JEE Main July 2022

Question Paper With Text Solution

25 July | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JULY 2022 | 25TH JULY SHIFT-1

SECTION – A

Question ID : 100001

Function

1. The number of functions, f : {1, 2, 3, 4} $\rightarrow$ {1, 2, 3, 4, 5, 6} such that f(1) + f(2) = f(3), is equal to :

फलनों f : {1, 2, 3, 4} $\rightarrow$ {1, 2, 3, 4, 5, 6}, जिनके लिए f(1) + f(2) = f(3) है, की कुल संख्या है :

(1) 60 (2) 90 (3) 108 (4) 126

Ans. Official Answer NTA (2)

Sol. A = {1, 2, 3, 4}

B = {1, 2, 3, 4, 5, 6}

Here f(3) can be 2, 3, 4, 5, 6

f(3) = 2, (f(1), f(2)) $\rightarrow$ (1,1) $\rightarrow$ 6 cases

f(3) = 3, (f(1), f(2)) $\rightarrow$ (1,2), (2,1)

$\rightarrow$ 2 × 6 = 12 cases

f(3) = 4, (f(1), f(2)) $\rightarrow$ (1,3), (3,1), (2,2)

$\rightarrow$ 3 × 6 = 18 cases

f(3) = 5, (f(1), f(2)) $\rightarrow$ (1,4), (4,1), (2,3), (3,2)

$\rightarrow$ 4 × 6 = 24 cases

f(3) = 6, (f(1), f(2)) $\rightarrow$ (1,5),(5,1),(2,4),(4,2),(3,3)

$\rightarrow$ 5 × 6 = 30 cases

Total number of cases = 6+ 12 + 18 + 24 + 30 = 90

Question ID : 100002

Complex Number

2. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 + x^3 + x^2 + x + 1 = 0$, then $\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021}$ is equal to :

यदि समीकरण $x^4 + x^3 + x^2 + x + 1 = 0$ के मूल $\alpha, \beta, \gamma, \delta$ हैं, तो $\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021}$ बराबर हैं: :

(1) –4 (2) –1 (3) 1 (4) 4

Ans. Official Answer NTA (2)

Sol. $x^4 + x^3 + x^2 + x + 1 = 0$ OR $\dfrac{x^5 - 1}{x - 1} = 0\left(x \neq 1\right)$

So roots are $e^{i2\pi/5}, e^{i4\pi/5}, e^{i6\pi/5}, e^{i8\pi/5}$

i.e. α, β, γ and δ

From properties of nth root of unity

$1^{2021} + \alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021} = 0$

$\Rightarrow \alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021} = -1$

Question ID : 100003

Complex Number

3. For $n \in N$, let $S_n = \left\{z \in C : |z - 3 + 2i| = \frac{n}{4}\right\}$ and $T_n = \left\{z \in C : |z - 2 + 3i| = \frac{1}{n}\right\}$. Then the number of elements in the set $\{n \in N : S_n \cap T_n = \phi\}$ is :

$n \in N$ के लिए, $S_n = \left\{z \in C : |z - 3 + 2i| = \frac{n}{4}\right\}$ तथा $T_n = \left\{z \in C : |z - 2 + 3i| = \frac{1}{n}\right\}$ हैं। तो समुच्चय $\{n \in N : S_n \cap T_n = \phi\}$ में अवयवों की संख्या है :

(1) 0 (2) 2 (3) 3 (4) 4

Ans. Official Answer NTA (4)

Sol. $S_n : |z - (3 - 2i)| = \frac{n}{4}$ is a circle center C_1 (3, –2) and radius n/4

$T_n : |Z - (2 - 3i)| = \frac{1}{n}$ is a circle center C_2 (2, –3) and radius 1/n

Here $S_n \cap T_n = \phi$

Both circles do not intersec each other

Case-1 : $C_1C_2 > n/4 + 1/n$

$\sqrt{2} > \frac{n}{4} + \frac{1}{n}$

then n = 1, 2, 3, 4

Case-2 : $C_1C_2 < \left|\frac{n}{4} - \frac{1}{n}\right|$

$\Rightarrow \sqrt{2} < \left|\frac{n^2 - 4}{4n}\right|$

$\Rightarrow$ n has infinite solutions for $n \in \mathbf{N}$

Question ID : 100004

Determinant

4. The number of $\theta \in (0, 4\pi)$ for which the system of linear equations

अंतराल $(0, 4\pi)$ में θ के मानों जिनके लिए रैखिक समीकरण निकाय

$$3 (\sin 3\theta) x - y + z = 2$$

$$3 (\cos 2\theta) x + 4y + 3z = 3$$

$6x + 7y + 7z = 9$

has no solution, is :

का कोई हल नहीं है, की संख्या है :

(1) 6 (2) 7 (3) 8 (4) 9

Ans. Official Answer NTA (2)

Sol. The system of equation has no solution.

$$D = \begin{vmatrix} 3\sin 3\theta & -1 & 1 \\ 3\cos 2\theta & 4 & 3 \\ 6 & 7 & 7 \end{vmatrix} = 0$$

$21 \sin 3\theta + 42 \cos 2\theta - 42 = 0$

$\sin 3\theta + 2 \cos 2\theta - 2 = 0$

Number of solution is 7 in $(0,4\pi)$

Question ID : 100005

Limit

5. If $\lim_{n\to\infty}\left(\sqrt{n^2 - n - 1} + n\alpha + \beta\right) = 0$, then $8(\alpha + \beta)$ is equal to :

यदि $\lim_{n\to\infty}\left(\sqrt{n^2 - n - 1} + n\alpha + \beta\right) = 0$ है, तो $8(\alpha + \beta)$ बराबर है :

(1) 4 (2) –8 (3) –4 (4) 8

Ans. Official Answer NTA (3)

Sol. $\lim_{n\to\infty}\left(\sqrt{n^2 - n - 1} + n\alpha + \beta\right) = 0$

$$= \lim_{n\to\infty}\left[\sqrt{1 - \frac{1}{n} - \frac{1}{n^2}} + \alpha + \frac{\beta}{n}\right] = 0$$

$\therefore \alpha = -1$

Now,

$$\lim_{n\to\infty} n\left[\left\{1 - \left(\frac{1}{n} + \frac{1}{n^2}\right)\right\}^{1/2} + \frac{\beta}{n} - 1\right] = 0$$

$$= \lim_{n\to\infty} \frac{\left(1-\frac{1}{2}\left(\frac{1}{n}+\frac{1}{n^2}\right)+.....\right)+\frac{\beta}{n}-1}{\frac{1}{n}} = 0$$

$$\Rightarrow \beta - \frac{1}{2} = 0$$

$$\therefore \beta = \frac{1}{2}$$

Now, $8(\alpha+\beta) = 8\left(-\frac{1}{2}\right) = -4$

Question ID : 100006

Maxima & Minima

6. If the absolute maximum value of the function $f(x) = (x^2 - 2x + 7)\, e^{(4x^3 - 12x^2 - 180x + 31)}$ in the interval $[-3, 0]$ is $f(\alpha)$, then :

यदि अंतराल $[-3, 0]$ में फलन $f(x) = (x^2 - 2x + 7)\, e^{(4x^3 - 12x^2 - 180x + 31)}$ का निरपेक्ष उच्चतम मान $f(\alpha)$ है, तो :

(1) $\alpha = 0$ (2) $\alpha = -3$ (3) $\alpha \in (-1, 0)$ (4) $\alpha \in (-3, -1]$

Ans. Official Answer NTA (2)

Sol. $f'(x) = e^{(4x^3 - 12x^2 - 180x + 31)}\left(12\left(x^2 - 2x + 7\right)(x+3)(x-5) + 2(x-1)\right)$

for $x \in [-3, 0]$

$\Rightarrow f'(x) < 0$

$f(x)$ is decreasing function on $[-3, 0]$

The absolute maximum value of the function $f(x)$ is at $x = -3$

$\Rightarrow \alpha = -3$

Question ID : 100007

7. The curve $y(x) = ax^3 + bx^2 + cx + 5$ touches the x-axis at the point $P(-2, 0)$ and cuts the y-axis at the point Q, where y' is equal to 3. Then the local maximum value of $y(x)$ is :

वक्र $y(x) = ax^3 + bx^2 + cx + 5$, x-अक्ष को बिन्दु $P(-2, 0)$ पर स्पर्श करता है तथा y-अक्ष को बिन्दु Q पर y' का मान 3 है। तो $y(x)$ का स्थानीय उच्चतम मान है :

(1) $\frac{27}{4}$ (2) $\frac{29}{4}$ (3) $\frac{37}{4}$ (4) $\frac{9}{2}$

Ans. Official Answer NTA (1)

Sol. $y(x) = ax^3 + bx^2 + cx + 5$ is passing through

(-2,0) then $8a - 4b + 2c = 5$........(1)

$y'(x) = 3ax^2 + 2bx + c$ touches x-axis at (-2,0)

$12a - 4b + c = 0$ ....(2)

again, for $x = 0$, $y'(x) = 3$

$c = 3$ ....(3)

Solving eq. (1), (2) & (3) $a = -\frac{1}{2}, b = -\frac{3}{4}$

$y'(x) = -\frac{3}{2}x^2 - \frac{3}{2}x + 3$

y(x) has local maxima at x = 1

$y(1) = \frac{27}{4}$

Question ID : 100008

Area Under Curve

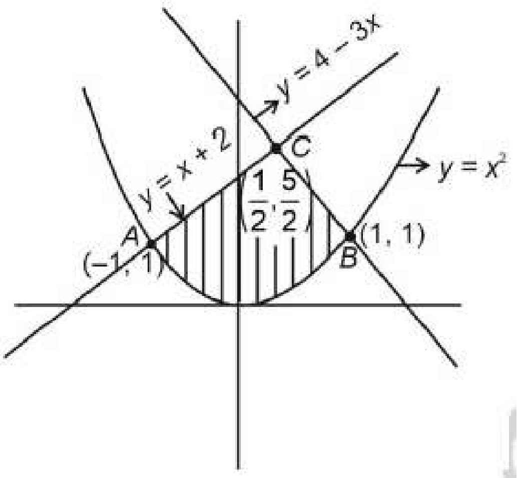
8. The area of the region given by $A = \{(x, y) : x^2 \le y \le \min\{x + 2, 4 - 3x\}$ is :

$A = \{(x, y) : x^2 \le y \le \min\{x + 2, 4 - 3x\}$ द्वारा दिए गए क्षेत्र का क्षेत्रफल है :

(1) $\frac{31}{8}$ (2) $\frac{17}{6}$ (3) $\frac{19}{6}$ (4) $\frac{27}{8}$

Ans. Official Answer NTA (2)

Sol. $A = \{(x, y) : x^2 \le y \le \min\{x + 2, 4 - 3x\}\}$



So area of requierd region

$$A=\int_{-1}^{\frac{1}{2}}\left(x+2-x^2\right)dx+\int_{\frac{1}{2}}^{1}\left(4-3x-x^2\right)dx$$

$$=\left[\frac{x^2}{2}+2x-\frac{x^3}{3}\right]_{-1}^{\frac{1}{2}}+\left[4x-\frac{3x^2}{2}-\frac{x^3}{3}\right]_{\frac{1}{2}}^{1}$$

$$=\left(\frac{1}{8}+1-\frac{1}{24}\right)-\frac{1}{2}-2+\frac{1}{3}+\left(1-\frac{3}{2}-\frac{1}{3}\right)-\left(2-\frac{3}{8}-\frac{1}{24}\right)$$

$$=\frac{17}{6}$$

Question ID : 100009

Definite Integration

9. For any real number x, let[x] denote the largest integer less than equal to x. Let f be a real valued function defined on the interval [–10, 10] by $f(x)=\begin{cases}x-[x], & \text{if } [x] \text{ is odd}\\ 1+[x]-x, & \text{if } [x] \text{ is even}\end{cases}$. Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos\pi x\,dx$ is :

किसी वास्तविक संख्या x के लिए, माना [x] महत्तम पूर्णांक ≤ x है। माना अंतराल [–10, 10] में एक वास्तविक मान फलन $f(x)=\begin{cases}x-[x], & \text{यदि } [x] \text{ विषम है}\\ 1+[x]-x, & \text{यदि } [x] \text{ सम है}\end{cases}$ द्वारा परिभाषित है। तो $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos\pi x\,dx$ का मान है :

(1) 4 (2) 2 (3) 1 (4) 0

Ans. Official Answer NTA (1)

Sol. $f(x)=\begin{cases}\{x\}, & 2n-1\le x<2n\\ 1-\{x\}, & 2n\le x<2n+1\end{cases}$

Clearly f(x) is a periodic function with period = 2

Hence f(x) . cos πx is also periodic with period = 2

$$\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos(\pi x)dx=\pi^2\int_0^2 f(x)\cos(\pi x)\,dx=\pi^2\int_0^1((1-\{x\})+\{-x\})\cos(\pi x)dx$$

$$=2\pi^2\int_0^1(-x\cos\pi x)dx=-2\pi^2\left[\frac{x\sin\pi x}{\pi}+\frac{\cos\pi x}{\pi^2}\right]_0^1=-2\pi^2\left(-\frac{2}{\pi^2}\right)=4$$

Question ID : 100010

Definite Integration

10. The slope of the tangent to a curve $C : y = y(x)$ at any point (x, y) on it is $\frac{2e^{2x} - 6e^{-x} + 9}{2 + 9e^{-2x}}$. If C passes through the points $\left(0, \frac{1}{2} + \frac{\pi}{2\sqrt{2}}\right)$ and $\left(\alpha, \frac{1}{2}e^{2\alpha}\right)$, then e^{α} is equal to :

वक्र $C : y = y(x)$ के किसी बिन्दु (x, y) पर स्पर्श रेखा की प्रवणता $\frac{2e^{2x} - 6e^{-x} + 9}{2 + 9e^{-2x}}$ है। यदि C, बिन्दुओं $\left(0, \frac{1}{2} + \frac{\pi}{2\sqrt{2}}\right)$ तथा $\left(\alpha, \frac{1}{2}e^{2\alpha}\right)$ से होकर जाता है, तो e^{α} बराबर है :

(1) $\frac{3+\sqrt{2}}{3-\sqrt{2}}$ (2) $\frac{3}{\sqrt{2}}\left(\frac{3+\sqrt{2}}{3-\sqrt{2}}\right)$ (3) $\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$ (4) $\frac{\sqrt{2}+1}{\sqrt{2}-1}$

Ans. Official Answer NTA (2)

Sol. $\frac{dy}{dx} = \frac{2e^{2x} - 3e^{-x} + 9}{2 + 9e^{-2x}}$

$\frac{dy}{dx} = e^{2x} - \frac{6e^{x}}{2e^{2x} + 9}$

$y = \frac{e^{2x}}{2} - \tan^{-1}\left(\frac{\sqrt{2}e^{x}}{3}\right) + c$

If C passes through the point $\left(0, \frac{1}{2} + \frac{\pi}{2\sqrt{2}}\right)$

$c = -\frac{\pi}{4} - \tan^{-1}\frac{\sqrt{2}}{3}$

Again C passes through the point $\left(\alpha, \frac{1}{2}e^{2\alpha}\right)$

then $e^{\alpha} = \frac{3}{\sqrt{2}}\left(\frac{3+\sqrt{2}}{3-\sqrt{2}}\right)$

Question ID : 100011

Differential Equation

11. The general solution of the differential equation $(x - y^2)dx + y(5x + y^2)dy = 0$ is :

अवकल समीकरण $(x - y^2)dx + y(5x + y^2)dy = 0$ का व्यापक हल है :

(1) $(y^2 + x)^4 = C|(y^2 + 2x)^3|$ (2) $(y^2 + 2x)^4 = C|(y^2 + x)^3|$

(3) $|(y^2 + x)^3| = C(2y^2 + x)^4$ (4) $|(y^2 + 2x)^3| = C(2y^2 + x)^4$

Ans. Official Answer NTA (1)

Sol. $\left(x - y^2\right)dx + y\left(5x + y^2\right)dy = 0$

$$y\frac{dy}{dx} = \frac{y^2 - x}{5x + y^2}$$

Let $y^2 = t$

$$\frac{1}{2}.\frac{dt}{dx} = \frac{t - x}{5x + t}$$

Now substitute, $t = vx$

$$\frac{dt}{dx} = v + x\frac{dv}{dx}$$

$$\frac{1}{2}\left\{v + x\frac{dv}{dx}\right\} = \frac{v - 1}{5 + v}$$

$$x\frac{dv}{dx} = \frac{2v - 2}{5 + v} - v = \frac{-3v - v^2 - 2}{5 + v}$$

$$\int\frac{5 + v}{v^2 + 3v + 2}dv = \int -\frac{dx}{x}$$

$$\int\frac{4}{v + 1}dv - \int -\frac{3}{v + 2}dv = -\int\frac{dx}{x}$$

$4 \ln|v + 1| - 3 \ln|v + 2| = -\ln x + \ln C$

$$\left|\frac{(v + 1)^4}{(v + 2)^3}\right| = \frac{C}{x}$$

$$\left|\frac{\left(\frac{y^2}{x} + 1\right)^4}{\left(\frac{y^2}{x} + 2\right)^3}\right| = \frac{C}{x}$$

$$\left|\left(y^2 + x\right)^4\right| = C\left|\left(y^2 + 2x\right)^3\right|$$

Question ID : 100012

Straight Line

12. A line, with the slope greater than one, passes through the point A(4, 3) and intersects the line $x - y - 2 = 0$ at the point B. If the length of the line segment AB is $\frac{\sqrt{29}}{3}$, then B also lies on the line :

एक रेखा, जिसकी प्रवणता एक से अधिक है, बिन्दु A(4, 3) से होकर जाती है तथा रेखा $x - y - 2 = 0$ को बिन्दु B पर काटती है। यदि रेखाखंड AB की लंबाई $\frac{\sqrt{29}}{3}$ है, तो बिन्दु निम्न में से किस रेखा पर भी स्थित है :

(1) $2x + y = 9$ (2) $3x - 2y = 7$ (3) $x + 2y = 6$ (4) $2x - 3y = 3$

Ans. Official Answer NTA (3)

Sol. Let $B(x_1, x_1 - 2)$

$\sqrt{(x_1-4)^2 + (x_1-2-3)^2} = \frac{\sqrt{29}}{3}$

Squaring on both side

$18x_1^2 - 162x_1 + 340 = 0$

$x_1 = \frac{51}{9}$ or $x_1 = \frac{10}{3}$

$y_1 = \frac{33}{9}$ or $y_1 = \frac{4}{3}$

Option (C) will satisfy $\left(\frac{10}{3}, \frac{4}{3}\right)$

Question ID : 100013

Circle

13. Let the locus of the centre (α, β), $\beta > 0$, of the circle which touches the circle $x^2 + (y - 1)^2 = 1$ externally and also touches the x-axis be L. Then the area bounded by L and the line $y = 4$ is :

माना एक वृत्त, जो वृत्त $x^2 + (y - 1)^2 = 1$ को बाह्यत: स्पर्श करता है तथा x - अक्ष को भी स्पर्श करता है, के केन्द्र (α, β), $\beta > 0$ का बिन्दुपथ L है। तो L तथा रेखा $y = 4$ द्वारा परिबद्ध क्षेत्र का क्षेत्रफल है :

(1) $\frac{32\sqrt{2}}{3}$ (2) $\frac{40\sqrt{2}}{3}$ (3) $\frac{64}{3}$ (4) $\frac{32}{3}$

Ans. Official Answer NTA (3)

Sol. $(\alpha - 0)^2 + (\beta - 1)^2 = (\beta + 1)^2$

$\alpha^2 = 4\beta$

$x^2 = 4y$

$$A = 2\int_0^4 \left(4 - \frac{x^2}{4}\right)dx = \frac{64}{3}$$

Question ID : 100014

3D Geometry

14. Let P be the plane containing the straight line $\frac{x-3}{9} = \frac{y+4}{-1} = \frac{z-7}{-5}$ and perpendicular to the plane containing the straight lines $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$ and $\frac{x}{3} = \frac{y}{7} = \frac{z}{8}$. If d is the distance of P from the point(2, –5, 11), then d^2 is equal to :

माना रेखा $\frac{x-3}{9} = \frac{y+4}{-1} = \frac{z-7}{-5}$, समतल P में स्थित है तथा समतल P, रेखाओं $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$ तथा $\frac{x}{3} = \frac{y}{7} = \frac{z}{8}$ को अंतर्विष्ट करने वाले समतल के लंबवत है। यदि P की बिन्दु (2, –5, 11) से दूरी d है, तो d^2 बराबर है :

(1) $\frac{147}{2}$ (2) 96 (3) $\frac{32}{3}$ (4) 54

Ans. Official Answer NTA (4)

Sol. Let <a, b, c> be direction ratios of plane containing

lines $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$ and $\frac{x}{3} = \frac{y}{7} = \frac{z}{8}$.

$\therefore$ 2a + 3b + 5c = 0 ....(i)

and 3a + 7b + 8c = 0 ....(ii)

from eq. (i) and (ii) : $\frac{a}{24-35} = \frac{b}{15-16} = \frac{c}{14-9}$

$\therefore$ D.R^S. of plane are < 11, 1, –5>

Let D.R^S of plane P be $<a_1, b_1, c_1>$ then.

$11a_1 + b_1 - 5c_1 = 0$ ....(iii)

and $9a_1 - b_1 - 5c_1 = 0$ ....(iv)

From eq. (iii) and (iv) :

$$\frac{a_1}{-5-5} = \frac{b_1}{-45-55} = \frac{c_1}{-11-9}$$

$\therefore$ D.A^5. of plane P are < 1, –1, 2>

Equation plane P is : 1(x –3) –1(y + 4) +2(z – 7) = 0

$\Rightarrow x - y + 2z - 21 = 0$

Distance from point (2, –5, 11) is $d = \frac{|2+5+22-2|}{\sqrt{6}}$

$\therefore \quad d^2 = \frac{32}{3}$

Question ID : 100015

Vectors

15. Let ABC be a triangle such that $\overrightarrow{BC} = \vec{a}$, $\overrightarrow{CA} = \vec{b}$, $\overrightarrow{AB} = \vec{c}$, $|\vec{a}| = 6\sqrt{2}$, $|\vec{b}| = 2\sqrt{3}$ and $\vec{b}.\vec{c} = 12$. Consider the statements :

(S1) : $\left|\left(\vec{a}\times\vec{b}\right)+\left(\vec{c}\times\vec{b}\right)\right|-|\vec{c}| = 6\left(2\sqrt{2}-1\right)$

(S2) : $\angle ACB = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

Then :

(1) Both (S1) and (S2) are true
(2) Only (S1) is true
(3) Only (S2) is true
(4) Both (S1) and (S2) are false

माना एक त्रिभुज ABC के लिए $\overrightarrow{BC} = \vec{a}$, $\overrightarrow{CA} = \vec{b}$, $\overrightarrow{AB} = \vec{c}$, $|\vec{a}| = 6\sqrt{2}$, $|\vec{b}| = 2\sqrt{3}$ तथा $\vec{b}.\vec{c} = 12$ हैं :

कथनों :

(S1) : $\left|\left(\vec{a}\times\vec{b}\right)+\left(\vec{c}\times\vec{b}\right)\right|-|\vec{c}| = 6\left(2\sqrt{2}-1\right)$

(S2) : $\angle ACB = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

में :

(1) (S1) तथा (S2) दोनों सत्य हैं
(2) केवल (S1) सत्य हैं
(3) केवल (S2) सत्य है
(4) दोनों (S1) तथा(S2) असत्य हैं

Ans. Official Answer NTA (4)

Sol. $\vec{a}+\vec{b}+\vec{c} = 0$

$\vec{b}+\vec{c} = -\vec{a}$

$|\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b}\cdot\vec{c} = |\vec{a}|^2$

$|\vec{c}|^2 = 36$

$|\vec{c}| = 6$

$S1 : |\vec{a}\times\vec{b}+\vec{c}\times\vec{b}| - |\vec{c}|$

$|(\vec{a}+\vec{c})\times\vec{b}|-|\vec{c}|$

$|-\vec{b}\times\vec{b}|-|\vec{c}|$

$0-6=-6$

$S2:\vec{a}+\vec{b}+\vec{c}=0$

$\vec{b}+\vec{c}=-\vec{a}$

$|\vec{a}|^2+|\vec{b}|^2-2|\vec{a}||\vec{b}|\cos(\angle ACB)=|\vec{c}|^2$

$\cos(\angle ACB)=\sqrt{\frac{2}{3}}$

Question ID : 100016

Probability

16. If the sum and the product of mean and variance of a binomial distribution are 24 and 128 respectively, then the probability of one or two successes is :

यदि एक द्विपद बंटन के माध्य तथा प्रसरण के योग और गुणनफल क्रमशः 24 तथा 128 हैं, तो एक या दो सफलताओं की प्रायिकता है :

(1) $\frac{33}{2^{32}}$ (2) $\frac{33}{2^{29}}$ (3) $\frac{33}{2^{28}}$ (4) $\frac{33}{2^{27}}$

Ans. Official Answer NTA (3)

Sol. $np + npq = 24$ ....(1)

$np \cdot npq = 128$ ....(2)

Solving (1) and (2) :

We get $p=\frac{1}{2}, q=\frac{1}{2}, n=32.$

Now,

$P(X = 1) + P(X = 2)$

$= {}^{32}C_1\, pq^{31} + {}^{32}C_2\, p^2 q^{30}$

$=\frac{33}{2^{28}}$

Question ID : 100017

Probability

17. If the numbers appeared on two throws of a fair six faced die are α and β, then the probability that $x^2 + \alpha x + \beta > 0$, for all $x \in R$, is :

यदि छः फलकों के एक न्यास पासे को दो बार फेंकने पर प्रकट होने वाली संख्याएँ α तथा β हैं, तो सभी $x \in R$ के लिए

$x^2 + \alpha x + \beta > 0$ होने की प्रायिकता है :

(1) $\frac{17}{36}$ (2) $\frac{9}{4}$ (3) $\frac{1}{2}$ (4) $\frac{19}{36}$

Ans. Official Answer NTA (1)

Sol. For $x^2 + \alpha x + \beta > 0 \ \forall \ x \in R$ to hold, we should have $\alpha^2 - 4\alpha\beta < 0$

If $\alpha = 1$, β can be 1, 2, 3, 4, 5, 6 i.e., 6 choices

If $\alpha = 2$, β can be 2, 3, 4, 5, 6 i.e., 5 choices

If $\alpha = 3$, β can be 3, 4, 5, 6 i.e., 4 choices

If $\alpha = 4$, β can be 5 or 6 i.e., 2 choices

If $\alpha = 6$, No possible value for β i.e., 0 choices

Hence total favourable outcomes

$= 6 + 5 + 4 + 2 + 0 + 0$

$= 17$

Total possible choices for α and $\beta = 6 \times 6 = 36$

Required probability $= \frac{17}{36}$

Question ID : 100018

Trigonometric Equation

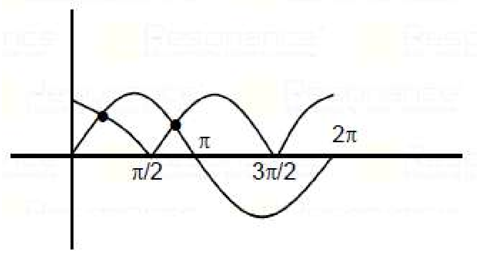
18. The number of solutions of $|\cos x| = \sin x$, $-4\pi \le x \le 4\pi$ is :

समीकरण $|\cos x| = \sin x$, $-4\pi \le x \le 4\pi$ के हलों की संख्या है :

(1) 4 (2) 6 (3) 8 (4) 12

Ans. Official Answer NTA (3)

Sol.



$\because$ $|\cos x|$ is periodic will period π and sinx is periodic with 2π

and as shown in figure $|\cos x| = \sin x$ is having 2 solutions

$\therefore$ Total number of solution of the equation $|\cos x| = \sin x$ is $[-4\pi, 4\pi]$ is equal to 4 5 2 = 8

Question ID : 100019

Heights & Distances

19. A tower PQ stands on a horizontal ground with base Q on the ground. The point R divides the tower in two

parts such that QR = 15m. If from a point A on the ground the angle of elevation of R is 60° and the parts PR of the tower subtends an angle of 15° at A, then the height of the tower is :

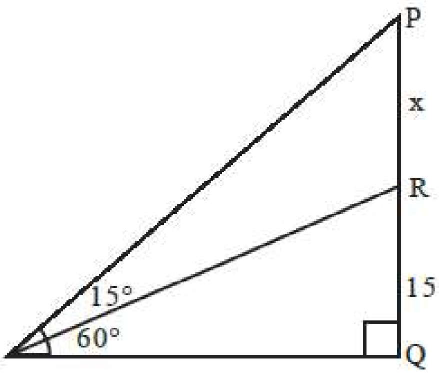
एक टावर PQ एक क्षैतिज पर खड़ा है। टावर का आधार Q धरातल पर है। बिन्दु R, टावर को दो भागों में बाँटता है जबकि QR = 15 मीटर है। यदि धरातल पर एक बिन्दु A से R का उन्नयन कोण 60° है, तथा टावर का भाग PR, बिन्दु A पर 15° का कोण बनाता है, तो टावर ऊँचाई (मीटर में) है:

(1) $5(2\sqrt{3}+3)$m (2) $5(\sqrt{3}+3)$m

(3) $10(\sqrt{3}+1)$m (4) $10(2\sqrt{3}+1)$m

Ans. Official Answer NTA (1)

Sol.



$$\frac{15}{AQ} = \tan 60^\circ \quad .....(1)$$

$$\frac{15+x}{AQ} = \tan 75^\circ \quad ......(2)$$

$$\frac{(1)}{(2)} \Rightarrow x = 10\sqrt{3}$$

So, $PQ = 5(2\sqrt{3}+3)$m

Question ID : 100020

Mathematical Reasoning

20. Which of the following statements is a tautology?

निम्न कथनों में कौनसा पुनरुक्ति है ?

(1) $((\sim p) \vee q) \Rightarrow p$ (2) $p \Rightarrow ((\sim p) \vee q)$

(3) $((\sim p) \vee q) \Rightarrow q$ (4) $q \Rightarrow ((\sim p) \vee q)$

Ans. Official Answer NTA (4)

Sol. Truth Table

p	q	~ p	~ q	$(\sim p)\vee q$	$((\sim p)\vee q)\rightarrow p$	$p\rightarrow((\sim p)\vee q)$	$(\sim p)\vee q\rightarrow q$	$q\rightarrow((\sim p)\vee q)$
T	T	F	F	T	T	T	T	T
T	F	F	T	F	T	F	T	T
F	T	T	F	T	F	T	T	T
F	F	T	T	T	F	T	F	T

SECTION – B

Question ID : 100021

Complex Number

21. Let $A=\begin{pmatrix}2 & -1 & -1\\ 1 & 0 & -1\\ 1 & -1 & 0\end{pmatrix}$ and B = A – I. If $\omega=\frac{\sqrt{3}i-1}{2}$, then the number of elements in the set $\left\{n\in\{1,2,....,100\}:A^n+(\omega B)^n=A+B\right\}$ is equal to ________.

माना $A=\begin{pmatrix}2 & -1 & -1\\ 1 & 0 & -1\\ 1 & -1 & 0\end{pmatrix}$ तथा $B=A-I$ हैं। यदि $\omega=\frac{\sqrt{3}i-1}{2}$ है, तो $\left\{n\in\{1,2,....,100\}:A^n+(\omega B)^n=A+B\right\}$ है, में अवयवों की संख्या है _______।

Ans. Official Answer NTA (17)

Sol. $A=\begin{bmatrix}2 & -1 & -1\\ 1 & 0 & -1\\ 1 & -1 & 0\end{bmatrix}\Rightarrow A^2=A\Rightarrow A^n=A.$

$\forall n\in\{1,2,.....,100\}$

Now, $B=A-I=\begin{bmatrix}1 & -1 & -1\\ 1 & -1 & -1\\ 1 & -1 & -1\end{bmatrix}$

$B^2=-B$

$\Rightarrow B^3=-B^2=B$

$\Rightarrow B^5=B$

$\Rightarrow B^{99}=B$

Also, $\omega^{3k} = 1$
So, n = common of {1, 3, 5, ., 99} and
{3, 6, 9, ., 99} = 17

Question ID : 100022

P & C

22. The letters of the word 'MANKIND' are written in all possible orders and arranged in serial order as in an English dictionary. Then the serial number of the word 'MANKIND' is _________.

शब्द 'MANKIND' के अक्षरों को सभी संभव क्रमों में लिखा जाता है तथा अंग्रेजी शब्दकोश की तरह क्रमानुसार व्यवस्थित किया जाता है। तो 'MANKIND' की क्रम संख्या है _________।

Ans. Official Answer NTA (1492)

Sol.

M	A	N	K	I	N	D

$$\left(\frac{4\times 6!}{2!}\right)+(5!\times 0)+\left(\frac{4!\times 3}{2!}\right)+(3!\times 2)+(2!\times 1)+(1!\times 1)+(0!\times 0)+1=1492$$

Question ID : 100023

Binomial Theorem

23. If the maximum value of the term independent of t in the expansion of $\left(t^2x^{\frac{1}{5}}+\frac{(1-x)^{\frac{1}{10}}}{t}\right)^{15}$, $x \geq 0$, is K, then 8 K is equal to ________.

यदि $\left(t^2x^{\frac{1}{5}}+\frac{(1-x)^{\frac{1}{10}}}{t}\right)^{15}$, $x \geq 0$ के प्रसार t से स्वतंत्र पद का अधिकतम K है, तो 8 K बराबर है ________.

Ans. Official Answer NTA (6006)

Sol. General Term = $15C_r\left(t^2\,x^{\frac{1}{5}}\right)^{15-r}\left(\frac{(1-x)^{\frac{1}{10}}}{t}\right)^{r}$

for tem independent on t
$2(15 - r) - r = 0$
$\Rightarrow r = 10$
$\therefore T_{11} = {}^{15}C_{10}x(1-x)$

Maximum value of x (1 – x) occur at $x = \frac{1}{2}$

i.e., $(x(1-x))_{max} = \frac{1}{4}$

$\Rightarrow K = {}^{15}C_{10} \times \frac{1}{4}$

$\Rightarrow 8K = 2\left({}^{15}C_{10}\right) = 6006$

Question ID : 100024

Sequence & progression

24. Let a, b be two non-zero real numbers. If p and r are the roots of the equation $x^2 - 8ax + 2a = 0$ and q and s are the roots of the equation $x^2 + 12bx + 6b = 0$, such that $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$ are in A.P., then $a^{-1} - b^{-1}$ is equal to _________.

माना a, b दो शून्येत्तर वास्तविक संख्याएँ हैं। यदि समीकरण $x^2 - 8ax + 2a = 0$ के मूल p मूल r हैं और समीकरण $x^2 + 12bx + 6b = 0$, के मूल है q तथा s हैं, जबकि $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$ A.P. में हैं, तो $a^{-1} - b^{-1}$ बराबर है _________।

Ans. Official Answer NTA (38)

Sol.

$x^2 - 8ax + 2a = 0$ $x^2 + 12bx + 6b = 0$

$p + r = 8a$ $q + s = -12b$

$pr = 2a$ $qs = 6b$

$\frac{1}{p} + \frac{1}{r} = 4$ $\frac{1}{q} + \frac{1}{s} = -2$

$\frac{2}{q} = 4$ $\frac{2}{r} = -2$

$q = \frac{1}{2}$ $r = -1$

$p = \frac{1}{5}$ $s = \frac{-1}{4}$

Now, $\frac{1}{a} - \frac{1}{b} = \frac{2}{pr} - \frac{6}{qs} = 38$

Question ID : 100025

Sequence & progression

25. Let $a_1 = b_1 = 1$, $a_n = a_{n-1} + 2$ and $b_n = a_n + b_{n-1}$ for every natural number $n \geq 2$. Then $\sum_{n=1}^{15} a_n.b_n$ is equal to _________.

माना $a_1 = b_1 = 1$ हैं तथा प्रत्येक $n \geq 2$ के लिए $a_n = a_{n-1} + 2$ और $b_n = a_n + b_{n-1}$ हैं। तो $\sum_{n=1}^{15} a_n.b_n$ बराबर है

________।

Ans. Official Answer NTA (27560)

Sol. $a_1 = b_1 = 1$

$a_2 = a_1 + 2 = 3$

$a_3 = a_2 + 2 = 5$

$a_4 = a_2 + 2 = 7$

$\Rightarrow an = 2n - 1$

$b_2 = a_1 + b_1 = 4$

$b_3 = a_3 + b_2 = 9$

$b_4 = a_4 + b_3 = 16$

$b_n = n^2$

$\sum_{n=1}^{15} a_n b_n$

$\sum_{n=1}^{15} (2n-1)n^2$

$\sum_{n=1}^{15} (2n^3 - n^2)$

$= 2\frac{n^2(n+1)^2}{4} - \frac{n(n+1)(2n+1)}{6}$

Put n = 15

$= \frac{2\times 225\times 16\times 16}{4} - \frac{15\times 16\times 31}{6}$

$= 27560$

Question ID : 100026

Continuity & Differentiability

26. $f(x) = \begin{cases} |4x^2 - 8x + 5|, & \text{if } 8x^2 - 6x + 1 \geq 0 \\ [4x^2 - 8x + 5] & \text{if } 8x^2 - 6x + 1 < 0 \end{cases}$ where $[\alpha]$ denotes the greatest integer less than or equal to α. Then the number of points in R where f is not differentiable is ________.

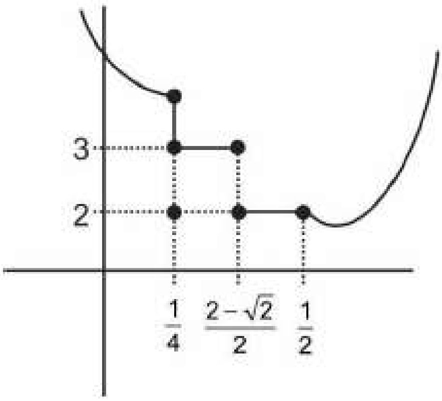
माना $f(x) = \begin{cases} |4x^2 - 8x + 5|, & \text{if } 8x^2 - 6x + 1 \geq 0 \\ [4x^2 - 8x + 5] & \text{if } 8x^2 - 6x + 1 < 0 \end{cases}$ जहाँ $[\alpha]$ महत्तम पूर्णांक α है। तो R में उन बिन्दुओं की संख्या, जहाँ f अवकलनीय नहीं है, है ________।

Ans. Official Answer NTA (3)

Sol. $f(x)=\begin{cases}\left|4x^2-8x+5\right|, & \text{if } 8x^2-6x+1\ge 0\\ \left[4x^2-8x+5\right], & \text{if } 8x^2-6x+1<0\end{cases}$

$$=\begin{cases}4x^2-8x+5, & \text{if } x\in\left[-\infty,\frac{1}{4}\right]\cup\left[\frac{1}{2},\infty\right)\\ \left[4x^2-8x+5\right] & \text{if } x\in\left(\frac{1}{4},\frac{1}{2}\right)\end{cases}$$

$$f(x)=\begin{cases}4x^2-8x+5 \text{ if } & x\in\left(-\infty\frac{1}{4}\right]\cup\left[\frac{1}{2},\infty\right)\\ 3 & x\in\left(\frac{1}{4},\frac{2-\sqrt{2}}{2}\right)\\ 2 & x\in\left[\frac{2-\sqrt{2}}{2},\frac{1}{2}\right)\end{cases}$$



$\therefore$ Non–diff at $x=\frac{1}{4},\frac{2-\sqrt{2}}{2},\frac{1}{2}$

Question ID : 100027

Definite Integration

27. If $\lim_{n\to\infty}\frac{(n+1)^{k-1}}{n^{k+1}}\left[(nk+1)+(nk+2)+\ldots+(nk+n)\right]=33.\ \lim_{n\to\infty}\frac{1}{n^{k+1}}.\left[1^k+2^k+3^k+\ldots+n^k\right]$, then the integral value of k is equal to _________.

यदि $\lim_{n\to\infty}\frac{(n+1)^{k-1}}{n^{k+1}}\left[(nk+1)+(nk+2)+\ldots+(nk+n)\right]=33.\ \lim_{n\to\infty}\frac{1}{n^{k+1}}.\left[1^k+2^k+3^k+\ldots+n^k\right]$ हैं, तो k का

पूर्णांक मान बराबर है _________।

Ans. Official Answer NTA (5)

Sol. LHS

$$\lim_{n\to\infty}\frac{(n+1)^{k-1}}{n^{k+1}}[nk.n+1+2+....+n]$$

$$=\lim_{n\to\infty}\frac{(n+1)^{k-1}}{n^{k+1}}.\left[n^2k+\frac{n(n+1)}{2}\right]$$

$$=\lim_{n\to\infty}\frac{(n+1)^{k-1}.n^2\left(k+\frac{\left(1+\frac{1}{n}\right)}{2}\right)}{n^{k+1}}$$

$$\Rightarrow\lim_{n\to\infty}\left(1+\frac{1}{n}\right)\left(k+\frac{\left(1+\frac{1}{n}\right)}{2}\right)$$

$$\Rightarrow\left(k+\frac{1}{2}\right)$$

RHS

$$\Rightarrow\lim_{n\to\infty}\frac{1}{n^{k+1}}\left(1^k+2^k+....+n^k\right)=\frac{1}{k+1}$$

LHS = RHS

$$\Rightarrow k+\frac{1}{2}=33.\frac{1}{k+1}$$

$\Rightarrow (2k + 1)(k + 1) = 66$

$\Rightarrow (k - 5)(2k + 13) = 0$

$$\Rightarrow k = 5 \text{ or } -\frac{13}{2}$$

Question ID : 100028

Hyperbola

28. Let the equation of two diameters of a circle $x^2 + y^2 - 2x + 2fy + 1 = 0$ be $2px - y = 1$ and $2x + py = 4p$. Then then slope $m \in (0, \infty)$ of the tangent to the hyperbola $3x^2 - y^2 = 3$ passing through the centre of the circle is equal to _________.

माना एक वृत्त $x^2 + y^2 - 2x + 2fy + 1 = 0$ के दो व्यासों के समीकरण $2px - y = 1$ तथा $2x + py = 4p$ हैं। तो अतिपरवलय $3x^2 - y^2 = 3$ की एक स्पर्श रेखा, जो वृत्त के केन्द्र होकर जाती है, की प्रवणता $m \in (0, \infty)$ बराबर है ________.

Ans. Official Answer NTA (2)

Sol. $2p + f - 1 = 0$ ......(1)

$2 - pf - 4p = 0$ ......(2)

$2 = p(f + 4)$

$p = \dfrac{2}{f+4}$

$2p = 1 - f$

$\dfrac{4}{f+4} = 1 - f$

$f^2 + 3f = 0$

$f = 0$ or -3

Hyperbola $3x^2 - y^2 = 3$, $x^2 - \dfrac{y^2}{3} = 1$

$y = mx \pm \sqrt{m^2 - 3}$

It passes (1, 0)

$o = m \pm \sqrt{m^2 - 3}$

m tends ∞

It passes (1, 3)

$3 = m \pm \sqrt{m^2 - 3}$

$(3 - m)^2 = m^2 - 3$

$m = 2$

Question ID : 100029

Parabola

29. The sum of diameters of the circles that touch (i) the parabola $75x^2 = 64(5y - 3)$ at the point $\left(\dfrac{8}{5}, \dfrac{6}{5}\right)$ and (ii) the y-axis, is equal to ________.

वृत्तों, जो (i) परवलय $75x^2 = 64(5y - 3)$ को बिन्दु $\left(\dfrac{8}{5}, \dfrac{6}{5}\right)$ पर तथा (ii) y-अक्ष को स्पर्श करते हैं, के व्यासों का योग है _______।

Ans. Official Answer NTA (10)

Sol. $x^2 = \frac{64.5}{75}\left(y - \frac{3}{5}\right)$

equation of tangent at $\left(\frac{8}{5}, \frac{6}{5}\right)$

$$x - \frac{8}{5} = \frac{64}{15}\left(\frac{y + \frac{6}{5}}{2} - \frac{3}{5}\right)$$

$3x - 4y = 0$

equation of family of circle is

$$\left(x - \frac{8}{5}\right)^2 + \left(y - \frac{6}{5}\right)^2 + \lambda(3x - 4y) = 0$$

It touches y axis so $f^2 = c$

$$x^2 + y^2 + x\left(3\lambda - \frac{16}{5}\right) + y\left(-4\lambda - \frac{12}{5}\right) + 4 = 0$$

$$\frac{\left(4\lambda + \frac{12}{5}\right)^2}{4} = 4$$

$\lambda = \frac{2}{5}$ or $\lambda = -\frac{8}{5}$

$\lambda = \frac{2}{5}$, $r = 1$

$\lambda = -\frac{8}{5}$, $r = 4$

$d_1 + d_2 = 10$

Question ID : 100030

3D Geometry

30. The line of shortest distance between the lines $\frac{x-2}{0} = \frac{y-1}{1} = \frac{z}{1}$ and $\frac{x-3}{2} = \frac{y-5}{2} = \frac{z-1}{1}$ makes an angle of $\cos^{-1}\left(\sqrt{\frac{2}{27}}\right)$ with the plane P : $ax - y - z = 0$, $(a > 0)$. If the image of the point (1, 1, –5) in the plane P is (α, β, γ), then $\alpha + \beta - \gamma$ is equal to _________.

रेखाओं $\frac{x-2}{0} = \frac{y-1}{1} = \frac{z}{1}$ तथा $\frac{x-3}{2} = \frac{y-5}{2} = \frac{z-1}{1}$ के बीच न्यूनतम दूरी की रेखा, समतल P : $ax - y - z = 0$,

$(a > 0)$ से $\cos^{-1}\left(\sqrt{\frac{2}{27}}\right)$ का कोण बनाती है। यदि बिन्दु $(1, 1, -5)$ P में प्रतिबिंब (α, β, γ) है, तो $\alpha + \beta - \gamma$ बराबर है

________।

Ans. Official Answer NTA (3)

Sol. DR's of line of shortest distance

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ 2 & 2 & 1 \end{vmatrix} = -\hat{i} + 2\hat{j} - 2\hat{k}$$

angle between line and plane is $\cos^{-1}\sqrt{\frac{2}{27}} = \alpha$

$$\cos\alpha = \left|\frac{-a - 2 + 2}{\sqrt{4+4+1}\sqrt{a^2+1+1}}\right| = \frac{5}{3\sqrt{3}}$$

$$\sqrt{3}\,|a| = 5\sqrt{a^2+2}$$

$$3a^2 = 25a^2 + 50$$

No value of (a)