JEE Main January 2023

Question Paper With Text Solution

25 January | Shift-2

MATHEMATICS

JEE MAIN JANUARY 2023 | 25TH JANUARY SHIFT-2

SECTION – A

Question ID : 7155051696

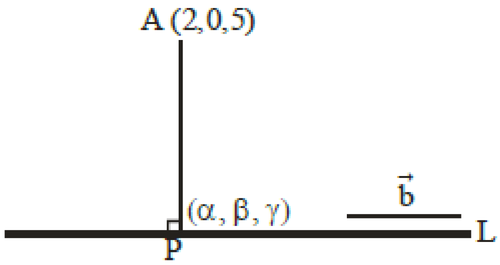
1. The foot of perpendicular of the point (2,0,5) on the line $\frac{x+1}{2}=\frac{y-1}{5}=\frac{z+1}{-1}$ is (α,β,γ). then :

बिंदु (2,0,5) से रेखा $\frac{x+1}{2}=\frac{y-1}{5}=\frac{z+1}{-1}$ पर लंब का पाद (α,β,γ) है। तो निम्न में से कौन सा सही नहीं है:

(1) $\frac{\beta}{\gamma}=-5$ (2) $\frac{\gamma}{\alpha}=\frac{5}{8}$ (3) $\frac{\alpha}{\beta}=-8$ (4) $\frac{\alpha\beta}{\gamma}=\frac{4}{15}$

Ans. Official Answer NTA (1)

Sol. $L:\frac{x+1}{2}=\frac{y-1}{5}=\frac{z+1}{-1}=\lambda$ (let)



Let food of perpendicular is

$P(2\lambda-1,5\lambda+1,-\lambda-1)$

$\overrightarrow{PA}=(3-2\lambda)\hat{i}-(5\lambda+1)\hat{j}+(6+\lambda)\hat{k}$

Direction ratio of line $\Rightarrow \vec{b}=2\hat{i}+5\hat{j}-\hat{k}$

Now, $\Rightarrow \overrightarrow{PA}\cdot\vec{b}=0$

$\Rightarrow 2(3-2\lambda)-5(5\lambda+1)-(6+\lambda)=0$

$\Rightarrow \lambda=\frac{-1}{6}$

$P(2\lambda-1,5\lambda+1,-\lambda-1)\equiv P(\alpha,\beta,\gamma)$

$\Rightarrow \alpha=2\left(-\frac{1}{6}\right)-1=-\frac{4}{3}\Rightarrow \alpha=-\frac{4}{3}$

$\Rightarrow \beta=5\left(-\frac{1}{6}\right)+1=\frac{1}{6}\Rightarrow \beta=\frac{1}{6}$

$\Rightarrow \gamma=-\lambda-1=\frac{1}{6}-1\Rightarrow \gamma=-\frac{5}{6}$

$\therefore$ Check options

Question ID : 7155051681

2. Let f: $\mathbb{R} \to \mathbb{R}$ be a function defined by

$f(x) = \log_{\sqrt{m}}\left\{\sqrt{2}(\sin x - \cos x) + m - 2\right\}$, for some m, such that the range of f is [0,2]. Then the value of m is ______.

माना फलन f: $\mathbb{R} \to \mathbb{R}$, किसी m के लिए

$f(x) = \log_{\sqrt{m}}\left\{\sqrt{2}(\sin x - \cos x) + m - 2\right\}$ द्वारा परिभाषित है तथा f का परिसर [0, 2] है। तो m का मान है।

(1) 3 (2) 2 (3) 5 (4) 4

Ans. Official Answer NTA (3)

Sol. $\log_{\sqrt{5}}(\sqrt{2}(\sin x - \cos x) + m - 2) \in [0,2] \in [\log_{\sqrt{5}} 1, \log_{\sqrt{5}} 5]$

$\Rightarrow \sqrt{2}(\sin x - \cos x) + m - 2 \in [1, m]$

Range of (sinx – cosx) is $[-\sqrt{2}, \sqrt{2}]$

So, from (i)

$[-4 + m, m] \to [1, m] \Rightarrow -4 + m = 1 \Rightarrow m = 5$

Question ID : 7155051693

3. The equations of two sides of a variable triangle are x = 0 and y = 3, and its third side is a tangent to the parabola $y^2 = 6x$. The locus of its circumcentre is :

एक चर त्रिभुज की दो भुजाओं के समीकरण x = 0 तथा y = 3 है तथा इसकी तीसरी भुजा परवलय $y^2 = 6x$ की एक स्पर्श रेखा है। तो इसके परिकेन्द्र का बिंदुपथ है :

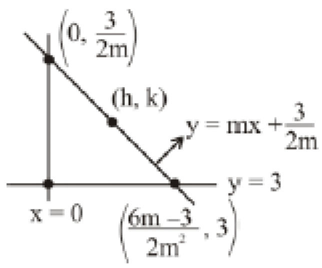
(1) $4y^2 - 18y - 3x + 18 = 0$ (2) $4y^2 + 18y + 3x + 18 = 0$

(3) $4y^2 - 18y - 3x - 18 = 0$ (4) $4y^2 - 18y + 3x + 18 = 0$

Ans. Official Answer NTA (4)

Sol. $y^2 = 6x$ & $y^2 = 4ax$

$\Rightarrow 4a = 6 \Rightarrow a = \frac{3}{2}$



$y = mx + \frac{3}{2m}; (m \neq 0)$

$h = \frac{6m-3}{4m^2}, k = \frac{6m+3}{4m}$, Now eliminating m and we get

$\Rightarrow 3h = 2(-2k^2 + 9k - 9)$

$\Rightarrow 4y^2 - 18y + 3x + 18 = 0$

Question ID : 7155051691

4. The integral $16\int_1^2 \frac{dx}{x^3(x^2+2)^2}$ is equal to

समाकलन $16\int_1^2 \frac{dx}{x^3(x^2+2)^2}$ बराबर है

(1) $\frac{11}{6} + \log_e 4$ (2) $\frac{11}{12} + \log_e 4$ (3) $\frac{11}{6} - \log_e 4$ (4) $\frac{11}{12} - \log_e 4$

Ans. Official Answer NTA (3)

Sol. $\int_1^2 \frac{dx}{x^3 x^4 \left(1+\frac{2}{x^2}\right)^2}$

Let $1 + \frac{2}{x^2} = t \Rightarrow \frac{-4}{x^3}dx = dt$

$I = \frac{-1}{4}\int_3^{\frac{3}{2}} \left(\frac{t-1}{2}\right)^2 \frac{dt}{t^2} = -\frac{1}{16}\int_3^{\frac{3}{2}} \frac{t^2-2t+1}{t^2}dt$

$= \frac{-1}{16}\int_3^{\frac{3}{2}} \left(1 - \frac{2}{t} + \frac{1}{t^2}\right)dt$

$= \frac{-1}{16}\left(t - 2\ln|t| - \frac{1}{t}\right)_3^{\frac{3}{2}}$

$= \frac{-1}{16}\left(\frac{3}{2} - 2\ln\frac{3}{2} - \frac{2}{3}\right) - (3-2)n2 - \frac{1}{3}\Big)$

$= \frac{-1}{16}\left(2\left(\ln 2 - \ln\frac{3}{2}\right) - \frac{3}{2} - \frac{1}{3}\right)$

$= \frac{-1}{16}\left(2\ln 3 - \frac{11}{6}\right) = \frac{-1}{16}\left(\ln 4 - \frac{11}{6}\right) = \frac{11 - 6\ln 4}{16 \times 6}$

$\frac{11-6\ln 4}{96}$

Question ID : 7155051688

5. Let $f(x) = 2x^n + \lambda, \lambda \in \mathbb{R}, n \in N$, and $f(4) = 133, f(5) = 255$. Then the sum of all the positive integer divisors of $(f(3)-f(2))$ is

माना $f(x) = 2x^n + \lambda, \lambda \in \mathbb{R}, n \in N$, और $f(4) = 133, f(5) = 255$ है। तो $(f(3)-f(2))$ के सभी धनात्मक पूर्णांक भाजकों का योग है

(1) 60 (2) 58 (3) 59 (4) 61

Ans. Official Answer NTA (1)

Sol. $f(x) = 2x^n + \lambda$

$f(4) = 133$

$f(5) = 255$

$133 = 2 \times 4^n + \lambda$ (1)

$255 = 2 \times 5^n + \lambda$ (2)

(2) – (1)

$122 = 2(5^n - 4^n)$

$\Rightarrow 5^n - 4^n = 61$

$\therefore n = 3 \;\&\; \lambda = 5$

Now, $f(3) - f(2) = 2(3^3 - 2^3) = 38$

divisors = 1,2,19,38

Sum = 60

Question ID : 7155051687

6. $\sum_{k=0}^{6} {}^{51-k}C_3$ is equal to

$\sum_{k=0}^{6} {}^{51-k}C_3$ बराबर है

(1) ${}^{51}C_4$ - ${}^{45}C_4$ (2) ${}^{52}C_4$ - ${}^{45}C_4$ (3) ${}^{51}C_3$ - ${}^{45}C_3$ (4) ${}^{52}C_3$ - ${}^{45}C_3$

Ans. Official Answer NTA (2)

Sol. ${}^{51}C_3 + {}^{50}C_3 + --------- + {}^{45}C_3$

$\Rightarrow {}^{45}C_4 + {}^{45}C_3 + {}^{46}C_3 + --------- + {}^{51}C_3 - {}^{45}C_4$

$\Rightarrow {}^{46}C_4 + {}^{46}C_3 + -------$

$= {}^{51}C_4 + {}^{51}C_3 - {}^{45}C_4 = {}^{52}C_4 - {}^{45}C_4$

Question ID : 7155051683

7. Let z be a complex number such that $\left|\frac{z-2i}{z+i}\right| = 2, z \neq -i$. Then z lies on the circle of radius 2 and centre

माना z एक सम्मिश्र संख्या है तथा $\left|\frac{z-2i}{z+i}\right| = 2, z \neq -i$ है। तो z त्रिज्या 2 के एक वृत्त पर है जिसका केन्द्र है

(1) (2,0) (2) (0,-2) (3) (0,2) (4) (0,0)

Ans. Official Answer NTA (2)

Sol. $(z-2i)(\overline{z}+2i) = 4(z+i)(\overline{z}-i)$

$z\overline{z}+4+2i(z-\overline{z}) = 4(z\overline{z}+1+i(\overline{z}-z))$

$3z\overline{z}-6i(z-\overline{z}) = 0$

$x^2+y^2-2i(2iy) = 0$

$x^2+y^2+4y = 0$

Question ID : 7155051692

8. Let $y = y(t)$ be a solution of the differential equation

$$\frac{dy}{dt}+\alpha y = \gamma e^{-\beta t}$$

where, $\alpha > 0, \beta > 0$ and $\gamma > 0$. Then $\lim_{t\to\infty} y(t)$

(1) is -1 (2) is 0 (3) does not exist (4) is 1

माना अवकल समीकरण

$$\frac{dy}{dt}+\alpha y = \gamma e^{-\beta t}$$

जहाँ $\alpha > 0, \beta > 0$ तथा $\gamma > 0$ है, का हल $y = y(t)$ है। तो $\lim_{t\to\infty} y(t)$

(1) –1 है (2) 0 है (3) का अस्तित्व नहीं है (4) 1 है

Ans. Official Answer NTA (2)

Sol. I.F. $= e^{\int a dt} = e^{at}$

Solution $y.e^{at} = \int e^{at}\cdot\gamma\cdot e^{-at}dt + C$

$\Rightarrow ye^{\alpha t} = \gamma\int e^{(\alpha-\beta)t}dt + c$

$$\Rightarrow y(t)\cdot e^{\alpha t}=\gamma\frac{e^{(\alpha-\beta)t}}{(\alpha-\beta)}+c$$

$$\Rightarrow y(t)=\gamma\frac{e^{-\beta t}}{(\alpha-\beta)}+ce^{-\alpha t}$$

$$\lim_{t\to\infty}y(t)=\lim_{t\to\infty}\left(\frac{\gamma\cdot e^{-\beta t}}{(\alpha-\beta)}-c\cdot e^{-at}\right)$$

$$=0-0=0$$

Question ID : 7155051695

9. The shortest distance between the lines x + 1 = 2y = -12z and x = y + 2 = 6z - 6 is

रेखाओं x + 1 = 2y = -12z तथा x = y + 2 = 6z - 6 के बीच न्यूनतम दूरी है

(1) $\frac{3}{2}$ (2) 3 (3) 2 (4) $\frac{5}{2}$

Ans. Official Answer NTA (3)

Sol. $$\frac{x+1}{1}=\frac{y}{\frac{1}{2}}=\frac{z}{\frac{-1}{12}}\text{ and }\frac{x}{1}=\frac{y+2}{1}=\frac{z-1}{\frac{1}{6}}$$

$$\Rightarrow \text{ Shortest distance }=\frac{(\vec{b}-\vec{a})\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}$$

$$\text{S.D. }=(-\hat{i}+2\hat{j}-\hat{k})\cdot\frac{(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}$$

$$\left\{\vec{p}\times\vec{q}\equiv\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&\frac{1}{2}&\frac{-1}{12}\\1&1&\frac{1}{6}\end{vmatrix}=\frac{1}{6}\hat{i}-\frac{1}{4}\hat{j}+\frac{1}{2}\hat{k}\text{ or }2\hat{i}-3\hat{j}+6\hat{k}\right\}$$

$$\text{S.D. }=\frac{(-\hat{i}+2\hat{j}-\hat{k})\cdot(2\hat{i}-3\hat{j}+6\hat{k})}{\sqrt{2^2+3^2+6^2}}=\left|\frac{-14}{7}\right|=2$$

Question ID : 7155051698

10. If the four points, whose position vectors are $3\hat{i}-4\hat{j}+2\hat{k}, \hat{i}+2\hat{j}-\hat{k}, -2\hat{i}-\hat{j}+3\hat{k}$ and $5\hat{i}-2\alpha\hat{j}+4\hat{k}$ are coplanar, then α is equal to

यदि चार बिंदु, जिनके स्थिति सदिश $3\hat{i}-4\hat{j}+2\hat{k}, \hat{i}+2\hat{j}-\hat{k}, -2\hat{i}-\hat{j}+3\hat{k}$ तथा $5\hat{i}-2\alpha\hat{j}+4\hat{k}$ है, सहतलीय है, तो α बराबर है

(1) $-\frac{73}{17}$ (2) $\frac{107}{17}$ (3) $\frac{73}{17}$ (4) $-\frac{107}{17}$

Ans. Official Answer NTA (3)

Sol. $$\begin{vmatrix} 3-5 & -4+2\alpha & 2-4 \\ 3-1 & -4-2 & 2+1 \\ 3+2 & -4+1 & 2-3 \end{vmatrix} = \begin{vmatrix} -2 & 2\alpha-4 & -2 \\ 2 & -6 & 3 \\ 5 & -3 & -1 \end{vmatrix} = 0$$

$$\begin{vmatrix} 0 & 2\alpha-4 & -2 \\ -1 & -6 & 3 \\ 6 & -3 & -1 \end{vmatrix} = 0 \Rightarrow -(2\alpha-4)(1-18)-2(3+36)=0$$

$$(\alpha-2)(-17)+39=0$$

$$\alpha-2=\frac{39}{17} \Rightarrow \alpha = 2+\frac{39}{17}=\frac{34+39}{17}$$

$$\alpha=\frac{73}{17}$$

Question ID : 7155051700

11. Let $\Delta, \nabla \in \{\wedge, \vee\}$ be such that $(p \to q)\Delta(p \nabla q)$ is a tautology. Then

माना $\Delta, \nabla \in \{\wedge, \vee\}$ इस प्रकार है कि $(p \to q)\Delta(p \nabla q)$ एक पुनरूक्ति है। तो

(1) $\Delta = \wedge, \nabla = \wedge$ (2) $\Delta = \wedge, \nabla = \vee$ (3) $\Delta = \vee, \nabla = \vee$ (4) $\Delta = \vee, \nabla = \wedge$

Ans. Official Answer NTA (3)

Sol. Given $(p \to q)\Delta(p \nabla q)$

Option I $\Delta = \wedge, \nabla = \vee$

p	q	$(p \to q)$	$(p \vee q)$	$(p \to q) \wedge (p \vee q)$
T	T	T	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

Option 2 $\Delta = \vee, \nabla = \wedge$

p	q	$(p \to q)$	$(p \wedge q)$	$(p \to q) \vee (p \wedge q)$
T	T	T	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

Option 3 $\Delta = \vee, \nabla = \vee$

p	q	$(p \to q)$	$(p \vee q)$	$(p \to q) \vee (p \wedge q)$
T	T	T	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

Hence, it is tautology.

Option 4 $\Delta = \wedge, \nabla = \wedge$

p	q	$(p \to q)$	$(p \wedge q)$	$(p \to q) \wedge (p \wedge q)$
T	T	T	T	T
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

Question ID : 7155051686

12. The number of numbers, strictly between 5000 and 10000 can be formed using the digits 1,3,5,7,9 without repetition, is

बिना पुनरावृति के अंकों 1, 3, 5, 7, 9 के प्रयोग से 5000 तथा 10000 के बीच बनाई जा सकने वाली संख्याओं की संख्या है

(1) 72 (2) 12 (3) 6 (4) 120

Ans. Official Answer NTA (1)

Sol. Digit in thousand place must be 5, 7 or 9.

= 3 ways

Now Total numbers between 5000 to 10000

are = $3 \times 4 \times 3 \times 2 = 72$

Question ID : 7155051684

13. Let $A = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}$, where $i = \sqrt{-1}$

If $M = A^T B A$, then the inverse of the matrix $AM^{2023}A^T$ is

माना $A = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$ तथा $B = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}$ है, जहाँ $i = \sqrt{-1}$ है।

यदि $M = A^T B A$ है, तो आव्युह $AM^{2023}A^T$ का व्युत्क्रम है

(1) $\begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 0 \\ 2023i & 1 \end{bmatrix}$ (3) $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & 0 \\ -2023i & 1 \end{bmatrix}$

Ans. Official Answer NTA (3)

Sol. $AA^T = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{-3}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$B^2 = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2i \\ 0 & 1 \end{bmatrix}$

$B^3 = \begin{bmatrix} 1 & -3i \\ 0 & 1 \end{bmatrix}$

.
.
.

$B^{2023} = \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

$M = A^T BA$

$M^2 = M \cdot M = A^T BAA^T BA = A^T B^2 A$

$M^3 = M^2 \cdot M = A^T B^2 A A^T BA = A^T B^3 A$

.

.

.

$M^{2023} = \ldots\ldots\ldots\ldots\ldots A^T B^{2023} A$

$AM^{2023} A^T = AA^T B^{2023} AA^T = B^{2023}$

$= \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

Inverse of $(AM^{2023} A^T)$ is $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

Question ID : 7155051690

14. Let the function $f(x) = 2x^3 + (2p - 7)x^2 + 3(2p - 9)x - 6$ have a maxima for some value of $x < 0$ and a minima for some value of $x > 0$. Then, the set of all values of p is

माना फलन $f(x) = 2x^3 + (2p - 7)x^2 + 3(2p - 9)x - 6$ का एक उच्चिष्ठ किसी $x < 0$ पर है तथा एक निम्निष्ठ किसी $x > 0$ पर है। तो p के सभी मानों का समुच्चय है

(1) $\left(\frac{9}{2}, \infty\right)$ (2) $\left(-\infty, \frac{9}{2}\right)$ (3) $\left(-\frac{9}{2}, \frac{9}{2}\right)$ (4) $\left(0, \frac{9}{2}\right)$

Ans. Official Answer NTA (2)

Sol. $f'(x) = 6x^2 + 2(2p - 7)x + 3(2p - 9)$

$f'(0) < 0$

$3(2p - 9) < 0$

$2p - 9 < 0$

$p - \frac{9}{2} \Rightarrow p \in \left(-\infty, \frac{9}{2}\right)$

Question ID : 7155051694

15. Let T and C respectively be the transverse and conjugate axes of the hyperbola $16x^2 - y^2 + 64x + 4y + 44 = 0$. Then the area of the region above the parabola $x^2 = y + 4$, below the transverse axis T and on the right of the conjugate axis C is :

माना अतिपरवलय $16x^2 - y^2 + 64x + 4y + 44 = 0$ के अनुप्रस्थ तथा संयुग्मी अक्ष क्रमशः T तथा C है। तो परवलय $x^2 = y + 4$ के ऊपर, अनुप्रस्थ अक्ष T के नीचे तथा संयुग्मी अक्ष की दांयी तरफ के क्षेत्र का क्षेत्रफल है :

(1) $4\sqrt{6}+\frac{44}{3}$ (2) $4\sqrt{6}+\frac{28}{3}$ (3) $4\sqrt{6}-\frac{44}{3}$ (4) $4\sqrt{6}-\frac{28}{3}$

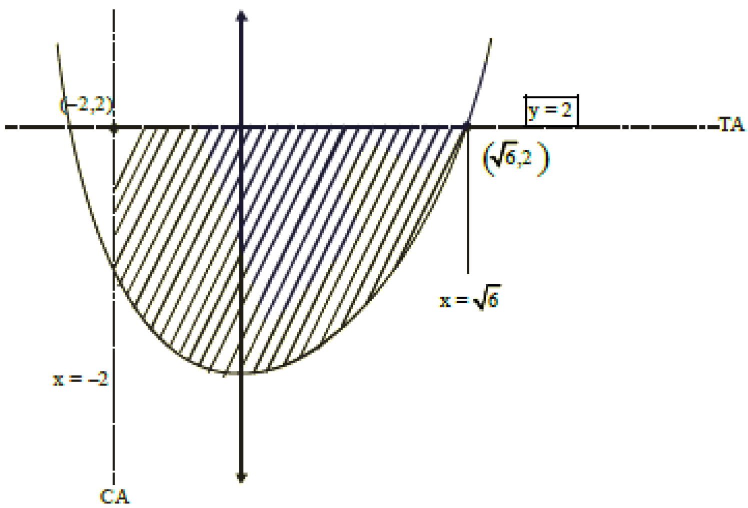
Ans. Official Answer NTA (2)

Sol. $16(x^2+4x)-(y^2-4y)+44=0$

$16(x+2)^2-64-(y-2)^2+4+44=0$

$16(x+2)^2-(y-2)^2=16$

$$\frac{(x+2)^2}{1}-\frac{(y-2)^2}{16}=1$$



$$A=\int_{-2}^{\sqrt{6}}\left(2-\left(x^2-4\right)\right)dx$$

$$A=\int_{-2}^{\sqrt{6}}\left(6-x^2\right)dx=\left(6x-\frac{x^3}{3}\right)_{-2}^{\sqrt{6}}$$

$$A=\left(6\sqrt{6}-\frac{6\sqrt{6}}{3}\right)-\left(-12+\frac{8}{3}\right)$$

$$A=\frac{12\sqrt{6}}{3}+\frac{28}{3}$$

$$A=4\sqrt{6}+\frac{28}{3}$$

Question ID : 7155051682

16. The number of functions

$f:\{1,2,3,4\}\rightarrow\{a\in:\mathbb{Z}|a|\le 8\}$

satisfying $f(n)+\frac{1}{n}f(n+1)=1, \forall n \in \{1,2,3\}$ is

$f(n)+\frac{1}{n}f(n+1)=1, \forall n \in \{1,2,3\}$

को संतुष्ट करने वाले फलनों

$f:\{1,2,3,4\} \rightarrow \{a \in \mathbb{Z} : |a| \leq 8\}$

की संख्या है

(1) 3 (2) 2 (3) 4 (4) 1

Ans. Official Answer NTA (2)

Sol. $nf(n) + f(n+1) = n$

$f(1) + f(2) = 1$

$2f(2) + f(3) = 2$

$3f(3) + f(4) = 3$

$2.(1-f(1)) + f(3) = 2$

$f(3) = 2f(1)$

$3.2f(1)+f(4) = 3$

$f(4) = 3 - 6\,f(1) \leq 8$

$-8 \leq f(4) \leq 8$

$-8 \leq 3 - 6\,f(1) \leq 8$

$-11 \leq -6f(1) \leq 5$

$-\frac{5}{6} \leq f(1) \leq \frac{11}{6}$

$f(1) = 0, 1$

C–I: $f(1) = 0 \Rightarrow f(2) = 1$

$f(3) = 0, f(4) = 3$

C–II: $f(1) = 1 \Rightarrow f(2) = 0$

$\Rightarrow f(3) = 2, f(4) = -3$

so '2' such f^n s.

Question ID : 7155051699

17. Let N be the sum of the numbers appeared when two fair dice are rolled and let the probability that $N-2$, $\sqrt{3N}$, $N+2$ are in geometric progression be $\frac{k}{48}$. Then the value of k is

माना दो न्याय पासे फेंकने पर प्राप्त संख्याओं का योग N है तथा माना कि $N-2, \sqrt{3N}, N+2$ के गुणोत्तर श्रेढ़ी में होने की प्रायिकता $\frac{k}{48}$ है। तो k बराबर है

(1) 4 (2) 16 (3) 2 (4) 8

Ans. Official Answer NTA (1)

Sol. $n(s) = 36$

Given : $N - 2, \sqrt{3N}, N + 2$ are in G.P.

$3N = (N - 2)(N + 2)$

$3N = N^2 - 4$

$\Rightarrow N^2 - 3N - 4 = 0$

$(N - 4)(N + 1) = 0 \Rightarrow N = 4$ or $N = -1$ rejected

$(\text{Sum} = 4) \equiv \{(1, 3), (3,1), (2,2)\}$

$n(A) = 3$

$P(A) = \frac{3}{36} = \frac{1}{12} = \frac{4}{48} \Rightarrow k = 4$

Question ID : 7155051689

18. If the function $f(x) = \begin{cases} (1+|\cos x|)^{\frac{\lambda}{|\cos x|}}, & 0 < x < \frac{\pi}{2} \\ \mu, & x = \frac{\pi}{2} \\ e^{\frac{\cot 6x}{\cot 4x}}, & \frac{\pi}{2} < x < \pi \end{cases}$

is continuous at $x = \frac{\pi}{2}$, then $9\lambda + 6\log_e \mu + \mu^6 - e^{6\lambda}$ is equal to

यदि फलन $f(x) = \begin{cases} (1+|\cos x|)^{\frac{\lambda}{|\cos x|}}, & 0 < x < \frac{\pi}{2} \\ \mu, & x = \frac{\pi}{2} \\ e^{\frac{\cot 6x}{\cot 4x}}, & \frac{\pi}{2} < x < \pi \end{cases}$

$x=\frac{\pi}{2}$ पर संतत है, तो $9\lambda+6\log_e\mu+\mu^6-e^{6\lambda}$ बराबर है

(1) 8 (2) 10 (3) $2e^4 + 8$ (4) 11

Ans. Official Answer NTA (Drop)

Sol. $f\left(\frac{\pi}{2}+\right)=\lim_{x\to\frac{\pi}{2}+} e^{\frac{\cot 6x}{\cot 4x}}=\lim_{x\to\frac{\pi}{2}+} e^{\frac{\sin 4x}{\sin 6x}\cdot\frac{\cos 6x}{\cos 4x}}$

$=\lim_{x\to\frac{\pi}{2}+} e^{\frac{4\cos 4x}{6\cos 6x}}=e^{\frac{2}{3}}$

$\text{LHL } f\left(\frac{\pi^-}{2}\right)=\lim_{x\to\frac{\pi^-}{2}}(1+|\cos x|)^{\frac{\lambda}{\cos x}}=e^{\lambda}\Rightarrow\lambda=\frac{2}{3},\mu=e^{\frac{2}{3}}$

So, $9\lambda+6\ln\mu+\mu^6-e^{6\lambda}$

$=9\left(\frac{2}{3}\right)+6\left(\frac{2}{3}\right)+e^4-e^4=6+4=10$

Question ID : 7155051697

19. Let $\vec{a}=-\hat{i}-\hat{j}+\hat{k},\vec{a}\cdot\vec{b}=1$ and $\vec{a}\times\vec{b}=\hat{i}-\hat{j}$

Then $\vec{a}-6\vec{b}$ is equal to

माना $\vec{a}=-\hat{i}-\hat{j}+\hat{k},\vec{a}\cdot\vec{b}=1$ तथा $\vec{a}\times\vec{b}=\hat{i}-\hat{j}$

तो $\vec{a}-6\vec{b}$ बराबर है

(1) $3(\hat{i}+\hat{j}+\hat{k})$ (2) $3(\hat{i}-\hat{j}-\hat{k})$ (3) $3(\hat{i}+\hat{j}-\hat{k})$ (4) $3(\hat{i}-\hat{j}+\hat{k})$

Ans. Official Answer NTA (1)

Sol. $\vec{a}\times\vec{b}=(\hat{i}-\hat{j})$

Taking cross product with $\vec{a}$

$\Rightarrow\ \vec{a}\times(\vec{a}\times\vec{b})=\vec{a}\times(\hat{i}-\hat{j})$

$\Rightarrow\ (\vec{a}\cdot\vec{b})\vec{a}-(\vec{a}\cdot\vec{a})\vec{b}=\hat{i}+\hat{j}+2\hat{k}$

$\Rightarrow\ \vec{a}-3\vec{b}=\hat{i}+\hat{j}+2\hat{k}$

$\Rightarrow\ 2\vec{a}-6\vec{b}=2\hat{i}+2\hat{j}+4\hat{k}$

$\Rightarrow\ \vec{a}-6\vec{b}=3\hat{i}+3\hat{j}+3\hat{k}$

Question ID : 7155051685

20. Let A, B, C be 3×3 matrices such that A is symmetric and B and C are skew-symmetric.

Consider the statements

(S1) $A^{13}B^{26} - B^{26}A^{13}$ is symmetric

(S2) $A^{26}C^{13} - C^{13}A^{26}$ is symmetric

Then

(1) Both S1 and S2 are true

(2) Only S1 is true

(3) Both S1 and S2 are false

(4) Only S2 is true

माना 3×3 के आव्यूहों A, B, C में A सममित है तथा B और C विषम सममित है। कथनों

(S1) $A^{13}B^{26} - B^{26}A^{13}$ सममित है

(S2) $A^{26}C^{13} - C^{13}A^{26}$ सममित है

का विचार कीजिए। तो

(1) S1 तथा S2 दोनों सत्य है

(2) केवल S1 सत्य है

(3) S1 तथा S2 दोनों असत्य है

(4) केवल S2 सत्य है

Ans. Official Answer NTA (4)

Sol. $S_1 : (A^{13}\,B^{26} - B^{26}\,A^{13})^T = (A^{13}\,B^{26})T - (B^{26}\,A^{13})^T$

$\Rightarrow (B^{26})^T\,(A^{13})^T - (A^{13})^T\,(B^{26})^T$

$\Rightarrow (BT)^{26}\,(A^T)^{13} - (A^T)^{13}\,(B^T)^{26}$

$\Rightarrow (-B)^{26}\,A^{13} - A^{13}\,(-B)26$

$\Rightarrow -(A^{13}B^{26} - B26\,A^{13})$ so skew symmetric

$S_2 : (A^{26}\,C^{13} - C^{13}\,A^{26})^T$

$= (C^T)^{13}\,(A^T)^{26} - (A^T)^{26}\,(C^T)^{13}$

$= -\,C^{13}\,A^{26} + A^{26}\,.\,C^{13}$

So, symmetric matrix

Hence S_2 is true and S_1 is false

SECTION – B

Question ID : 7155051704

21. For the two positive numbers a, b, if a, b and $\frac{1}{18}$ are in geometric progression, while $\frac{1}{a}$, 10 and $\frac{1}{b}$ are in an arithmetic progression, then 16a + 12b is equal to _________.

दो घनात्मक संख्याओं a, b के लिए, यदि a, b तथा $\frac{1}{18}$ एक गुणोत्तर श्रेढ़ी में है और $\frac{1}{a}$, 10 तथा $\frac{1}{b}$ एक समांतर श्रेढ़ी में है, तो 16a + 12b बराबर है।

Ans. Official Answer NTA (3)

Sol. $a, b, \frac{1}{18} \rightarrow GP$

$\frac{a}{18} = b^2$ .....(i)

$\frac{1}{a}, 10, \frac{1}{b} \rightarrow AP$

$\frac{1}{a} + \frac{1}{b} = 20$

$\Rightarrow a + b = 20ab$, from eq. (i) ; we get

$\Rightarrow 18b^2 + b = 360b^3$

$\Rightarrow 360b^2 - 18b - 1 = 0 \quad \{\because b \neq 0\}$

$\Rightarrow b = \frac{18 \pm \sqrt{324 + 1440}}{720}$

$\Rightarrow b = \frac{18 + \sqrt{1764}}{720} \quad \{\because b > 0\}$

$\Rightarrow b = \frac{1}{12}$

$\Rightarrow a = 18 \times \frac{1}{144} = \frac{1}{8}$

Now, $16a + 12b = 16 \times \frac{1}{8} + 12 \times \frac{1}{12} = 3$

Question ID : 7155051710

22. If m and n respectively are the numbers of positive and negative values of θ in the interval $[-\pi, \pi]$ that satifsy the equation $\cos 2\theta \cos\frac{\theta}{2} = \cos 3\theta \cos\frac{9\theta}{2}$, then mn is equal to ______.

यदि समीकरण $\cos 2\theta \cos\frac{\theta}{2} = \cos 3\theta \cos\frac{9\theta}{2}$ को संतुष्ट करने वाले अंतराल $[-\pi, \pi]$ में θ के धनात्मक तथा ऋणात्मक मानों की संख्या क्रमशः m तथा n है, तो mn बराबर है।

Ans. Official Answer NTA (25)

Sol. $\cos\frac{5\theta}{2} + \cos\frac{3\theta}{2} = \cos\frac{15\theta}{2} + \cos\frac{3\theta}{1} \Rightarrow \frac{15\theta}{2} = 2n\pi \pm \frac{5\theta}{2}$

$5\theta = 2n\pi \Rightarrow \frac{2n\pi}{5}$

$10\theta = 2n\pi \Rightarrow \frac{n\pi}{5}$

$\theta = \frac{n\pi}{5}$

$-\pi \le \frac{p\pi}{5} \le \pi$

$-5 \le p \le 5$

$m = 5$

$n = 5$

$m\,n = 25$

Question ID : 7155051706

23. If $\int_{\frac{1}{3}}^{3} |\log_e x|\,dx = \frac{m}{n}\log_e\left(\frac{n^2}{e}\right)$, where m and n are coprime natural numbers, then $m^2 + n^2 - 5$ is equal ______.

यदि $\int_{\frac{1}{3}}^{3} |\log_e x|\,dx = \frac{m}{n}\log_e\left(\frac{n^2}{e}\right)$ है, जहाँ m तथा n असहभाज्य धन पूर्णांक है, तो $m^2 + n^2 - 5$ बराबर है।

Ans. Official Answer NTA (20)

Sol. $\int_{\frac{1}{3}}^{3} |\ell nx|\,dx = \int_{\frac{1}{3}}^{1} (-\ell nx)dx + \int_{1}^{3} (\ell nx)dx$

$= -[x\ell nx - x]_{1/3}^{1} + [x\ell nx - x]_{1}^{3}$

$= -\left[-1 - \left(\frac{1}{3}\ln\frac{1}{3} - \frac{1}{3}\right)\right] + [3\ln 3 - 3 - (-1)]$

$= \left[-\frac{2}{3} - \frac{1}{3}\ln\frac{1}{3}\right] + [3\ln 3 - 2]$

$= -\frac{4}{3} + \frac{8}{3}\ln 3$

$= \frac{4}{3}(2\ln 3 - 1)$

$= \frac{4}{3}\left(\ell n \frac{9}{2}\right)$

$\therefore$ m = 4, n = 3

Now, $m^2 + n^2 - 5 = 16 + 9 - 5 = 20$

Question ID : 7155051709

24. 25% of the population are smokers. A smoker has 27 times more chances to develop lung cancer than a non smoker. A person is diagnosed with lung cancer and the probability that this person is a smoker is $\frac{k}{10}$. Then the value of k is ___________.

किसी जनसंख्या का 25% धुम्रपान करते है। एक धुम्रपान करने वाले को फेफड़े में कैंसर होने की संभावना धुम्रपान न करने वाले की अपेक्षा 27 गुना है। एक व्यक्ति के फेफड़े में कैंसर होने का पता चलता है तथा इस व्यक्ति के धुम्रपान करने की प्रायिकता $\frac{k}{10}$ है। तो k का मान है।

Ans. Official Answer NTA (9)

Sol. $P(S) = \frac{1}{4}, P(N) = \frac{3}{4}$

$$P\left(\frac{C}{S}\right) = 27P\left(\frac{C}{N}\right); P\left(\frac{S}{C}\right) = \frac{P(S).P\left(\frac{C}{S}\right)}{P(S).P\left(\frac{C}{S}\right) + P(N).P\left(\frac{C}{N}\right)}$$

$$= \frac{\frac{1}{4} \times 27 \times P\left(\frac{C}{N}\right)}{\frac{1}{4} \times 27 \times P\left(\frac{C}{N}\right) + \frac{3}{4} \times P\left(\frac{C}{N}\right)} = \frac{27}{27+3} = \frac{27}{30} = \frac{9}{10} = \frac{k}{10} \Rightarrow k = 9$$

Question ID : 7155051705

25. Ponts P(-3,2), Q(9,10) and $R(\alpha, 4)$ lie on a circle C with PR as its diameter. The tangents to C at the points Q and R intersect at the point. If S lies on the line $2x - ky = 1$, then k is equal to ________.

बिंदु P(-3,2), Q(9,10) तथा $R(\alpha, 4)$ एक वृत्त पर है, जिसका एक व्यास PR है। बिंदुओं Q तथा R पर वृत्त C की स्पर्श

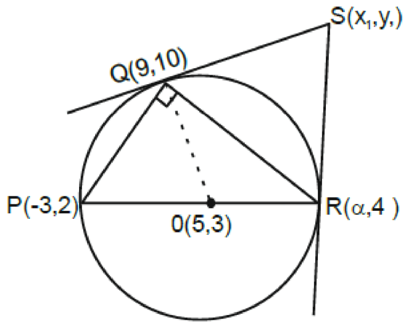
रेखाएँ बिंदु S पर मिलती है। यदि बिंदु S रेखा $2x - ky = 1$ पर है, तो k बराबर है।

Ans. Official Answer NTA (3)

Sol. $m_{PQ} \cdot m_{QR} = -1$

$$\Rightarrow \frac{10-2}{9+3} \times \frac{10-4}{9-\alpha} = -1 \Rightarrow \alpha = 13$$

$$m_{0P} \cdot m_{QS} = -1 \Rightarrow m_{QS} = -\frac{4}{7}$$



Equation of QS

$$y - 10 = -\frac{4}{7}(x - 9)$$

$\Rightarrow 4x + 7y = 106$ .....(1)

$m_{0R} - m_{RS} = -1 \Rightarrow m_{RS} = -8$

Equation of RS

$y - 4 = -8(x - 13)$

$\Rightarrow 8x + y = 108$ .....(2)

Solving eq. (1) & (2)

$$x_1 = \frac{25}{2} y_1 = 8$$

$S(x_1,y_1)$ lies on $2x - ky = 1$

$25 - 8k = 1$

$\Rightarrow 8k = 24$

$\Rightarrow k = 3$

Question ID : 7155051703

26. The remainder when $(2023)^{2023}$ is divided by 35 is __________.

$(2023)^{2023}$ को 35 से विभाजित करने पर शेषफल है।

Ans. Official Answer NTA (7)

Sol. $-72023 = -7 \times 7^{2022}$

$$= \frac{-7^{2022}}{5}$$

$= \frac{-[50-1]^{1011}}{5} = \frac{[5\lambda-1]}{5}$

remainder = + 1

when divided by 5 now remainder is 7 when divided 35

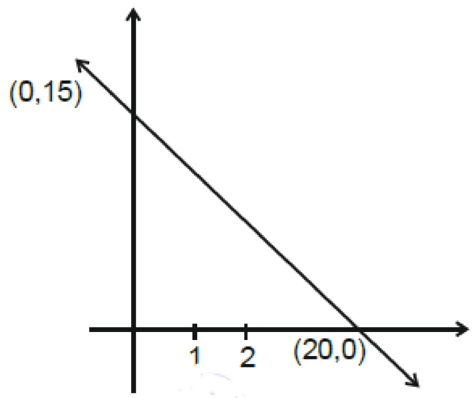
Question ID : 7155051707

27. A triangle is formed by X-axis, Y-axis and the line $3x + 4y = 60$. Then the number f points P(a,b) which lie strictly inside the triangle, where a is an integer and b is a multiple of a, is ______.

X-अक्ष, Y-अक्ष तथा रेखा $3x + 4y = 60$ एक त्रिभुज बनाते है। तो ऐसे बिंदुओं P(a,b) जहाँ a एक पूर्णांक है तथा b, a का एक गुणज है, जो त्रिभुज के अंदर है, की संख्या है।

Ans. Official Answer NTA (31)

Sol. If $x = 1, y = \frac{57}{4} = 14.25$



(1, 1) (1, 2) – (1, 14) $\Rightarrow$ 14 pts.

If $x = 2,\ y = \frac{27}{2} = 13.5$

(2, 2) (2, 4) ¡K. (2, 12) $\Rightarrow$ 6 pts.

If $x = 3,\ y = \frac{51}{4} = 12.75$

(3, 3) (3, 6) – (3, 12) $\Rightarrow$ 4 pts.

If x = 4, y = 12

(4, 4) (4, 8) $\Rightarrow$ pts.

If $x = 5.\ y = \frac{45}{4} = 11.25$

(5, 5), (5, 10) $\Rightarrow$ 2 pts.

If $x = 6,\ y = \frac{21}{2} = 10.5$

(6, 6) $\Rightarrow$ 1 pt.

If $x = 7,\ y = \frac{39}{4} = 9.75$

(7, 7) $\Rightarrow$ 1 pt.

If $x = 8, y = 9$

(8, 8) $\Rightarrow$ 1 pt.

If $x = 9\ \ y = \frac{33}{4} = 8.25$ $\Rightarrow$ no pt.

Total = 31 pts.

Question ID : 7155051702

28. Suppose Anil's mother wants to give 5 whole fruits to Anil from a basket of 7 red apples, 5 white apples snf 8 oranges. If in the selected 5 fruits, at least 2 oranges, at least 2 oranges, at least one red apple and at least one white apple must be given, then the numbers of ways, Anil's mother can offer 5 fruits to Anil is _______.

माना अनिल की माँ एक टोकरी, जिसमें 7 लाल सेब, 5 सफेद सेब तथा 8 संतरे हैं, में से 5 फल अनिल को देना चाहती है। यदि टोकरी से लिए गए 5 फलों में कम से कम 2 संतरे, कम से कम एक लाल सेब तथा कम से कम एक सफेद सेब अवश्य होने चाहिए, तो अनिल की माँ द्वारा अनिल को 5 फल देने के तरीकों की संख्या है।

Ans. Official Answer NTA (6860 or 3)

Sol.

	7 Red Apple	**5 white Apple**	**8 oranges**
	x +	y +	z = 5
	≥ 1	≥ 1	≥ 2
C-I	2	1	2
C-II	1	2	2
C-III	1	1	3

$^7C_2 \cdot {}^5C_1 \cdot {}^8C_2 + {}^7C_1 \cdot {}^5C_2 \cdot {}^8C_2 + {}^7C_1 \cdot {}^5C_1 \cdot {}^8C_3$

$= 2940 + 1960 + 1960 = 6860$

Question ID : 7155051708

29. If the shortest distance between the line joining the points (1,2,3) and (2,3,4), and the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{0}$ is α, then $28\alpha^2$ is equal to _______.

यदि बिंदुओं (1,2,3) तथा (2,3,4) को मिलाने वाली रेखा तथा रेखा $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-2}{0}$ के बीच न्यूनतम दूरी α है, तो $28\alpha^2$ बराबर है।

Ans. Official Answer NTA (18)

Sol. $\frac{x+1}{1}=\frac{y}{\frac{1}{2}}=\frac{z}{\frac{-1}{12}}$ and $\frac{x}{1}=\frac{y+2}{1}=\frac{z-1}{\frac{1}{6}}$

$\Rightarrow$ Shortest distance $=\frac{(\vec{b}-\vec{a})\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}$

S.D. $=(-\hat{i}+2\hat{j}-\hat{k})\cdot\frac{(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}$

$$\left\{\vec{p}\times\vec{q}\equiv\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & \frac{1}{2} & \frac{-1}{12}\\ 1 & 1 & \frac{1}{6}\end{vmatrix}=\frac{1}{6}\hat{i}-\frac{1}{4}\hat{j}+\frac{1}{2}\hat{k} \text{ or } 2\hat{i}-3\hat{j}+6\hat{k}\right\}$$

S.D. $=\frac{(-\hat{i}+2\hat{j}-\hat{k})\cdot(2\hat{i}-3\hat{j}+6\hat{k})}{\sqrt{2^2+3^2+6^2}}=\left|\frac{-14}{7}\right|=2$

Question ID : 7155051701

30. Let $a\in\mathbb{R}$ and let α,β be the roots of the equation $x^2+60^{\frac{1}{4}}x+a=0$. If $\alpha^4+\beta^4=-30$, then the product of all possile values of a is ________.

माना $a\in\mathbb{R}$ तथा समीकरण $x^2+60^{\frac{1}{4}}x+a=0$ के मूल α,β है। यदि $\alpha^4+\beta^4=-30$ है, तो a के सभी संभव मानों का गुणनफल है।

Ans. Official Answer NTA (45)

Sol. Since, $\alpha^4+\beta^4=-30$

$\Rightarrow\left[(\alpha+\beta)^2-2\alpha\beta\right]^2-2\alpha^2\beta^2=-30$

$\Rightarrow\left[\left(-60^{1/4}\right)^2-2a\right]^2-2a^2=-30$

$\Rightarrow(\sqrt{60}-2a)^2=2a^2-30$

$\Rightarrow 60+4a^2-4a\sqrt{60}=2a^2-30$

$\Rightarrow 2a^2-4a(\sqrt{60})+90=0$

$\Rightarrow a^2-2a(\sqrt{60})+45=0$

So, product of value of a = 45

Product = 45