JEE Main August 2021

Question Paper With Text Solution

26 August. | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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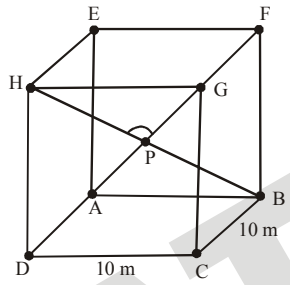
JEE MAIN AUGUST 2021 | 26TH AUGUST SHIFT-2

SECTION – A

1. A hall a square floor of dimension 10 m × 10 m (see the figure) and vertical walls. If the angle GPH between the diagonals AG and BH is $\cos^{-1}\frac{1}{5}$, then the height of the hall (in meters) is:

एक हाल के वर्गाकार फर्श की विमा 10 m × 10 m है तथा इसकी दीवारें ऊर्ध्वाधर हैं (चित्र देखिए) । यदि इसके विकर्णों AG तथा BH के बीच न्यून कोण GPH = $\cos^{-1}\frac{1}{5}$ है, तो हाल की ऊँचाई (मीटर में) है :



(1) $5\sqrt{3}$ (2) 5 (3) $2\sqrt{10}$ (4) $5\sqrt{2}$

Question ID : 86435120239
Option 1 ID : 86435167308
Option 2 ID : 86435167309
Option 3 ID : 86435167310
Option 4 ID : 86435167307

Ans. Official Answer NTA (4)

Sol. Let $A(0,0,0)$

$B(10,0,0)$

$G(10,h,10)$

$H(0,h,10)$

$\overrightarrow{BH} = -10\hat{i} + h\hat{j} + 10\hat{k}$

$\overrightarrow{AG} = 10\hat{i} + h\hat{j} + 10\hat{k}$

$$\cos\theta = \frac{\overrightarrow{AG}.\overrightarrow{BH}}{|\overrightarrow{AG}||\overrightarrow{BH}|} = \frac{h^2}{\sqrt{200+h^2}\sqrt{200+h^2}}$$

$$\frac{1}{5} = \frac{h^2}{200+h^2}$$

$$4h^2 = 200$$

$h^2 = 50$

$h = 5\sqrt{2}$

2. A fair die is tossed until six is obtained on it. Let X be the number of required tosses, then the conditional probability $P(X \geq 5)\ (X > 2)$ is :

एक न्याय पासे को छः प्राप्त होने तक उछाला जाता है। माना पासे को उछालने की आवश्यक संख्या X है, तो सप्रतिबंध प्रायिकता $P(X \geq 5)\ (X > 2)$ है :

(1) $\frac{11}{36}$ (2) $\frac{125}{216}$ (3) $\frac{25}{36}$ (4) $\frac{5}{6}$

Question Type : MCQ
Question ID : 86435120247
Option 1 ID : 86435167339
Option 2 ID : 86435167342
Option 3 ID : 86435167340
Option 4 ID : 86435167341

Ans. Official Answer NTA (3)

Sol. $P(\text{six is obtained}) = \frac{1}{6} = P(A)$

$P(\overline{A}) = \frac{5}{6}$

$$\text{Req. Probability} = \frac{\left(\frac{5}{6}\right)^5 + \left(\frac{5}{6}\right)^5 \frac{1}{6} + ----}{\left(\frac{5}{6}\right)^2 . \frac{1}{6} + \left(\frac{5}{6}\right)^3 + \frac{1}{6} + ----}$$

$$= \frac{25}{36}$$

3. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$. Then $A^{2025} - A^{2020}$ is equal to :

माना $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ है तो $A^{2025} - A^{2020}$ बराबर है :

(1) A^6 (2) $A^5 - A$ (3) A^5 (4) $A^6 - A$

Question Type : MCQ
Question ID : 86435120233
Option 1 ID : 86435167283
Option 2 ID : 86435167284
Option 3 ID : 86435167286
Option 4 ID : 86435167285

Ans. Official Answer NTA (4)

Sol. $A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ $A^6 = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$A^{2025} = \begin{bmatrix} 1 & 0 & 0 \\ 2024 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $A^{2020} = \begin{bmatrix} 1 & 0 & 0 \\ 2019 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$A^{2025} - A^{2020} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = A^6 - A$

4. If the value of the integeral $\int_0^5 \frac{x+[x]}{e^{x-[x]}}\,dx = \alpha e^{-1} + \beta$, where $\alpha, \beta, \in R$, $5\alpha + 6\beta = 0$, and $[x]$ denotes the greatest integer less than or equal to x; then the value of $(\alpha+\beta)^2$ is equal to :

यदि समाकलन $\int_0^5 \frac{x+[x]}{e^{x-[x]}}\,dx = \alpha e^{-1} + \beta$ है, जहाँ $\alpha, \beta, \in R$, $5\alpha + 6\beta = 0$ तथा $[x]$ महत्तम पूर्णांक हैं, तो $\leq x$ हैं, तो $(\alpha+\beta)^2$ का मान बराबर है :

(1) 100 (2) 16 (3) 36 (4) 25

Question Type : MCQ
Question ID : 86435120243
Option 1 ID : 86435167325
Option 2 ID : 86435167326
Option 3 ID : 86435167324
Option 4 ID : 86435167323

Ans. Official Answer NTA (4)

Sol. $\int_0^1 \frac{x}{e^x}dx+\int_1^2 \frac{x+1}{e^{x-1}}dx+\int_2^3 \frac{x+2}{e^{x-2}}+\int_3^4 \frac{x+3}{e^{x-3}}dx+\int_4^5 \frac{x+4}{e^{x-4}}dx$

$x=a+1 \quad x=b+2 \quad x=c+3 \quad x=d+4$

$\int_0^1 \frac{x}{e^x}dx+\int_0^1 \frac{a+2}{e^a}da+\int_0^1 \frac{b+4}{e^b}bd+\int_0^1 \frac{c+6}{e^c}dc+\int_0^1 \frac{d+8}{e^d}ed$

$=\int_0^1 \frac{5x+20}{ex}=5\int_0^1 \frac{x+4}{e^x}dx=-30e^{-1}+25$

$\alpha=-30$

$\beta=25$

$(\alpha+\beta)^2=25$

5. Let P be the plane passing through the point (1, 2, 3) and the line of intersection of the planes $\vec{r}.\left(\hat{i}+\hat{j}+4\hat{k}\right)=16$ and $\vec{r}.\left(-\hat{i}+\hat{j}+\hat{k}\right)=6$. Then which of the following points does **NOT** lie on P ?

माना P एक समतल है जो बिन्दु (1, 2, 3) तथा समतलों $\vec{r}.\left(\hat{i}+\hat{j}+4\hat{k}\right)=16$ और $\vec{r}.\left(-\hat{i}+\hat{j}+\hat{k}\right)=6$ की प्रतिच्छेदन रेखा से होकर जाता है तो निम्न में से कौनसा बिन्दु P पर स्थित **नहीं** है?

(1) (3, 3, 2) (2) (6, – 6, 2) (3) (– 8, 8, 6) (4) (4, 2, 2)

Question Type : MCQ
Question ID : 86435120248
Option 1 ID : 86435167346
Option 2 ID : 86435167344
Option 3 ID : 86435167343
Option 4 ID : 86435167345

Ans. Official Answer NTA (4)

Sol. Equation of plane

$(x+y+4x-16)+\lambda(-x+y+z-6)=0$

$(1,2,3)$

$\lambda=\frac{-1}{2}$

equation of Req. plane $3x + y + 7z = 26$

6. The value of is $2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$:

$2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$ का मान है :

(1) $\frac{1}{4}$ (2) $\frac{1}{8}$ (3) $\frac{1}{8\sqrt{2}}$ (4) $\frac{1}{4\sqrt{2}}$

Question Type : MCQ
Question ID : 86435120249
Option 1 ID : 86435167349
Option 2 ID : 86435167347
Option 3 ID : 86435167348
Option 4 ID : 86435167350

Ans. Official Answer NTA (2)

Sol. $2\sin^2\left(\frac{\pi}{8}\right)\sin^2\left(\frac{2\pi}{8}\right)\sin^2\left(\frac{3\pi}{8}\right)$

$= \sin^2\frac{\pi}{8}.\sin^2\left(\frac{\pi}{2}-\frac{\pi}{8}\right)$

$= \sin^2\frac{\pi}{8}.\cos^2\frac{\pi}{8}$

$= \frac{1}{4}\sin^2\frac{\pi}{4} = \frac{1}{8}$

7. Let [t] denote the greatest integer less than or equal to t. Let $f(x) = x - [x]$, $g(x) = 1 - x + [x]$, and $h(x) = \min\{f(x), g(x)\}$. $x \in [-2, 2]$. Then h is :

(1) Continuous in $[-2, 2]$ but not differentiable at exactly three points in $(-2, 2)$

(2) Not continuous at exactly four points in $[-1, 2]$

(3) Not continuous at exactly three points in $[-1, 2]$

(4) Continuous in $[-2, 2]$ but not differentiable at more than four points in $(-2, 2)$

माना [t] महत्तम पूर्णांक $\leq$ t है। माना $f(x) = x - [x]$, $g(x) = 1 - x + [x]$ तथा $h(x) = \min\{f(x), g(x)\}$, $x \in [-2, 2]$ है, तो h :

(1) $[-2, 2]$ में संतत है परन्तु $(-2, 2)$ में ठीक तीन बिन्दुओं से अधिक पर अवकलनीय नहीं है।

(2) [–1, 2] में ठीक तीन बिन्दुओं पर संतत नहीं है।

(3) [–1, 2] में ठीक चार बिन्दुओं पर संतत नहीं है।

(4) [– 2, 2] में संतत है परन्तु (–2, 2) में चार बिन्दुओं से अधिक पर अवकलनीय नहीं है।

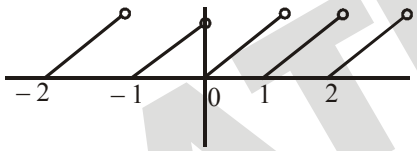
Question Type : MCQ
Question ID : 86435120245
Option 1 ID : 86435167334
Option 2 ID : 86435167331
Option 3 ID : 86435167332
Option 4 ID : 86435167333

Ans. Official Answer NTA (4)

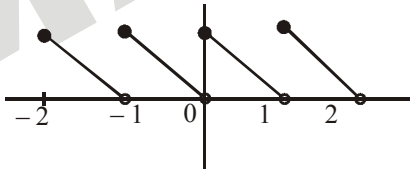
Sol. $f(x) = \{x\}$

$g(x) = 1 - \{x\}$

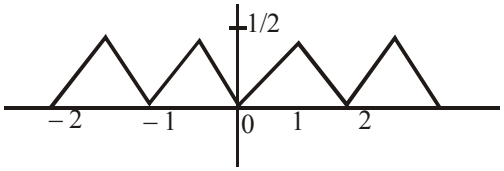
$y = \{x\}$



$g(x) = 1 - \{x\}$



$h(x) = \min\left(f(x), g(x)\right)$



non differentiable at 7 points

8. Two fair dice are thrown. The numbers on them are taken as λ and μ, and a system of linear equations

$x + y + z = 5$

$x + 2y + 3z = \mu$

$x + 3y + \lambda z = 1$

is constructed. If p the probability that the system has a unique solution and q is the probability that the system has no solution, then :

दो न्यास पासे फेंके जाते है। उनमें प्राप्त अंको को λ तथा μ लेकर रैखिक समीकरण निकाय

$x + y + z = 5$

$x + 2y + 3z = \mu$

$x + 3y + \lambda z = 1$

बनाया जाता है। यदि इस निकाय का अद्वितीय हल होने की प्रायिकता p है तथा इस निकाय का कोई भी हल न होने की प्रायिकता q है, तो :

(1) $p = \frac{1}{6}$ and $q = \frac{1}{36}$

(2) $p = \frac{5}{6}$ and $q = \frac{5}{36}$

(3) $p = \frac{5}{6}$ and $q = \frac{1}{36}$

(4) $p = \frac{1}{6}$ and $q = \frac{5}{36}$

Question Type : MCQ
Question ID : 86435120232
Option 1 ID : 86435167279
Option 2 ID : 86435167282
Option 3 ID : 86435167280
Option 4 ID : 86435167281

Ans. Official Answer NTA (2)

Sol. $$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} = \lambda - 5$$

$$D_x = \begin{vmatrix} 1 & 1 & 5 \\ 2 & 3 & 4 \\ 3 & \lambda & 1 \end{vmatrix} = 10\lambda + 3\mu - \lambda\mu - 44$$

$$D_y = \begin{vmatrix} 1 & 1 & 5 \\ 2 & 3 & \mu \\ 1 & \lambda & 1 \end{vmatrix} = 5\lambda + \mu - \lambda\mu - 13$$

$$x^3 + 2y^2 - y^2x = 0$$

$$D_z = \begin{vmatrix} 1 & 1 & 5 \\ 2 & 3 & \mu \\ 1 & 3 & 1 \end{vmatrix} = -2\mu + 6$$

for unique solution

$\lambda \pm 5 \quad \mu \in \{1,2,3,4,5,6\}$

$$P = \frac{5}{6} \times \frac{6}{6} = \frac{5}{6}$$

for no solution

$\lambda = 5 \quad \mu \pm 3$

$$q = \frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$$

9. $\lim\limits_{x \to 2} \left(\sum\limits_{n=1}^{9} \frac{x}{n(n+1)x^2 + 2(2n+1)x + 4} \right)$ is equal to :

$\lim\limits_{x \to 2} \left(\sum\limits_{n=1}^{9} \frac{x}{n(n+1)x^2 + 2(2n+1)x + 4} \right)$ के बराबर है :

(1) $\frac{9}{44}$ (2) $\frac{5}{24}$ (3) $\frac{1}{5}$ (4) $\frac{7}{36}$

Question Type : MCQ
Question ID : 86435120234
Option 1 ID : 86435167289
Option 2 ID : 86435167290
Option 3 ID : 86435167288
Option 4 ID : 86435167287

Ans. Official Answer NTA (1)

Sol. $\lim\limits_{x \to 2} \sum\limits_{x=1}^{9} \left(\frac{x}{x(x+1)x^2 + 2(2x+1)x + 2} \right)$

$\lim\limits_{x \to 2} \sum\limits_{x=1}^{9} \left(\frac{x}{(hx+2)x^2 + (x+1)x + 2} \right)$

$\lim\limits_{x \to 2} \sum\limits_{x=1}^{9} \left(\frac{1}{hx+2} - \frac{1}{(x+1)x+2} \right)$

$\Rightarrow \lim\limits_{x \to 2} \left(\frac{1}{x+2} - \frac{1}{10x+2} \right) = \frac{9}{44}$

10. The locus of the mid points of the chord of the hyperbola $x^2 - y^2 = 4$, which touch the parabola $y^2 = 8x$, is :

अतिपरवलय $x^2 - y^2 = 4$ की उन जीवाओं, जो परवलय $y^2 = 8x$ को स्पर्श करती है, के मध्य बिन्दुओं का बिन्दुपथ है :

(1) $x^3(x-2) = y^2$ (2) $y^3(x-2) = x^2$ (3) $x^2(x-2) = y^3$ (4) $y^2(x-2) = x^3$

Question Type : MCQ
Question ID : 86435120242
Option 1 ID : 86435167319
Option 2 ID : 86435167322
Option 3 ID : 86435167320
Option 4 ID : 86435167321

Ans. Official Answer NTA (4)

Sol. Tangent to $y^2 = 8x$ is $y = mx + \frac{2}{m}$

$y = \left(\frac{h}{k}\right)x + \frac{k^2 - h^2}{k}$ is the equation of chord with mid point (h, k)

Comparing above equations we get

$m = \frac{h}{k}$ and $\frac{2}{m} = \frac{k^2 - h^2}{k}$

$h^3 + 2k^2 - k^2h = 0$

Equation of locus is $x^3 + 2y^2 - y^2x = 0$

11. Consider the two statements :

(S1) : $(p \to q) \vee (\sim q \to p)$ is a tautology.

(S2) : $(p \wedge \sim q) \wedge (\sim q \vee p)$ is a fallacy.

Then :

(1) both (S1) and (S2) are false (2) only (S2) is true

(3) only (S1) is true (4) both (S1) and (S2) are true

Question Type : MCQ
Question ID : 86435120244
Option 1 ID : 86435167329
Option 2 ID : 86435167328
Option 3 ID : 86435167327
Option 4 ID : 86435167330

Ans. Official Answer NTA (4)

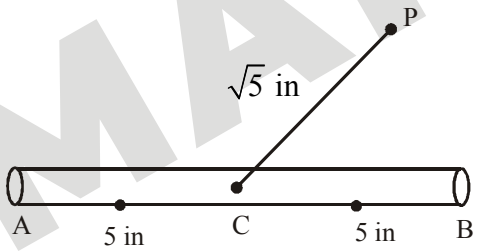
Sol.

p	q	~ p	~ q	p → q	~ q → p	(p → q) ∨ (~ q → p)	p ∧ ~ q	~ p ∨ q	(p ∧ ~ q) ∧ (~ p ∨ q)
T	T	F	F	T	T	T	F	T	F
T	F	F	T	F	T	T	T	F	F
F	T	T	F	T	T	T	F	T	F
F	F	T	T	T	F	T	F	T	F

12. A 10 inches long pencil AB with mid point C and a small eraser P are placed on the horizontal top of a table such that PC = $\sqrt{5}$ inches and $\angle PCB = \tan^{-1}(2)$.

The acute angle through which the pencil must be rotated about C so that the perpendicular distance between eraser and pencil becomes exactly 1 inch is :

10 इंच लम्बी पेन्सिल AB जिसका मध्य बिन्दु C है, तथा एक छोटा सा रबड़ P एक मेज की क्षैतिज ऊपरी सतह पर इस प्रकार रखे हैं कि PC = $\sqrt{5}$ इंच तथा $\angle PCB = \tan^{-1}(2)$ हैं। पेंसिल को C के सापेक्ष निम्न में से किस न्यून कोण तक घुमाया जाए कि रबड़ तथा पेंसिल के बीच लम्बवत् दूरी ठीक 1 इंच हो जाए?

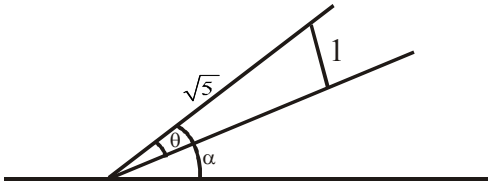


(1) $\tan^{-1}(1)$ (2) $\tan^{-1}\left(\frac{4}{3}\right)$ (3) $\tan^{-1}\left(\frac{3}{4}\right)$ (4) $\tan^{-1}\left(\frac{1}{2}\right)$

Question Type : MCQ
Question ID : 86435120238
Option 1 ID : 86435167305
Option 2 ID : 86435167303
Option 3 ID : 86435167304
Option 4 ID : 86435167306

Ans. Official Answer NTA (3)

Sol.



$$\sin\beta = \frac{1}{\sqrt{5}}$$

$$\Rightarrow \tan\beta = \frac{1}{2}$$

$$\tan\alpha = 2$$

$$\tan(\alpha-\beta) = \frac{2-\frac{1}{2}}{1+1} = \frac{3}{4}$$

$$\alpha-\beta = \tan^{-1}\left(\frac{3}{4}\right)$$

13. The domain of the function $\text{cosec}^{-1}\left(\frac{1+x}{x}\right)$ is :

फलन $\text{cosec}^{-1}\left(\frac{1+x}{x}\right)$ का प्रांत है :

(1) $\left(-\frac{1}{2},\infty\right]-\{0\}$

(2) $\left[-\frac{1}{2},0\right)\cup[0,\infty)$

(3) $\left[-\frac{1}{2},0\right)\cup[1,\infty)$

(4) $\left(-\frac{1}{2},\infty\right)-\{0\}$

Question Type : MCQ
Question ID : 86435120230
Option 1 ID : 86435167271
Option 2 ID : 86435167274
Option 3 ID : 86435167273
Option 4 ID : 86435167272

Ans. Official Answer NTA (1)

Sol. $\frac{1+x}{x} \le -1$ or $\frac{1+x}{x} \ge 1$

$\frac{1+2x}{x} \le 0$ or $\frac{1}{x} \ge 1$

$x \in \left[\frac{-1}{2}0\right)$ or $x \in (0,\infty)$

$x \in \left[-\frac{1}{2}\infty\right)-\{0\}$

14. If $\left(\sqrt{3}+i\right)^{100} = 2^{99}\ (p+iq)$, then p and q are roots of the equation :

यदि $\left(\sqrt{3}+i\right)^{100} = 2^{99}\ (p+iq)$ है, तो p तथा q किस समीकरण के मूल है?

(1) $x^2+\left(\sqrt{3}-1\right)x-\sqrt{3}=0$ (2) $x^2+\left(\sqrt{3}+1\right)x+\sqrt{3}=0$

(3) $x^2-\left(\sqrt{3}+1\right)x+\sqrt{3}=0$ (4) $x^2-\left(\sqrt{3}-1\right)x-\sqrt{3}=0$

Question Type : MCQ
Question ID : 86435120231
Option 1 ID : 86435167275
Option 2 ID : 86435167278
Option 3 ID : 86435167277
Option 4 ID : 86435167276.

Ans. Official Answer NTA (4)

Sol. $(\sqrt{3}+i)^{100} = 2^{99}(p+iq)$

$$2^{100}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^{100} = 2^{99}(p+iq)$$

$$2.e^{i\frac{50\pi}{3}} = p+iq$$

$$2\left(\cos\frac{50\pi}{3}+i\sin\frac{50\pi}{3}\right) = p+iq$$

$$2\left(\frac{-1}{2}+i\frac{\sqrt{3}}{2}\right) = p+iq$$

$$p=-1 \quad \text{and} \quad q=\sqrt{3}$$

$$x^2-(\sqrt{3}-1)x-\sqrt{3}=0$$

15. Let y(x) be the solution of the differential equation $2x^2\,dy+(e^y-2x)\,dx=0, x>0$. If $y(e)=1$, then $y(1)$ is equal to :

माना अवकल समीकरण $2x^2\,dy+(e^y-2x)\,dx=0, x>0$ का हल y(x) है। यदि $y(e)=1$ है, तो $y(1)$ बराबर है :

(1) 2 (2) $\log_e(2e)$ (3) 0 (4) $\log_e 2$

Question Type : MCQ
Question ID : 86435120237
Option 1 ID : 86435167299
Option 2 ID : 86435167300
Option 3 ID : 86435167302
Option 4 ID : 86435167301

Ans. Official Answer NTA (4)

Sol. $\frac{dy}{dx}-\frac{-e^{y}}{2x^{2}}+\frac{1}{x}$

$\frac{e^{-y}dy}{dx}=\frac{e^{y}}{x}+\frac{-1}{2x^{2}}$

Let $e^{-y}=t$

$\frac{dt}{dx}+\frac{t}{x}=\frac{1}{2x^{2}}$ $\left\{\text{I.F.}=e^{\int\frac{1}{x}dx}=x\right\}$

$tx=\int\frac{1}{2x}dx$

$e^{-y}x=\frac{1}{2}\ln x+c$

$c=\frac{1}{2}$

$e_{x}^{-y}=\frac{1}{2}(1+\ln x)$

16. The point $P(-2\sqrt{6},\sqrt{3})$ lies on the hyperbola $\frac{x^{2}}{a^{2}}-\frac{y^{2}}{b^{2}}=1$ having eccentricity $\frac{\sqrt{5}}{2}$. If the tangent and normal at P to the hyperbola intersect its conjugate axis at points Q and R respectively, then QR is equal to :

अतिपरवलय $\frac{x^{2}}{a^{2}}-\frac{y^{2}}{b^{2}}=1$ जिसकी उत्केन्द्रता $\frac{\sqrt{5}}{2}$ है, एक बिन्दु $P(-2\sqrt{6},\sqrt{3})$ है। यदि इस अतिपरवलय के बिन्दु P पर स्पर्श रेखा तथा अभिलम्ब अतिपरवलय के संयुग्मी अक्ष को क्रमशः बिन्दुओं Q तथा R पर काटते हैं, तो QR बराबर है :

(1) 6 (2) $3\sqrt{6}$ (3) $4\sqrt{3}$ (4) $6\sqrt{3}$

Question Type : MCQ
Question ID : 86435120241
Option 1 ID : 86435167317
Option 2 ID : 86435167316
Option 3 ID : 86435167315
Option 4 ID : 86435167318

Ans. Official Answer NTA (4)

Sol.

17. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{1+\sin^2 x}{1+\pi^{\sin x}}\right)dx$ is :

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{1+\sin^2 x}{1+\pi^{\sin x}}\right)dx$ का मान है :

(1) $\frac{3\pi}{4}$ (2) $\frac{3\pi}{2}$ (3) $\frac{\pi}{2}$ (4) $\frac{5\pi}{2}$

Question Type : MCQ
Question ID : 86435120236
Option 1 ID : 86435167296
Option 2 ID : 86435167295
Option 3 ID : 86435167297
Option 4 ID : 86435167298

Ans. Official Answer NTA (1)

Sol.

18. A circle C touches the line $x = 2y$ at the point (2, 1) and itnersects the circle $C_1 : x^2 + y^2 + 2y - 5 = 0$ at two points P and Q such that PQ is a diameter of C_1. Then the diameter of C is :

एक वृत्त C रेखा $x = 2y$ को बिन्दु (2, 1) पर स्पर्श करता है तथा वृत्त $C_1 : x^2 + y^2 + 2y - 5 = 0$ को दो बिन्दुओं P तथा Q पर इस प्रकार काटता है कि PQ वृत्त C_1 का एक व्यास है, तो C का व्यास है :

(1) $7\sqrt{5}$ (2) $\sqrt{285}$ (3) 15 (4) $4\sqrt{15}$

Question Type : MCQ
Question ID : 86435120240
Option 1 ID : 86435167314
Option 2 ID : 86435167313
Option 3 ID : 86435167311
Option 4 ID : 86435167312

Ans. Official Answer NTA (1)

Sol.

19. The local maximum value of the fucntion $f(x)=\left(\frac{2}{x}\right)^{x^2}$, $x>0$, is :

फलन $f(x)=\left(\frac{2}{x}\right)^{x^2}$, $x > 0$, का स्थानीय अधिकतम मान है :

(1) $\left(\frac{4}{\sqrt{e}}\right)^{\frac{e}{4}}$ (2) $e^{\frac{2}{e}}$ (3) 1 (4) $\left(2\sqrt{e}\right)^{\frac{1}{e}}$

Question Type : MCQ
Question ID : 86435120235
Option 1 ID : 86435167293
Option 2 ID : 86435167292
Option 3 ID : 86435167294
Option 4 ID : 86435167291

Ans. Official Answer NTA (2)

Sol.

20. If $\sum_{r=1}^{50} \tan^{-1}\frac{1}{2r^2} = p$, then the value of tan p is :

यदि $\sum_{r=1}^{50} \tan^{-1}\frac{1}{2r^2} = p$ है, तो tan p का मान है :

(1) $\frac{50}{51}$ (2) 100 (3) $\frac{101}{102}$ (4) $\frac{51}{50}$

Question Type : MCQ
Question ID : 86435120246
Option 1 ID : 86435167335
Option 2 ID : 86435167338
Option 3 ID : 86435167337
Option 4 ID : 86435167336

Ans. Official Answer NTA (1)

Sol.

SECTION - B

1. The least positive integer n such that $\frac{(2i)^n}{(1-i)^{n-2}}, i = \sqrt{-1}$, is a positive integer, is _____.

निम्नतम धनात्मक पूर्णांक n जिसके लिए $\frac{(2i)^n}{(1-i)^{n-2}}, i = \sqrt{-1}$, एक धनात्मक पूर्णांक है, है_____।

Question Type : SA

Question ID : 86435120257

Ans. Official Answer NTA (6)

Sol.

2. Let Q be the foot of the perpendicular from the point P(7, –2, 13) on the plane containing the lines $\frac{x+1}{6}=\frac{y-1}{7}=\frac{z-3}{8}$ and $\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-3}{7}$ .Then $(PQ)^2$, is equal to _____.

माना बिन्दु P(7, –2, 13) से समतल, जिसमें रेखाएँ $\frac{x+1}{6}=\frac{y-1}{7}=\frac{z-3}{8}$ तथा $\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-3}{7}$ स्थित है, पर डाले गये लम्ब का पाद Q है तो $(PQ)^2$ बराबर _____ है।

Question Type : SA

Question ID : 86435120258

Ans. Official Answer NTA (96)

Sol.

3. Let a and b respectively be the points of local maximum and local minimum of the function $f(x)=2x^3-3x^2-12x$. If A is the total area of the region bounded by $y=f(x)$, the x-axis and lines $x=a$ and $x=b$, then 4A is equal to _____.

माना फलन $f(x)=2x^3-3x^2-12x$ के स्थानीय उच्चतम तथा स्थानीय निम्नतम बिन्दु क्रमशः a तथा b है। यदि $y=f(x)$, x-अक्ष, तथा रेखाओं $x=a$ और $x=b$ से घिरे क्षेत्र का क्षेत्रफल A है, तो 4A बराबर है_____।

Question Type : SA

Question ID : 86435120254

Ans. Official Answer NTA (114)

Sol.

4. The sum of all 3-digit numbers less than or equal to 500, that are formed without using the digit "1" and they all are multiple of 11, is _____.

अंक 1 का प्रयोग किए बिना 500 के बराबर या उससे कम 3-अंको की सभी संख्याऐं, जो 11 की गुणज है, का योग_____।

Question Type : SA

Question ID : 86435120253

Ans. Official Answer NTA (7744)

Sol.

5. Let $\binom{n}{k}$ denote ${}^nC_k$ and $\left[\begin{matrix} n \\ k \end{matrix}\right] = \begin{cases} \binom{n}{k}, & \text{if } 0 \le k \le n \\ 0, & \text{otherwise} \end{cases}$.

If $A_k = \sum_{i=0}^{9} \binom{9}{i}\left[\begin{matrix} 12 \\ 12-k+i \end{matrix}\right] + \sum_{i=0}^{8} \binom{8}{i}\left[\begin{matrix} 13 \\ 13-k+i \end{matrix}\right]$ and $A_4 - A_3 = 190\,p$, then p is equal to ____.

माना $\binom{n}{k}$, ${}^nC_k$ को दर्शाता है तथा $\left[\begin{matrix} n \\ k \end{matrix}\right] = \begin{cases} \binom{n}{k}, & \text{यदि } 0 \le k \le n \\ 0, & \text{अन्यथा} \end{cases}$ है।

यदि $A_k = \sum_{i=0}^{9} \binom{9}{i}\left[\begin{matrix} 12 \\ 12-k+i \end{matrix}\right] + \sum_{i=0}^{8} \binom{8}{i}\left[\begin{matrix} 13 \\ 13-k+i \end{matrix}\right]$ तथा $A_4 - A_3 = 190\,p$ है, तो p बराबर है____।

Question Type : SA

Question ID : 86435120252

Ans. Official Answer NTA (49)

Sol.

6. If the proejction of the vector $\hat{i}+2\hat{j}+\hat{k}$ on the sum of the two vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $-\lambda\hat{i}+2\hat{j}+3\hat{k}$ is 1, then λ is equal to _____.

यदि दो सदिशों $2\hat{i}+4\hat{j}-5\hat{k}$ तथा $-\lambda\hat{i}+2\hat{j}+3\hat{k}$ के योग पर सदिश $\hat{i}+2\hat{j}+\hat{k}$ का प्रक्षेप 1 है, तो λ बराबर है_____।

Question Type : SA

Question ID : 86435120256

Ans. Official Answer NTA (5)

Sol.

7. Let the mean and variance of the four numbers 3, 7, x and y (x > y) be 5 and 10 respectively. Then the mean of four numbers 3 + 2x, 7 + 2y, x + y and x – y is _____.

माना चार संख्याओं 3, 7, x तथा y (x > y) के माध्य तथा प्रसरण क्रमशः 5 तथा 10 है तो चार संख्याओं 3 + 2x, 7 + 2y, x + y तथा x – y का माध्य है_____।

Question Type : SA

Question ID : 86435120255

Ans. Official Answer NTA (12)

Sol.

8. Let $\lambda \neq 0$ be in R. If α and β are the roots of the equation $x^2 - x + 2\lambda = 0$ and α and γ are the roots of the equation $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to _____.

माना $\lambda \neq 0$, R में है। यदि समीकरण $x^2 - x + 2\lambda = 0$ के मूल α तथा β हैं और समीकरण $3x^2 - 10x + 27\lambda = 0$ के मूल α तथा γ हैं, तो $\frac{\beta\gamma}{\lambda}$ बराबर है_____।

Question Type : SA

Question ID : 86435120250

Ans. Official Answer NTA (18)

Sol.

9. Let A be a 3×3 real matrix. If $\det(2\,\text{Adj})\,(2\,\text{Adj}(\text{Adj})\,(2A))) = 2^{41}$, then the value of $\det(A^2)$ equals ____.

माना A एक 3×3 वास्तविक आव्यूह है। यदि $\det(2\,\text{Adj})\,(2\,\text{Adj}(\text{Adj})\,(2A))) = 2^{41}$ है, तो $\det(A^2)$ का मान बराबर है____।

Question Type : SA

Question ID : 86435120251

Ans. Official Answer NTA (4)

Sol.

10. Let $a_1, a_2, \ldots\ldots a_{10}$ be an AP with common difference -3 and $b_1, b_2, \ldots b_{10}$ be a GP with common ratio 2.

Let $c_k = a_k + b_k$, $k = 1, 2, \ldots\ldots, 10$. If $c_2 = 12$ and $c_3 = 13$, then $\sum_{k=1}^{10} c_k$ is equal to ____.

माना $a_1, a_2, \ldots\ldots a_{10}$ एक AP है, जिसका सार्वअंतर -3 है तथा $b_1, b_2, \ldots b_{10}$ एक GP है, जिसका सार्व अनुपात 2 है।

माना $c_k = a_k + b_k$, $k = 1, 2, \ldots\ldots, 10$ है। यदि $c_2 = 12$ तथा $c_3 = 13$ है, तो $\sum_{k=1}^{10} c_k$ बराबर है____।

Question Type : SA

Question ID : 86435120259

Ans. Official Answer NTA (2021)

Sol.