JEE Main February 2021

Question Paper With Text Solution

25 Feb.| Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN FEB 2021 | 25TH FEB SHIFT-1

1. The value of $\int_{-1}^{1} x^2 e^{[x^3]} dx$, where [t] denotes the greatest integer $\leq t$, is :

$\int_{-1}^{1} x^2 e^{[x^3]} dx$, जहाँ [t] महत्तम पूर्णांक $\leq t$ है, का मान है :

(1) $\frac{e+1}{3e}$ (2) $\frac{e+1}{3}$ (3) $\frac{e-1}{3e}$ (4) $\frac{1}{3e}$

Question Type : MCQ

Question ID : 70819117651

Option 1 ID : 70819158112

Option 2 ID : 70819158109

Option 3 ID : 70819158111

Option 4 ID : 70819158110

Ans. Official Answer NTA : (1)

Sol. $\int_{-1}^{1} x^2 e^{[x^3]} dx$

$= \int_{-1}^{0} x^2 e^{-1} dx + \int_{0}^{1} x^2 e^{0} dx$

$= \frac{1}{e}\int_{-1}^{0} x^2 dx + \int_{0}^{1} x^2 dx$

$= \frac{1}{e}\left(\frac{x^3}{3}\right)_{-1}^{0} + \left(\frac{x^3}{3}\right)_{1}^{0}$

$= \frac{1}{e}\left(0+\frac{1}{3}\right)+\frac{1}{3}$

$= \frac{e+1}{3e}$

2. The total number of positive integral solutions (x, y, z) such that xyz = 24 is :

xyz = 24 के धन पूर्णांक हलों (x, y, z) की कुल संख्या है :

(1) 24 (2) 36 (3) 45 (4) 30

Question Type : MCQ

Question ID : 70819117661

Option 1 ID : 70819158149

Option 2 ID : 70819158151

Option 3 ID : 70819158152

Option 4 ID : 70819158150

Ans. Official Answer NTA : (4)

Sol. $xyz = 24 = 2^3 \times 3$

Power of 2 & 3 are to be distributed among x, y, z

Powers of 2 :

Number of ways = $5_{C_2} = 10$

Powers of 3 :

Number of ways = $3_{C_2} = 3$

Required number of ways = $10 \times 3 = 30$

3. If $0 < \theta, \phi < \frac{\pi}{2}$, $x = \sum_{n=0}^{\infty} \cos^{2n}\theta$, $y = \sum_{n=0}^{\infty} \sin^{2n}\phi$ and $z = \sum_{n=0}^{\infty} \cos^{2n}\theta \cdot \sin^{2n}\phi$ then :

यदि $0 < \theta, \phi < \frac{\pi}{2}$, $x = \sum_{n=0}^{\infty} \cos^{2n}\theta$, $y = \sum_{n=0}^{\infty} \sin^{2n}\phi$ तथा $z = \sum_{n=0}^{\infty} \cos^{2n}\theta \cdot \sin^{2n}\phi$ हैं, तो :

(1) xy + yz + zx = z (2) xy – z = (x + y)z (3) xyz = 4 (4) xy + z = (x + y)z

Question Type : MCQ

Question ID : 70819117647

Option 1 ID : 70819158095

Option 2 ID : 70819158093

Option 3 ID : 70819158096

Option 4 ID : 70819158094

Ans. Official Answer NTA : (4)

Sol. $x = 1 + \cos^2\theta + \cos^4\theta + ...\infty$

$$x = \frac{1}{1-\cos^2\theta} = \frac{1}{\sin^2\theta}$$

$y = 1 + \sin^2\phi + \sin^4\phi + ...\infty$

$$y = \frac{1}{1-\sin^2\phi} = \frac{1}{\cos^2\phi}$$

$z = 1 + \cos^2\theta\ \sin^2\phi + \cos^4\theta\ \sin^4\phi + ...$

$$z = \frac{1}{1-\cos^2\theta\sin^2\phi}$$

$$z = \frac{1}{1-\left(1-\sin^2\theta\right)\left(1-\cos^2\phi\right)}$$

$$z = \frac{1}{1-\left(1-\frac{1}{x}\right)\left(1-\frac{1}{y}\right)}$$

By simplification

$(x + y)z = xy + z$

4. The value of the integral $\int \frac{\sin\theta.\sin 2\theta(\sin^6\theta+\sin^4\theta+\sin^2\theta)\sqrt{2\sin^4\theta+3\sin^2\theta+6}}{1-\cos 2\theta}d\theta$ is :

(where c is a constant of integration)

समाकलन $\int \frac{\sin\theta.\sin 2\theta(\sin^6\theta+\sin^4\theta+\sin^2\theta)\sqrt{2\sin^4\theta+3\sin^2\theta+6}}{1-\cos 2\theta}d\theta$ बराबर हैं :

(जहाँ c एक समाकलन अचर है)

(1) $\frac{1}{18}\left[11-18\sin^2\theta+9\sin^4\theta-2\sin^6\theta\right]^{\frac{3}{2}}+c$

(2) $\frac{1}{18}\left[9-2\cos^6\theta-3\cos^4\theta-6\cos^2\theta\right]^{\frac{3}{2}}+c$

(3) $\frac{1}{18}\left[11-18\cos^2\theta+9\cos^4\theta-2\cos^6\theta\right]^{\frac{3}{2}}+c$

(4) $\frac{1}{18}\left[9-2\sin^6\theta-3\sin^4\theta-6\sin^2\theta\right]^{\frac{3}{2}}+c$

Question Type : MCQ

Question ID : 70819117650

Option 1 ID : 70819158107

Option 2 ID : 70819158106

Option 3 ID : 70819158108

Option 4 ID : 70819158105

Ans. Official Answer NTA : (3)

Sol. $I = \int \frac{\sin\theta.\sin 2\theta(\sin^6\theta+\sin^4\theta+\sin^2\theta)\sqrt{2\sin^4\theta+3\sin^2\theta+6}}{1-\cos 2\theta}d\theta$

$I = \int \frac{\sin\theta(2\sin\theta\cos\theta)(\sin^6\theta+\sin^4\theta+\sin^2\theta)\sqrt{2\sin^4\theta+3\sin^2\theta+6}}{2\sin^2\theta}d\theta$

$I = \int (\sin^6\theta+\sin^4\theta\sin^2\theta)\cos\theta\sqrt{2\sin^4\theta+3\sin^2\theta+6}\,d\theta$

$I = \int (\sin^5\theta+\sin^3\theta\sin\theta)\cos\theta\sqrt{2\sin^6\theta+3\sin^4\theta+6\sin^2\theta}\,d\theta$

Let, $2\sin^6\theta + 3\sin^4\theta + 6\sin^2\theta = t$

$12(\sin^5\theta + \sin^3\theta + \sin\theta)\cos\theta\, d\theta = dt$

$(\sin^5\theta + \sin^3\theta + \sin\theta)\cos\theta\, d\theta = \frac{dt}{12}$

$I = \frac{1}{12}\int t^{\frac{1}{2}}dt$

$= \frac{1}{18}t^{3/2}+C$

$= \frac{1}{18}(2\sin^6\theta+3\sin^4\theta+6\sin^2\theta)^{\frac{3}{2}}+C$

$= \frac{1}{18}\left(2(1-\cos^2\theta)^3+3(1-\cos^2\theta)^2+6(1-\cos^2\theta)\right)+C$

$= \frac{1}{18}(11-2\cos^6\theta+9\cos^4\theta-18\cos^2\theta)^{\frac{3}{2}}+C$

5. The coefficients a, b and c of the quadratic equation, $ax^2 + bx + c = 0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is :

द्विघातीय समीकरण $ax^2 + bx + c = 0$ के गुणांक a, b तथा c एक पासे को तीन बार उछाल कर प्राप्त किए जाते हैं। इस समीकरण के मूल बराबर होने की प्रायिकता है :

(1) $\frac{1}{72}$ (2) $\frac{1}{36}$ (3) $\frac{1}{54}$ (4) $\frac{5}{216}$

Question Type : MCQ

Question ID : 70819117659

Option 1 ID : 70819158143

Option 2 ID : 70819158141

Option 3 ID : 70819158142

Option 4 ID : 70819158144

Ans. Official Answer NTA : (4)

Sol. $ax^2 + bx + c = 0$

Roots are equal, hence

$D = 0$

$b^2 = 4ac$

$a, b, c \in \{1, 2, 3, 4, 5, 6\}$

$b = 2 \quad a = 1 \quad c = 1$

$b = 4 \quad a = 2 \quad c = 2$

$b = 4 \quad a = 1 \quad c = 4$

$b = 4 \quad a = 4 \quad c = 1$

$b = 6 \quad a = 3 \quad c = 3$

$p = \frac{5}{6 \times 6 \times 6}$

$p = \frac{5}{216}$

6. All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in :

$\theta \in [0, 2\pi]$ के सभी संभव मान, जिनके लिए $\sin 2\theta + \tan 2\theta > 0$ है, निम्न में से किस में है ?

(1) $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

(2) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

(3) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

(4) $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

Question Type : MCQ

Question ID : 70819117660

Option 1 ID : 70819158145

Option 2 ID : 70819158146

Option 3 ID : 70819158148

Option 4 ID : 70819158147

Ans. Official Answer NTA : (3)

Sol. $\sin 2\theta + \tan 2\theta > 0$

$$\sin 2\theta + \frac{\sin 2\theta}{\cos 2\theta} > 0$$

$$\sin 2\theta \left(\frac{\cos 2\theta + 1}{\cos 2\theta}\right) > 0$$

$\tan 2\theta \, (2\cos^2 \theta) > 0$

$\Rightarrow \tan 2\theta > 0 \quad (\cos^2 \theta \neq 0)$

$\theta \in [0, 2\pi]$

$2\theta \in [0, 4\pi]$

$$2\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right) \cup \left(2\pi, \frac{5\pi}{2}\right) \cup \left(3\pi, \frac{7\pi}{2}\right)$$

$$\theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$$

7. If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is $\frac{x^2-4x+y+8}{x-2}$, then this curve also passes through the point :

यदि एक वक्र मूलबिंदू से होकर जाता है तथा इसके किसी बिंदु (x, y) पर स्पर्श रेखा की प्रवणता $\frac{x^2-4x+y+8}{x-2}$ है, तो यह वक्र निम्न में से किस बिंदु से भी होकर जाता है ?

(1) (4, 5) (2) (5, 5) (3) (5, 4) (4) (4, 4)

Question Type : MCQ

Question ID : 70819117652

Option 1 ID : 70819158113

Option 2 ID : 70819158114

Option 3 ID : 70819158115

Option 4 ID : 70819158116

Ans. Official Answer NTA : (2)

Sol. $\frac{dy}{dx}=\frac{x^2-4x+y+8}{x-2}$

$\frac{dy}{dx}=\frac{(x-2)^2+y+4}{(x-2)}$

Let, $x-2=X$

$y+4=Y \qquad \left[\frac{dY}{dX}=\frac{dy}{dx}\right]$

$\frac{dY}{dX}=\frac{X^2+Y}{X}$

$\frac{dY}{dX}=Y\left(\frac{-1}{X}\right)=X$

Linear D.E.

I.F. $= e^{\int\frac{-1}{X}dX}=\frac{1}{X}$

Now,

$$\frac{1}{X}\frac{dY}{dX}+Y\left(\frac{-1}{X^2}\right)=1$$

$$\frac{d}{dX}\left(\frac{Y}{X}\right)=1$$

By intergrating

$$\frac{Y}{X}=X+c$$

$Y = X^2 + cX$

$(y + 4) = (x – 2)^2 + c (x – 2)$

Assign through (0, 0)

Hence, $c = 0$

then, $(y + 4) = (x – 2)^2$

Which also passes through (5, 5)

8. $\lim\limits_{n\to\infty}\left(1+\frac{1+\frac{1}{2}+.....+\frac{1}{n}}{n^2}\right)^n$ is equal to :

$\lim\limits_{n\to\infty}\left(1+\frac{1+\frac{1}{2}+.....+\frac{1}{n}}{n^2}\right)^n$ बराबर है :

(1) $\frac{1}{e}$ (2) $\frac{1}{2}$ (3) 0 (4) 1

Question Type : MCQ

Question ID : 70819117649

Option 1 ID : 70819158103

Option 2 ID : 70819158102

Option 3 ID : 70819158101

Option 4 ID : 70819158104

Ans. Official Answer NTA : (4)

Sol. $$L = e\lim_{n\to\infty}\left(\frac{1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n}}{n}\right) = e^k \quad \text{(say)}$$

Let $n = 2^p + \lambda \qquad \lambda \in \{0,1,2.....2^p - 1\}$

$$1+\left(\frac{1}{2}+\frac{1}{3}\right)+\left(\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}\right)+.....+\left(\frac{1}{2^{p-1}}+\frac{1}{2^{p-1}+1}+.....+\frac{1}{2^p-1}\right)+\left(\frac{1}{2^p}+\frac{1}{2^p+1}+.....+\frac{1}{2^p+\lambda}\right)$$

$$S<1+\left(\frac{1}{2}+\frac{1}{2}\right)+\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right)+.....+\left(\underbrace{\frac{1}{2^p}+\frac{1}{2^p}+...+\frac{1}{2^p}}_{(\lambda+1)\text{time}}\right)$$

$$S<1+1+1+1+\frac{\lambda+1}{2p}<p+1$$

$$k \le \lim_{p\to\infty}\frac{p+1}{2p} = 0$$

$$S > \frac{\frac{1}{n}+\frac{1}{n}+.....+\frac{1}{n}}{n} = 1$$

$$k \ge \lim_{n\to\infty}\frac{1}{n} = 0$$

So, L = 1

9. A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x + y = 1$. Which of the following points does NOT lie on it ?

परवलय, $y^2 = 6x$ पर एक स्पर्श रेखा खींची गई है जो रेखा $2x + y = 1$ के लंबवत है। तो निम्न में से कौन सा बिंदु इस पर स्थित नहीं हैं ?

(1) (4, 5) (2) (–6, 0) (3) (5, 4) (4) (0, 3)

Question Type : MCQ

Question ID : 70819117654

Option 1 ID : 70819158124

Option 2 ID : 70819158122

Option 3 ID : 70819158123

Option 4 ID : 70819158121

Ans. Official Answer NTA : (3)

Sol. $y = mx + \frac{a}{m}$

$m = \frac{1}{2}$

$y = \frac{x}{2} + \frac{3}{2 \times \frac{1}{2}}$

$y = \frac{x}{2} + 3$

$2y = x + 6$

Clearly point (5, 4) does not lie on it.

10. If the curves, $\frac{x^2}{a} + \frac{y^2}{b} = 1$ and $\frac{x^2}{c} + \frac{y^2}{d} = 1$ intersect each other at an angle of 90°, then which of the following relations is TRUE ?

यदि वक्र $\frac{x^2}{a} + \frac{y^2}{b} = 1$ तथा $\frac{x^2}{c} + \frac{y^2}{d} = 1$ एक दूसरे को 90° के कोण पर काटते हैं, तो निम्न में से कौन सा संबंध सत्य है ?

(1) $a - b = c - d$ (2) $ab = \frac{c+d}{a+b}$ (3) $a + b = c + d$ (4) $a - c = b + d$

Question Type : MCQ

Question ID : 70819117655

Option 1 ID : 70819158126

Option 2 ID : 70819158127

Option 3 ID : 70819158125

Option 4 ID : 70819158128

Ans. Official Answer NTA : (1)

Sol. $\frac{x^2}{a} + \frac{y^2}{b} = 1$ ...(1)

$\frac{x^2}{c}+\frac{y^2}{d}=1$ ...(2)

Let P(x, y) be the point of intersection

$\frac{2x}{a}+\frac{2y}{b}\frac{dy}{dx}=0$

$m_1=\frac{dy}{dx}=\frac{-bx}{ay}$ ...(1)

$\frac{2x}{c}+\frac{2y}{d}\frac{dy}{dx}=0$

$m_2=\frac{dy}{dx}=\frac{-dx}{cy}$ ...(2)

$m_1m_2=-1$

$\frac{bdx^2}{acy^2}=-1$

$\frac{x^2}{y^2}=\frac{-ac}{bd}$ ...(3)

By (1) – (2)

$x^2\left(\frac{1}{a}-\frac{1}{c}\right)+y^2\left(\frac{1}{b}-\frac{1}{d}\right)=0$

$\frac{x^2(c-a)}{ac}=\frac{y^2(b-d)}{bd}$

$\frac{x^2}{y^2}=\frac{ac(b-d)}{bd(c-a)}$ ...(4)

By (3) and (4)

$\frac{-ac}{bd}=\frac{ac(b-d)}{bd(c-a)}$

$-c+a=b-d \Rightarrow a-b=c-d$

11. The image of the point (3, 5) in the line $x-y+1=0$, lies on :

रेखा $x-y+1=0$ में बिंदु (3, 5) का प्रतिबिंब निम्न में से किस पर स्थित है ?

(1) $(x-4)^2+(y+2)^2=16$ (2) $(x-2)^2+(y-4)^2=4$

(3) $(x-2)^2+(y-2)^2=12$ (4) $(x-4)^2+(y-4)^2=8$

Question Type : MCQ

Question ID : 70819117653

Option 1 ID : 70819158117

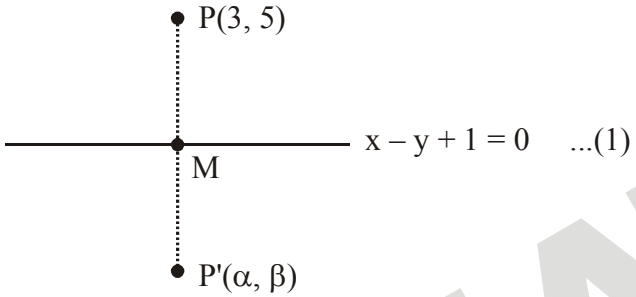
Option 2 ID : 70819158118

Option 3 ID : 70819158120

Option 4 ID : 70819158119

Ans. Official Answer NTA : (2)

Sol.



Equation of PP'

$y - 5 = -1(x - 3)$

$y - 5 = -x + 3$

$x + y - 8 = 0$ ...(2)

Point of intersection of (1) and (2)

$x = \frac{7}{2}$

$y = \frac{9}{2}$

$\frac{\alpha + 3}{2} = \frac{7}{2}$

$\alpha = 4$

$\frac{\beta + 5}{2} = \frac{9}{2}$

$\beta = 4$

Clearly (α , β) lies on curve $(x - 2)^2 + (y - 4)^2 = 4$

12. The equation of the line through the point (0, 1, 2) and perpendicular to the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{-2}$ is :

बिंदु (0, 1, 2) से होकर जाने वाली तथा रेखा $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{-2}$ के लबंवत रेखा का समीकरण है :

(1) $\frac{x}{-3}=\frac{y-1}{4}=\frac{z-2}{3}$ (2) $\frac{x}{3}=\frac{y-1}{4}=\frac{z-2}{3}$

(3) $\frac{x}{3}=\frac{y-1}{-4}=\frac{z-2}{3}$ (4) $\frac{x}{3}=\frac{y-1}{4}=\frac{z-2}{-3}$

Question Type : MCQ

Question ID : 70819117657

Option 1 ID : 70819158134

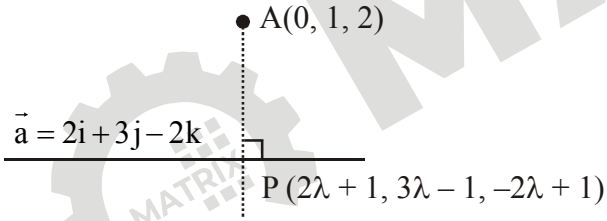
Option 2 ID : 70819158136

Option 3 ID : 70819158135

Option 4 ID : 70819158133

Ans. Official Answer NTA : (1)

Sol.



$\vec{AP}=(2\lambda+1)i+(3\lambda-2)j-(2\lambda+1)k$

Here, $\vec{AP}\cdot\vec{a}=0$

$2(2\lambda+1)+3(3\lambda-2)+2(2\lambda+1)=0$

$\lambda=\frac{2}{17}$

$\vec{AP}=\frac{21i}{17}-\frac{28j}{17}-\frac{21k}{17}$

Dr's < 21, –28, –21>

<3, –4, –3>

Hence equation of line

$\frac{x-0}{3}=\frac{y-1}{-4}=\frac{z-2}{-3}$

$\frac{x}{-3}=\frac{y-1}{4}=\frac{z-2}{3}$

13. Let f, g : N → N such that f(n + 1) = f(n) + f(1) $\forall$ n∈N and g be any arbitrary function. Which of the following statements is NOT true ?

(1) If g is onto, then fog is one–one

(2) If f is onto, then f(n) = n $\forall$ n∈N

(3) If fog is one–one, then g is one–one

(4) f is one–one

माना f, g : N → N हैं, जिनके लिए f(n + 1) = f(n) + f(1) $\forall$ n∈N है तथा g एक स्वेच्छ फलन है ? निम्न में से कौन सा कथन सत्य नहीं है ?

(1) यदि g आच्छादक है, तो fog एकैकी है

(2) यदि f आच्छादक है, तो f(n) = n $\forall$ n∈N है

(3) यदि fog एकैकी है, तो g एकैकी है

(4) f एकैकी है

Question Type : MCQ

Question ID : 70819117644

Option 1 ID : 70819158083

Option 2 ID : 70819158081

Option 3 ID : 70819158084

Option 4 ID : 70819158082

Ans. Official Answer NTA : (1)

Sol. f (n + 1) = f (n) + f (1)

n = 1

f (2) = f (1) + f (1)

f (2) = 2f (1)

f (3) = 3f (1)

f (4) = 4 + (1)

Option (1) : If 'g' is onto them fog might not be one - one, So it is wrong

Option (2) : f (n) = n

f (1) = 1

f (2) = 2

f (3) = 3

hence onto which is correct.

Option (3) : Correct, because fog is one - one them 'g' should by one - one.

Option (4) : f is one - one, hence this is also correct.

14. Let the lines $(2-i)z = (2+i)\,\bar{z}$ and $(2+i)z + (i-2)\,\bar{z} - 4i = 0$, (here $i^2 = -1$) be normal to a circle C. If the line $iz + \bar{z} + 1 + i = 0$ is tangent to this circle C, then its radius is :

(1) $3\sqrt{2}$ (2) $\frac{3}{2\sqrt{2}}$ (3) $\frac{3}{\sqrt{2}}$ (4) $\frac{1}{2\sqrt{2}}$

माना रेखाऐं $(2-i)z = (2+i)\,\bar{z}$ तथा $(2+i)z + (i-2)\,\bar{z} - 4i = 0$, (यहाँ $i^2 = -1$) एक वृत्त C पर अभिलम्ब हैं। यदि रेखा $iz + \bar{z} + 1 + i = 0$, वृत्त C की स्पर्श रेखा है, तो इसकी त्रिज्या है :

Question Type : MCQ

Question ID : 70819117645

Option 1 ID : 70819158085

Option 2 ID : 70819158087

Option 3 ID : 70819158088

Option 4 ID : 70819158086

Ans. Official Answer NTA : (2)

Sol. $z = x + iy$

$\bar{z} = x - iy$

$(2-i)z = (2+i)\bar{z}$

$(2-i)(x+iy) = (2+i)(x-iy)$

$x - 2y = 0 \quad ... (1)$

$(2+i)z + (i-2)\bar{z} - 4i = 0$

$(2+i)(x+iy) + (i-2)(x-iy) - 4i = 0$

$x+2y-2=0$ ...(2)

Point of intersection of (1) and (2) is centre of circle which is $\left(1,\frac{1}{2}\right)$

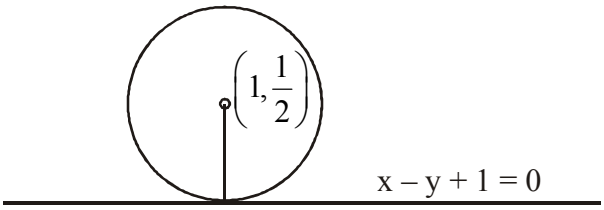
Now, tangent

$iz+\bar{z}+1+i=0$

$i(x+iy)+x-iy+1+i=0$

$(x-y+1)+i(x-y+1)=0$

$x-y+1=0$



$$r=\left|\frac{1-\frac{1}{2}+1}{\sqrt{2}}\right|$$

$$r=\frac{3}{2\sqrt{2}}$$

15. Let α be the angle between the lines whose direction cosines satisfy the equations $\ell+m-n=0$ and $\ell^2+m^2-n^2=0$. Then the value of $\sin^4\alpha+\cos^4\alpha$ is :

(1) $\frac{3}{8}$ (2) $\frac{1}{2}$ (3) $\frac{5}{8}$ (4) $\frac{3}{4}$

माना दो रेखाऐं जिनकी दिक्कोज्यायें समीकरणों $\ell+m-n=0$ तथा $\ell^2+m^2-n^2=0$ को सन्तुष्ट करती हैं, के बीच एक कोण α है। तो $\sin^4\alpha+\cos^4\alpha$ का मान है :

Question Type : MCQ

Question ID : 70819117656

Option 1 ID : 70819158130

Option 2 ID : 70819158131

Option 3 ID : 70819158129

Option 4 ID : 70819158132

Ans. Official Answer NTA : (3)

Sol. $\ell + m - n = 0$ ....(1)

$\ell^2 + m^2 - n^2 = 0$ ....(2)

We know $\ell^2 + m^2 + n^2 = 1$ ....(3)

by (3) – (1)

$2n^2 = 1$

$n = \pm\frac{1}{\sqrt{2}}$

if $n = \frac{1}{\sqrt{2}}$

by (1) & (2)

$\ell^L + m^L + (m+\ell)^2 = 0$

$\ell m = 0$

If $\ell = 0$ $m = n$

$\left\langle 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$

If, $m = 0$ $\ell = n$

$\left\langle \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right\rangle$

hence DC's of first line $\left\langle 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$

DC's second line $\left\langle \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right\rangle$

$\cos\alpha = 0 + 0 + \frac{1}{2}$ $\alpha = \frac{\pi}{3}$

$\sin^4\alpha + \cos^4\alpha = \left(\frac{\sqrt{3}}{2}\right)^4 + \left(\frac{1}{2}\right)^4 = \frac{5}{8}$

16. When a missile is fired from a ship, the probability that it is intercepted is $\frac{1}{3}$ and the probability that the missible hits the target, given that it is not intercepted, is $\frac{3}{4}$. If three missiles are fired independently from the ship, then the probability that all three hit the target, is :

(1) $\frac{3}{4}$ (2) $\frac{1}{8}$ (3) $\frac{3}{8}$ (4) $\frac{1}{27}$

जब एक प्रक्षेपास्त्र किसी जहाज से दागा जाता है, तो इसके अवरूद्ध होने की प्रायिकता $\frac{1}{3}$ है तथा यह दिए होने पर कि यह अवरूद्ध नहीं होता, इसके निशाने पर लगने की प्रायिकता $\frac{3}{4}$ है। यदि जहाज से तीन प्रक्षेपास्त्र स्वतंत्र रूप से दागे जाते हैं, तो सभी तीनों के निशाने पर लगनेष की प्रायिकता है :

Question Type : MCQ

Question ID : 70819117658

Option 1 ID : 70819158140

Option 2 ID : 70819158138

Option 3 ID : 70819158139

Option 4 ID : 70819158137

Ans. Official Answer NTA : (2)

Sol. P(When target got) = P(Missile not intercept) × P (hit the target)

$$= \frac{2}{3} \times \frac{3}{4}$$

$$= \frac{1}{2}$$

P(all three hit target) $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$$= \frac{1}{8}$$

17. A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At that point, angle of depression of the boat with the man's eye is 30° (Ignore man's height). After sailing for 20 seconds, towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45°. Then the time taken (in seconds) by the boat from B to reach the base of the tower is :

(1) $10\left(\sqrt{3}+1\right)$ (2) 10 (3) $10\left(\sqrt{3}-1\right)$ (4) $10\sqrt{3}$

एक स्तंभ के शीर्ष से एक पुरूष देख रहा है कि एक निश्चित बिंदु A से एक नाव एक समान गति से स्तंभ की ओर आ रही है। उस समय पुरूष की आँख से नाव का अवनमन कोण 30° है (पुरूष की ऊँचाई का ध्यान न दें) स्तंभ के आधार (जो पानी की सतह पर है) की तरफ नाव 20 सेकण्ड चलने के पश्चात् एक बिंदु B पर पहुँचती है, जहाँ अवनमन कोण 45° है। नाव के B से स्तंभ के आधार पर पहुँचने में लिया गया समय (सेकण्ड में) है :

Question Type : MCQ

Question ID : 70819117662

Option 1 ID : 70819158154

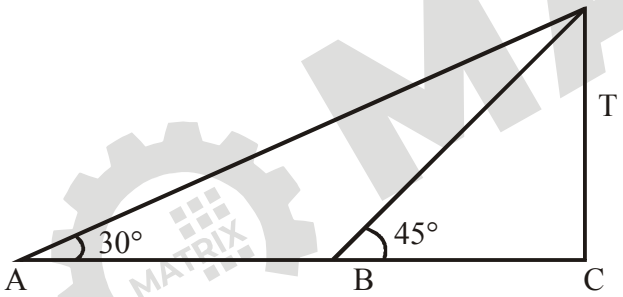
Option 2 ID : 70819158153

Option 3 ID : 70819158156

Option 4 ID : 70819158155

Ans. Official Answer NTA : (1)

Sol. Velocity = V



distance AB = v × t = 20v

distance BC = vt

$$\tan 30° = \frac{t}{20v + vt} \quad .....(1)$$

$$\tan 45° = \frac{t}{vt} \quad .....(2)$$

by 1 ÷ (2)

$$\frac{1}{\sqrt{3}} = \frac{t}{20 + t}$$

$$t\sqrt{3} = 20 + t$$

$t\left(\sqrt{3}-1\right)=20$

$t=10\left(\sqrt{3}+1\right)$

18. The statement $A \rightarrow (B \rightarrow A)$ is equivalent to :

कथन $A \rightarrow (B \rightarrow A)$ निम्न में से किसके तुल्य है ?

(1) $A \rightarrow (A \vee B)$ (2) $A \rightarrow (A \rightarrow B)$ (3) $A \rightarrow (A \leftrightarrow B)$ (4) $A \rightarrow (A \wedge B)$

Question Type : MCQ

Question ID : 70819117663

Option 1 ID : 70819158158

Option 2 ID : 70819158157

Option 3 ID : 70819158160

Option 4 ID : 70819158159

Ans. Official Answer NTA : (1)

Sol. $A \rightarrow (B \rightarrow A)$

$(\sim A) \vee (B \rightarrow A)$

$(\sim A) \vee ((\sim B) \vee A) = t$

(i) $A \rightarrow (A \vee B)$

$(-A) \vee (A \vee B) \equiv t$

(ii) $A \rightarrow (A \rightarrow B)$

$(\sim A) \vee (A \rightarrow B)$

$(\sim A) \vee ((\sim A) \vee B)$

$\equiv (\sim A) \vee B$

(iii) $\left((\wedge A) \vee A\right) \wedge \left((\wedge A) \vee B\right)$

$(\sim A) \vee B$

(iv) $A \rightarrow (A \wedge B)$

$(\sim A) \vee (A \wedge B)$

Hence, in option (1) there is tautology, so it is correct option.

19. If Rolle's theorem holds for the function $f(x) = x^3 - ax^2 + bx - 4, x \in [1, 2]$ with $f'\left(\frac{4}{3}\right) = 0$, then ordered pair (a, b) is equal to :

यदि $f'\left(\frac{4}{3}\right) = 0$ के साथ फलन $f(x) = x^3 - ax^2 + bx - 4, x \in [1, 2]$ के लिए रोले का प्रमेय लागू होता है, तो क्रमित युग्म (a, b) बराबर है :

(1) (– 5, – 8) (2) (– 5, 8) (3) (5, – 8) (4) (5, 8)

Question Type : MCQ

Question ID : 70819117648

Option 1 ID : 70819158100

Option 2 ID : 70819158099

Option 3 ID : 70819158098

Option 4 ID : 70819158097

Ans. Official Answer NTA : (4)

Sol. $f(1) = f(2)$

By Rolley's theorem

$1 - a + b - 4 = 8 - 4a + 2b - 4$

$3a - b - 7 = 0$ ... (1)

$f'\left(\frac{4}{3}\right) = 0$

$3x^2 - 2ax + b = 0$

$3\left(\frac{4}{3}\right)^2 - 2a\left(\frac{4}{3}\right) + b = 0$ ... (2)

By (1) and (2)

$a = 5$

$b = 8$

20. The integer 'k', for which the inequality $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$ is valid for every x in R, is :

(1) 0 (2) 4 (3) 2 (4) 3

पूर्णांक 'k', जिसके लिए असमिका $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$, R में प्रत्येक x के लिए मान्य है, है :

Question Type : MCQ

Question ID : 70819117646

Option 1 ID : 70819158092

Option 2 ID : 70819158091

Option 3 ID : 70819158089

Option 4 ID : 70819158090

Ans. Official Answer NTA : (4)

Sol. $D < 0$

$4(3k - 1)^2 - 4(8k^2 - 7) < 0$

$9k^2 - 6k + 1 - 8k^2 + 7 < 0$

$k^2 - 6k + 8 < 0$

$(k - 4)(k - 2) < 0$

$k \in (2, 4)$

$\Rightarrow k = 3$

1. Let f(x) be a polynomial of degree 6 in x, in which the coefficient of x^6 is unity and it has extrema at x $= -1$ and $x = 1$. If $\lim\limits_{x\to 0}\frac{f(x)}{x^3} = 1$, then 5.f(2) is equal to _____.

माना x में एक बहुपद f(x) की घात 6 है, तथा पद x^6 का गुणांक है और $x = -1$ तथा $x = 1$ इसके चरम बिन्दु हैं।

यदि $\lim\limits_{x\to 0}\frac{f(x)}{x^3} = 1$ है, तो 5.f(2) बराबर है :

Question Type : SA

Question ID : 70819117670

Ans. Official Answer NTA : (144)

Sol. $f(x) = x^6 + bx^5 + cx^4 + x^3 + ex^2 + fx + g$

$\lim\limits_{x\to 0}\frac{f(x)}{x^3} \Rightarrow d = 1$

$e = f = g = 0$

$f(x) = x^6 + bx^5 + cx^4 + x^3$

$f'(1) = f'(-1) = 0$

$f'(x) = 6x^5 + 5bx^4 + 4cx^3 + 3x^2$

$f'(1) = 6 + 5b + 4c + 3 = 0$ ... (1)

$f'(-1) = -6 + 5b - 4c + 3 = 0$ ... (2)

By (1) and (2)

$b = -\frac{3}{5}$

$c = -\frac{3}{2}$

$f(x) = x^6 - \frac{3}{5}x^5 - \frac{3}{2}x^4 + x^3$

$f(2) = \frac{144}{5}$

$5f(2) = 5\times\frac{144}{5} = 144$

2. The graphs of sine and cosine functions, intersect each other at a number of points and between two consecutive points of intersection, the two graphs enclose the same area A. Then A^4 is equal to _____.

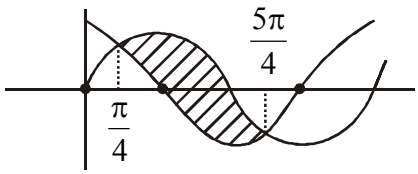
साइन तथा कोसाइन फलनों के ग्राफ एक दूसरे को बहुत से बिन्दुओं पर काटते हैं, तथा इनके दो क्रमागत प्रतिच्छेदन बिन्दुओं के बीच में ये दोनों ग्राफ एक समान क्षेत्रफल A घेरते हैं। तो A^4 बराबर है :

Question Type : SA

Question ID : 70819117671

Ans. Official Answer NTA : (64)

Sol.



$$A = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (\sin x - \cos x)\,dx$$

$$A = -\cos x - \sin x \Big]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$$

$$A = 2\sqrt{2}$$

$$A^4 = 64$$

3. The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4, 5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5, is _____.

अंकों 1, 2, 3, 4, 5 से 100 तथा 1000 के बीच की बनाई जा सकने वाली संख्याओं, यदि कोई भी अंक दोहराया नहीं जाता है तथा संख्याऐं या तो 3 से या 5 से विभाज्य हैं, की कुल संख्या है :

Question Type : SA

Question ID : 70819117667

Ans. Official Answer NTA : (32)

Sol. Numbers divisible by 3

Case-I	1, 2, 3	$\underline{\vert 3} = 6$
Case-II	1, 3, 5	$\underline{\vert 3} = 6$
Case-III	2, 3, 4	$\underline{\vert 3} = 6$
Case-IV	3, 4, 5	$\underline{\vert 3} = 6$

Numbers divisible by 5

		5

$4 \times 3 \times 1 = 12$

Common : 1, 3, 5 3, 1, 5

3, 4, 5 4, 3, 5

Ans = 24 + 12 – 4 = 32

4. If the system of equations

$kx + y + 2z = 1$

$3x - y - 2z = 2$

$-2x - 2y - 4z = 3$

has infinitely many solutions, then k is equal to _____.

यदि समीकरण निकाय

$kx + y + 2z = 1$

$3x - y - 2z = 2$

$-2x - 2y - 4z = 3$

के अनन्त हल हैं, तो k बराबर है :

Question Type : SA

Question ID : 70819117666

Ans. Official Answer NTA : (21)

Sol. Here, $D = Dx = Dy = Dz = 0$

$$D = \begin{vmatrix} k & 1 & 2 \\ 3 & -1 & -2 \\ -2 & -2 & 4 \end{vmatrix} = 0 \quad k \in R$$

$$Dx = \begin{vmatrix} 1 & 1 & 2 \\ 2 & -1 & -2 \\ 3 & -2 & -4 \end{vmatrix} = 0$$

$$Dy = \begin{vmatrix} k & 1 & 2 \\ 3 & 2 & -2 \\ -2 & 3 & -4 \end{vmatrix} = 0 \Rightarrow k = 21$$

$$Dz = \begin{vmatrix} k & 1 & 1 \\ 3 & -1 & 2 \\ -2 & -2 & 3 \end{vmatrix} = 0 \quad k = 21$$

Hence, $k = 21$

5. Let $A_1, A_2, A_3, \ldots\ldots\ldots$ be squares such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If the length of A_1 is 12 cm, then the smallest value of n for which area of A_n is less than one, is _____.

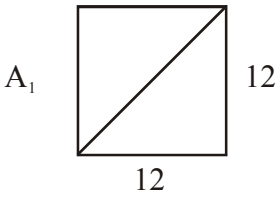
माना $A_1, A_2, A_3, \ldots\ldots\ldots$ वर्ग हैं जबकि प्रत्येक $n \geq 1$ के लिए A_n की भुजा की लम्बाई A_{n+1} के विकर्ण की लम्बाई के बराबर है। यदि A_1 की भुजा की लम्बाई 12 cm है, तो n का न्युनतम मान, जिसके लिए A_n का क्षेत्रफल एक से कम है, है :

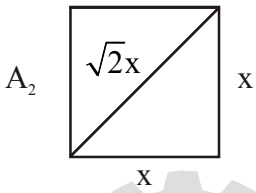
Question Type : SA

Question ID : 70819117668

Ans. Official Answer NTA : (9)

Sol.





given,

$\sqrt{2}x = 12$

$x = 6\sqrt{2}$

Square	Side	Area
A_1	12	144
A_2	$6\sqrt{2}$	72
–	–	–
–	–	–
–	–	–
A_8	$\frac{3}{\sqrt{2}}$	$\frac{9}{8}$
A_9	$\frac{3}{4}$	$\frac{9}{16} < 1$

$n = 9$

6. Let $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$, where x, y and z are real numbers such that $x + y + z > 0$ and $xyz = 2$. If $A^2 = I_3$, then the value of $x^3 + y^3 + z^3$ is _____.

माना $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$ है, जहाँ x, y तथा z वास्तविक संख्याऐं हैं, जिनके लिए $x + y + z > 0$ तथा $xyz = 2$ हैं।

यदि $A^2 = I_3$ है, तो $x^3 + y^3 + z^3$ का मान है :

Question Type : SA

Question ID : 70819117665

Ans. Official Answer NTA : (7)

Sol. $$\begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}\begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x^2+y^2+z^2 & xy+yz+zx & xy+yz+zx \\ xy+yz+zx & x^2+y^2+z^2 & xy+yz+zx \\ xy+yz+zx & xy+yz+zx & x^2+y^2+z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$x^2 + y^2 + z^2 = 1$

$xy + yz + zx = 0$

$(x + y + z)^2 = \sum x^2 + 2\sum xy$

$(x + y + z)^2 = 1 + 0$

$x + y + z = \pm 1$

$\Rightarrow x + y + z = 1$

$x^3 + y^3 + z^3 = x^3 + y^3 + z^3 - 3xyz + 3xyz$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) + 3xyz$

$= (1)(1) + 3 \times 2$

$= 7$

7. The number of points, at which the function $f(x) = |2x + 1| - 3|x + 2| + |x^2 + x - 2|, x \in R$ is not differentiable is _____.

उन बिन्दुओं की संख्या, जिन पर फलन $f(x) = |2x + 1| - 3|x + 2| + |x^2 + x - 2|, x \in R$ अवकलनीय नहीं है, है :

Question Type : SA

Question ID : 70819117669

Ans. Official Answer NTA : (2)

Sol. $f(x) = |2x+1| - 3|x+2| + |x^2 + x - 2|$

$f(x) = |2x+1| - 3|x+2| + |(x+2)(x-1)|$

$$f(x) = \begin{cases} x^2 - 7 & : \quad x \geq 1 \\ -x^2 - 2x - 3 & : \quad -\frac{1}{2} \leq x < 1 \\ -x^2 - 6x - 5 & : \quad -2 \leq x < -\frac{1}{2} \\ x^2 + 2x + 3 & : \quad x \leq -2 \end{cases}$$

Function is always continuous, hence

$$f^1(x) = \begin{cases} 2x & : \quad x \geq 1 \\ -2x - 2 & : \quad -\frac{1}{2} \leq x < 1 \\ -2x - 6 & : \quad -2 \leq x < -\frac{1}{2} \\ 2x + 2 & : \quad x \leq -2 \end{cases}$$

At $x = 1$, LHD $= -4$, RHD $= 2$

$x = -\frac{1}{2}$, LHD $= -5$, RHD $= -1$

$x = -2$, LHD $= -2$, RHD $= -2$

Hence, f (x) is not differential at two points.

8. If $A=\begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ and $(I_2+A)(I_2-A)^{-1}=\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then $13(a^2+b^2)$ is equal to _____.

यदि $A=\begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ तथा $(I_2+A)(I_2-A)^{-1}=\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ हैं, तो $13(a^2+b^2)$ बराबर है :

Question Type : SA

Question ID : 70819117664

Ans. Official Answer NTA : (13)

Sol. $A=\begin{bmatrix} 0 & -\tan\theta \\ \tan\frac{\theta}{2} & 0 \end{bmatrix}$

$$(I_2+A)(I_2-A)^{-1}=\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$(I_2+A)=\begin{bmatrix} a & -b \\ b & a \end{bmatrix}(I_2-A)$$

$$\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}=\begin{bmatrix} a & -b \\ b & a \end{bmatrix}\begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}=\begin{bmatrix} a+b\tan\frac{\theta}{2} & a\tan\frac{\theta}{2}-b \\ b-a\tan\frac{\theta}{2} & b\tan\frac{\theta}{2}+a \end{bmatrix}$$

By comparison

$$a+b\tan\frac{\theta}{2}=1 \quad ...(1)$$

$$a\tan\frac{\theta}{2}-b=-\tan\frac{\theta}{2} \quad ...(2)$$

$$b-a\tan\frac{\theta}{2}=\tan\frac{\theta}{2} \quad ...(3)$$

$b\tan\frac{\theta}{2}+a=1$ ...(4)

By, $(1)^2 + (2)^2$

$a^2 + b^2 = 1$

$13(a^2 + b^2) = 13$

9. रेखाओं $\left(\sqrt{3}\right)kx + ky - 4\sqrt{3} = 0$ तथा $\sqrt{3}\,x - y - 4\left(\sqrt{3}\right)k = 0$ के प्रतिच्छेदन बिंदु का बिंदुपथ एक शांकव है, जिसकी उत्केन्द्रता है :

The locus of the point of intersection of the lines $\left(\sqrt{3}\right)kx + ky - 4\sqrt{3} = 0$ and $\sqrt{3}\,x - y - 4\left(\sqrt{3}\right)k = 0$ is a conic, whose eccentricity is _____.

Question Type : SA

Question ID : 70819117672

Ans. Official Answer NTA : (2)

Sol. $k\left(\sqrt{3}x + y\right) = 4\sqrt{3}$ ...(1)

$\left(\sqrt{3}x - y\right) = 4\sqrt{3}k$ ...(2)

(1) × (2)

$3x^2 - y^2 = 48$

$\frac{x^2}{16} - \frac{y^2}{48} = 1$

$e = \sqrt{1+\frac{b^2}{a^2}}$

$e = \sqrt{1+\frac{48}{16}}$

$e = 2$

10. Let $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}, \vec{b} = \hat{i} - \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{a} = \vec{c}\times\vec{a}$ and $\vec{r}.\vec{b} = 0$, then $\vec{r}.\vec{a}$ is equal to _____.

माना तीन सदिश $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}, \vec{b} = \hat{i} - \hat{j}$ तथा $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ दिए गए हैं। यदि $\vec{r}$ एक सदिश है, जिसके लिए $\vec{r}\times\vec{a} = \vec{c}\times\vec{a}$ तथा $\vec{r}.\vec{b} = 0$ हैं, तो $\vec{r}.\vec{a}$ बराबर है :

Question Type : SA

Question ID : 70819117673

Ans. Official Answer NTA : (12)

Sol. $\vec{r}\times\vec{a}=\vec{c}\times\vec{a}$

$\vec{r}\times\vec{a}-\vec{c}\times\vec{a}=0$

$\left(\vec{r}-\vec{c}\right)\times\vec{a}=0$

$\vec{r}-\vec{c}=k\vec{a}$

$\vec{r}=\vec{c}+k\vec{a}$

$\vec{r}\cdot\vec{b}=\vec{b}\cdot\vec{c}+k\,\vec{a}\cdot\vec{b}$

$0=2+k\,(-1)$

$k=2$

$\vec{r}=\vec{c}+2\vec{a}$

$\vec{r}\cdot\vec{a}=\vec{a}\cdot\vec{c}+2\left|\vec{a}\right|^2$

$=0+2\left(\sqrt{6}\right)^2$

$=12$