio = r.

Given $ar + ar^5 = \frac{25}{2}$ .....(1)

and $ar^2 . ar^4 = 25 \Rightarrow ar^3 = 5$ .....(2)

From (1) & (2) ; $r = \sqrt{2}$ and $a = \frac{5}{2\sqrt{2}}$.

$t_4 + t_6 + t_8 = ar^3 + ar^5 + ar^7$
$= ar^3 (1 + r^2 + r^4)$

$= 5 (1 + 2 + 4) = 35$

7. If $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \vec{b}\right)\right)\right)$ is equal to :

(1) $\frac{1}{2}|\vec{a}|^4 \vec{b}$ (2) $|\vec{a}|^4 \vec{b}$ (3) $\vec{0}$ (4) $\vec{a} \times \vec{b}$

Ans. Official Answer NTA : (2)

Ans. (2)

Sol. $\vec{a} \times (\vec{a} \times (\vec{a} \times (\vec{a} \times \vec{b})))$

$= \vec{a} \times (\vec{a} \times [(\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b}]$

Since $\vec{a} \perp \vec{b} \Rightarrow = \vec{a} \cdot \vec{b} = 0$

Given expression $= \vec{a} \times (\vec{a} \times (-(\vec{a} \cdot \vec{a})\vec{b}))$

$= |\vec{a}|^4 \vec{b}.$

8. Let R = {(P, Q) | P and Q are at the same distance from the origin} be a relation, then the equivalence class of (1, –1) is the set :

(1) $S=\{(x,y)\,|\,x^2+y^2=\sqrt{2}\}$ (2) $S=\{(x,y)\,|\,x^2+y^2=2\}$

(3) $S=\{(x,y)\,|\,x^2+y^2=4\}$ (4) $S=\{(x,y)\,|\,x^2+y^2=1\}$

Ans. Official Answer NTA : (2)

Ans. (2)

Sol. R (1, –1) P (h, k), O (0, 0)

OR = OP

locus of P is $h^2 + k^2 = 2$

$x^2 + y^2 = 2$

9. The value of $\int_{-\pi/2}^{\pi/2}\frac{\cos^2 x}{1+3^x}dx$ is :

(1) $\frac{\pi}{4}$ (2) 4π (3) $\frac{\pi}{2}$ (4) 2π

Ans. Official Answer NTA : (1)

Ans. (1)

Sol. $I=\int_{-\pi/2}^{\pi/2}\frac{\cos^2 x}{1+3^x}dx.$ .....(1)

using I (x) = I (a + b – x)

$\Rightarrow I=\int_{\pi/2}^{\pi/2}\frac{\cos^2 x\,dx}{1+3^{-x}}$

$I=\int_{-\pi/2}^{\pi/2}\frac{3^x\cos^2 x\ dx}{1+3^x}$ .....(2)

Adding (1) & (2)

$2I=\int_{-\pi/2}^{\pi/2}\cos^2 x\ dx=\int_{-\pi/2}^{\pi/2}\frac{1+\cos 2x}{2}dx$

$\Rightarrow 2I=\frac{\pi}{2}$

$\Rightarrow I=\frac{\pi}{4}.$

10. If $\frac{\sin^{-1}x}{a}=\frac{\cos^{-1}x}{b}=\frac{\tan^{-1}y}{c}$; $0 < x < 1$, then the value of $\cos\left(\frac{\pi c}{a+b}\right)$ is :

(1) $\frac{1-y^2}{y\sqrt{y}}$ (2) $1-y^2$ (3) $\frac{1-y^2}{2y}$ (4) $\frac{1-y^2}{1+y^2}$

Ans. Official Answer NTA : (4)

Ans. (4)

Sol. Let $\frac{\sin^{-1}x}{a}=\frac{\cos^{-1}x}{b}=\frac{\tan^{-1}y}{c}=K.$

$$\cos\left(\frac{\pi c}{a+b}\right)= \cos\left(\frac{\pi\frac{\tan^{-1}y}{k}}{\frac{\sin^{-1}x}{K}+\frac{\cos^{-1}x}{K}}\right)$$

$$=\cos\left(\frac{\pi\tan^{-1}y}{\frac{\pi}{2}}\right)$$

$$=\cos(2\tan^{-1}y)$$

$$\frac{1-y^2}{1+y^2}$$

11. Let f be any function defined on R and let it satisfy the condition :

$|f(x)-f(y)|\le|(x-y)^2|, \forall(x,y)\in R$

If $f(0)=1$, then :

(1) $f(x)<0, \forall x\in R$ (2) $f(x)>0, \forall x\in R$

(3) $f(x)=0, \forall x\in R$ (4) $f(x)$ can take any value in R

Ans. Official Answer NTA : (2)

Ans. (2)

Sol. $\left|f(x)-f(y)\right|\le\left|(x-y)^2\right|\forall(x,y)\in R$

$$\Rightarrow \left|\frac{f(x)-f(y)}{x-y}\right|\le|x-y|$$

$$\Rightarrow \lim_{y\to x}\left|\frac{f(x)-f(y)}{x-y}\right| \le \lim_{y\to x}|x-y|$$

$$\Rightarrow |f'(x)| \le 0$$

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow f(x) = \text{constant}$$

and sinc e $f(0) = 1 = f(x) = 1 > 0 \; \forall \; x \in R$

12. The value of $\sum_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]}\,dx$, where [x] is the greatest integer $\le$ x, is :

(1) 100e (2) 100(1 – e) (3) 100(1 + e) (4) 100(e – 1)

Ans. Official Answer NTA : (4)

Ans. ()

Sol. $S = \sum_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]}dx \quad = \quad \sum_{n=1}^{100} \int_{n-1}^{n} e^{\{x\}}dx.$

Since $\{x\}$ is periodic with
Fundalmental period = 1,

$$S = 100\int_0^1 e^{\{x\}}dx = 100\int_0^1 e^{x}dx$$

$$= 100\left[e^x\right]_0^1$$

$$= 100(e-1)$$

13. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is :

(1) 1 (2) 12 (3) 6 (4) 4

Ans. Official Answer NTA : (4)

Ans. (4)

Sol. Let $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$; a, b, c, d $\in$ Integer.

$$A^2 = \begin{bmatrix} a & b \\ b & c \end{bmatrix}\begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} a^2+b^2 & ab+bc \\ ab+bc & b^2+c^2 \end{bmatrix}$$

Sum of diagonal entries $= a^2 + b^2 + b^2 + c^2 = 1$

$$\Rightarrow a^2 + 2b^2 + c^2 = 1$$

$\Rightarrow a = \pm 1, b = 0 = C$

O R $a = b = 0, c = \pm 1.$

$\Rightarrow$ 4 possible matrices

14. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is :

(1) $\frac{15}{2^{13}}$ (2) $\frac{15}{2^{14}}$ (3) $\frac{15}{2^{8}}$ (4) $\frac{15}{2^{12}}$

Ans. Official Answer NTA : (1)

Ans. (1)

Sol. P (7 Heads) = P (9 Heads)

$${}^{n}C_{7}\left(\frac{1}{2}\right)^{n} = {}^{n}C_{9}\left(\frac{1}{2}\right)^{n} \Rightarrow n = 7 + 9 = 16.$$

$$P\text{ (2 Heads)} = {}^{16}C_{2}.\left(\frac{1}{2}\right)^{16}$$

$$= \frac{16 \times 15}{2} \times \frac{1}{2^{16}} = \frac{15}{2^{13}}$$

15. Consider the three planes

$P_1 : 3x + 15y + 21z = 9,$

$P_2 : x - 3y - z = 5,$ and

$P_3 : 2x + 10y + 14z = 5$

Then, which of the following is true ?

(1) P_1 and P_2 are parallel (2) P_1 and P_3 are parallel

(3) P_2 and P_3 are parallel (4) P_1, P_2 and P_3 all are parallel

Ans. Official Answer NTA : (2)

Ans. (2)

Sol. $P_1 = 3x + 15y + 21z = 9$

$P_2 = x - 3y - 3 = 5$

$P_3 = 2x + 10y + 14z = 5$

For P_1 & P_3, $\frac{a_1}{a_3}=\frac{b_1}{b_3}=\frac{c_1}{c_3}\neq\frac{d_1}{d_3}$

$$\frac{3}{2}=\frac{15}{10}=\frac{21}{14}\neq\frac{9}{5}$$

So, P_1 & P_3 are parallel.

16. The maximum slope of the curve $y=\frac{1}{2}x^4-5x^3+18x^2-19x$ occurs at the point :

(1) (0, 0) (2) $\left(3,\frac{21}{2}\right)$ (3) (2, 2) (4) (2, 9)

Ans. Official Answer NTA : (3)

Ans. (3)

Sol. Slope $=\frac{dy}{dx}=2x^3-15x^2+36x-19=m$

To find maximum : differentiating

$$\frac{dm}{dx}=6x^2-30x+36=0\Rightarrow x=2\ \text{OR}\ 3$$

+ − +
2 3
max min

x = 2 ; point of maxima
x = 3 ; point of minima

So point of maximum ; x = 2 ;

$$y=\frac{1}{2}(2)^4-5(2)^3+18(2)^2-19(2)$$

$=8-40+72-38$
$=2$

$=(2, 2)$

17. The value of $\lim_{h\to 0}2\left\{\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\sqrt{3}\cos h-\sin h\right)}\right\}$ is :

(1) $\frac{2}{3}$ (2) $\frac{4}{3}$ (3) $\frac{2}{\sqrt{3}}$ (4) $\frac{3}{4}$

Ans. Official Answer NTA : (2)

Ans. (2)

Sol. $\lim_{h\to 0} 2\left\{\frac{\frac{\sqrt{3}}{2}\sin\left(\frac{\pi}{6}+h\right)-\frac{1}{2}\cos\left(\frac{\pi}{6}-h\right)}{\sqrt{3}\ h\left(\frac{\sqrt{3}}{2}\cos h-\frac{1}{2}\sin\ h\right)}\right\}$

$=\lim_{h\to 0} 2\left\{\frac{\sin\left(h+\frac{\pi}{6}-\frac{\pi}{6}\right)}{\sqrt{3}\ h\ \cos\left(h+\frac{\pi}{6}\right)}\right\}$

$=\frac{2}{\sqrt{3}}\ \lim_{h\to 0}\frac{\sin\ h}{h\ \cos\left(h+\frac{\pi}{6}\right)}$

$=\frac{2}{\sqrt{3}}\times\frac{1}{\frac{\sqrt{3}}{2}}=\frac{4}{3}.$

18. The intersection of three lines x – y = 0, x + 2y = 3 and 2x + y = 6 is a :

(1) Isosceles triangle (2) Right angled triangle

(3) None of the above (4) Equilateral triangle

Ans. Official Answer NTA : (1)

Ans. (1)

Sol. Slopes of the 3 given lines in order are

$m_1 = 1,\ m_2 = -\frac{1}{2},\ m_3 = -2$

So the internal anlges of the triangle will be

$\tan A = \frac{m_1 - m_2}{1+m_2m_3} = 3$

$\tan C = \frac{m_3 - m_1}{1+m_2m_3} = 3$

$\tan B = \frac{m_2 - m_3}{1+m_2m_3} = -3$

So, it is an Isosceles Triangle.

19. The value of $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$ is :

(1) 0 (2) –2

(3) (a + 2) (a + 3) (a + 4) (4) (a + 1) (a + 2) (a + 3)

Ans. Official Answer NTA : (2)

Ans. (2)

Sol. $\Delta = \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$

Apply Row Transformations $R_3 \to R_3 - R_2,\ R_1 \to R_2 - R_1$

$\Delta = \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ 2(a+2) & 1 & 0 \\ 2(a+3) & 1 & 0 \end{vmatrix}$

$= 2(a+2) - 2(a+3) = -2$

20. The rate of growth of bacteria in a culture is proportional to the number of bacteria present and the bacteria count is 1000 at initial time t = 0. the number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after $\dfrac{k}{\log_e\left(\frac{6}{5}\right)}$ hours, then $\left(\dfrac{k}{\log_e 2}\right)^2$ is equal to :

(1) 16 (2) 8 (3) 2 (4) 4

Ans. Official Answer NTA : (4)

Ans. (4)

Sol. Let number of bacteriat at any time t is N(t)

$\dfrac{dN}{dt} = \lambda N$ Where λ is a constant.

$\Rightarrow \int \dfrac{dN}{N} = \int \lambda dt$

$\Rightarrow \log_e N = \lambda t + C$

given at t = 0 ; N = 1000

$\Rightarrow C = \log_e 1000$

Also in t = 2 ; N = 1200

So $\log_e 1200 = \lambda \times 2 + \log_e 1000$

$\Rightarrow \lambda = \frac{1}{2} \log_e \frac{6}{5}$

If N = 2000

$\Rightarrow \log_e 2000 = \frac{1}{2} \log_e \frac{6}{5}(t) + \log_e 1000$

$\Rightarrow t = \frac{2\log_e 2}{\log_e\left(\frac{6}{5}\right)} = \frac{k}{\log_e\left(\frac{6}{5}\right)} \Rightarrow k = \log_e 4$

$\left(\frac{k}{\log_e 2}\right)^2 = \left(\frac{\log_e 4}{\log_e 2}\right)^2 = (2)^2 = 4$

SECTION – B

1. The sum of 162[th] power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is :

Ans. Official Answer NTA : (3)

Ans. (3)

Sol. The roots of the given equation

$x^3 - 2x^2 + 2x - 1 = 0$

$\Rightarrow (x-1)(x^2 - x + 1) = 0$

are $1, -\omega$ and $-\omega^2$,

$1^{162} + (-\omega)^{162} + (-\omega^2)^{162} = 3.$

2. The value of the integral $\int_0^{\pi} |\sin 2x|\, dx$ is :

Ans. Official Answer NTA : (2)

Ans. (2)

Sol. $\int_0^{\pi} |\sin 2x|\, dx = 2\int_0^{\pi/2} |\sin 2x|\, dx$ (Since $f(\pi - x)$: $f(x)$)

$$= 2\int_0^{\pi/2} \sin 2x \, dx$$

$$= 2\left[\frac{-\cos 2x}{2}\right]_0^{\pi/2} = 2$$

3. The difference between degree and order of a differential equation that represents the family of curves given by $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right)$, $a > 0$ is :

Ans. Official Answer NTA : (2)

Ans. (2)

Sol. $y^2 = ax + \frac{a\sqrt{a}}{2}$, ...(1)

Differentiating

$2y\frac{dy}{dx} = a.$

Substituting in (1)

$y^2 = 2y\frac{dy}{dx} \times x + \frac{1}{2}\left(2y\frac{dy}{dx}\right)^{3/2}$

$\Rightarrow \left(2y\frac{dy}{dx}\right)^{3/2} = 2y^2 - 4xy\frac{dy}{dx}.$

Squaring $\left(2y\frac{dy}{dx}\right)^3 = \left(2y^2 - 4xy\frac{dy}{dx}\right)^2$

So Order = 1

Deglce = 3

Differrance = 2

4. The number of integral values of 'k' for which the equation 3sin x + 4cos x = k + 1 has a solution, $k \in R$ is :

Ans. Official Answer NTA : (11)

Ans. (11)

Sol. 3 sin x + 4 cos x = k + 1

Since Range of $3\sin x + 4\cos x \in [-5, 5]$

for solution $k + 1 \in [-5, 5]$

$\Rightarrow k \in [-6, 4]$

No. of integral values = 11

5. The number of solutions of the equation $\log_4(x-1) = \log_2(x-3)$ is :

Ans. Official Answer NTA : (1)

Ans. (1)

Sol. $\log_4(x-1) = \log_2(x-3)$

$x - 1 > 0$ and $x - 3 > 0 \Rightarrow x > 3$

$\log_2(x-1) = \log_2(x-3) \Rightarrow \frac{1}{2}\log_2(x-1) = \log_2(x-3)$

$\Rightarrow \log_2(x-1) = 2\log_2(x-3) = \log_2(x-3)^2$

$\Rightarrow x - 1 = (x-3)^2$

$\Rightarrow x^2 - 7x + 10 = 0$

$\Rightarrow s = 2$(Rejected ; as $x > 3$)

So no. of values = 1

6. If $\sqrt{3}(\cos^2 x) = (\sqrt{3}-1)\cos x + 1$,the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ is :

Ans. Official Answer NTA : (1)

Ans. (1)

Sol. $\sqrt{3}\left(\cos^2 x\right) = \left(\sqrt{3}-1\right)\cos x + 1 \quad ; \; x + \left[0, \pi/2\right]$

$\Rightarrow \sqrt{3}\cos^2 x - \sqrt{3}\cos x + \cos x - 1 = 0$

$\Rightarrow \left(\sqrt{3}\cos x + 1\right)\left(\cos x - 1\right) = 0$

$\Rightarrow \cos x = \frac{-1}{\sqrt{3}}$ OR $\cos x = 1$

For $x \in \left[0, \pi/2\right]$; only 1 solution ; x = 0

7. Let (λ, 2, 1) be a point on the plane which passes through the point (4, –2, 2). If the plane is perpendicular lo the line joining the points (–2, –21, 29) and (–1, –16, 23), then $\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$ is equal

to :

Ans. Official Answer NTA : (8)

Ans. (8)

Sol. Direction cosines of normal vector of a plane

will be : $\left(-1-(-2),-16-(-21),\ 23-29\right)$

$=(1,5,-6)$

Equation of plane

$x+5y-6z=k$

Passes through (4, –2, 2)

$\Rightarrow 4-10-12;\ k-y\ \ k=-18$

$\Rightarrow x+5y-6z+18=0.$

$(\lambda,2,1)$ Also lies or plane

$\Rightarrow \lambda+10-6+18=0 \Rightarrow \lambda=-22$

$\frac{\lambda}{11}=-2$

$\Rightarrow \left(\frac{\lambda}{11}\right)^2-4\left(\frac{\lambda}{11}\right)-4$

$=(-2)^2-4(-2)-4$

$=4+8-4=8$

8. Let m, n $\in$ N and gcd (2, n) = 1. If $30\binom{30}{0}+29\binom{30}{1}+........+2\binom{30}{28}+1\binom{30}{29}=n\cdot 2^m$, then n + m is equal to :

$\left(\text{Here}\binom{n}{k}={}^nC_k\right)$

Ans. Official Answer NTA : (45)

Ans. (45)

Sol. $S=30.{}^{30}C_0+29.{}^{30}C_1+28.{}^{30}C_1+....2.{}^{30}C_{22}+1.{}^{30}C_{29}+0.{}^{30}C_{30}$ .....(1)

$\Rightarrow S=0.{}^{30}C_0+1.{}^{30}C_1+2.{}^{30}C_2+....28.{}^{30}C_{28}+29.{}^{30}C_{29}+30.{}^{30}C_{30}$ .....(2)

Adding (1) & (2)

$\Rightarrow 2\ \ S=30\left[{}^{30}C_0+{}^{30}C_1+{}^{30}C_2+......{}^{30}C_{30}\right]$

$\Rightarrow S = 15.2^{30}$

$d(d(2,n)=1) \Rightarrow n$ is odd

$S = 15.2^{30} = n.2^{m} \Rightarrow n = 15; m = 30$

$n + m = 45.$

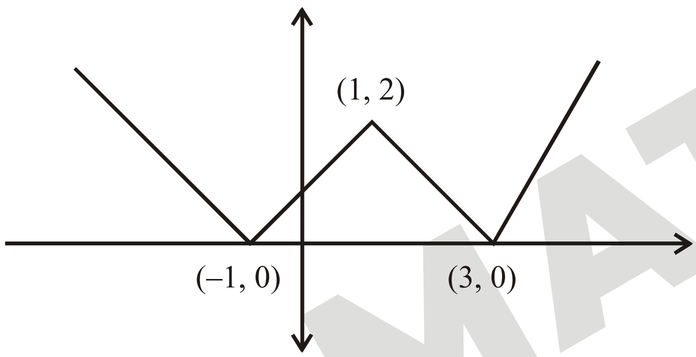
9. The area bounded by the lines y = || x – 1 | –2 | is :

Ans. Official Answer NTA : (8)

Ans. (8)

Sol. $y = ||x-1|-2|$

Graph



language not complete

10. If y = y(x) is the solution of the equation $e^{\sin y}\cos y\frac{dy}{dx} + e^{\sin y}\cos x = \cos x, y(0) = 0$; then

$1 + y\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right) + \frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)$ is equal to :

Ans. Official Answer NTA : (1)

Ans. (1)

Sol. $e^{\sin y}\cos y\frac{dy}{dx} + e^{\sin y}\cos x = \cos x$

Let $e^{\sin y} = t$

$\Rightarrow e^{\sin y}\cos y\frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \dfrac{dt}{dx} + t\cos x = \cos x$

Integrating Factors

I.F. $= e^{\int \cos x dx} = e^{\sin x}$.

$\Rightarrow \int d(t.e^{\sin x})\,dx = \int e^{\sin x} \cos x\,dx$

$\Rightarrow t.e^{\sin x} = e^{\sin x} + c$

$\Rightarrow e^{\sin y}.e^{\sin x} = e^{\sin x} + c$

$x = 0;\ y = 0\ \Rightarrow c = 0$

$\Rightarrow e^{\sin y}.e^{\sin x} = e^{\sin x}$

$\Rightarrow e^{\sin y} = 1\ \Rightarrow\ \sin y = 0\ \Rightarrow\ y = 0$

$\Rightarrow 1 + y\left(\dfrac{\pi}{6}\right) + \dfrac{\sqrt{3}}{2}y\left(\dfrac{\pi}{3}\right) + \dfrac{1}{\sqrt{2}}y\left(\dfrac{\pi}{4}\right) = 1$