JEE Main June 2022

Question Paper With Text Solution

25 June | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JUNE 2022 | 25TH JUNE SHIFT-2

SECTION – A

Question ID : 161

Set & Relations

1. Let $A = \{x \in R : |x + 1| < 2\}$ and $B = \{x \in R : |x - 1| \geq 2\}$. Then which one of the following statements is NOT true?

माना $A = \{x \in R : |x + 1| < 2\}$ तथा $B = \{x \in R : |x - 1| \geq 2\}$ हैं। तो निम्न कथनों में से कौनसा एक सत्य नहीं है:

(1) $A - B = (-1, 1)$ (2) $B - A = R - (-3, 1)$

(3) $A \cap B = (-3, -1]$ (4) $A \cup B = R - [1, 3)$

Ans. Official Answer NTA (2)

Sol. A : $|x + 1| < 2$

$-2 < x + 1 < 2$

$x \in (-3, 1)$

B $|x - 1| \geq 2$

$x - 1 \in (-\infty, -2] \cup [2, \infty)$

$x \in (-\infty, -1] \cup [3, \infty)$

$A - B = (1, 1)$

$B - A = (-\infty, -3] \cup [3, \infty) = R - (-3, 3)$

$A \cap B = (-3, -1]$

$A \cup B = P(-\infty, 1) \cup [3, \infty) = R - [1, 3)$

Question ID : 162

Quadratic Equation

2. Let $a, b \in R$ be such that the equation $ax^2 - 2bx + 15 = 0$ has a repeated root α. If α and β are the roots of the equation $x^2 - 2bx + 21 = 0$, then $\alpha^2 + \beta^2$ is equal to :

माना $a, b \in R$ इस प्रकार है कि समीकरण $ax^2 - 2bx + 15 = 0$ का एक पुनरावृत्त मूल (repeated root) α है। यदि समीकरण $x^2 - 2bx + 21 = 0$ के मूल α तथा β हैं, तो $\alpha^2 + \beta^2$ बराबर है :

(1) 37 (2) 58 (3) 68 (4) 92

Ans. Official Answer NTA (2)

Sol. $ax^2 - 2bx + 15 = a(x - \alpha)^2$

$x^2 - 2bx + 21 = (x - \alpha)(x - \beta)$

$2\alpha = \frac{2b}{a}$, $\alpha^2 = \frac{15}{a}$

$\alpha + \beta = 2b$, $\alpha\beta = 21$

again, $a\alpha^2 - 2b\alpha + 15 = 0$

$\alpha^2 - 2b\alpha + 21 = 0$

$(a-1)\alpha^2 - 6 = 0$

$$\alpha^2 = \frac{6}{a-1} = \frac{15}{a} \quad \Rightarrow a = \frac{5}{3}$$

$$\alpha^2 = \frac{15}{5/3} = 9$$

$$\beta^2 = \frac{(21)^2}{\alpha^2} = 49$$

$\alpha^2 + \beta^2 = 58$

Question ID : 163

Complex Number

3. Let z_1 and z_2 be two complex numbers such that $\overline{z}_1 = i\overline{z}_2$ and $\arg\left(\frac{z_1}{\overline{z}_2}\right) = \pi$. Then :

माना दो सम्मिश्र संख्याओं z_1 तथा z_2 के लिए $\overline{z}_1 = i\overline{z}_2$ तथा $\arg\left(\frac{z_1}{\overline{z}_2}\right) = \pi$ हैं। तो :

(1) $\arg z_2 = \frac{\pi}{4}$ (2) $\arg z_2 = -\frac{3\pi}{4}$ (3) $\arg z_1 = \frac{\pi}{4}$ (4) $\arg z_1 = -\frac{3\pi}{4}$

Ans. Official Answer NTA (3)

Sol. $z_1 = i\overline{z}_2 \Rightarrow z_1 = -i\overline{z}_2$

$z_2 = iz_1$

$\arg\left(\frac{z_1}{\overline{z}_2}\right) = \pi \Rightarrow \quad \arg(z_1) - \arg(\overline{z}_2) = \pi$

$\arg(z_1) + \arg(z_2) = \pi$ _____(i)

$\arg(\overline{z}_1) = \arg(i\overline{z}_2)$

$-\arg(z_1) = \arg(i) + \arg(\overline{z}_2)$

$-\arg(z_1) = \frac{\pi}{2} - \arg(z_2)$

$-\arg(z_1) + \arg(z_2) = \frac{\pi}{2}$ _____(ii)

From (i) – (ii)

$2\arg(z_1) = \frac{\pi}{2}$

$$\arg(z_1) = \frac{\pi}{4}$$

Question ID : 164

Determinant

4. The system of equations

$-kx + 3y - 14z = 25$

$-15x + 4y - kz = 3$

$-4x + y + 3z = 4$

is consistent for all k in the set :

समीकरण निकाय

$-kx + 3y - 14z = 25$

$-15x + 4y - kz = 3$

$-4x + y + 3z = 4$

क्रसंगत है, समुच्चय सभी k के लिए :

(1) R (2) R – {–11, 13} (3) R – {13} (4) R – {–11, 11}

Ans. Official Answer NTA (4)

Sol. $-kx + 3y - 14z = 25$

$-15x + 4y - kz = 3$

$-4x + y + 3z = 4$

After eleminating y

$(k - 12)x + 23z = -13$

$-x + (k + 12)z = 13$

For no solution

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{k-12}{-1} = \frac{23}{k+12} \neq \frac{-13}{13}$$

$k = 11, -11$

If system of equation is consistant

$k = R - \{-11, 11\}$

Question ID : 165

Limits

5. $\lim\limits_{x\to\frac{\pi}{2}}\left(\tan^2 x\left(\left(2\sin^2 x+3\sin x+4\right)^{\frac{1}{2}}-\left(\sin^2 x+6\sin x+2\right)^{\frac{1}{2}}\right)\right)$ is equal to :

$\lim\limits_{x\to\frac{\pi}{2}}\left(\tan^2 x\left(\left(2\sin^2 x+3\sin x+4\right)^{\frac{1}{2}}-\left(\sin^2 x+6\sin x+2\right)^{\frac{1}{2}}\right)\right)$ बराबर है :

(1) $\frac{1}{12}$ (2) $-\frac{1}{18}$ (3) $-\frac{1}{12}$ (4) $\frac{1}{6}$

Ans. Official Answer NTA (1)

Sol. $\lim\limits_{x\to\frac{\pi}{2}}\tan^2 x\left(\sqrt{2\sin^2+3\sin x+4}-\sqrt{\sin^2 x+6\sin x+2}\right)$

$\lim\limits_{x\to\frac{\pi}{2}}\tan^2 x\left(\frac{\left(2\sin^2 x+3\sin x+4\right)-\left(\sin^2 x+6\sin xx+2\right)}{\sqrt{2\sin^2 x+3\sin x+14}+\sqrt{\sin^2 x+6\sin x+2}}\right)$

$\lim\limits_{x\to\frac{\pi}{2}}\frac{\tan^2 x.\left(\sin x-1\right)\left(\sin x-2\right)}{\left(3+3\right)}$

$\lim\limits_{x\to\frac{\pi}{2}}\frac{-1}{6}.\tan^2 x\left(\sin x-1\right)$

put $x=\frac{\pi}{2}-h$

$\lim\limits_{h\to0}\frac{-1}{6}\cot^2 h\left(\cos h-1\right)$

$\lim\limits_{h\to0}\frac{1}{6}\frac{\left(1-\cos h\right)}{\tan^2 h}$

$\lim\limits_{h\to0}\frac{1}{6}\frac{\left(1-\cos h\right)}{h^2}=\frac{1}{6}.\frac{1}{2}=\frac{1}{12}$

Question ID : 166

Area Under Curve

6. The area of the region enclosed between the parabolas $y^2 = 2x - 1$ and $y^2 = 4x - 3$ is :

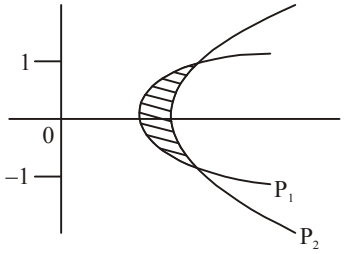
परवलयों $y^2 = 2x - 1$ तथा $y^2 = 4x - 3$ के बीच घिरे क्षेत्र का क्षेत्रफल है :

(1) $\frac{1}{3}$ (2) $\frac{1}{6}$ (3) $\frac{2}{3}$ (4) $\frac{3}{4}$

Ans. Official Answer NTA (1)

Sol. $P_1 : y^2 = 2x - 1$

$P_2 : y^2 = 4x - 3$



$$A = 2.\int_0^1 \left(x_{P_2} - x_{P_1}\right) dy$$

$$A = 2\int_0^1 \left(\frac{y^2+3}{4}\right) - \left(\frac{y^2+1}{2}\right) dy$$

$$A = \frac{1}{3}$$

Question ID : 167

Binomial Theorem

7. The coefficient of x^{101} in the expression $(5 + x)^{500} + x(5 + x)^{499} + x^2(5 + x)^{498} + ... + x^{500}$, $x > 0$, is :

व्यंजक $(5 + x)^{500} + x(5 + x)^{499} + x^2(5 + x)^{498} + ... + x^{500}$, $x > 0$ में x^{101} का गुणांक है :

(1) ${}^{501}C_{101}\,(5)^{399}$ (2) ${}^{501}C_{101}\,(5)^{400}$ (3) ${}^{501}C_{100}\,(5)^{400}$ (4) ${}^{500}C_{101}\,(5)^{399}$

Ans. Official Answer NTA (1)

Sol. $(5 + x)^{500} + x\,(5 + x)^{499} + x^2(5 + x)^{498} + ... + x^{500}$

$$= (5+x)^{500}\left\{\frac{1-\left(\frac{x}{5+x}\right)^{501}}{1-\frac{x}{5+x}}\right\}$$

$$= \frac{1}{5}\left\{(5+x)^{501} - x^{501}\right\}$$

coff. at $x^{101} = \frac{1}{5}\,{}^{501}C_{101} \cdot 5^{400}$

$= {}^{501}C_{101}\,5^{399}$

Question ID : 168

Sequence & progression

8. The sum $1+2\cdot3+3\cdot3^2+.....+10\cdot3^9$ is equal to :

योगफल $1+2\cdot3+3\cdot3^2+.....+10\cdot3^9$ बराबर है :

(1) $\frac{2\cdot3^{12}+10}{4}$ (2) $\frac{19\cdot3^{10}+1}{4}$ (3) $5\cdot3^{10}-2$ (4) $\frac{9\cdot3^{10}+1}{2}$

Ans. Official Answer NTA (2)

Sol. $S = 1 + 2.3 + 3.3^2 + ... + 10.3^9$

$3S = 1.3 + 2.3^2 + ... + 9.3^9 + 10.3^{10}$

$-2S = 1 + 3 + 3^2 + ...+ 3^9 - 10.3^{10}$

$-2S = \frac{1.\left(3^{10}-1\right)}{3-1} - 10.3^{10}$

$S = \frac{19.3^{10}+1}{4}$

Question ID : 169

3D Geometry

9. Let P be the plane passing through the intersection of the planes $\vec{r}.\left(\hat{i}+3\hat{j}-\hat{k}\right)=5$ and $\vec{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)=3$ and the point (2, 1, –2). Let the position vectors of the points X and Y be $\hat{i}-2\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+2\hat{k}$ respectively. Then the points :

(1) X and X + Y are on the same side of P

(2) Y and Y – X are on the opposite side of P

(3) X and Y are on the opposite side of P

(4) X + Y and X – Y are on the same side of P

माना समतलों $\vec{r}.\left(\hat{i}+3\hat{j}-\hat{k}\right)=5$ तथा $\vec{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)=3$ के प्रतिच्छेदन तथा बिन्दु (2, 1, –2) से होकर जाने वाला समतल P है। माना बिन्दुओं X तथा Y के स्थिति सदिश क्रमशः $\hat{i}-2\hat{j}+4\hat{k}$ तथा $5\hat{i}-\hat{j}+2\hat{k}$ हैं। तो बिन्दु :

(1) X तथा X + Y समतल P के एक ही ओर हैं

(2) Y तथा Y – X, समतल P के भिन्न ओर हैं

(3) X तथा Y, समतल P के भिन्न ओर हैं

(4) X + Y तथा X – Y, समतल P के एक ही ओर हैं

Ans. Official Answer NTA (3)

Sol. $P_1 : x + 3y - z - 5 = 0$

$P_2 : 2x - y + z - 3 = 0$

$P : P_1 + \lambda P_2 = O$

$(x + 3y - z - 5) + \lambda(2x - y + z - 3) = 0$

Passes through (2, 1, –2)

$\Rightarrow \lambda = 1$

$P : 3x + 2y - 8 = O$

X= (1, –2, 4)

Y = (5, –1, 2)

Putting value in plane P

$X \Rightarrow 3 - 4 - 8 < 0$

$Y \Rightarrow 15 - 2 - 8 > 0$

X and Y are on the opposite side of P.

Question ID : 1610

Circle

10. A circle touches both the y-axis and the line $x + y = 0$. Then the locus of its center is :

एक वृत्त, y-अक्ष तथा रेखा $x + y = 0$ दोनों की स्पर्श करता है। तो इसके केन्द्र का बिंदुपथ है :

(1) $y = \sqrt{2}x$ (2) $x = \sqrt{2}y$ (3) $y^2 - x^2 = 2xy$ (4) $x^2 - y^2 = 2xy$

Ans. Official Answer NTA (4)

Sol. Let center of circle be (h, k)

circle touches y-axis $\Rightarrow r = |h|$

cricle also touches $x + y = 0$

$$\Rightarrow \left|\frac{h+k}{\sqrt{2}}\right| = |h|$$

$\Rightarrow h^2 - k^2 = 2hk$

$\Rightarrow x^2 - y^2 = 2xy$

Question ID : 1611

Tangent and normal

11. Water is being filled at the rate of 1 cm^3/sec in a right circular conical vessel (vertex downwards) of height 35 cm and diameter 14 cm. When the height of the water level is 10cm, the rate (in cm^2/sec) at which the wet conical surface area of the vessel increases is :

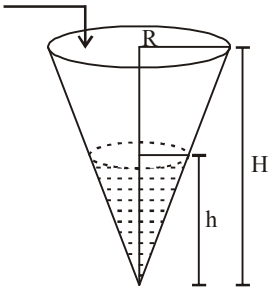
35 सेमी ऊँचाई तथा 14 सेमी व्यास के एक लंब वृत्तीय शंक्वाकार बर्तन, जिसका शीर्ष नीचे की ओर है, में 1 cm^3/sec की दर से पानी भरा जा रहा है। जब पानी की सतह की ऊँचाई 10 सेमी है, तब बर्तन के भीगे हुए शंक्वाकार पृष्ठीय क्षेत्रफल

की वृद्धि दर (सेमी²/सै. में) है :

(1) 5 (2) $\frac{\sqrt{21}}{5}$ (3) $\frac{\sqrt{26}}{5}$ (4) $\frac{\sqrt{26}}{10}$

Ans. Official Answer NTA (3)

Sol.



$$\frac{R}{r}=\frac{H}{h}\Rightarrow\frac{7}{r}=\frac{35}{h}$$

$$\Rightarrow h=5r$$

$$V=\frac{1}{3}\pi r^2h=\frac{5\pi}{3}r^3$$

$$\frac{dV}{dt}=5\pi r^2\frac{dr}{dt}=1$$

$$\frac{dr}{dt}=\frac{1}{5\pi r^2}$$

$$S=\pi r\ell=\pi r\sqrt{r^2+h^2}=\pi r.\sqrt{r^2+25r^2}$$

$$S=\sqrt{26}\pi r^2$$

$$\frac{ds}{dt}=\sqrt{26}\pi.2r\frac{dr}{dt}$$

$$\frac{ds}{dt}=2\sqrt{26}\,\pi.r.\frac{1}{5\pi r^2}$$

$$\frac{ds}{dt}=\frac{2\sqrt{26}}{5r}$$ (when h = 10cm, r = 2cm)

$$\frac{ds}{dt}=\frac{\sqrt{26}}{5}$$

Question ID : 1612

Definite Integration

12. If $b_n = \int_0^{\frac{\pi}{2}} \frac{\cos^2 nx}{\sin x} dx,\ n \in N$, then :

(1) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in an A.P. with common difference – 2

(2) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A.P. with common difference 2

(3) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in a G.P.

(4) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A.P. with common difference – 2

यदि $b_n = \int_0^{\frac{\pi}{2}} \frac{\cos^2 nx}{\sin x} dx,\ n \in N$, है, तो :

(1) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ एक A.P. में हैं, जिसका सार्वअंतर – 2 है

(2) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ एक A.P. में हैं, जिसका सार्वअंतर 2 है

(3) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ एक G.P. में हैं

(4) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ एक A.P. में हैं, जिसका सार्वअंतर – 2 है

Ans. Official Answer NTA (4)

Sol. $b_n - b_{n-1} = \int_0^{\frac{\pi}{2}} \frac{\cos^2 nx - \cos^2 (n-1)x}{\sin x} dx$

$= \int_0^{\frac{\pi}{2}} \frac{\sin(2n-1)x \sin(-x)}{\sin x} dx$

$= \left[\frac{\cos(2n-1)x}{2n-1}\right]_0^{\frac{\pi}{2}}$

$b_n - b_{n-1} = \frac{-1}{2n-1}$

$\frac{1}{b_n - b_{n-1}} = -2n+1$, A. P. with common difference "–2".

Question ID : 1613

Differential Equation

13. If y = y(x) is the solution of the differential equation $2x^2\frac{dy}{dx}-2xy+3y^2=0$ such that $y(e)=\frac{e}{3}$, then y(1) is equal to :

यदि y = y(x), अवकल समीकरण $2x^2\frac{dy}{dx}-2xy+3y^2=0$, $y(e)=\frac{e}{3}$ का हल है, तो y(1) बराबर है :

(1) $\frac{1}{3}$ (2) $\frac{2}{3}$ (3) $\frac{3}{2}$ (4) 3

Ans. Official Answer NTA (2)

Sol. $2x^2\frac{dy}{dx}-2xy+3y^2=0$

$\frac{1}{y^2}\frac{dy}{dx}-\frac{1}{y}.\frac{1}{x}=\frac{-3}{2x^2}$

Put $-\frac{1}{y}=z$

$\frac{1}{y^2}.\frac{dy}{dx}=\frac{dz}{dx}$

$\frac{dz}{dx}+\frac{1}{x}.z=\frac{-3}{2x^2}$

I.F. = $e^{\int\frac{1}{x}dx}$

Solution $z.x=\int\frac{-3}{2x^2}.x\,dx$

$-\frac{1}{y}.x=\frac{-3}{2}\ln x-C$

$\frac{x}{y}=\frac{3}{2}\ln x+C$

$y(e)=\frac{e}{3}\Rightarrow C=\frac{3}{2}$

$y=\frac{2}{3}.\frac{x}{(\ln x+1)}$

$y(1)=\frac{2}{3}$

Question ID : 1614

Tangent and normal

14. If the angle made by the tangent at the point (x_0, y_0) on the curve $x = 12(t + \sin t \cos t)$, $y = 12(1 + \sin t)^2$, $0 < t < \frac{\pi}{2}$, with the positive x-axis is $\frac{\pi}{3}$, then y_0 is equal to :

यदि वक्र $x = 12(t + \sin t \cos t)$, $y = 12(1 + \sin t)^2$, $0 < t < \frac{\pi}{2}$ के बिन्दु (x_0, y_0) पर स्पर्श रेखा धनात्मक x-अक्ष से $\frac{\pi}{3}$ का कोण बनाती है, तो y_0 बराबर है :

(1) $6\left(3+2\sqrt{2}\right)$ (2) $3\left(7+4\sqrt{3}\right)$ (3) 27 (4) 48

Ans. Official Answer NTA (3)

Sol. $$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{12.2(1+\sin t).\cos t}{12\left(1+\sin t.(-\sin t)+\cos t.\cos t\right)}$$

$$= \frac{2(1+\sin t)\cos t}{2\cos^2 t} = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$

$$\Rightarrow 1 + \sin t = \sqrt{3}\cos t$$

$$\frac{\sqrt{3}}{2}\cos t - \frac{1}{2}.\sin t = \frac{1}{2}$$

$$\cos\left(t + \frac{\pi}{6}\right) = \cos\frac{\pi}{3}$$

$$t = \frac{\pi}{6}$$

$$y_0 = 12\left(1+\sin\frac{\pi}{6}\right)^2 = 27$$

Question ID : 1615

Trigonometric Ratio and Identities

15. The value of $2\sin(12°) - \sin(72°)$ is :

$2\sin(12°) - \sin(72°)$ का मान है :

(1) $\frac{\sqrt{5}\left(1-\sqrt{3}\right)}{4}$ (2) $\frac{1-\sqrt{5}}{8}$ (3) $\frac{\sqrt{3}\left(1-\sqrt{5}\right)}{2}$ (4) $\frac{\sqrt{3}\left(1-\sqrt{5}\right)}{4}$

Ans. Official Answer NTA (4)

Sol. $\sin12° - (\sin 72° - \sin 12°)$

$\Rightarrow \sin12° - 2 \cdot \cos 42° \cdot \sin 30°$

$\Rightarrow \cos78° - \cos 42°$

$\Rightarrow 2\sin60° \sin(-18°)$

$\Rightarrow -\sqrt{3}.\left(\frac{\sqrt{5}-1}{4}\right) = \frac{\sqrt{3}}{4}\left(1-\sqrt{5}\right)$

Question ID : 1616

Probability

16. A biased die is marked with numbers 2, 4, 8, 16, 32, 32 on its faces and the probability of getting a face with mark n is $\frac{1}{n}$. If the die is thrown thrice, then the probability, that the sum of the numbers obtained is 48, is :

एक अभिनत पासे के फलकों पर संख्याएँ 2, 4, 8, 16, 32, 32 अंकित हैं तथा संख्या n के प्रकट होने की प्रायिकता $\frac{1}{n}$ है। इस पासे को तीन बार फेंकने पर प्रकट होने वाली संख्याओं का योग 48 होने की प्रायिकता है :

(1) $\frac{7}{2^{11}}$ (2) $\frac{7}{2^{12}}$ (3) $\frac{3}{2^{10}}$ (4) $\frac{13}{2^{12}}$

Ans. Official Answer NTA (4)

Sol. S = 48

16, 16, 16 or 32, 8, 8

$P = \left(\frac{1}{16}\right)^3 + \left(\frac{2}{32}\right)\left(\frac{1}{8}\right)^2 . \frac{3!}{2!}$

$P = \frac{13}{2^{12}}$

Question ID : 1617

Mathematical Reasoning

17. The negation of the Boolean expression $\left((\sim q) \wedge p\right) \Rightarrow \left((\sim p) \vee q\right)$ is logically equivalent to :

बूलीय व्यंजक $\left((\sim q) \wedge p\right) \Rightarrow \left((\sim p) \vee q\right)$ का निषेधन निम्न में से किसके तर्कतः तुल्य है :

(1) $p \Rightarrow q$ (2) $q \Rightarrow p$ (3) $\sim(p \Rightarrow q)$ (4) $\sim(q \Rightarrow p)$

Ans. Official Answer NTA (3)

Sol. $\sim((\sim q \wedge p) \rightarrow (\sim p \vee q))$

$\sim(\sim(\sim q \wedge p) \vee (\sim p \vee q))$

$\sim((q \vee \sim p) \vee (\sim p \vee q))$

$\sim(\sim p \vee q)$

$\sim(p \rightarrow q)$

Question ID : 1618

Parabola

18. If the line $y = 4 + kx$, $k > 0$, is the tangent to the parabola $y = x - x^2$ at the point P and V is the vertex of the parabola, then the slope of the line through P and V is :

यदि परवलय $y = x - x^2$ के बिन्दु P पर स्पर्श रेखा का समीकरण $y = 4 + kx$, $k > 0$ है तथा परवलय का शीर्ष V है, तो P तथा V से होकर जाने वाली रेखा की प्रवणता है :

(1) $\frac{3}{2}$ (2) $\frac{26}{9}$ (3) $\frac{5}{2}$ (4) $\frac{23}{6}$

Ans. Official Answer NTA (3)

Sol. $L : y = 4 + kx$

$P : y = x - x^2$

$4 + kx = x - x^2$

$x^2 + (k - 1)x + 4 = 0$

$D = 0$

$(k - 1)^2 - 16 = 0 \Rightarrow k = -3, 5 \ (k > 0)$

$k = 5$

$x^2 + 4x + 4 = 0 \Rightarrow x = 2$

$y = 2 - 4 = -2$

$P(2, -2)$

$P : y = -\left(x^2 - x + \frac{1}{4} - \frac{1}{4}\right)$

$$y=\frac{1}{4}-\left(\frac{x-1}{2}\right)^2 \Rightarrow \left(x-\frac{1}{2}\right)^2=-\left(y-\frac{1}{4}\right)$$

$$V=\left(\frac{1}{2},\frac{1}{4}\right)$$

$$m_{PV}=\frac{5}{2}$$

Question ID : 1619

ITF

19. The value of $\tan^{-1}\left(\frac{\cos\left(\frac{15\pi}{4}\right)-1}{\sin\left(\frac{\pi}{4}\right)}\right)$ is equal to :

$\tan^{-1}\left(\frac{\cos\left(\frac{15\pi}{4}\right)-1}{\sin\left(\frac{\pi}{4}\right)}\right)$ का मान बराबर है :

(1) $-\frac{\pi}{4}$ (2) $-\frac{\pi}{8}$ (3) $-\frac{5\pi}{12}$ (4) $-\frac{4\pi}{9}$

Ans. Official Answer NTA (2)

Sol. $\tan^{-1}\left(\frac{\cos\left(4\pi-\frac{\pi}{4}\right)-1}{\sin\frac{\pi}{4}}\right)$

$$-\tan^{-1}\left(\frac{1-\cos\frac{\pi}{4}}{\sin\frac{\pi}{4}}\right)$$

$$-\tan^{-1}\left(\tan\frac{\pi}{8}\right)=-\frac{\pi}{8}$$

Question ID : 1620

Ellipse

20. The line $y = x + 1$ meets the ellipse $\frac{x^2}{4}+\frac{y^2}{2}=1$ at two points P and Q. If r is the radius of the circle with PQ as diameter then $(3r)^2$ is equal to :

रेखा $y = x + 1$, दीर्घवृत्त $\frac{x^2}{4}+\frac{y^2}{2}=1$ को दो बिन्दुओं P तथा Q पर मिलती है। यदि वृत्त, जिसका एक व्यास PQ है, की त्रिज्या r है, तो $(3r)^2$ बराबर है :

(1) 20 (2) 12 (3) 11 (4) 8

Ans. Official Answer NTA (1)

Sol. $L : y = x + 1$

$E : \frac{x^2}{4}+\frac{y^2}{2}=1$

$x^2 + 2y^2 = 4$

Quadratic in x

$x^2 + 2(x + 1)^2 = 4$

$3x^2 + 4x - 2 = 0$

Quadratic in y

$(y - 1)^2 + 2y^2 = 4$

$3y^2 - 2y - 3 = 0$

Eq. of circle

$(3x^2 + 4x - 2) + (3y^2 - 2y - 3) = 0$

$$x^2+y^2+\frac{4}{3}x-\frac{2}{3}y-\frac{5}{3}=0$$

$$r=\sqrt{\frac{4}{9}+\frac{1}{9}-\frac{5}{3}}=\frac{\sqrt{20}}{3}$$

$(3r)^2 = 20$

SECTION – B

Question ID : 1621

Matrices

21. Let $A=\begin{pmatrix}2 & -2\\ 1 & -1\end{pmatrix}$ and $B=\begin{pmatrix}-1 & 2\\ -1 & 2\end{pmatrix}$. Then the number of elements in the set$\{(n, m) : n, m \in \{1, 2, ...,10\}$ and $nA^n + mB^m = I\}$ is _____.

माना $A=\begin{pmatrix}2 & -2\\ 1 & -1\end{pmatrix}$ तथा $B=\begin{pmatrix}-1 & 2\\ -1 & 2\end{pmatrix}$ हैं। तो समुच्चय $\{(n, m) : n, m \in \{1, 2, ...,10\}$ तथा $nA^n + mB^m = I\}$ है में अवयवों की संख्या है _____

Ans. Official Answer NTA (1)

Sol. Here $A^2 = A$ and $B^2 = B$

$\Rightarrow A^n = A$ and $B^m = B$

$$n\begin{pmatrix}2 & -2\\ 1 & -1\end{pmatrix}+m\begin{pmatrix}-1 & 2\\ -1 & 2\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$\left.\begin{aligned}2n-m&=1\\ -2n+2m&=0\\ n-m&=0\\ -n+2m&=1\end{aligned}\right\}\Rightarrow n=1\ \&\ m=1$$

$(n, m) = (1, 1)$

Question ID : 1622

Continuity & Differentiability

22. Let $f(x) = [2x^2 + 1]$ and $g(x)=\begin{cases}2x-3, & x<0\\ 2x+3, & x\geq 0\end{cases}$, where [t] is the greatest integer $\leq t$. Then, in the open interval $(-1, 1)$, the number of points where fog is discontinuous is equal to _____.

माना $f(x) = [2x^2 + 1]$ तथा $g(x)=\begin{cases}2x-3, & x<0\\ 2x+3, & x\geq 0\end{cases}$ हैं, जहाँ [t] महत्तम पूर्णांक $\leq t$ है। तो विवृत अंतराल $(-1, 1)$ में उन बिंदुओं, जहाँ fog असंतत है, की संख्या है _____

Ans. Official Answer NTA (62)

Sol. $f(x) = [2x^2] + 1$

$f(g(x))=\left[2g^2(x)\right]+1$

$$f(g(x))=\begin{cases}\left[2(2x-3)^2\right]+1 & , \quad x\in(-1,0)\\ \left[2(2x+3)^2\right]+1 & , \quad x\in[0,1)\end{cases}$$

(i) $x \in (-1, 0)$

$2x \in (-2, 0)$

$2x - 3 \in (-5, -3)$

$(2x - 3)^2 \in (9, 25)$

$2(2x - 3)^2 \in (18, 50)$

f(g(x)) is DC when

$2(2x - 3)^2 \in \{19, 20, ...,49\}$

Total = 31

(ii) $x \in [0, 1)$

$2x \in [0, 2)$

$2x + 3 \in [3, 5)$

$(2x + 3)^2 \in [9, 25)$

$2(2x - 3)^2 \in [18, 50)$

f(g(x)) is DC when

$2(2x + 3)^2 \in \{19, 20, ...,49\}$

Total = 31

Total point of discontinuity = 31 + 31 = 62

Question ID : 1623

Definite Integration

23. The value of $b > 3$ for which $12\times\int_3^b \frac{1}{(x^2-1)(x^2-4)}dx = \log_e\left(\frac{49}{40}\right)$, is equal to _____.

$b > 3$ का वह मान, जिसके लिए $12\times\int_3^b \frac{1}{(x^2-1)(x^2-4)}dx = \log_e\left(\frac{49}{40}\right)$ है, है _____

Ans. Official Answer NTA (6)

Sol. $12.\int_3^b \frac{1}{(x^2-1)(x^2-4)}dx = \ln\left(\frac{49}{40}\right)$

$$12.\int_3^b \left(\frac{-1}{3}.\frac{1}{x^2-1}+\frac{1}{3}.\frac{1}{x^2-4}\right)dx = \ln\left(\frac{49}{40}\right)$$

$$\left[-4.\frac{1}{2}\ln\left(\frac{x-1}{x+1}\right)+4.\frac{1}{4}\ln\left(\frac{x-2}{x+2}\right)\right]_3^b = \ln\left(\frac{49}{40}\right)$$

$$\left[\ln\left\{\frac{x-2}{x+2}.\left(\frac{x+1}{x-1}\right)^2\right\}\right]_3^b = \ln\left(\frac{49}{50}\right)$$

$$\ln\left\{\frac{b-2}{b+2}.\left(\frac{b+1}{b-1}\right)^2\right\}-\ln\left\{\frac{1}{5}.4\right\} = \ln\left(\frac{49}{40}\right)$$

$$\ln\left\{\frac{b-2}{b+2}.\left(\frac{b+1}{b-1}\right)^2\right\} = \ln\left\{\frac{49}{40}.\frac{4}{5}\right\}$$

$$\ln\left\{\frac{b-2}{b+2}.\left(\frac{b+1}{b-1}\right)^2\right\} = \ln\left\{\frac{4}{8}.\left(\frac{7}{5}\right)^2\right\}$$

$b = 6$

Question ID : 1624

Binomial Theorem

24. If the sum of the co-efficients of all the positive even power of x in the binomial expansion of $\left(2x^3+\frac{3}{x}\right)^{10}$ is $5^{10}-\beta\cdot 3^9$, then β is equal to _____.

यदि $\left(2x^3+\frac{3}{x}\right)^{10}$ के द्विपद प्रसार में x की सभी धनात्मक सम घातों के गुणांकों का योगफल $5^{10}-\beta\cdot 3^9$ है, तो β बराबर है _____

Ans. Official Answer NTA (83)

Sol. $T_{r+1} = {}^{10}C_r\,(2x^3)^{10-r}\left(\frac{3}{x}\right)^r$

$T_{r+1} = {}^{10}C_r\,2^{10-r}\,.\,3^r\,x^{30-4r}$

for positive even powers

$r \neq 8, 9, 10$

Sum of coeff. $= (2 + 3)^{10} = 5^{10}$

Sum of coeff. of positive even powers

$= \text{Total} - (\text{sum for } r = 8, 9, 10)$

$= 5^{10} - ({}^{10}C_8\, 2^2 \,.\, 3^8 + {}^{10}C_9\, 2.3^9 + {}^{10}C_{10}\, 3^{10})$

$= 5^{10} - 83.3^9$

So, $\beta = 83$

Question ID : 1625

Statistics

25. If the mean deviation about the mean of the number 1, 2, 3, ..., n where n is odd, is $\frac{5(n+1)}{n}$, then n is equal to _____.

यदि संख्याओं 1, 2, 3, ...n, जहाँ n विषम है, का माध्य के सापेक्ष माध्य विचलन $\frac{5(n+1)}{n}$ है, तो n बराबर है _____

Ans. Official Answer NTA (21)

Sol. $\bar{x} = \frac{1+2+3+.....+n}{n} = \frac{n+1}{2}$

$$\sum |x_i - \bar{x}| = |1-\bar{x}| + |2-\bar{x}| + ... + \left|\frac{n+1}{2} - \bar{x}\right| + ... + |n-\bar{x}|$$

$$= 2.\left\{|1-\bar{x}| + |2-\bar{x}| + ... + \left|\frac{n-1}{2} - \bar{x}\right|\right\}$$

$$= 2\left\{|\bar{x}-1| + |\bar{x}-2| + ... + \left(\bar{x} - \frac{n-1}{2}\right)\right\}$$

$$= 2\left\{\frac{n-1}{2}.\bar{x} - \frac{\left(\frac{n-1}{2}.\frac{n+1}{2}\right)}{2}\right\}$$

$$= \frac{(n-1).(n+1)}{2} - \frac{(n-1).(n+1)}{4}$$

$$\sum |x_i - \bar{x}| = \frac{n^2-1}{4}$$

$$\frac{\sum|x_i-\bar{x}|}{n}=\frac{n^2-1}{4n}=\frac{5n+5}{n}$$

$\Rightarrow$ $n^2-20n-21=0$

$(n+1)(n-21)=0$

$n=21$

Question ID : 1626

Vectors

26. Let $\vec{b}=\hat{i}+\hat{j}+\lambda\hat{k},\lambda\in R$. If $\vec{a}$ is a vector such that $\vec{a}\times\vec{b}=13\hat{i}-\hat{j}-4\hat{k}$ and $\vec{a}.\vec{b}+21=0$, then $(\vec{b}-\vec{a}).(\hat{k}-\hat{j})+(\vec{b}+\vec{a}).(\hat{i}-\hat{k})$ is equal to _____.

माना $\vec{b}=\hat{i}+\hat{j}+\lambda\hat{k},\lambda\in R$ है। यदि एक सदिश $\vec{a}$ के लिए $\vec{a}\times\vec{b}=13\hat{i}-\hat{j}-4\hat{k}$ तथा $\vec{a}.\vec{b}+21=0$ हैं, तो, $(\vec{b}-\vec{a}).(\hat{k}-\hat{j})+(\vec{b}+\vec{a}).(\hat{i}-\hat{k})$ बराबर है _____

Ans. Official Answer NTA (14)

Sol. Let $\vec{a}=xi+yj+zk$

$\vec{b}=i+j+\lambda k$

$\vec{a}.\vec{b}=x+y+\lambda z=-21$

$$\vec{a}\times\vec{b}=\begin{vmatrix} i & j & k \\ x & y & z \\ 1 & 1 & \lambda \end{vmatrix}$$

$=(\lambda y-z)\,i-(\lambda x-z)j+(x-y)k=13i-j-4k$

$$\Rightarrow \left.\begin{matrix} \lambda y-z=13 \\ \lambda x-z=1 \\ x-y=-4 \end{matrix}\right\}\Rightarrow \begin{matrix} \lambda=3 \\ x=-2 \\ y=2 \\ z=-7 \end{matrix}$$

$\vec{a}=-2i+2j-7k$

$\vec{b}=i+j+3k$

$\vec{b}-\vec{a}=3i-j+10k$

$\vec{b}+\vec{a}=-i+3j-4k$

$(\vec{b}.\vec{a}).(k-j)+(\vec{b}+\vec{a}).(i-k)=(10+1)+(-1+4)$

$=14$

Question ID : 1627

P & C

27. The total number of three-digit numbers, with one digit repeated two times, is _____.

तीन अंकों की संख्याओं, जिनमें एक अंक ठीक दो बार हो, की कुल संख्या है _____

Ans. Official Answer NTA (243)

Sol. without zero + with zero (100 or 110)

$= {}^{9}C_{2}.{}^{2}C_{1}.\frac{3!}{2!}+{}^{9}C_{1}.\{1+2\}$

$= 243$

Question ID : 1628

Maxima & Minima

28. Let $f(x) = |(x-1)(x^2-2x-3)| + x - 3$, $x \in R$. If m and M are respectively the number of points of local minimum and local maximum of f in the interval (0, 4), then m + M is equal to _____.

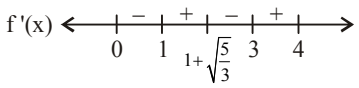
माना $f(x) = |(x-1)(x^2-2x-3)| + x - 3$, $x \in R$ है। यदि अंतराल (0, 4) में f के स्थानीय निम्नतम तथा स्थानीय उच्चतम बिन्दुओं की संख्या क्रमशः m + M हैं, तो m + M बराबर है _____

Ans. Official Answer NTA (3)

Sol. $f(x) = |(x-1)(x-3)(x+1)| + (x-3)$

$$f(x)=\begin{cases} x^3-3x^2 & , \ x\in(0,1) \\ -x^3+3x^2+2x-6 & , \ x\in(1,3) \\ x^3-3x^2 & , \ x\in(3,4) \end{cases}$$

$$f'(x)=\begin{cases} 3x^2-6x & , \ x\in(0,1) \\ -3x^2+6x+2 & , \ x\in(1,3) \\ 3x^2-6x & , \ x\in(3,4) \end{cases}$$



local minima at x = 1, 3

local maxima at $x = 1+\sqrt{\frac{5}{3}}$

m = 2, M = 1

m + M = 3

Question ID : 1629

Hyperbola

29. Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be $\frac{5}{4}$. If the equation of the normal at the point $\left(\frac{8}{\sqrt{5}},\frac{12}{5}\right)$ on the hyperbola is $8\sqrt{5}x+\beta y=\lambda$, then $\lambda-\beta$ is equal to _____.

माना अतिपरवलय $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ की उत्केन्द्रता $\frac{5}{4}$ है। यदि अतिपरवलय के बिन्दु $\left(\frac{8}{\sqrt{5}},\frac{12}{5}\right)$ पर अभिलम्ब का समीकरण $8\sqrt{5}x+\beta y=\lambda$ है, तो $\lambda-\beta$ बराबर है _____

Ans. Official Answer NTA (85)

Sol. H : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$$e^2=1+\frac{b^2}{a^2}=\frac{25}{16}\Rightarrow\frac{b^2}{a^2}=\frac{9}{16}$$

eq. of normal at (x_1, y_1)

$$\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2+b^2$$

$$\frac{x}{x_1}+\frac{\frac{b^2}{a^2}}{y_1}y=1+\frac{b^2}{a^2}$$

$$\frac{x}{x_1}+\frac{9y}{16y_1}=\frac{25}{16}$$

$$\frac{x}{\frac{8}{\sqrt{5}}}+\frac{9y}{16.\frac{12}{5}}=\frac{25}{16}$$

$$8\sqrt{5}x+15y=100$$

$$\beta=15,\ \gamma=100$$

$$\gamma-\beta=85$$

Question ID : 1630

3D Geometry

30. Let l_1 be the line in xy-plane with x and y intercepts $\frac{1}{8}$ and $\frac{1}{4\sqrt{2}}$ respectively and l_2 be the line in zx-plane with x and z intercepts $-\frac{1}{8}$ and $-\frac{1}{6\sqrt{3}}$ respectively. If d is the shortest distance between the line l_1 and l_2, then d^{-2} is equal to _____.

माना रेखा l_1 xy-समतल में है तथा इसके x तथा y अंतःखंड $\frac{1}{8}$ और $\frac{1}{4\sqrt{2}}$ है तथा रेखा l_2 zx-समतल में है तथा इसके x तथा z अंतःखंड क्रमशः $-\frac{1}{8}$ और $-\frac{1}{6\sqrt{3}}$ हैं। यदि रेखाओं l_1 तथा l_2 के बीच न्यूनतम दूरी d है, तो d^{-2} बराबर है _____

Ans. Official Answer NTA (51)

Sol. $l_1 : \frac{x}{\frac{1}{8}} + \frac{y}{\frac{1}{4\sqrt{2}}} = 1$ and $z = 0$

$l_1 : \frac{x-\frac{1}{8}}{\sqrt{2}} = \frac{y-0}{-2} = \frac{z-0}{0}$, $A = \left(\frac{1}{8}, 0, 0\right)$, $\vec{p} = \sqrt{2}i - 2j$

$l_2 : \frac{x}{-\frac{1}{8}} + \frac{z}{\frac{-1}{6\sqrt{3}}} = 1$ and $y = 0$

$l_2 : \frac{x+\frac{1}{8}}{3\sqrt{3}} = \frac{y-0}{0} = \frac{z-0}{4}$, $B = \left(\frac{-1}{8}, 0, 0\right)$, $\vec{q} = 3\sqrt{3}i + 4k$

d = Projection of $\overrightarrow{AB}$ on $\vec{p} \times \vec{q}$

$\overrightarrow{AB} = \frac{-1}{4}i$

$$\vec{p} \times \vec{q} = \begin{vmatrix} i & j & k \\ \sqrt{2} & -2 & 0 \\ 3\sqrt{3} & 0 & 4 \end{vmatrix}$$

$\vec{p} \times \vec{q} == -2\left(4i + 2\sqrt{2}j - 3\sqrt{3}\right)$

$d = \left|\frac{\overrightarrow{AB}.(\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|}\right|$

$d = \frac{1}{\sqrt{51}}$

$d^{-2} = 51$