JEE Main August 2021

Question Paper With Text Solution

26 August. | Shift-1

MATHEMATICS

JEE MAIN AUGUST 2021 | 26TH AUGUST SHIFT-1

SECTION – A

1. Let $f(x)=\cos\left(2\tan^{-1}\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right), 0<x<1$. Then :

माना Let $f(x)=\cos\left(2\tan^{-1}\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right), 0<x<1$ है, तो :

(1) $(1+x)^2 f'(x)-2(f(x))^2=0$

(2) $(1-x)^2 f'(x)+2(f(x))^2=0$

(3) $(1+x)^2 f'(x)+2(f(x))^2=0$

(4) $(1-x)^2 f'(x)-2(f(x))^2=0$

Question ID : 86435120057
Option 1 ID : 86435166762
Option 2 ID : 86435166761
Option 3 ID : 86435166760
Option 4 ID : 86435166759

Ans. Official Answer NTA (2)

Sol. Put $x=\sin^2\theta$

$$f(x)=\cos\left(2\tan^{-1}\left(\sin\left(\cot^{-1}\sqrt{\frac{\cos^2\theta}{\sin^2\theta}}\right)\right)\right)$$

$$=\cos(2\tan^{-1}\sin\theta)$$

let $\tan^{-1}(\sin\theta)=0$

$\tan\theta=\sin\theta$

$f=\cos(2\theta)$

$$=\frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$f(x)=\frac{1-x}{1+x}$$

$$f'(x)=\frac{-2}{(1+x)^2}$$

$$(1-x)^2 f'(x) = -2\left(\frac{1-x}{1+x}\right)^2$$

$$(1-x)^2 f'(x) = -2(f(x))^2$$

2. The value of $\lim\limits_{n\to\infty}\frac{1}{n}\sum\limits_{r=0}^{2n-1}\frac{n^2}{n^2+4r^2}$ is :

$\lim\limits_{n\to\infty}\frac{1}{n}\sum\limits_{r=0}^{2n-1}\frac{n^2}{n^2+4r^2}$ का मान है :

(1) $\frac{1}{4}\tan^{-1}(4)$

(2) $\frac{1}{2}\tan^{-1}(2)$

(3) $\frac{1}{2}\tan^{-1}(4)$

(4) $\tan^{-1}(4)$

Question ID : 86435120059
Option 1 ID : 86435166767
Option 2 ID : 86435166770
Option 3 ID : 86435166768
Option 4 ID : 86435166769

Ans. Official Answer NTA (3)

Sol. $\lim\limits_{x\to\infty}\frac{1}{n}\int\limits_{r=0}^{2x}\frac{n^2}{n^2+4x^2}$

$\lim\limits_{x\to\infty}\frac{1}{n}\int\limits_{r=0}^{2x}\frac{1}{1+4\left(\frac{r}{n}\right)^2}$

$=\int\limits_0^2\frac{dx}{1+dx^2}$

$=\frac{1}{2}\left(\tan^{-1}(2x)\right)_0^2$

$=\frac{\tan^{-1}(4)}{2}$

3. The mean and standard deviation of 20 observations were calculated as 10 and 2.5 respectively. It was found that by mistake one data value owas taken as 25 instead of 35. If α and $\sqrt{\beta}$ are the mean and standard deviation respectively for correct data, then (α, β) is :

20 प्रेक्षणों के माध्य तथा मानक विचलन क्रमशः 10 तथा 2.5 निकाले गये। यह पाया गया कि गलती से एक आंकड़ा 35 की जगह 25 लिया गया था। यदि सही आंकड़ों का माध्य तथा मानक विचलन क्रमशः α तथा $\sqrt{\beta}$ हैं, तो (α, β) है :

(1) (11, 25)

(2) (10.5, 25)

(3) (11, 26)

(4) (10.5, 26)

Question ID : 86435120069
Option 1 ID : 86435166807
Option 2 ID : 86435166810
Option 3 ID : 86435166808
Option 4 ID : 86435166809

Ans. Official Answer NTA (4)

Sol. $n = 20 \qquad \bar{x} = 10 \qquad \sigma = 2.5$

$\Sigma\bar{x} = n\bar{x} = 200$

$\sigma^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2$

$6.25 = \frac{\Sigma x_i^2}{10} - 100$

$\frac{\Sigma x_i^2}{20} = 106.25$

$\Sigma x_i^2 = 2125$

For new distribution

$\Sigma x_i = 200 - 25 + 35 = 210 \quad \Rightarrow \quad \bar{x} = \frac{\Sigma x_i}{20} = \frac{210}{20} = 10.5$

$\Sigma x_i^2 = 2125 - (25)^2 + (35)^2 = 2725$

$\beta = \sigma^2 = \frac{\Sigma x_i^2}{20} - (\bar{x})^2 = \frac{2725}{20} - (10.5)^2$

$= 136.25 - 110.25$

$= 26$

$(\alpha, \beta) \equiv (10.5, 26)$

4. If ${}^{20}C_r$ is the co–efficient of x^r in the expansion of $(1+x)^{20}$, then the value of $\sum_{r=0}^{20} r^2\ {}^{20}C_r$ is equla to :

यदि $(1+x)^{20}$ के प्रसार में x^r का गुणांक ${}^{20}C_r$ है, तो $\sum_{r=0}^{20} r^2\ {}^{20}C_r$ का मान बराबर है :

(1) 380×2^{18}

(2) 420×2^{19}

(3) 380×2^{19}

(4) 420×2^{18}

Question ID : 86435120055
Option 1 ID : 86435166753
Option 2 ID : 86435166751
Option 3 ID : 86435166752
Option 4 ID : 86435166754

Ans. Official Answer NTA (4)

Sol. $\sum_{r=0}^{20} r^2\ {}^{20}C_r = \sum_{r=0}^{20} r^2 \left(\frac{20}{r}\right)({}^{19}C_{r-1})$

$= \Sigma\, 20(r-1)\ {}^{19}C_{r-1} + \Sigma\, 20\ {}^{19}C_{r-1}$

$= \Sigma\,(r-1)\left(\frac{19}{r-1}\right){}^{18}C_{r-2} + (20)\,\Sigma\ {}^{19}C_{r-1}$

$= (20)(19)\Sigma\ {}^{19}C_{r-2} + 20\,\Sigma\ {}^{19}C_{r-1}$

$= (20)(19)2^{18} + (20)(2^{19})$

$= 2^{19}(190+20) = 210\,(2^{19}) = 420(2^{18})$

5. Let $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ and $\vec{b} = \hat{j}-\hat{k}$. If $\vec{c}$ is a vector such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}\,.\,\vec{c} = 3$, then $\vec{a}.(\vec{b}\times\vec{c})$ is equal to :

माना $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ तथा $\vec{b} = \hat{j}-\hat{k}$ है। यदि एक सदिश $\vec{c}$ इस प्रकार है कि $\vec{a} \times \vec{c} = \vec{b}$ तथा $\vec{a}\,.\,\vec{c} = 3$ है, तो $\vec{a}.(\vec{b}\times\vec{c})$ बराबर है :

(1) – 6 (2) 6 (3) 2 (4) – 2

Question ID : 86435120066
Option 1 ID : 86435166797
Option 2 ID : 86435166798
Option 3 ID : 86435166796
Option 4 ID : 86435166795

Ans. Official Answer NTA (4)

Sol. $\vec{a}\times\vec{c}=\vec{b}$

cross with $\vec{a}$

$\vec{a}\times(\vec{a}\times\vec{c})=\vec{a}\times\vec{b}$

$3\vec{a}-3\vec{c}=\vec{a}\times\vec{b}$

$3\vec{c}=3\vec{a}-\vec{a}\times\vec{b}$

dot with $\vec{a}\times\vec{b}$

$3(\vec{a}\times\vec{b}).\vec{c}=0-(\vec{a}\times\vec{b})^2$

$3(\vec{a}\times\vec{b}).\vec{c}=-\ 6$

$(\vec{a}\times\vec{b}).\vec{c}=-2=\vec{a}.(\vec{b}\times\vec{c})$

$\vec{a}\times\vec{b}=-2\hat{i}+\hat{j}+\hat{k}$

$|\vec{a}\times\vec{b}|=\sqrt{6}$

6. A plane P contains the line $x+2y+3z+1=0=x-y-z-6$, and is perpendicular to the plane $-2x+y+z+8=0$. Then which of the following points lies on P ?

एक समतल P में रेखा $x+2y+3z+1=0=x-y-z-6$ स्थित है तथा P, समतल $-2x+y+z+8=0$ के लम्बवत् है तो निम्न में से कौनसा बिन्दु समतल P पर है ?

(1) (2, – 1, 1) (2) (1, 0, 1) (3) (–1, 1, 2) (4) (0, 1, 1)

Question ID : 86435120064
Option 1 ID : 86435166787
Option 2 ID : 86435166789
Option 3 ID : 86435166790
Option 4 ID : 86435166788

Ans. Official Answer NTA (4)

Sol. Equation of required plane

$x + 2y + 3z + 1 + \lambda\ (x - y - z - 6) = 0$

It is perpendicular to $-2x + y + z + 8 = 0$

$(1 + \lambda, 2 - \lambda, 3 - \lambda).\ (-2, 1, 1) = 0$

$-2 - 2\lambda + 2 - \lambda + 3 - \lambda = 0$

$\lambda = \frac{3}{4}$

$x + 2y + 3z + 1 + \frac{3}{4}(x - y - z - 6) = 0$

$7x + 5y + 9z - 14 = 0$

(0, 1, 1) lies in this plane

7. $A = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}$, $i = \sqrt{-1}$ and $Q = A^T BA$, then the inverse of the matrix $A\,Q^{2021}\,A^T$ is equal to :

यदि $A = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}$, $i = \sqrt{-1}$ तथा $Q = A^T BA$ है, तो आव्यूह $A\,Q^{2021}\,A^T$ का व्युत्क्रम बराबर है–

(1) $\begin{pmatrix} \frac{1}{\sqrt{5}} & -2021 \\ 2021 & \frac{1}{\sqrt{5}} \end{pmatrix}$ (2) $\begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$ (3) $\begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$ (4) $\begin{pmatrix} 1 & -2021i \\ 0 & 1 \end{pmatrix}$

Question ID : 86435120052
Option 1 ID : 86435166740
Option 2 ID : 86435166739
Option 3 ID : 86435166742
Option 4 ID : 86435166741

Ans. Official Answer NTA (2)

Sol. $A = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$; $AA^T = I$

$Q = A^T BA$

$Q^2 = A^T BAA^T BA = A^T B^2 A$

||ly $Q^{2021} = A^T\ B^{2021} A$

Let $P = A\,Q^{2021}\,A^T$

$= AA^T B^{2021} AA^T$

$P = B^{2021}$

$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix} = I + C$

$C^2 = O$

$B^{2021} = (I + C)^{2021} = I + 2021\ C$

$P = B^{2021} = \begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$; $|P| = 1$

$P^{-1} = \dfrac{\text{adj}(P)}{|P|} = \begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$

8. Out of all the patients in a hospital 89% are found to be suffering from heart ailment and 98% are suffering from lungs infection. If K% of them are suffering from both ailments, then K can not belong to the set :

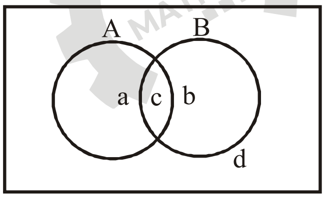
एक अस्पताल के सभी मरीजों में से 89% दिल की बीमारी से ग्रसित पाये गये तथा 98% के फेफड़े संक्रमित पाये गये। यदि K% दोनों बीमारियों से ग्रसित हैं, तो निम्न में से किस समुच्चय में K नहीं हो सकता?

(1) {84, 87, 90, 93} (2) {79, 81, 83, 85} (3) {84, 86, 88, 90} (4) {80, 83, 86, 89}

Question ID : 86435120050
Option 1 ID : 86435166734
Option 2 ID : 86435166733
Option 3 ID : 86435166731
Option 4 ID : 86435166732

Ans. Official Answer NTA (2)

Sol.



$A \equiv$ Heart ailment ; $B \equiv$ lung ailment

$a + b + c + d = 100$ ....(1)

$a + c = 89$ ....(2)

$b + c = 98$ ....(3)

$a + b + 2c = 187$

$a + b + c \leq 100$

$\Rightarrow c \geq 87$

9. If a line along a chord of the circle $4x^2 + 4y^2 + 120x + 675 = 0$, passes through the point (–30, 0) and is tangent to the parabola $y^2 = 30x$, then the length of this chord is :

यदि वृत्त $4x^2 + 4y^2 + 120x + 675 = 0$ की एक जीवा के अनुदिश एक रेखा बिन्दु (–30, 0) से होकर जाती है तथा परवलय $y^2 = 30x$ की स्पर्श रेखा है, तो इस जीवा की लम्बाई है :

(1) 7 (2) 5 (3) $3\sqrt{5}$ (4) $5\sqrt{3}$

Question ID : 86435120062
Option 1 ID : 86435166779
Option 2 ID : 86435166781
Option 3 ID : 86435166780
Option 4 ID : 86435166782

Ans. Official Answer NTA (3)

Sol. Let line y = m (x + 30)

This is tangent to $y^2 = 30x$

apply COT

$c = \frac{a}{m}$

$30\,m = \frac{\frac{30}{4}}{m} \Rightarrow m^2 = \frac{1}{4} \Rightarrow m = \frac{1}{2}$ or $m = \frac{1}{2}$

$2y = x + 30$

length of chord $= 2\sqrt{r^2 - p^2}$ $\qquad r = \frac{15}{2}$

$= 2\sqrt{\left(\frac{15}{2}\right)^2 - \left(\frac{15}{\sqrt{5}}\right)^2}$ $\qquad p = \left|\frac{30-15}{\sqrt{5}}\right| = \frac{15}{\sqrt{5}}$

$= 3\sqrt{5}$

10. The value of $\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \left(\left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x-1}{x+1}\right)^2 - 2\right)^{\frac{1}{2}} dx$ is :

$\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \left(\left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x-1}{x+1}\right)^2 - 2\right)^{\frac{1}{2}} dx$ का मान है :

(1) $4\log_e(3+2\sqrt{2})$ (2) $2\log_e 16$ (3) $\log_e 4$ (4) $\log_e 16$

Question ID : 86435120058
Option 1 ID : 86435166763
Option 2 ID : 86435166766
Option 3 ID : 86435166764
Option 4 ID : 86435166765

Ans. Official Answer NTA (4)

Sol. $I = \int_{\frac{-1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} f(x)\,dx$

$$f(x) = \left(\left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x-1}{x+1}\right)^2 - 2\right)^{\frac{1}{2}}.$$

$$= \sqrt{\left(\frac{x+1}{x-1} - \frac{x-1}{x+1}\right)^2}$$

$$f(x) = \left|\frac{x+1}{x-1} - \frac{x-1}{x+1}\right|$$

$$f(x) = f(-x)$$

$$I = \int_{\frac{-1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} f(x)\,dx$$

$$= 2\int_{0}^{\frac{1}{\sqrt{2}}} f(x)\,dx$$

$$= 2\int_{0}^{\frac{1}{\sqrt{2}}} \left|\frac{x+1}{x-1} - \frac{x-1}{x+1}\right| dx$$

$$= 2\int_{0}^{\frac{1}{\sqrt{2}}} \left(\frac{x-1}{1+x} - \frac{x+1}{x-1}\right) dx$$

$$= 2\int_{0}^{\frac{1}{\sqrt{2}}} \left(1 - \frac{2}{1+x}\right) - \left(1 + \frac{2}{x-1}\right) dx$$

$$= 2\left[-2\ln|1+x| - 2\ln|1-x|\right]_0^{\frac{1}{\sqrt{2}}}$$

$= -4\ln\left(1+\frac{1}{\sqrt{2}}\right) - 4\ln\left(1-\frac{1}{\sqrt{2}}\right)$

$= -4\ln\left(1-\frac{1}{2}\right) = -4\ln\left(\frac{1}{2}\right) = 4\ln 2$

$= \ln 16$

11. If the sum of an infinite GP a, ar, ar^2, ar^3, ... is 15 and the sum of the squares of its each term is 150, then the sum of ar^2, ar^4, ar^6, ... is

एक अनंत GP a, ar, ar^2, ar^3, ... का योग 15 है तथा इसके प्रत्येक पद का वर्ग करने पर योग 150 है, तो ar^2, ar^4, ar^6, ... का योग है :

(1) $\frac{25}{2}$ (2) $\frac{1}{2}$ (3) $\frac{5}{2}$ (4) $\frac{9}{2}$

Question ID : 86435120056
Option 1 ID : 86435166758
Option 2 ID : 86435166756
Option 3 ID : 86435166755
Option 4 ID : 86435166757

Ans. Official Answer NTA (2)

Sol. $\frac{a}{1-r} = 15$ $\frac{a^2}{1-r^2} = 150$

$\frac{(15)^2(1-r)^2}{1-r^2} = 150$

$\frac{1-r}{1+r} = \frac{2}{3}$

$3 - 3r = 212r$

$r = \frac{1}{5}$

$\Rightarrow a = 15(1-r) = 12$

$S = ar^2 + ar^4 + ar^6 + ...\infty$

$= \frac{ar^2}{1-r^2} = \frac{12\left(\frac{1}{5}\right)^2}{1-\left(\frac{1}{5}\right)^2} = \frac{12}{24} = \frac{1}{2}$

12. The equation $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{4}$ represents a circle with :

(1) centre at (0, 0) and radius $\sqrt{2}$

(2) centre at (0, – 1) and radius $\sqrt{2}$

(3) centre at (0, 1) and radius 2

(4) centre at (0, 1) and radius $\sqrt{2}$

समीकरण $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{4}$ एक वृत्त को निरूपित करता है जिसका :

(1) केन्द्र (0, 0) है तथा त्रिज्या $\sqrt{2}$ है

(2) केन्द्र (0, – 1) है तथा त्रिज्या $\sqrt{2}$

(3) केन्द्र (0, 1) है तथा त्रिज्या 2 है

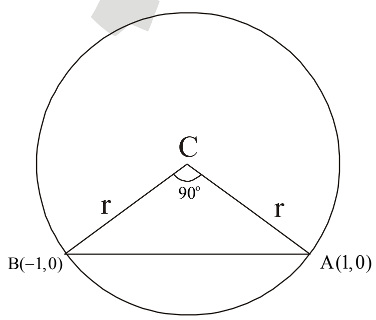
(4) केन्द्र (0, 1) है तथा त्रिज्या $\sqrt{2}$ है

Question ID : 86435120051
Option 1 ID : 86435166735
Option 2 ID : 86435166738
Option 3 ID : 86435166736
Option 4 ID : 86435166737

Ans. Official Answer NTA (4)

Sol. $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{4}$

Chord AB will subtend 90^o at center.



$2r^2=(AB)^2 \Rightarrow \quad r=\sqrt{2}$

Center = (0, 1)

13. On the ellipse $\frac{x^2}{8}+\frac{y^2}{4}=1$ let P be a point in the second quadrant such that the tangent at P to ellipse is perpendicular to the line x + 2y = 0. Let S and S' be the foci of the ellipse and e be its eccentricity. If A is the area of the triangle SPS' then, the value of $(5 - e^2)$. A is :

माना दीर्घवृत्त $\frac{x^2}{8}+\frac{y^2}{4}=1$ पर दूसरे चतुर्थांश में एक बिन्दु P इस प्रकार है कि P पर दीर्घवृत्त की स्पर्श रेखा, रेखा x + 2y = 0 के लम्बवत् है। माना दीर्घवृत्त की नाभियाँ S तथा S' है तथा इसकी उत्केन्द्रता e है। यदि त्रिभुज SPS' का क्षेत्रफल A है, तो A $(5 - e^2)$ का मान है :

(1) 24

(2) 14

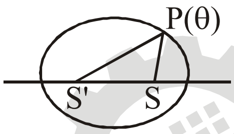
(3) 12

(4) 6

Question ID : 86435120063
Option 1 ID : 86435166783
Option 2 ID : 86435166784
Option 3 ID : 86435166785
Option 4 ID : 86435166786

Ans. Official Answer NTA (4)

Sol.



E $\frac{x^2}{8}+\frac{y^2}{4}=1$

$e^2 = 1-\frac{b^2}{a^2} = 1-\frac{4}{8} = \frac{1}{2}$

$P(2\sqrt{2}\cos\theta, 2\sin\theta)$

$A = \left|\frac{1}{2}(2ae)(b\sin\theta)\right|$

$= |abe\sin\theta|$

$= |4\sin\theta|$

Tangent at P

$\frac{x\cos\theta}{2\sqrt{2}}+\frac{y\sin\theta}{2}=1$

slope of tangent = 2

$$-\frac{\cos\theta}{\sin\theta\sqrt{2}} = 2$$

$$\cot\theta = -2\sqrt{2}$$

$$\Rightarrow |\sin\theta| = \frac{1}{3}$$

$$A = |4\sin\theta| = \frac{4}{3}$$

$$(5-e^2)A = \left(5-\frac{1}{2}\right)\frac{4}{3}$$

$$= \frac{9}{2} + \frac{4}{3} = 6$$

14. Let $\theta \in \left(0, \frac{\pi}{2}\right)$. If the system of linear equations.

$$(1+\cos^2\theta)x + \sin^2\theta\, y + 4\sin 3\theta\, z = 0$$
$$\cos^2\theta\, x + (1+\sin^2\theta)\, y + 4\sin 3\theta\, z = 0$$
$$\cos^2\theta\, x + \sin^2\theta\, y + (1+4\sin 3\theta)z = 0$$

माना $\theta \in \left(0, \frac{\pi}{2}\right)$ है। यदि रैखिक समीकरण निकाय

$$(1+\cos^2\theta)x + \sin^2\theta\, y + 4\sin 3\theta\, z = 0$$
$$\cos^2\theta\, x + (1+\sin^2\theta)\, y + 4\sin 3\theta\, z = 0$$
$$\cos^2\theta\, x + \sin^2\theta\, y + (1+4\sin 3\theta)z = 0$$

का अतुल्छ हल है, तो θ का मान है :

(1) $\frac{\pi}{18}$

(2) $\frac{7\pi}{18}$

(3) $\frac{4\pi}{9}$

(4) $\frac{5\pi}{18}$

Question ID : 86435120053
Option 1 ID : 86435166743

Option 2 ID : 86435166744

Option 3 ID : 86435166746

Option 4 ID : 86435166745

Ans. Official Answer NTA (2)

Sol. $D = 0$

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} 2+4\cos 3\theta & \sin^2\theta & 4\sin 3\theta \\ 2+4\sin 3\theta & 1+\sin^2\theta & 4\sin 3\theta \\ 2+4\sin 3\theta & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$$

$\Rightarrow 2 + 4\sin 3\theta = 0$

$$\sin 3\theta = \frac{-1}{2}$$

$$3\theta = \frac{7\pi}{6}$$

$$\theta = \frac{7\pi}{18}$$

15. Let A and B be independent events such that P(A) = p, P(B) = 2p. The largest value of p, for which P (exactly one of A, B occurs) $= \frac{5}{9}$, is :

माना स्वतंत्र घटनाओं A तथा B के लिए P(A) = p तथा P(B) = 2p है तो p का अधिकतम मान, जिसके लिए P (A तथा B में से ठीक एक घटित होती है) $= \frac{5}{9}$ है, है :

(1) $\frac{2}{9}$

(2) $\frac{1}{3}$

(3) $\frac{5}{12}$

(4) $\frac{4}{9}$

Question ID : 86435120068
Option 1 ID : 86435166806
Option 2 ID : 86435166803
Option 3 ID : 86435166805
Option 4 ID : 86435166804

Ans. Official Answer NTA (3)

Sol. P (Exactly one of A or B occurs) $= \frac{5}{9}$

$$P(A\overline{B}) + P(\overline{A}B) = \frac{5}{9}$$

$$p(1-2p) + 2p(1-p) = \frac{5}{9}$$

$$3p - 4p^2 = \frac{5}{9}$$

$$36p^2 - 27p + 5 = 0$$

$$(12p-5)(3p-1) = 0$$

$$p = \frac{5}{12} \quad \text{or} \quad p = \frac{1}{3}$$

16. The sum of the series $\frac{1}{x+1} + \frac{2}{x^2+1} + \frac{2^2}{x^4+1} + ... + \frac{2^{100}}{x^{2^{100}}+1}$ when $x = 2$ is :

श्रेणी $\frac{1}{x+1} + \frac{2}{x^2+1} + \frac{2^2}{x^4+1} + ... + \frac{2^{100}}{x^{2^{100}}+1}$ का योग, जब $x = 2$ है, है :

(1) $1 + \frac{2^{101}}{4^{101}-1}$

(2) $1 - \frac{2^{101}}{4^{101}-1}$

(3) $1 + \frac{2^{100}}{4^{101}-1}$

(4) $1 - \frac{2^{100}}{4^{101}-1}$

Question ID : 86435120054
Option 1 ID : 86435166749

Option 2 ID : 86435166747
Option 3 ID : 86435166750
Option 4 ID : 86435166748

Ans. Official Answer NTA (2)

Sol. $S = \frac{1}{x+1} + \frac{2}{x^2+1} + \frac{2^2}{x^4+1} + ...$

$S - \frac{1}{x-1} = \frac{1}{x+1} - \frac{1}{x-1} + \frac{2}{x^2+1} + ...$

$S - \frac{1}{x-1} = \frac{2}{x^2+1} - \frac{2}{x^2-1} + \frac{2}{x^4+1}$

$S - \frac{1}{x-1} = -\frac{2^{101}}{x^{2^{101}}-1}$

put x = 2

$S = 1 - \frac{2^{101}}{2^{2^{101}}-1}$ Bonus

17. The sum of solutions of the equations $\frac{\cos x}{1+\sin x} = |\tan 2x|,\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$ is :

समीकरण $\frac{\cos x}{1+\sin x} = |\tan 2x|,\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$ के हलों का योग है :

(1) $-\frac{11\pi}{30}$

(2) $-\frac{7\pi}{30}$

(3) $-\frac{\pi}{15}$

(4) $\frac{\pi}{10}$

Question ID : 86435120060
Option 1 ID : 86435166772
Option 2 ID : 86435166773
Option 3 ID : 86435166774
Option 4 ID : 86435166771

Ans. Official Answer NTA (1)

Sol. $\frac{\cos x}{1+\sin x} = |\tan 2x|$

$$\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)=|\tan 2x|$$

C–1 $\tan 2x \geq 0$

$$\tan 2x = \tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$$

$$2x = n\pi + \frac{\pi}{4}-\frac{x}{2}$$

$$\frac{5x}{2} = n\pi + \frac{\pi}{4}$$

$$x = \frac{2n\pi}{5}+\frac{\pi}{10} = \frac{(4n+1)\pi}{10}$$

$$\Rightarrow x = \frac{\pi}{10} \quad , \quad -\frac{3\pi}{10}$$

C–2 $\tan 2x < 0 \quad \tan 2x = -\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$

$$\tan 2x = \tan\left(\frac{x}{2}-\frac{\pi}{4}\right)$$

$$2x = n\pi + \frac{x}{2}-\frac{\pi}{4}$$

$$\frac{3x}{2} = n\pi - \frac{\pi}{4} = \frac{(4n-1)\pi}{4}$$

$$x = \frac{(4n-1)\pi}{6}$$

$$x = \frac{-\pi}{6}$$

$$\text{Sum of solution} = \frac{\pi}{10}-\frac{3\pi}{10}-\frac{\pi}{6} = -\frac{11\pi}{30}$$

18. If the truth value of the Boolean expression $((p\vee q)\wedge(q\to r)\wedge(\sim r))\to(p\wedge q)$ is false, then the truth values of the statements p, q, r respectively can be :

यदि बूलीय व्यंजक $((p\vee q)\wedge(q\to r)\wedge(\sim r))\to(p\wedge q)$ का सत्य मान असत्य है, तो कथन p, q, r के सत्यमान क्रमशः है :

(1) F T F

(2) T F T

(3) F F T

(4) T F F

Question ID : 86435120067
Option 1 ID : 86435166801
Option 2 ID : 86435166799
Option 3 ID : 86435166800
Option 4 ID : 86435166802

Ans. Official Answer NTA (4)

Sol. $E \equiv \left((p \vee q) \wedge (q \to r) \wedge (\sim r)\right) \to (p \wedge q)$

E is False

$\Rightarrow (p \wedge q)$ is False

$(p \vee q) \wedge (q \to r) \wedge (\sim r)$ is True

$\Rightarrow \sim r$ is True $\Rightarrow$ r is False

$(q \to r)$ is True $\Rightarrow$ q is False

$(p \vee q)$ is True $\Rightarrow$ p is True

19. Let $y = y(x)$ be a solution curve of the differential equation $(y+1)\tan^2 x\, dx + \tan x\, dy + y dx = 0$, $x \in \left(0, \frac{\pi}{2}\right)$.

If $\lim\limits_{x \to 0+} xy(x) = 1$, then the value of $y\left(\frac{\pi}{4}\right)$ is :

माना अवकल समीकरण $(y+1)\tan^2 x\, dx + \tan x\, dy + y dx = 0$, $x \in \left(0, \frac{\pi}{2}\right)$ का हल $y = y(x)$ है। यदि $\lim\limits_{x \to 0+} xy(x) = 1$ है, तो $y\left(\frac{\pi}{4}\right)$ का मान है :

(1) $\frac{\pi}{4}$

(2) $-\frac{\pi}{4}$

(3) $\frac{\pi}{4} + 1$

(4) $\frac{\pi}{4} - 1$

Question ID : 86435120065
Option 1 ID : 86435166794
Option 2 ID : 86435166793
Option 3 ID : 86435166792
Option 4 ID : 86435166791

Ans. Official Answer NTA (1)

Sol. $\tan x \frac{dy}{dx} + y + (y+1)\tan^2 x = 0$

$\tan x \frac{dy}{dx} + y\sec^2 x = -\tan^2 x$

$\int d(y\tan x) = -\int \tan^2 x\, dx$

$y\tan x = x - \tan x + C$

$y = x\cot x - 1 + C\cot x$

$\lim_{x\to 0^+} xy = \lim_{x\to 0}(x^2\cot x - x + xC\cot x) = 1$

$y = x\cot x - 1 + \cot x$

Put $x = \frac{\pi}{4}$

$y\left(\frac{\pi}{4}\right) = \frac{\pi}{4} - 1 + 1 = \frac{\pi}{4}$

20. Let ABC be a triangle with A (– 3, 1) and $\angle ACB = \theta, 0 < \theta < \frac{\pi}{2}$. If the equation of the median through B is 2x + y – 3 = 0 and the equation of angle bisector of C is 7x – 4y – 1 = 0, then $\tan\theta$ is equal to :

माना ABC एक त्रिभुज है जिसमें A (– 3, 1) तथा $\angle ACB = \theta, 0 < \theta < \frac{\pi}{2}$ है। यदि B से माध्यिका रेखा का समीकरण 2x + y – 3 = 0 है तथा कोण C की समद्विभाजक रेखा का समीकरण 7x – 4y – 1 = 0 है, तो $\tan\theta$ बराबर है :

(1) $\frac{3}{4}$

(2) $\frac{1}{2}$

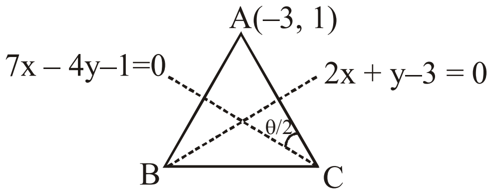
(3) $\frac{4}{3}$

(4) 2

Question ID : 86435120061

Option 1 ID : 86435166776
Option 2 ID : 86435166775
Option 3 ID : 86435166777
Option 4 ID : 86435166778

Ans. Official Answer NTA (3)

Sol.


$C\left(h, \frac{7h-1}{4}\right)$

Mid point of AC will lie on median.

$= \text{Mid point } \left(\frac{h-3}{2}, \frac{7h+3}{8}\right)$

$2\left(\frac{h-3}{2}\right) + \frac{7h+3}{8} - 3 = 0$

$8h - 24 + 7h + 3 - 24 = 0$

$15h = 45 \quad \Rightarrow h = 3$

$\Rightarrow C(3, 5)$

$m_{AC} = \frac{5-1}{3+3} = \frac{2}{3}$

$\tan\frac{\theta}{2} = \left|\frac{\frac{7}{4} - \frac{2}{3}}{1 + \frac{7}{6}}\right| = \frac{1}{2}$

$\tan\theta = \frac{2\tan\frac{\theta}{2}}{1 - \tan^2\frac{\theta}{2}} = \frac{2\left(\frac{1}{2}\right)}{1 - \frac{1}{4}} = \frac{4}{3}$

SECTION - B

1. The number of three – digit even numbers, formed by the digits 0, 1, 3, 4, 6, 7 if the repetition of digits is not allowed, is ________.

अंको 0, 1, 3, 4, 6, 7 से बनने वाली तीन अंको की सम संख्याओं, जबकि अंको की पुनरावृत्ति की अनुमति नहीं है, की संख्या है________ ।

Question ID : 86435120070

Ans. Official Answer NTA (52)

Sol. C–1 ending with 0

$--0 \quad 4\times 5 = 20$

C–2 Ending with non–zero digit

$= 4\times 4\times 2 = 32$

Ans = 20 + 32 = 52

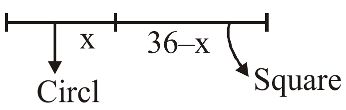
2. A wire of length 36 m is cut into two pieces, one of the pieces is bent to form a square and the other is bent to form a circle. If the sum of the areas of the two figures is minimum, and the circumference of the circle is k (meter), then $\left(\frac{4}{\pi}+1\right)k$ is equal to _________.

36 m की एक तार दो भागों में काटा गया है। एक भाग को मोड़कर एक वर्ग बनाया गया है तथा दूसरे भाग को मोड़कर एक वृत्त बनाया गया है। यदि दोनों आकृतियों के क्षेत्रफल का योग निम्नतम है तथा वृत्त की परिधि k है, तो $\left(\frac{4}{\pi}+1\right)k$ बराबर है __________ ।

Question ID : 86435120073

Ans. Official Answer NTA (36)

Sol.



$2\pi r = x \qquad 36 - x = 4a$

$r = \frac{x}{2\pi} \qquad a = 9 - \frac{x}{4}$

$S = \pi r^2 + a^2$

$S = \pi\left(\frac{x}{2\pi}\right)^2 + \left(9 - \frac{x}{4}\right)^2$

$$S = \frac{x^2}{4\pi} + \left(9 - \frac{x}{4}\right)^2$$

$$\frac{ds}{dx} = \frac{x}{2\pi} - \frac{1}{2}\left(9 - \frac{x}{4}\right) = 0$$

$$\frac{x}{2\pi} + \frac{x}{8} = \frac{9}{2}$$

$$\frac{x}{8}\left(\frac{4}{\pi} + 1\right) = \frac{9}{2}$$

Ans. $= \left(\frac{4}{\pi} + 1\right)k = \left(\frac{4}{\pi} + 1\right)x$

$$= \left(\frac{9}{2}\right)8 = 36$$

3. Let the line L be the projection of the line

$$\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{2}$$

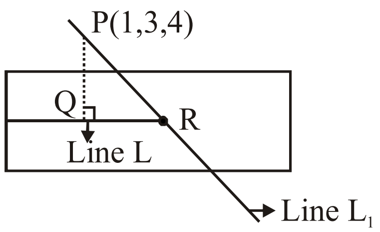
in the plane $x - 2y - z = 3$. If d is the distance of the point (0, 0, 6) from L, then d^2 is equal to ________.

माना रेखा $\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{2}$ का समतल $x - 2y - z = 3$ में प्रक्षेप रेखा L है। यदि बिन्दु (0, 0, 6) की L से दूरी d है, तो d^2 बराबर है________।

Question ID : 86435120077

Ans. Official Answer NTA (26)

Sol.



R is POI of line L_1 & Plane

$R(1 + 2\gamma, 3 + \gamma, 4 + 2\gamma)$

R will lie on $x - 2y - z = 3$

$1 + 2\gamma - 2(3 + \gamma) - (4 + 2\gamma) = 3$

$2\gamma = 1 - 6 - 4 - 3 \quad \Rightarrow \gamma = -6$

$R(-11,-3,-8)$

$Q \equiv$ Foot of perpendicular from P on P line

$Q(1+\mu, 3-2\mu, 4-\mu)$

$1+\mu-2(3-2\mu)-(4-\mu)=3$

$6\mu = 6+4+3-1$

$\mu = 2$ $\qquad\qquad \overrightarrow{QR} = (-14,-2,-10)$

$Q(3,-1,2)$

Equation of line L

$\vec{r} = (3,-1,2) + t(7,1,5)$

$S(0,0,6)$

distance of S from line $= \dfrac{|\overrightarrow{SQ} \times \overrightarrow{QR}|}{|\overrightarrow{QR}|}$

$= \dfrac{|\hat{i} + 43\hat{j} - 10\hat{k}|}{|7\hat{i} + \hat{j} + 5\hat{k}|}$

$d = \sqrt{26}$

$\Rightarrow d^2 = 26$

4. The sum of all integral values of $k(k \neq 0)$ for which the equation $\dfrac{2}{x-1} - \dfrac{1}{x-2} = \dfrac{2}{k}$ in x has no real roots, is __________.

$k(k \neq 0)$ के सभी पूर्णांक मानों, जिनके लिए x समीकरण $\dfrac{2}{x-1} - \dfrac{1}{x-2} = \dfrac{2}{k}$ का कोई वास्तविक मूल नहीं है, का योग है __________ ।

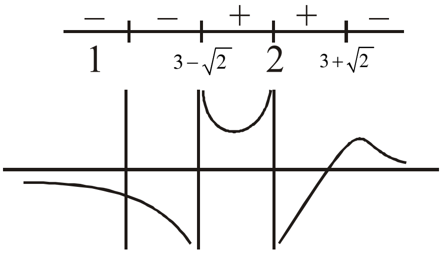
Question ID : 86435120072

Ans. Official Answer NTA (66)

Sol. $\dfrac{2}{x-1} - \dfrac{1}{x-2} = \dfrac{2}{k}$

$f(x) = \dfrac{2}{x-1} - \dfrac{1}{x-2}$

$f'(x) = \dfrac{-(x^2-6x+7)}{(x-1)^2(x-6)^2}$



$$f(3+\sqrt{2}) < \frac{2}{k} < f(3-\sqrt{2})$$

$$\frac{\sqrt{2}-1}{\sqrt{2}+1} < \frac{2}{k} < \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$\frac{\sqrt{2}-1}{\sqrt{2}+1} < \frac{k}{2} < \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$3-2\sqrt{2} < \frac{k}{2} < 3+2\sqrt{2}$$

$$6-4\sqrt{2} < k < 6+4\sqrt{2}$$

$$0.344 < k < 11.656$$

$$k \in \{1, 2, 3, ...11\}$$

Sum = 66

5. The area of the region $S = \{(x, y) : 3x^2 \le 4y \le 6x + 24\}$ is __________.

क्षेत्र $S = \{(x, y) : 3x^2 \le 4y \le 6x + 24\}$ का क्षेत्रफल है__________ ।

Question ID : 86435120074

Ans. Official Answer NTA (27)

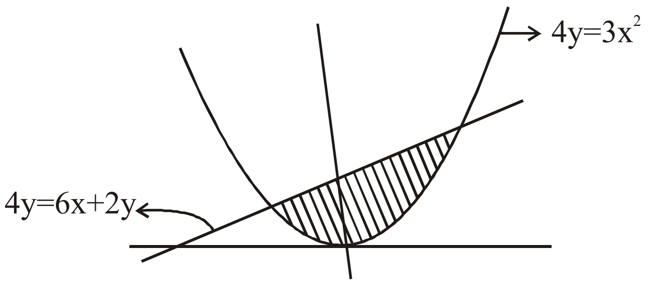
Sol. POM of intersection

$3x^2 = 6x + 24$

$\Rightarrow x = 4, x = -2$

$$\text{Area} = \int_{-2}^{4}\left(\frac{6x+24}{4} - \frac{3x^2}{4}\right)dx$$

$= 27$



6. Let $a, b \in R, b \neq 0$. Define a function

$$f(x)=\begin{cases} a\sin\frac{\pi}{2}(x-1), & \text{for } x\le 0 \\ \frac{\tan 2x-\sin 2x}{bx^3}, & \text{for } x>0. \end{cases}$$

If *f* is continuous at x = 0, then 10 – ab is equal to _________.

माना $a, b \in R, b \neq 0$ है। एक फलन

$$f(x)=\begin{cases} a\sin\frac{\pi}{2}(x-1), & \text{के लिए } x\le 0 \\ \frac{\tan 2x-\sin 2x}{bx^3}, & \text{के लिए } x>0. \end{cases}$$

द्वारा परिभाषित है। यदि x = 0 पर f संतत है, तो 10 – ab बराबर है_________।

Question ID : 86435120071

Ans. Official Answer NTA (14)

Sol. f(0) = –a = LHL

$$RHL=\lim_{x\to 0}\frac{\tan 2x-\sin 2x}{bx^3}=\frac{4}{b}$$

$$-a=\frac{4}{b}\Rightarrow ab=-4$$

10 – ab = 10 + 4 = 14

7. The locus of a point, which moves such that the sum of squares of its distances from the points (0, 0), (1, 0), (0, 1), (1, 1) is 18 units, is a circle of diameter d. Then d^2 is equal to

एक बिन्दु, जो इस प्रकार चलता है कि इसकी बिन्दुओं (0, 0), (1, 0), (0, 1), (1, 1) के दूरियों के वर्गों का योग 18 इकाई है, का बिन्दुपथ d व्यास का एक वृत्त है तो d^2 बराबर है_______।

Question ID : 86435120079

Ans. Official Answer NTA (16)

Sol. P(x, y)

$(PA)^2 + (PB)^2 + (PC)^2 + (PD)^2 = 18$

$x^2 + y^2 + (x-1)^2 + y^2 + x^2 + (y-1)^2 + (x-1)^2 + (y-1)^2 = 18$

$4x^2 + 4y^2 - 4x - 4y - 14 = 0$

$x^2 - y^2 - x - y - 7/2 = 0$

$$r = \sqrt{\frac{1}{4} + \frac{1}{4} + \frac{7}{2}} = 2$$

$2r = 4 \Rightarrow$ Ans = 16

8. Let $z = \frac{1 - i\sqrt{3}}{2}, i = \sqrt{-1}$. Then the value of $21 + \left(z + \frac{1}{z}\right)^3 + \left(z^2 + \frac{1}{z^2}\right)^3 + \left(z^3 + \frac{1}{z^3}\right)^3 + ... + \left(z^{21} + \frac{1}{z^{21}}\right)^3$ is __________.

माना $z = \frac{1 - i\sqrt{3}}{2}, i = \sqrt{-1}$ है। तो $21 + \left(z + \frac{1}{z}\right)^3 + \left(z^2 + \frac{1}{z^2}\right)^3 + \left(z^3 + \frac{1}{z^3}\right)^3 + ... + \left(z^{21} + \frac{1}{z^{21}}\right)^3$ का मान बराबर है __________ ।

Question ID : 86435120078

Ans. Official Answer NTA (13)

Sol. $z = \frac{1 - i\sqrt{3}}{2} = e^{-i\frac{\pi}{3}}$

$$z^n + \frac{1}{z^n} = e^{-i\frac{n\pi}{3}} + e^{i\frac{n\pi}{3}} = 2\cos\frac{n\pi}{3}$$

$z^n + \frac{1}{z^n}$
- → 2   n = 6K
- → 1   n = 6K+1
- → -1   n = 6K + 2
- → -2   n = 6K + 3
- → -1   n = 6K + 4
- → 1   n = 6K + 5

Sum $= 21 + 4(1)^3 + 4(-1)^3 + 4(-2)^3 + 3(-1)^3 + 3(1)^3 + 3(2)^3 = 21 + 1 - 1 - 8 = 13$

9. If y = y(x) is an implicit function of x such that $\log_e(x + y) = 4xy$, then $\frac{d^2y}{dx^2}$ at x = 0 is equal to _______.

यदि y = y(x) का एक अस्पष्ट फलन इस प्रकार है कि $\log_e(x + y) = 4xy$ है, तो x = 0 पर $\frac{d^2y}{dx^2}$ बराबर है_______ ।

Question ID : 86435120075

Ans. Official Answer NTA (40)

Sol. $l\,n(x+y) = 4xy \qquad x = 0 \;\Rightarrow\; y = 1$

$x + y = e^{4xy}$

$$1 + \frac{dy}{dx} = e^{4xy}\left(4y + 4x\frac{dy}{dx}\right) \qquad ....(1)$$

$1 + y_1 = 1(4 + 0)$

$y_1 = 3$

Diff... (1)

$$\frac{d^2y}{dx^2} = e^{4yx}\left(4y + \frac{4xdy}{dx}\right)^2$$

$$+e^{4xy}\left(4\frac{dy}{dx} + 4\frac{dy}{dx} + 4x\frac{d^2y}{dx}\right)$$

Put $x = 0, y = 1, \frac{dy}{dx} = 3$

$y_2 = (4)^2 + (12 + 12 + 0)$

$y_2 = 40$

$$\left.\frac{d^2y}{dx^2}\right)_{x=0} = 40$$

10. If ${}^1P_1 + 2.{}^2P_2 + 3.{}^3P_3 + ...+15.{}^{15}P_{15} = {}^q p_r - s,\ 0 \le s \le 1$, then ${}^{q+s}C_{r-s}$ is equal to __________.

यदि ${}^1P_1 + 2.{}^2P_2 + 3.{}^3P_3 + ...+15.{}^{15}P_{15} = {}^q p_r - s,\ 0 \le s \le 1$ है, तो ${}^{q+s}C_{r-s}$ बराबर है__________ ।

Question ID : 86435120076

Ans. Official Answer NTA (136)

Sol. $\sum_{k=1}^{15} k\,{}^kP_k = {}^qP_r - S$

$$\text{LHS} = \sum_{k=1}^{15} k(k!)$$

$$= \sum_{k=1}^{15}\left((k+1)! - k!\right)$$

$= \lfloor 16 - 1 = {}^qp_r - s$

$\Rightarrow {}^qp_r = \lfloor 16 \;\Rightarrow\; q = r = 16 \qquad s = 1$

$^{q+s}C_{r-s} = {}^{17}C_{15} = 136$