JEE Main February 2021

Question Paper With Text Solution

25 Feb.| Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911
Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in

JEE MAIN FEB 2021 | 25TH FEB SHIFT-2

SECTION – A

1. $\operatorname{cosec}\left[2\cot^{-1}(5)+\cos^{-1}\left(\frac{4}{5}\right)\right]$ is equal to :

(1) $\frac{56}{33}$ (2) $\frac{65}{56}$ (3) $\frac{65}{33}$ (4) $\frac{75}{56}$

Ans. Official answer NTA : (2)

Sol. $\operatorname{cosec}\left[2\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{3}{4}\right)\right]$

$\operatorname{cosec}\left[\tan^{-1}\frac{5}{12}+\tan^{-1}\frac{3}{4}\right]$

$\operatorname{cosec}\left[\tan^{-1}\frac{56}{13}\right]=\frac{65}{56}$

2. Let α and β be the roots of $x^2-6x-2=0$. If $a_n=\alpha^n-\beta^n$ for $n\geq 1$, then the value of $\frac{a_{10}-2a_8}{3a_9}$ is :

(1) 1 (2) 3 (3) 4 (4) 2

Ans. Official answer NTA : (4)

Sol. $\frac{\alpha^{10}-\beta^{10}-2\left(\alpha^8-\beta^8\right)}{3\left(\alpha^9-\beta^9\right)}=\frac{\alpha^8\left(\alpha^2-2\right)-\beta^8\left(\beta^2-2\right)}{3\left(\alpha^9-\beta^9\right)}$

$=\frac{\alpha^8(6\alpha)-\beta^8(6\beta)}{3\left(\alpha^9-\beta^9\right)}$

$\because\ \alpha^2=6\alpha-2=0$

$\alpha^2-2=6\alpha$

$\beta^2-2=6\beta$

$=\frac{6}{3}=2$

3. $\lim\limits_{n\to\infty}\left[\frac{1}{n}+\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+.........+\frac{n}{(2n-1)^2}\right]$ is equal to :

(1) 1 (2) $\frac{1}{4}$ (3) $\frac{1}{3}$ (4) $\frac{1}{2}$

Ans. Official Answer NTA : (4)

Sol. $\lim_{n\to\infty}\left[\frac{n}{n^2}+\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+.........+\frac{n}{(n+(n-1))^2}\right]$

$\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{n}{(n+r)^2}=\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{n-1}\frac{1}{\left(1+\frac{r}{n}\right)^2}$

$\int_0^1\frac{1}{(1+x)^2}dx=\left(\frac{-1}{1+x}\right)_0^1=\frac{-1}{2}+1=\frac{1}{2}$

4. A function $f(x)$ is given by $f(x)=\frac{5^x}{5^x+5}$, then the sum of the series

$f\left(\frac{1}{20}\right)+f\left(\frac{2}{20}\right)+f\left(\frac{3}{20}\right)+......+f\left(\frac{39}{20}\right)$

(1) $\frac{39}{2}$ (2) $\frac{29}{2}$ (3) $\frac{19}{2}$ (4) $\frac{49}{2}$

Ans. Official answer NTA : (1)

Sol. $f(x)=\frac{5^x}{5^x+5}$

$f(2-x)=\frac{5^{2-x}}{5^{2-x}+5}=\frac{1}{1+5^{x-1}}=\frac{5}{5+5^x}$

$f(x)+f(2-x)=1$

$f\left(\frac{1}{20}\right)+f\left(\frac{39}{20}\right)+f\left(\frac{2}{20}\right)+f\left(\frac{38}{20}\right)+......+f\left(\frac{20}{20}\right)$

$19+f(1)=19+\frac{1}{2}=\frac{39}{2}$

5. A plane passes through the points A(1, 2, 3), B(2, 3, 1) and C(2, 4, 2). If O is the origin and P is (2, –1, 1), then the projection of $\overrightarrow{OP}$ on this plane is of length :

(1) $\sqrt{\frac{2}{3}}$ (2) $\sqrt{\frac{2}{7}}$ (3) $\sqrt{\frac{2}{11}}$ (4) $\sqrt{\frac{2}{5}}$

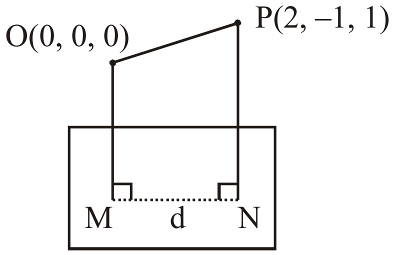
Ans. Official answer NTA : (3)

Sol. $\overrightarrow{AB} = \hat{i} + \hat{j} - 2\hat{k}$

$\overrightarrow{AC} = \hat{i} + 2\hat{j} - \hat{k}$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 3\hat{i} - \hat{j} + \hat{k}$$

Equation of plane 3x – y + z = 4]



$M\left(\frac{12}{11}, \frac{-4}{11}, \frac{4}{11}\right)$]

$N\left(\frac{10}{11}, \frac{-7}{11}, \frac{7}{11}\right)$

d = MN

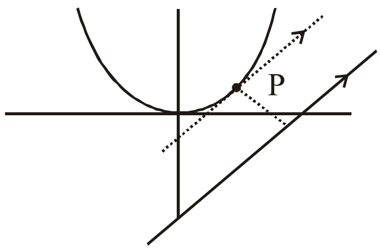
$d = \sqrt{\frac{2}{11}}$

6. The shortest distance between the line x – y = 1 and the curve $x^2 = 2y$ is :

(1) $\frac{1}{2}$ (2) 0 (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{2\sqrt{2}}$

Ans. Official answer NTA : (4)

Sol.



$\frac{dy}{dx} = m = 1$

$x^2 = 2y$

$2x = 2\frac{dy}{dx}$

2x = 2

$x = 1$

$P\left(1, \frac{1}{2}\right)$

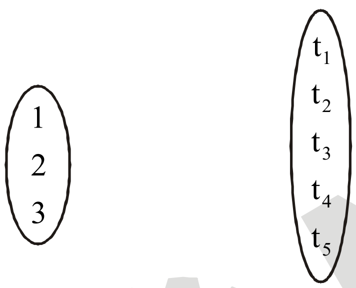
$$d = \left|\frac{1 - \frac{1}{2} - 1}{\sqrt{2}}\right| = \frac{1}{2\sqrt{2}}$$

7. Let x donate the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y donate the total number of one-one functions from the set A to the set A × B. Then :

(1) $y = 273x$ (2) $y = 91x$ (3) $2y = 91x$ (4) $2y = 273x$

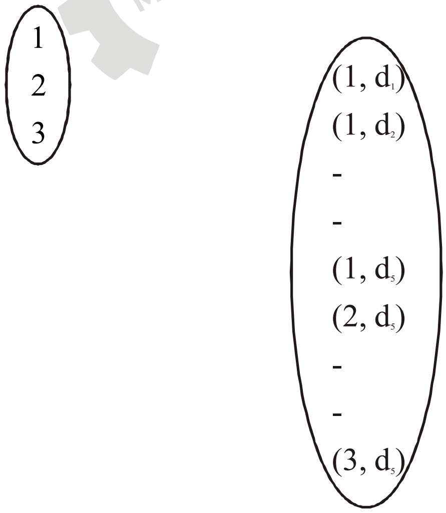
Ans. Official answer NTA : (3)

Sol.



$x = 5 \times 4 \times 3 = 60$

A A × B



$\because$ n (A × B) = 15

y = 15 × 14 × 13

2y = 91x

8. If for the matrix, $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$, $AA^T = I_2$, then the value of $\alpha^4 + \beta^4$ is

(1) 1 (2) 3 (3) 4 (4) 2

Ans. Official answer NTA : (1)

Sol. $AA^T = I_2$

$$\begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}\begin{bmatrix} 1 & \alpha \\ -\alpha & \beta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1+\alpha^2 & \alpha-2\beta \\ \alpha-\alpha\beta & \alpha^{2+}\beta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\alpha = 0 \quad \beta = 1$

$\alpha^4 + \beta^4 = 1$

9. If the curve $x^2 + 2y^2 = 2$ intersects the line x + y = 1 at two points P and Q, then the angle subtended by the line segment PQ at the origin is :

(1) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{3}\right)$ (2) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$ (3) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{3}\right)$ (4) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)$

Ans. Official answer NTA : (2)

Sol. H.F. = $\frac{x+y}{1}$

$x^2+2y^2 = 2\,(x + y)^2$

$x^2 + 4xy = 0$

$L_1 : x = 0$ $\qquad$ $L_2 : x + 4y = 0$

$m \to \infty$ $\qquad$ $m_2 = \frac{-1}{4}$

$$\tan\theta = \left|\frac{1}{\frac{-1}{4}}\right| = 4$$

$\theta = \tan^{-1}(4)$

obtuse angle $= \pi - \tan^{-1}(4)$

$= \pi - \left(\cot^{-1}\left(\frac{1}{4}\right)\right)$

$= \pi - \left(\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)\right)$

$= \frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$

10. The minimum value of $f(x) = a^{a^x} + a^{1-a^x}$, where a, x $\in$ R and a > 0, is equal to :

(1) $a + \frac{1}{a}$ (2) a + 1 (3) $2\sqrt{a}$ (4) 2a

Ans. Official answer NTA : (3)

Sol. $f(x) = a^{a^x} + \frac{a}{a^{a^x}}$

A.M. ≥ G.M.

$\frac{a^{a^x} + \frac{a}{a^{a^x}}}{2} \geq \sqrt{a}$

$a^{a^x} + a^{1-a^x} \geq 2\sqrt{a}$

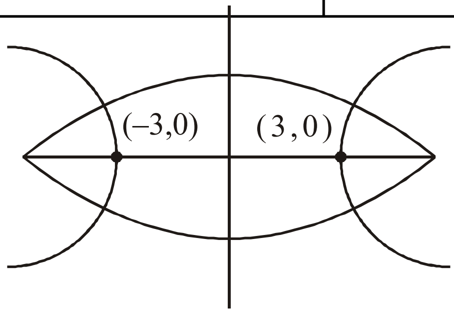
11. A hyperbola passes through the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its transverse and conjugate axes coincide with major and minor axes of the ellipse respectively. If the product of their eccentricities is one, then the equation of the hyperbola is :

अ

(1) $\frac{x^2}{9} - \frac{y^2}{4} = 1$ (2) $x^2 - y^2 = 9$ (3) $\frac{x^2}{9} - \frac{y^2}{16} = 1$ (4) $\frac{x^2}{9} - \frac{y^2}{25} = 1$

Ans. Official answer NTA : (3)

Sol. $e_1 = \sqrt{1 - \frac{16}{25}} = \frac{3}{5} \Rightarrow e_1 e_2 = 1 \Rightarrow e_2 = \frac{5}{3}$



a = 6

$e_2^2 = 1 + \frac{b^2}{a^2}$

$b^2 = 16$

$\frac{x^2}{9} - \frac{y^2}{16} = 1$

12. If $I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^n x\, dx$, then :

(1) $I_2 + I_4$, $I_3 + I_5$, $I_4 + I_6$ are in A.P.

(2) $\frac{1}{I_2 + I_4}, \frac{1}{I_3 + I_5}, \frac{1}{I_4 + I_6}$ are in A.P.

(3) $I_2 + I_4$, $(I_3 + I_5)^2$, $I_4 + I_6$ are in G.P.

(4) $\frac{1}{I_2 + I_4}, \frac{1}{I_3 + I_5}, \frac{1}{I_4 + I_6}$ are in G.P.

Ans. Official answer NTA : (2)

Sol. $I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\cot x)^{n-2} (\operatorname{cosec}^2 x - 1)$

$I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\cot x)^{n-2} \operatorname{cosec}^2 x\, dx - I_{n-2}$

cot x = t

$I_n + I_{n-2} = \int_1^0 t^{n-2} dt = \int_0^1 t^{n-2} dt$

$$= \left[\frac{t^{n-1}}{n-1}\right]_0^1 = \frac{1}{n-1}$$

13. Let A be a 3 × 3 matrix with det(A) = 4. Let R_i denote the i^{th} of A. If a matrix B is obtained by performing the operation $R_2 \to 2R_2 + 5R_3$ on 2A, then det(B) is equal to :

(1) 80 (2) 64 (3) 16 (4) 128

Ans. Official answer NTA : (2)

Sol. $$2A = \begin{bmatrix} 2a_1 & 2a_2 & 2a_3 \\ 2b_1 & 2b_2 & 2b_3 \\ 2c_1 & 2c_2 & 2c_3 \end{bmatrix}$$

$$B = \begin{bmatrix} 2a_1 & 2a_2 & 2a_3 \\ 4b_1 + 10c_1 & 4b_2 + 10c_2 & 4b_3 + 10c_3 \\ 2c_1 & 2c_2 & 2c_3 \end{bmatrix}$$

$$|B| = 16 \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + 40 \begin{vmatrix} a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$= 16 \times 4 = 64$

14. The contrapositive of the statement "If you will work, you will earn money" is :

(1) If you will earn money, you will work

(2) To earn money, you need to work

(3) If you willn not earn money, you will not work

(4) You will earn money, if you will not work

Ans. Official answer NTA : (3)

Sol. $P \to q$

$\sim q \to \sim p$

If you will not earn money, you will not work

15. In a group of 400 people, 160 are smokers and non-vegetarian;100 are smokers and vegetarian and the

remaining 140 are non-smokers and vegetarian. Their chances of getting a particular chest disorder are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the chest disorder. The probability that the selected person is a smoker and non-vegetarian is :

(1) $\frac{8}{45}$ (2) $\frac{14}{45}$ (3) $\frac{7}{45}$ (4) $\frac{28}{45}$

Ans. Official answer NTA : (4)

Sol. $$\frac{160\times\frac{35}{100}}{160\times\frac{35}{100}+100\times\frac{20}{100}+140\times\frac{10}{100}}=\frac{28}{45}$$

16. If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$, then $\sin x + \cos y$ is equal to :

(1) $\frac{1+\sqrt{3}}{2}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{1-\sqrt{3}}{2}$ (4) $\frac{1}{2}$

Ans. Official answer NTA : (1)

Sol. $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$

$$2\cos\frac{x+y}{2}\cos\frac{x-y}{2}-2\cos^2\frac{x+y}{2}+1=\frac{3}{2} \qquad ....(1)$$

$$2\cos\frac{x+5}{2}\cdot 2\sin\frac{x}{2}\sin\frac{y}{2}=\frac{1}{2} \qquad \begin{Bmatrix}0<x<\pi\\0<y<\pi\end{Bmatrix}$$

$$\left.\begin{matrix}\sin\frac{x}{2}\rightarrow\text{positive}\\ \sin\frac{y}{2}\rightarrow\text{positive}\end{matrix}\right\}\text{ so }\cos\frac{x+y}{2}\text{ is also positive}$$

$$1\ge\cos\frac{x-y}{2}$$

$$2\cos\frac{x+y}{2}\ge 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}\quad\left(\because\cos\frac{x+y}{2}>0\right)$$

$$2\cos\frac{x+y}{2}-2\cos^2\left(\frac{x+y}{2}\right)+1\ge 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}-2\cos^2\left(\frac{x+y}{2}\right)+1$$

$$2\cos\frac{x+y}{2} - 2\cos^2\left(\frac{x+y}{2}\right) + 1 \geq \frac{3}{2}$$

$$\left(2\cos\left(\frac{x+y}{2}\right) - 1\right)^2 \leq 0$$

$$\cos\left(\frac{x+y}{2}\right) = \frac{1}{2}$$

$$\frac{x+y}{2} = \frac{\pi}{3}$$

$$x + y = \frac{2\pi}{3}$$

from equation (1)

$$2\cos\frac{\pi}{3}\cos\frac{x-y}{2} - 2\cos^2\frac{\pi}{3} + 1 = \frac{3}{2}$$

$$\cos\frac{x-y}{2} = 1$$

$$x - y = 0$$

$$x = y$$

put $x = y = \frac{\pi}{3}$

17. Let A be a set of all 4-digit natural numbers whose exactly one digit is 7. Then the probability that a randomly chosen element of A leaves remainder 2 when divided by 5 is :

(1) $\frac{1}{5}$ (2) $\frac{97}{297}$ (3) $\frac{2}{9}$ (4) $\frac{122}{297}$

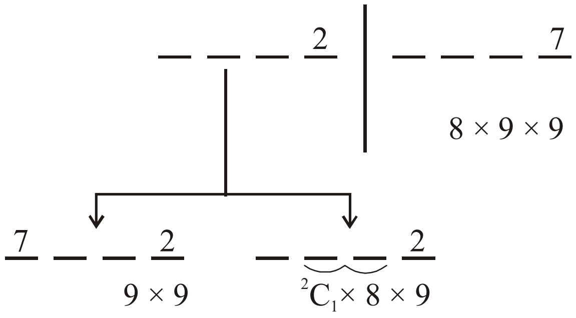
Ans. Official answer NTA : (2)

Sol. Sample space

7 _ _ _	_ _ _ _
$1 \times 9 \times 9 \times 9$	${}^3C_1 \times 9 \times 9 \times 9$

n(s) = 729 + 1944 = 2673

Event



h(E) = 873

$$p(E) = \frac{873}{2673} = \frac{97}{297}$$

18. The following system of linear equations :

$2x + 3y + 2z = 9$
$3x + 2y + 2z = 9$
$x - y + 4z = 8$

(1) Has infinitely many solutions

(2) Has a solution (α, β, γ) satisfying $\alpha + \beta^2 + \gamma^2 = 12$

(3) Does not have any solution

(4) Has a unique solution

Ans. Official answer NTA : (4)

Sol. $$\Delta = \begin{vmatrix} 2 & 3 & 2 \\ 3 & 2 & 2 \\ 1 & -1 & 4 \end{vmatrix}$$

$= 2(8 + 2) - 3(12 - 2) + 2(-3 - 2)$

$= 32 - 30 - 10 \qquad = -8$

$D \neq 0$

has a unique solution

19. If $\alpha, \beta \in R$ are such that $1 - 2i$ (here $i^2 = -1$) is a root of $z^2 + \alpha z + \beta = 0$, then $(\alpha - \beta)$ is equal to :

(1) –3 (2) –7 (3) 3 (4) 7

Ans. Official answer NTA : (2)

Sol. $(1-2i)^2 + \alpha(1-2i) + \beta = 0$

$(\alpha + \beta - 3) - i(4 + 2\alpha) = 0$

$\alpha = -2 \qquad \beta = 5$

20. The integral $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}} dx$, x > 0 is equal to :

(where c is a constant of integration)

(1) $\log_e \sqrt{x^2 + 5x - 7} + c$ (2) $\log_e |x^2 + 5x - 7| + c$

(3) $\frac{1}{4}\log_e |x^2 + 5x - 7| + c$ (4) $4\log_e |x^2 + 5x - 7| + c$

Ans. Official answer NTA : (4)

Sol. $\int \frac{8x^3 + 20x^2}{x^4 + 5x^3 - 7x^2} dx$

$= 4\int \frac{2x + 5}{x^2 + 5x - 7} dx$

$= 4\ln |x^2 + 5x - 7| + C$

SECTION – B

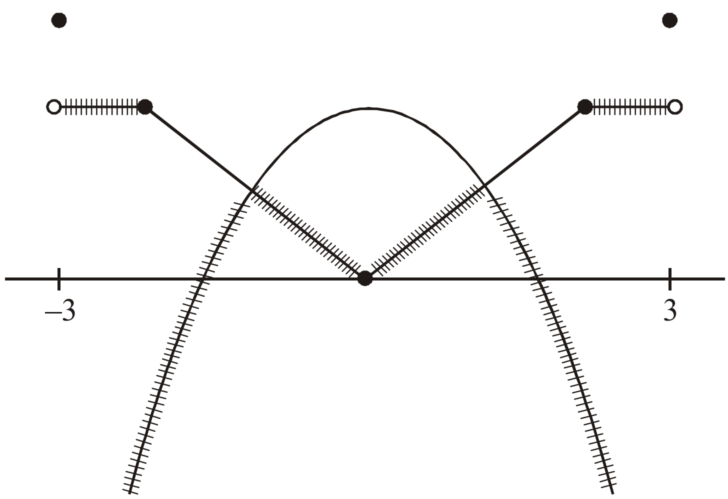
1. A function f is defined on [–3, 3] as

$$f(x) = \begin{cases} \min\{|x|, 2 - x^2\}, & -2 \le x \le 2 \\ [|x|], & 2 < |x| \le 3 \end{cases}$$

where [x] denotes the greatest r $\le$ x. The number of points, where f is not differentiable in (–3, 3) is :

Ans. Official answer NTA : (5)

Sol.



not differentiable at 5 point

2. If $\lim\limits_{x\to 0}\dfrac{ax-(e^{4x}-1)}{ax(e^{4x}-1)}$ exist and is equal to b, then the value of a – 2b is :

Ans. Official answer NTA : (5)

Sol. $\lim\limits_{x\to 0}\dfrac{ax-\left(a+4x+\dfrac{16x^2}{\lfloor 2}+-----1\right)}{4ax^2}$

$\lim\limits_{x\to 0}\dfrac{(a-4)x-8x^2-----}{4ax^2}=b$

$a = 4$

$b = \dfrac{-1}{2}$

$a - 2b = 5$

3. Let $\vec{a}=\hat{i}+\alpha\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}-\alpha\hat{j}+\hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $8\sqrt{3}$ square units, then $\vec{a}\cdot\vec{b}$ is equal to :

Ans. Official answer NTA : (2)

Sol. $\left\|\begin{matrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & \alpha & 3\\ 3 & -\alpha & 1\end{matrix}\right\|=8\sqrt{3}$

$\left|4\alpha\hat{i}+8\hat{j}-4\alpha\hat{k}\right|=8\sqrt{3}$

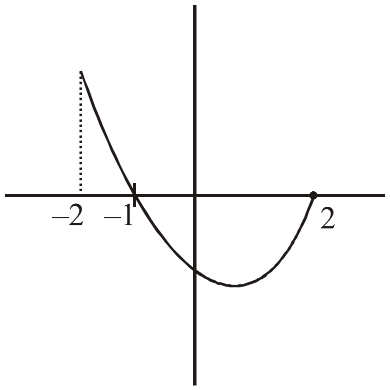
$\alpha^2 = 4$

$\vec{a}\cdot\vec{b}=3-\alpha^2+3=2$

4. The value of $\int_{-2}^{2}|3x^2-3x-6|\,dx$ is :

Ans. Official answer NTA : (19)

Sol. $3\int_{-2}^{2}|x^2-x-2|\,dx$



$$3\int_{-2}^{-1}(x^2-x-2)\,dx-3\int_{-1}^{2}(x^2-x-2)\,dx$$

$$=3\left(\frac{11}{6}+\frac{27}{6}\right)=19$$

5. If the curve, y = y(x) represented by the solution of the differential equation $(2xy^2 - y)dx + xdy = 0$, passes through the intersection of the lines, $2x - 3y = 1$ and $3x + 2y = 8$, then $|y(1)|$ is equal to :

Ans. Official answer NTA : (1)

Sol. $2xy^2\,dx - y\,dx + x\,dy = 0$

$$2x\,dx - \frac{1}{y}\,dx + \frac{x}{y^2}\,dy = 0$$

$$2x\,dx = \frac{1}{y}\,dx - \frac{x}{y^2}\,dy$$

$$\int 2x\,dx = \int d\left(\frac{x}{y}\right)$$

$x^2 = \frac{x}{y} + c$ put P.O.I. of $2x - 3y = 1$ and $3x + 2y = 8$

$c = 2$ i.e. (2, y)

$y(1) = -1$

$|y(1)| = 1$

6. The total number of two digit numbers 'n', such that $3^n + 7^n$ is a multiple of 10, is ________ .

Ans. Official answer NTA : (45)

Sol. $3^n + 7^n$ is divisible by 3 + 7 iff n is odd

11, 13, 15, ..., 99

number of two digit numbers 'n' = 45

7. A line is a common tangent to the circle $(x-3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$. If two points of contact (a, b) and (c, d) are distinct and lie in the first quadrant, then 2(a + c) is equal to _______ .

Ans. Official answer NTA : (9)

Sol. Equationo of tangent to the parabola $y^2 = 4x$

$y = mx + \frac{1}{m}$

C.O.T. for circle $(x-3)^2 + y^2 = 9$

$$\left|\frac{3m+\frac{1}{m}}{\sqrt{m^2+1}}\right| = 3$$

$9m^2 + \frac{1}{m^2} + 6 = 9(m^2 + 1)$

put $m^2 = t$

$9t^2 + 1 + 6t = 9t^2 + 9t$

$3t = 1$

$t = \frac{1}{3}$

$m = \pm\frac{1}{\sqrt{3}}$

$y = \frac{x}{\sqrt{3}} + \sqrt{3}$ or $y = -\frac{x}{\sqrt{3}} - \sqrt{3}$ Rejected

$x - \sqrt{3}y + 3 = 0$

$x - \sqrt{3}y + 3 = 0$ $\quad$ $x - \sqrt{3}y + 3 = 0$

$y^2 = 4x$ $\quad$ $(x-3)^2 + y^2 = 9$

$T = 0$ $\quad$ $T = 0$

$(3, 2\sqrt{3})$ $\quad$ $\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$

$2(a + c) = 2(3 + \frac{3}{2}) = 9$

8. If the remainder when x is divided by 4 is 3, then the remainder when $(2020 + x)^{2022}$ is divided by 8 is ____ .

Ans. Official answer NTA : (1)

Sol. $x = 4k + 3$

$(2020 + x)^{2022} = (2023 + 4k)^{2022}$

$(2024 + (4k - 1))^{2022}$

$= {}^{2022}C_0 (2024)^{2022} + ... + {}^{2022}C_{2021} (2024)(4k-1)^{2021} + {}^{2022}C_{2022} (4k-1)^{2022}$

$= 8I + {}^{2022}C_{2022} (4k-1)^{2022}$

$= 8I + (4k-1)^{2022}$

$= 8I + {}^{2022}C_0 (4k)^{2022} + ... + {}^{2022}C_{2022} (-1)^{2022}$

$= 8I + 8I_1 + 1$

Remainder =1

9. If the curves $x = y^4$ and $xy = k$ cut at right angles, then (4k)6 is equal to _______ .

Ans. Official answer NTA : (4)

Sol. let common point is $(x_1\ y_1)$

$x_1 = y_1^4$

$3y^3 \frac{dy}{dx} = 1$

$m_1 = \frac{dy}{dx} = \frac{1}{3y_1^3}$

$x_1 y_1 = k$

$x\frac{dy}{dx} + y = 0$

$m_2 = \frac{dy}{dx} = \frac{-y_1}{x_1}$

$m_1 m_2 = -1$

$4y_1^2 x_1 = 1$

Put $x_1 = k^{4/5}$ and $y_1 = k^{1/5}$

$4k^{6/5} = 1$

$k^6 = \left(\frac{1}{4}\right)^5$

$(4k)^6 = 4$

10. A line 'l' passing through origin is perpendicular to the lines

$l_1 : \vec{r} = (3+t)\,\hat{i} + (-1+2t)\,\hat{j} + (4+2t)\hat{k}$

$l_2 : \vec{r} = (3+2s)\hat{i} + (3+2s)\,\hat{j} + (2+s)\hat{k}$

If the co–ordinates of the point in the first octant on 'l_2' at a distance of $\sqrt{17}$ from the point of intersection of 'l' and 'l_1' are (a, b, c), then 18(a + b + c) is equal to ________ .

Ans. Official answer NTA : (44)

Sol. $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix}$

$\vec{n} = 2\hat{i} + 3\hat{j} - 2\hat{k}$

equation of line $\ell \rightarrow \frac{x}{2} = \frac{y}{-3} = \frac{z}{2} = p$

2p = 3 + t and –3p = –1 + 2t

p = 1

P.O.I. A (2, –3, 2)

Let point on ℓ_2 B (2s + 3, 2s + 3, s + 2)

distance between A and B

$\sqrt{(2s+1)^2 + (2s+6)^2 + s^2} = 17$

s = –2 (Rejected) and s = $-\frac{10}{9}$

$B\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$

18 (a + b + c) = 44