JEE Main January 2023

Question Paper With Text Solution

25 January | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JANUARY 2023 | 25TH JANUARY SHIFT-1

SECTION – A

Question ID : 3666941241

1. Let y = y(x) be the solution curve of the differential equation $\frac{dy}{dx}=\frac{y}{x}\left(1+xy^2(1+\log_e x)\right)$, x > 0, y(1) =3. Then $\frac{y^2(x)}{9}$ is equal to :

माना अवकल समीकरण $\frac{dy}{dx}=\frac{y}{x}\left(1+xy^2(1+\log_e x)\right)$, $x>0$, $y(1)=3$ का हल $y=y(x)$ है। तो $\frac{y^2(x)}{9}$ बराबर है :

(1) $\frac{x^2}{7-3x^2\left(2+\log_e x^2\right)}$

(2) $\frac{x^2}{2x^3\left(2+\log_e x^3\right)-3}$

(3) $\frac{x^2}{3x^3\left(1+\log_e x^2\right)-2}$

(4) $\frac{x^2}{5-2x^3\left(2+\log_e x^3\right)}$

Ans. Official Answer NTA (4)

Sol. $\frac{dy}{dx}=\frac{y}{x}\left(1+xy^2+1+\log_e x\right), x>0$

$\frac{dy}{dx}=\frac{y}{x}+y^3\cdot\left(1+\log_e x\right)$

$\frac{1}{y^3}\cdot\frac{dy}{dx}=\frac{1}{x\cdot y^2}+\left(1+\log_e x\right)$

$\frac{1}{y^3}\cdot\frac{dy}{dx}-\frac{1}{x}\cdot\frac{1}{y^2}=\left(1+\ln x\right)$

Let $\frac{1}{y^2}=t$

$-\frac{2}{y^3}\cdot\frac{dy}{dx}=\frac{dt}{dx}$

$\therefore -\frac{1}{2}\frac{dt}{dx}-\frac{t}{x}=\left(1+\ln x\right)$

$\frac{dt}{dx}+\frac{2t}{x}=-2\left(1+\ln x\right)$

if $e^{\int \frac{2}{x}dx} = x^2$

$\therefore tx^2 = \int -2(1+\ln x)x^2 \cdot dx$

$$tx^2 = -2\left[(1+\ln x)\frac{x^3}{3} - \int \frac{x^2}{3}dx\right] + C$$

$$\frac{x^2}{y^2} = -2\left[\frac{x^3}{3}(1+\ln x) - \frac{x^3}{9}\right] + C \quad ...(1)$$

$\because y(1) = 3$

$$\frac{1}{9} = -2\left[\frac{1}{3}(1+0) - \frac{1}{9}\right] + C$$

$c = \frac{5}{9}$

put $c = \frac{5}{9}$

$$\Rightarrow \frac{y^2(x)}{9} = \frac{x^2}{5 - 2x^3\left(2 + \log_e x^3\right)}$$

Question ID : 3666941247

2. The mean and variance of the marks obtained by the students in a test are 10 and 4 respectively. Later, the marks of one of the students is increased from 8 to 12. If the new mean of the marks is 10.2, then their new variance is equal to :

छात्रों द्वारा एक परीक्षा में प्राप्त अंकों के माध्य तथा प्रसरण क्रमशः 10 तथा 4 है। बाद में एक छात्र के अंक 8 से बढ़ाकर 12 किए जाते है। यदि अंकों का नया माध्य 10.2 है, तो उनका नया प्रसरण है :

(1) 4.04 (2) 3.96 (3) 3.92 (4) 4.08

Ans. Official Answer NTA (2)

Sol. Let number of observations is n

$(10.2)n = 10n - 8 + 12$

$\Rightarrow (10.2)n = 10n + 4$

$\Rightarrow n = 20$

For earlier observation set

$$\frac{\Sigma x_i^2}{20} - (10)^2 = 4$$

$\Sigma x_i^2 = (104)(20) = 2080$

After change

$\left(\Sigma x_i^2\right)_{new} = 2080 - 8^2 + 12^2$

$= 2160$

New variance $= \dfrac{2160}{20} - (10.2)^2$

$= 108 - (10.2)^2$

$= 3.96$

Question ID : 36669412443

3. The points of intersection of the line $ax + by = 0$, $(a \neq b)$ and the circle $x^2 + y^2 - 2x = 0$ are $A(\alpha, 0)$ and $B(1, \beta)$. The image of the circle with AB as a diameter in the line $x + y + 2 = 0$ is :

रेखा $ax + by = 0$, $(a \neq b)$ तथा वृत्त $x^2 + y^2 - 2x = 0$ के प्रतिच्छेदन बिंदु $A(\alpha, 0)$ तथा $B(1, \beta)$ है। वृत्त, जिसका एक व्यास AB है, का रेखा $x + y + 2 = 0$ में प्रतिबिंब है :

(1) $x^2 + y^2 + 3x + 3y + 4 = 0$
(2) $x^2 + y^2 + 3x + 5y + 8 = 0$
(3) $x^2 + y^2 - 5x - 5y + 12 = 0$
(4) $x^2 + y^2 + 5x + 5y + 12 = 0$

Ans. Official Answer NTA (4)

Sol. If the points of intersection of the line $ax + by = 0$, $(a \neq b)$ and the circle $x^2 + y^2 - 2x = 0$ are $A(\alpha, 0)$ and

$B(1, \beta) \Rightarrow \alpha = 0 \quad \beta = \dfrac{-a}{b}$ and $\beta^2 = 1$

$\Rightarrow a^2 = b^2 (a \neq b) \Rightarrow a + b = 0$

$\Rightarrow \beta = 1$

So centre of circle with diameter AB is $\left(\dfrac{1}{2} \dfrac{1}{2}\right)$ and radius $= \dfrac{1}{\sqrt{2}}$, where A(0, 0) B (1, 1)

Now image of $\left(\dfrac{1}{2} \dfrac{1}{2}\right)$ about the line $x + y + 2 = 0$

$$\frac{x - \frac{1}{2}}{1} = \frac{y - \frac{1}{2}}{1} = -2\left(\frac{3}{2}\right)$$

$\Rightarrow$ Image $\left(-\dfrac{5}{2}, -\dfrac{5}{2}\right)$

equation of Image circle

$$\left(x + \frac{5}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = \frac{1}{2}$$

Question ID : 3666941237

4. Let x = 2 be local minima of the function $f(x) = 2x^4 - 18x^2 + 8x + 12, x \in (-4, 4)$. If M is local maximum value of the function f in (–4, 4), then M = :

माना फलन $f(x) = 2x^4 - 18x^2 + 8x + 12, x \in (-4, 4)$ का एक स्थानीय निम्नतम x = 2 पर है। यदि (–4, 4) में फलन f का स्थानीय उच्चतम M है, तो M =

(1) $12\sqrt{6} - \frac{33}{2}$ (2) $18\sqrt{6} - \frac{31}{2}$ (3) $18\sqrt{6} - \frac{33}{2}$ (4) $12\sqrt{6} - \frac{31}{2}$

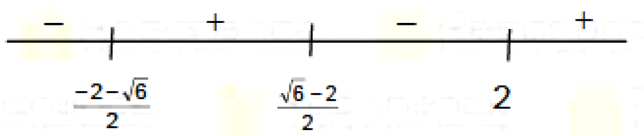
Ans. Official Answer NTA (1)

Sol. $f(x) = 2x^4 - 18x^2 + 8x + 12$

$f'(x) = 8x^3 - 36x + 8$

$= 4(x - 2)(2x^2 + 4x - 1)$

Number line of f'(x) is given by



so maxima point is $x = \frac{\sqrt{6}-2}{2}$

and maximum value of $f(x) = 12\sqrt{6} - \frac{33}{2}$

Question ID : 3666941232

5. Let S_1 and S_2 be respectively the sets of all $a \in R - \{0\}$ for which the system of linear equations :

$ax + 2ay - 3az = 1$
$(2a + 1)x + (2a + 3)y + (a + 1)z = 2$
$(3a + 5)x + (a + 5)y + (a + 2)z = 3$

has unique solution and infinitely many solutions. Then :

(1) S_1 is an infinite set and $n(S_2) = 2$ (2) $S_1 = R - \{0\}$ and $S_2 = \phi$

(3) $n(S_1) = 2$ and S_2 is an infinite set (4) $S_1 = \phi$ and $S_2 = R - \{0\}$

माना सभी $a \in R - \{0\}$, जिनके लिए रैखिक समीकरण निकाय

$ax + 2ay - 3az = 1$
$(2a + 1)x + (2a + 3)y + (a + 1)z = 2$
$(3a + 5)x + (a + 5)y + (a + 2)z = 3$

का केवल एक हल है तथा अनंत हल है, के समुच्चय क्रमशः S_1 तथा S_2 है। तो

(1) S_1 एक अपरिमित समुच्चय है तथा $n(S_2) = 2$ है

(2) $S_1 = R - \{0\}$ तथा $S_2 = \phi$

(3) $n(S_1) = 2$ तथा S_2 एक अपरिमित समुच्चय है

(4) $S_1 = \phi$ तथा $S_2 = R - \{0\}$

Ans. Official Answer NTA (2)

Sol. $$D = \begin{bmatrix} a & 2a & -3a \\ 2a+1 & 2a+3 & 2a+1 \\ 3a+5 & a+5 & a+2 \end{bmatrix} = a\left(15a^2 + 31a + 37\right)$$

$\Rightarrow D \neq a$ for all $a \in R - \{0\}$

Question ID : 3666941240

6. The minimum value of the function $f(x) = \int_0^2 e^{|x-t|}\,dt$ is :

फलन $f(x) = \int_0^2 e^{|x-t|}\,dt$ का निम्नतम मान है :

(1) 2 (2) e (e – 1) (3) 2 (e – 1) (4) 2e – 1

Ans. Official Answer NTA (3)

Sol. For $x > 2$

$f(x) = \int_0^2 e^{x-t}dt$

$= e^x\left(-e^{-t}\right)\Big|_0^2$

$= e^x\left(1 - e^{-2}\right)$

For $x < 0$

$f(x) = \int_0^2 e^{t-x}dt = e^{-x}e^t\Big|_0^2 = e^{-x}\left(e^2 - 1\right)$

For $0 \le x \le 2$

$f(x) = \int_0^x e^{x-t}dt + \int_x^2 e^{t-x}dt$

$= -e^x e^{-t}\Big|_0^x + e^{-x}e^t\Big|_x^2$

$\Rightarrow -e^x\left(e^{-x} - 1\right) + e^{-x}\left(e^2 - e^x\right)$

$\Rightarrow -1 + e^x + e^{2-x} - 1$

$= e^{2-x} + e^x - 2$

$$f(x)=\begin{cases} e^x\left(1-e^{-2}\right); & x>2 \\ e^{2-x}+e^x-2; & 0\le x\le 2 \\ e^{-x}\left(e^2-1\right); & x<0 \end{cases}$$

For $x > 2$

$f(x)_{min} = e^2 - 1$

For $0 \le x \le 2$

$f'(x) = -e^{2-x} + e^x = 0 \Rightarrow e^x = e^{2-x} \Rightarrow e^{2x} = e^2 \Rightarrow x = 1$

$f(x) = 2e - 2 = 2(e-1)$

For $x < 0$

$f(x)_{min} = e^2 - 1$

Question ID : 3666941245

7. The distance of the point P(4, 6, –2) from the line passing through the point (–3, 2, 3) and parallel to a line with direction ratios 3, 3, –1 is equal to :

बिंदु (–3, 2, 3) से होकर जाने वाली तथा दिक् अनुपात 3, 3, –1 की एक रेखा के समांतर रेखा से बिंदु P(4, 6, –2) की दूरी है :

(1) $2\sqrt{3}$ (2) $\sqrt{14}$ (3) $\sqrt{6}$ (4) 3

Ans. Official Answer NTA (2)

Sol. Equation of line passing through (–3, 2, 3) with direction ratios 3, 3, –1 is

$$\frac{x+3}{3}=\frac{y-2}{3}=\frac{z-3}{-1} \quad \text{\_\_\_\_\_\_ (1)}$$

Consider Q (3r – 3, 3r + 2, –r + 3) on the line (1) such that PQ is $\perp$ to line (1)

D.R. of PQ = 3r – 7, 3r – 4, – r + 5

PQ $\perp$ line (1)

So (3r – 7) (3) + (3r – 4) (3) + (– r + 5) (–1) = 0

$\Rightarrow$ r = 2

So Q (3, 8, 1)

$\Rightarrow \quad PQ = \sqrt{1+4+4} = \sqrt{14}$

Question ID : 3666941249

8. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non zero vectors such that $\vec{b}.\vec{c} = 0$ and $\vec{a}\times\left(\vec{b}\times\vec{c}\right) = \frac{\vec{b}-\vec{c}}{2}$. If $\vec{d}$ be a vector such that

$\vec{b}\cdot\vec{d}=\vec{a}\cdot\vec{b}$, then $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})$ is equal to :

माना $\vec{a}, \vec{b}$ और $\vec{c}$ तीन शून्येत्तर सदिश हैं जबकि $\vec{b}.\vec{c}=0$ और $\vec{a}\times(\vec{b}\times\vec{c})=\frac{\vec{b}-\vec{c}}{2}$ । यदि $\vec{d}$ एक सदिश ऐसा है कि $\vec{b}\cdot\vec{d}=\vec{a}\cdot\vec{b}$ तब $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})$ बराबर होगा–

(1) $\frac{1}{4}$ (2) $\frac{3}{4}$ (3) $-\frac{1}{4}$ (4) $\frac{1}{2}$

Ans. Official Answer NTA (1)

Sol. $\vec{a}\times(\vec{b}\times\vec{c})=\frac{\vec{b}-\vec{c}}{2}\Rightarrow(\vec{a},\vec{c})\vec{b}-(\vec{a},\vec{b})\vec{c}=\frac{\vec{b}}{2}-\frac{\vec{c}}{2}$

$\Rightarrow \vec{a},\vec{c}=\frac{1}{2},\vec{a}\cdot\vec{b}=\frac{1}{2}(\because \vec{b}\cdot\vec{c}=0)$

Given $\vec{b}\cdot\vec{d}=\vec{a}\cdot\vec{b}$

Now $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})=(\vec{a}\cdot\vec{c})(\vec{b}\cdot\vec{d})-(\vec{a}\cdot\vec{d})(\vec{b}\cdot\vec{c})=\frac{1}{4}$

Question ID : 3666941236

9. Consider the lines L_1 and L_2 given by

$L_1:\frac{x-1}{2}=\frac{y-3}{1}=\frac{z-2}{2}$

$L_2:\frac{x-2}{1}=\frac{y-2}{2}=\frac{z-3}{3}$

A line L_3 having direction ratios 1, –1, –2, intersects L_1 and L_2 at the points P and Q respectively. Then the length of line segment PQ is :

निम्न रेखाओं L_1 तथा L_2 का विचार कीजिए।

$L_1:\frac{x-1}{2}=\frac{y-3}{1}=\frac{z-2}{2}$

$L_2:\frac{x-2}{1}=\frac{y-2}{2}=\frac{z-3}{3}$

दिक् अनुपात 1, –1, –2 की एक रेखा L_3 रेखाओं L_1 तथा L_2 को क्रमशः बिंदुओं P तथा Q पर काटती है। तो रेखाखंड PQ की लंबाई है :

(1) $2\sqrt{6}$ (2) $4\sqrt{3}$ (3) $3\sqrt{2}$ (4) 4

Ans. Official Answer NTA (1)

Sol. Let P on L_1 be P $(2r_1+1, r_1+3, 2r_1+2)$

and Q on L_2 be Q $(r_2 + 2, 2r_2 + 2, 3r_2 + 3)$

$\Rightarrow$ D.R. of PQ = $2r_1 - r_2 - 1, r_1 - 2r_2 + 1, 2r_1 - 3r_2 - 1$

which are equivalent to $-1, 1, 2$

i.e. $$\frac{2r_1 - r_2 - 1}{-1} = \frac{r_1 - 2r_2 + 1}{1} = \frac{2r_1 - 3r_2 - 1}{2}$$

$\Rightarrow r_1 = 3 = r_2$

$\Rightarrow$ P(7, 6, 8) and Q (5, 8, 12) $\Rightarrow$ length PQ = $2\sqrt{6}$

Question ID : 3666941234

10. If a_r is the coefficient of x^{10-r} in the Binomial expansion of $(1+x)^{10}$, then $\sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2$ is equal to :

यदि $(1+x)^{10}$ के द्विपद प्रसार में x^{10-r} का गुणांक a_r है, तो $\sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2$ बराबर है :

(1) 4895 (2) 5445 (3) 3025 (4) 1210

Ans. Official Answer NTA (4)

Sol. Coeff. of x^{10-r} in $(1+x)^{10}$

$a_r = {}^{10}C_r$

$$\frac{a_r}{a_{r-1}} = \frac{{}^{10}C_r}{{}^{10}C_{r-1}} = \frac{10-r+1}{r} = \frac{11-r}{r}$$

$$\sum_{r=0}^{10} r^3 \left(\frac{11-r}{r}\right)^2 = \sum_{r=0}^{10} r(11-r)^2$$

$$= \sum_{r=0}^{10} \left(r^3 - 22r^2 + 121r\right)$$

$$= \sum_{r=0}^{10} r^3 - 22\sum_{r=0}^{10} r^2 + 121\sum_{r=0}^{10} r = \left(\frac{10(10+1)}{2}\right)^2 - 22 \times \frac{10(10+1)(20+1)}{6} + 121 \times \frac{10 \cdot 11}{2}$$

$$= 55^2 - 11 \times 11 \times 70 + 5 \times 11^3 = 11^2[25 - 70 + 55] = 11^2 \times 10 = 1210$$

Question ID : 3666941248

11. Let M be the maximum value of the product of two positive integers when their sum is 66. Let the sample space $S = \left\{x \in Z : x(66-x) \ge \frac{5}{9}M\right\}$ and the event $A = \{x \in S : x \text{ is a multiple of } 3\}$. Then P(A) is equal to :

माना 66 योगफल के दो धनात्मक पूर्णांकों का अधिकतम गुणनफल M है। माना प्रतिदर्श समष्टि

$S=\left\{x\in Z: x(66-x)\geq \frac{5}{9}M\right\}$ तथा घटना $A=\{x\in S: x, 3$ का एक गुणज है$\}$ है। तो $P(A)$ बराबर है :

(1) $\frac{7}{22}$ (2) $\frac{1}{5}$ (3) $\frac{1}{3}$ (4) $\frac{15}{44}$

Ans. Official Answer NTA (3)

Sol. $x+y=66, x, y\in N$

Now Maximum Value of xy is $M=(33)^2$ by using $A.M\geq G.M$

$\frac{x+y}{2}\geq (xy)^{\frac{1}{2}} \Rightarrow xy\leq (33)^2$

Now $S=\left\{x\in z, x(66-x)\geq \frac{5}{9}M\right\}$

$\Rightarrow S=\{x\in z, \quad x(66-x)\geq 55\times 11\}$

$S=\{11,12,13,\ldots,55\}$

$A=\{x\in s, x \text{ is multiple of } 3\}$

$A=\{12,15,18,\ldots,54\}$

$P(A)=\frac{n(A)}{n(S)}=\frac{15}{45}=\frac{1}{3}$

Question ID : 3666941238

12. Let $y(x)=(1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})$. Then $y'-y''$ at $x=-1$ is equal to :

माना $y(x)=(1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})$ है। तो $x=-1$ पर $y'-y''$ बराबर है :

(1) 464 (2) 944 (3) 496 (4) 976

Ans. Official Answer NTA (3)

Sol. $y=(1+x)\left(1+x^2\right)\left(1+x^4\right)\left(1+x^8\right)\left(1+x^{16}\right)$

$\Rightarrow y=\frac{(1-x)(1+x)\left(1+x^2\right)\left(1+x^4\right)\left(1+x^8\right)\left(1+x^{16}\right)}{(1-x)}$

$\Rightarrow y=\frac{1-x^{32}}{(1-x)}$

$\Rightarrow (1-x)y=1-x^{32}$

$\Rightarrow (1-x)y'-y=-32x^{31}$

$\Rightarrow (1-x)y''-2y'=-32.31x^{30}$

Put $x=-1$

$\Rightarrow y' - y'' = 496$

Question ID : 3666941246

13. The vector $\vec{a} = -\hat{i} + 2\hat{j} + \hat{k}$ is rotated through a right angle, passing through the y-axis in its way and the resulting vector is $\vec{b}$. Then the projection of $3\vec{a} + \sqrt{2}\vec{b}$ on $\vec{c} = 5\hat{i} + 4\hat{j} + 3\hat{k}$ is :

सदिश $\vec{a} = -\hat{i} + 2\hat{j} + \hat{k}$ को y-अक्ष से होकर ले जाते हुए एक समकोण तक घुमाया जाता है और इसके परिणापस्वरूप सदिश $\vec{b}$ प्राप्त होता है। तो $3\vec{a} + \sqrt{2}\vec{b}$ का सदिश $\vec{c} = 5\hat{i} + 4\hat{j} + 3\hat{k}$ पर प्रक्षेप है :

(1) $\sqrt{6}$ (2) $2\sqrt{3}$ (3) 1 (4) $3\sqrt{2}$

Ans. Official Answer NTA (4)

Sol. $\vec{b} = \lambda\vec{a} \times (\vec{a} \times \hat{j})$

$\Rightarrow \vec{b} = \lambda(-2\hat{i} - 2\hat{j} + 2\hat{k})$

$|\vec{b}| = |\vec{a}| \quad \therefore \sqrt{6} = \sqrt{12}\,|\lambda| \Rightarrow \lambda = \pm\frac{1}{\sqrt{2}}$

$\left(\lambda = \frac{1}{\sqrt{2}} \text{ rejected } \because \vec{b} \text{ makes acute angle with y axis}\right)$

$\vec{b} = -\sqrt{2}(-\hat{i} - \hat{j} + \hat{k})$

$\frac{(3\vec{a} + \sqrt{2}\vec{b}) \cdot \vec{c}}{|\vec{c}|} = 3\sqrt{2}$

Question ID : 3666941239

14. Let $f(x) = \int \frac{2x}{(x^2+1)(x^2+3)}dx$. If $f(3) = \frac{1}{2}(\log_e 5 - \log_e 6)$, then f(4) is equal to :

माना $f(x) = \int \frac{2x}{(x^2+1)(x^2+3)}dx$ है। यदि $f(3) = \frac{1}{2}(\log_e 5 - \log_e 6)$ तो f(4) बराबर है :

(1) $\frac{1}{2}(\log_e 17 - \log_e 19)$ (2) $\frac{1}{2}(\log_e 19 - \log_e 17)$

(3) $\log_e 17 - \log_e 18$ (4) $\log_e 19 - \log_e 20$

Ans. Official Answer NTA (1)

Sol. $f(x) = \int \frac{2x}{(x^2+1)(x^2+3)}dx$

Let $x^2 = t$

$2xdx = dt$

$$\int \frac{dt}{(t+1)(t+3)}$$

$$= \frac{1}{2}\int \frac{(t+3)-(t+1)}{(t+1)(t+3)}dt$$

$$= \frac{1}{2}[\ln|t+1| - \ln|t+3|] + \frac{C}{2}$$

$$= \frac{1}{2}\left[\ln\left|x^2+1\right| - \ln\left|x^2+3\right|\right] + \frac{C}{2}$$

$\because \quad f(3) = \frac{1}{2}[\ln 5 - \ln 6]$

$\therefore \quad \frac{1}{2}[\ln 5 - \ln 6] = \frac{1}{2}[\ln 10 - \ln 12] + \frac{C}{2}$

$\Rightarrow \quad C = 0$

$\therefore \quad f(x) = \frac{1}{2}\left[\ln\left|x^2+1\right| - \ln\left|x^2+3\right|\right]$

$f(4) = \frac{1}{2}[\ln 17 - \ln 19]$

Question ID : 3666941235

15. The value of $\lim\limits_{n\to\infty} \frac{1+2-3+4+5-6+.......+(3n-2)+(3n-1)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$ is :

$\lim\limits_{n\to\infty} \frac{1+2-3+4+5-6+.......+(3n-2)+(3n-1)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$ का मान है :

(1) $3\left(\sqrt{2}+1\right)$ (2) $\frac{3}{2}\left(\sqrt{2}+1\right)$ (3) $\frac{\sqrt{2}+1}{2}$ (4) $\frac{3}{2\sqrt{2}}$

Ans. Official Answer NTA (2)

Sol. $$\text{lt}_{n\to\infty} \frac{\sum_{r=1}^{n}((3r-2)+(3r-1)-3r)}{\sqrt{2n^4+3n+1}-\sqrt{n^4+n+3}}$$

$$lt_{n\to\infty}\frac{\sum_{r=1}^{n}3(r-1)}{\sqrt{2n^4+3n-1}-\sqrt{n^4+n+3}}$$

$$=lt_{n\to\infty}\frac{3\frac{n(n-1)}{2}\left(\sqrt{2n^4+3n-1}+\sqrt{n^4+n+3}\right)}{\left(2n^4+3n-1\right)-\left(n^4+n+3\right)}$$

$$=\frac{3}{2}(\sqrt{2}+1)$$

Question ID : 3666941242

16. The distance of the point $\left(6,-2\sqrt{2}\right)$ from the common tangent $y = mx + c, m > 0$, of the curves $x = 2y^2$ and $x = 1 + y^2$ is :

वक्रों $x = 2y^2$ तथा $x = 1 + y^2$ की उभयनिष्ठ स्पर्श रेखा $y = mx + c, m > 0$, से बिंदु $\left(6,-2\sqrt{2}\right)$ की दूरी बराबर है :

(1) $\frac{1}{3}$ (2) $\frac{14}{3}$ (3) $5\sqrt{3}$ (4) 5

Ans. Official Answer NTA (4)

Sol. For

$$y^2=\frac{x}{2}, T: y=mx+\frac{1}{8m}$$

For tangent to $y^2 + 1 = x$

$$\Rightarrow\left(mx+\frac{1}{8m}\right)^2+1=x$$

$$D=0\Rightarrow m=\frac{1}{2\sqrt{2}}$$

$$\therefore T: x-2\sqrt{2}y+1=0$$

$$d=\left|\frac{6+8+1}{\sqrt{9}}\right|=5$$

Question ID : 3666941243

17. Let $f:(0,1)\to R$ be a function defined by $f(x)=\frac{1}{1-e^{-x}}$, and $g(x) = (f(-x) - f(x))$. Consider two statements

(I) g is an increasing function in (0, 1)
(II) g is one-one in (0, 1)

Then, :

(1) Both (I) and (II) are true　　(2) Only (I) is true

(3) Neither (I) nor (II) is true　　(4) Only (II) is true

माना $f:(0,1)\to R,\ f(x)=\frac{1}{1-e^{-x}}$ द्वारा परिभाषित है तथा $g(x)=(f(-x)-f(x))$ है। दो कथनों का विचार कीजिए।

(I) (0, 1) में g एक वर्धमान फलन है

(II) (0, 1) में g एकैकी है

तो, :

(1) (I) तथा (II) दोनों सत्य है　　(2) केवल (I) सत्य है

(3) न तो (I) न ही (II) सत्य है　　(4) केवल (II) सत्य है

Ans. Official Answer NTA (1)

Sol. $g(x)=f(-x)-f(x)=\frac{1}{1-e^{x}}-\frac{1}{1-e^{-x}}$

$\Rightarrow g(x)=\frac{1+e^{x}}{1-e^{x}}$

$g'(x)=\frac{(1-e^{x})(e^{x})-(1+e^{x})(-e^{x})}{(1-e^{x})^{2}}$

$g'(x)=\frac{2e^{x}}{(1-e^{x})^{2}}>0$

g (x) is increasing as well as one-one.

Question ID : 3666941231

18. Let $z_1=2+3i$ and $z_2=3+4i$. The set $S=\left\{z\in C:|z-z_1|^2-|z-z_2|^2=|z_1-z_2|^2\right\}$ represents a :

(1) straight line with the sum of its intercepts on the coordinate axes equals –18

(2) hyperbola with eccentricity 2

(3) straight line with the sum of its intercepts on the coordinate axes equals 14

(4) hyperbola with the length of the transverse axis 7

माना $z_1=2+3i$ तथा $z_2=3+4i$ है। तो समुच्चय $S=\left\{z\in C:|z-z_1|^2-|z-z_2|^2=|z_1-z_2|^2\right\}$ किस को निरूपित करता है :

(1) एक सरल रेखा जिसके निर्देशांक अक्षों पर अंतःखंडों का योग –18 है

(2) एक अतिपरवलय जिसकी उत्केन्द्रता 2 है

(3) एक सरल रेखा जिसके निर्देशांक अक्षों पर अंतःखंडों का योग 14 है

(4) अतिपरवलय जिसके अनुप्रस्थ अक्ष की लंबाई 7 है

Ans. Official Answer NTA (3)

Sol. $\left((x-2)^2+(y-3)^2\right)-\left((x-3)^2-(y-4)^2\right)=1+1$

$\Rightarrow x+y=7$

Question ID : 3666941250

19. The statement $(p\wedge(\sim q))\Rightarrow(p\Rightarrow(\sim q))$ is :

(1) a tautology
(2) equivalent to $p\vee q$
(3) equivalent to $(\sim p)\vee(\sim q)$
(4) a contradiction

कथन $(p\wedge(\sim q))\Rightarrow(p\Rightarrow(\sim q))$

(1) एक पुनरुक्ति है
(2) $p\vee q$ के तुल्य है
(3) $(\sim p)\vee(\sim q)$ के तल्य है
(4) एक विरोधोक्ति है

Ans. Official Answer NTA (1)

Sol. Case I q = T

So $(P\wedge\sim q)\Rightarrow(P\Rightarrow\sim q)\equiv T$

Case II q = F

$\Rightarrow (P\wedge\sim q)\Rightarrow(P\Rightarrow\sim q)=T$

So $(P\wedge\sim q)\Rightarrow(P\Rightarrow\sim q)=T$

Question ID : 3666941233

20. Let x, y, z > 1 and $A\begin{bmatrix}1 & \log_x y & \log_x z\\ \log_y x & 2 & \log_y z\\ \log_z x & \log_z y & 3\end{bmatrix}$. Then |adj (ajd A^2)| is equal to :

माना x, y, z > 1 है तथा $A\begin{bmatrix}1 & \log_x y & \log_x z\\ \log_y x & 2 & \log_y z\\ \log_z x & \log_z y & 3\end{bmatrix}$ है। तो |adj (ajd A^2)| बराबर है :

(1) 4^8
(2) 2^8
(3) 6^4
(4) 2^4

Ans. Official Answer NTA (2)

Sol. $A=\begin{bmatrix}1 & \log_x y & \log_x z\\ \log_y x & 2 & \log_y z\\ \log_z x & \log_z y & 3\end{bmatrix}$

$$|A| = \frac{1}{\log x \log y \log z}\begin{vmatrix} \log x & \log y & \log z \\ \log x & 2\log y & \log z \\ \log x & \log y & 3\log z \end{vmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{vmatrix}$$

$|A| = 2$

$\left|\text{adj}\left(\text{adj}\, A^2\right)\right| = |A|^8$

$= 2^8$

SECTION – B

Question ID : 3666941260

21. If the sum of all the solution of $\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3}, -1 < x < 1, x \neq 0$, is $\alpha - \frac{4}{\sqrt{3}}$, then α is equal to ________.

यदि $\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3}, -1 < x < 1, x \neq 0$, के सभी हलों का योग $\alpha - \frac{4}{\sqrt{3}}$ है, तो α बराबर है।

Ans. Official Answer NTA (2)

Sol. **Case I :** $x > 0$

$$\tan^{-1}\frac{2x}{1-x^2} + \tan^{-1}\frac{2x}{1-x^2} = \frac{\pi}{3}$$

$x = 2 - \sqrt{3}$

Case II : $x < 0$

$$\tan^{-1}\frac{2x}{1-x^2} + \tan^{-1}\frac{2x}{1-x^2} + \pi = \frac{\pi}{3}$$

$x = \frac{-1}{\sqrt{3}} \Rightarrow \alpha = 2$

Question ID : 3666941259

22. Let the equation of the plane passing through the line $x - 2y - z - 5 = 0 = x + y + 3z - 5$ and parallel to the line $x + y + 2z - 7 = 0 = 2x + 3y + z - 2$ be $ax + by + cz = 65$. Then the distance of the point(a, b, c) from the plane

$2x + 2y - z + 16 = 0$ is ________.

माना रेखा $x - 2y - z - 5 = 0 = x + y + 3z - 5$ से होकर जाने वाले तथा रेखा $x + y + 2z - 7 = 0 = 2x + 3y + z - 2$ के समांतर समतल का समीकरण $ax + by + cz = 65$ है। तो बिंदु (a, b, c) की समतल $2x + 2y - z + 16 = 0$ से दूरी है।

Ans. Official Answer NTA (9)

Sol. since plane passing through the line

$x - 2y - z - 5 = 0 = x + y + 3z - 5$

So equation of plane is

$(x - 2y - z - 5) + \lambda (x + y + 3z - 5) = 0$

$(1 + \lambda) x + (\lambda - 2) y + (3\lambda - 1) z - 5 - 5\lambda = 0$ ___ (1)

And this plane parallel to the line $x + y + 2z - 7 = 0 = 2x + 3y + z - 2$ with direction ratio (–5, 3, 1)

Now

$-5 (1 + \lambda) + (\lambda - 2). 3 + (3\lambda - 1). 1 = 0$ [since plane & line are parallel]

$\Rightarrow \lambda = 12$

Hence equation of plane is $13 x + 10y + 35z = 65$

Given equation of place is $ax + by + cz = 65$

so, (a, b, c) = (13, 10, 35)

Now we have to find the distance of (13, 10, 35) from $2x + 2y - z + 16 = 0$

$$d = \frac{|26 + 20 - 35 + 16|}{\sqrt{4 + 4 + 1}} = 9$$

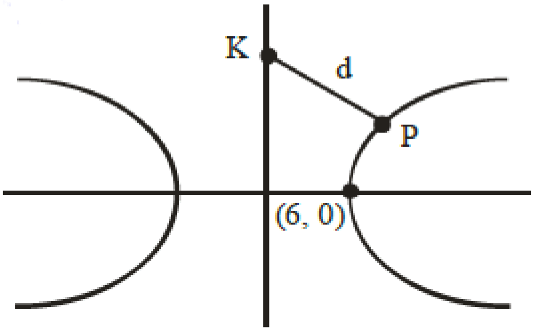
Question ID : 3666941256

23. The vertices of hyperbola H are (±6, 0) and its eccentricity is $\frac{\sqrt{5}}{2}$. Let N be the normal to H at a point in the first quadrant and parallel to the line $\sqrt{2}x + y = 2\sqrt{2}$. If d is the length of the line segment of N between H and the y-axis then d^2 is equal to ________.

एक अतिपरवलय H के शीर्ष (±6, 0) है तथा उत्केन्द्रता $\frac{\sqrt{5}}{2}$ है। माना प्रथम चतुर्थांश में H के एक बिंदु पर रेखा $\sqrt{2}x + y = 2\sqrt{2}$ के समांतर अभिलंब N है। यदि N के H तथा y-अक्ष के बीच रेखाखंड की लंबाई d है, तो d^2 बराबर है।

Ans. Official Answer NTA (216)

Sol.



$H : \frac{x^2}{36} - \frac{y^2}{9} = 1$

equation of normal is $6x\cos\theta + 3y\cot\theta = 45$

slope $= -2\sin\theta = -\sqrt{2}$

$\Rightarrow \theta = \frac{\pi}{4}$

Equation of normal is $\sqrt{2}x + y = 15$

$P : (a\sec\theta, b\tan\theta)$

$\Rightarrow P(6\sqrt{2}, 3)$ and K(0, 15)

$d^2 = 216$

Question ID : 3666941252

24. Let $S = \left\{\alpha : \log_2\left(9^{2\alpha-4} + 13\right) - \log_2\left(\frac{5}{2}.3^{2\alpha-4} + 1\right) = 2\right\}$. Then the maximum value of β for which the equation $x^2 - 2\left(\sum_{\alpha\in s}\alpha\right)^2 x + \sum_{\alpha\in s}(\alpha+1)^2\beta = 0$ has real roots, is ________.

माना $S = \left\{\alpha : \log_2\left(9^{2\alpha-4} + 13\right) - \log_2\left(\frac{5}{2}.3^{2\alpha-4} + 1\right) = 2\right\}$ है। तो β अधिकतम मान, जिसके लिए समीकरण $x^2 - 2\left(\sum_{\alpha\in s}\alpha\right)^2 x + \sum_{\alpha\in s}(\alpha+1)^2\beta = 0$ के वास्तविक मूल है।

Ans. Official Answer NTA (25)

Sol. $S = \left\{\alpha : \log_2\left(9^{2\alpha-4} + 13\right) - \log_2\left(\frac{5}{2}\cdot 3^{2\alpha-4} + 1\right) = 2\right\}$

$\log_2\left(9^{2\alpha-4}+13\right)-\log_2\left(\frac{5}{2}\cdot 3^{2\alpha-4}+1\right)=2$

$\Rightarrow \log_2\left(\frac{9^{2\alpha-4}+13}{\frac{5}{2}9^{\alpha-2}+4}\right)=2$

$\Rightarrow 9^{2\alpha-4}+13=4\left(\frac{5}{2}9^{\alpha-2}+1\right)$

$\Rightarrow 9^{2(\alpha-2)}+13=109^{\alpha-2}+4$

$\Rightarrow 9^2-10y+9=0$ [where $y=9^{\alpha\text{-}2}$]

$\Rightarrow (y-9)(y-1)=0$

$y=9$ or $y=1$

$9^{\alpha-2}=9$ or $9^{a-2}=1$

so $\alpha=2,3$

$\Rightarrow s=\{2,3\}$

Now

$x^2-2\left(\sum_{\alpha\in s}\alpha\right)^2 x+\sum_{\alpha=s}(\alpha+1)^2\beta=0$

$x^2-50x+25\beta=0$

for real roots

$D\ge 0$

$50^2-100\beta\ge 0$ $\left[D=b^2-4ac\right]$

$2500-100\beta\ge 0$

$25-\beta\ge 0$

$\beta\le 25$

$\beta_{max}=25$

Question ID : 3666941251

25. For some a, b, c $\in$ N, let f(x) = ax – 3 and g(x) = x^b + c, x $\in$ R. If $(fog)^{-1}(x)=\left(\frac{x-7}{2}\right)^{\frac{1}{3}}$, then (fog) (ac) + (gof) (b) is equal to ________.

किसी a, b, c ∈ N के लिए, माना $f(x) = ax - 3$ तथा $g(x) = x^b + c, x \in R$ है। यदि $(fog)^{-1}(x) = \left(\frac{x-7}{2}\right)^{\frac{1}{3}}$ है, तो

(fog) (ac) + (gof) (b) बराबर है।

Ans. Official Answer NTA (2039)

Sol. Let fog(x) = h(x)

$$\Rightarrow h^{-1}(x) = \left(\frac{x-7}{2}\right)^{\frac{1}{3}}$$

$\Rightarrow h(x) = fog(x) = 2x^3 + 7$

$fog(x) = a(xb + c) - 3$

$\Rightarrow a = 2, b = 3, c = 5$

$\Rightarrow fog(ac) = fog(10) = 2007$

$g(f(x) = (2x - 3)^3 + 5$

$\Rightarrow gof(b) = gof(3) = 32$

$\Rightarrow sum = 2039$

Question ID : 3666941253

26. Let S = {1, 2, 3, 5, 7, 10, 11}. The number of non-empty subsets of S that have the sum of all elements a multiple of 3, is ________.

माना S = {1, 2, 3, 5, 7, 10, 11} है। S के अतिरिक्त उपसमुच्चयों, जिनके सभी अवयवों का योग 3 का एक गुणज है, की संख्या है।

Ans. Official Answer NTA (43)

Sol. Out of the given numbers one is 3k type and 3 of 3k + 1 type and remaining three are 3k + 2 type.

Number of subsets with 0 elements = 1

[Considering the sum of elements of empty set equal to zero]

Number of subsets with 1 element = 1

1 of 3k type

Number of subsets with 2 elements

1 of (3k + 1) type + 1 of (3k + 2) type = 9

Number of subsets with 3 elements

1 of 3k type + 1 of (3k + 1) type + 1 of (3k + 2)

type = 9

3 of (3k + 1) type = 1

3 of (3k + 2) type = 1

Number of subsets with 4 elements
1 of 3k type + 3 of (3k + 1) type = 1
1 of 3k type + 3 of (3k + 2) type = 1
2 of (3k + 1) type + 2 of (3k + 2) = 9
Number of subsets with 5 elements
1 of 3k type + 2 of (3k + 1) type + 2 of (3k + 2)
type = 9
Number of subsets with 6 elements
3 of 3k + 1 type + 3 of 3k + 2 type = 1
The set itself = 1
Total = 44.

Question ID : 3666941254

27. The constant term in the expansion of $\left(2x+\frac{1}{x^7}+3x^2\right)^5$ is ________.

$\left(2x+\frac{1}{x^7}+3x^2\right)^5$ के प्रसार में अचर पद है।

Ans. Official Answer NTA (1080)

Sol. General term is $\sum\frac{5!(2x)^{n_1}\left(x^{-7}\right)^{n_2}\left(3x^2\right)^{n_3}}{n_1!n_2!n_3!}$

For constant term,
$n_1 + 2n_3 = 7n_2$
& $n_1 + n_2 + n_3 = 5$
Only possibility $n_1 = 1, n_2 = 1, n_3 = 3$
$\Rightarrow$ constant term = 1080

Question ID : 3666941255

28. Let A_1, A_2, A_3, be the three A.P. with the same common difference d and having their first terms as A, A + 1, A + 2, respectively. Let a, b, c be the 7th, 9th, 17th terms of A1, A2, A3, respectively such that $\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix}+70=0$. If a = 29, then the sum of first 20 terms of an AP whose first term is c–a–b and common

difference is $\frac{d}{12}$, is equal to ________.

माना A_1, A_2, A_3 तीन A.P. है, जिनका सार्वअंतर d है तथा जिनके पहले पद क्रमशः $A, A+1, A+2,$ है। माना A1, A2, A3, के 7वाँ, 9वाँ, 17वाँ पद क्रमशः a, b, c हैं तथा $\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix} + 70 = 0$ है। यदि $a = 29$ है, तो उस AP जिसका पहला पद c–a–b है तथा सार्वअंतर $\frac{d}{12}$ है, के प्रथम 20 पदों का योग बराबर है।

Ans. Official Answer NTA (495)

Sol. $A + 6d = a$ ____ (1)

$A + 1 + 8d = b$ _____ (2)

$A + 2 + 16d = c$ _____ (3)

Now

$$\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix} + 70 = 0$$

$\Rightarrow c - 2b = 7$

Using equation (2) & (3), we get $A = -7$

Now $a = -7 + 6d = 29 \Rightarrow d = 6$

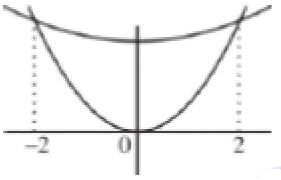
Now first term = c–a–b = 20 and common difference = $\frac{d}{12}$ = 1/2 and the sum of first 20 terms of an AP = 495

Question ID : 3666941258

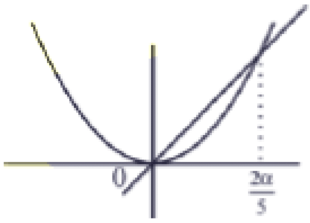
29. If the area enclosed by the parabolas $P_1 : 2y = 5x^2$ and $P_2 : x^2 - y + 6 = 0$ is equal to the area enclosed by P_1 and $y = \alpha x, \alpha > 0$, then α^3 is equal to ________.

यदि परवलयों $P_1 : 2y = 5x^2$ तथा $P_2 : x^2 - y + 6 = 0$ से घिरे क्षेत्र का क्षेत्रफल, परवलय P_1 तथा $y = \alpha x, \alpha > 0$ से घिरे क्षेत्र के क्षेत्रफल के बराबर है, तो α^3 है।

Ans. Official Answer NTA (600)

Sol. 

Abscissa of point of intersection of $2y = 5x^2$
and $y = x^2 + 6$ is ± 2



$$\text{Area } = 2\int_0^2 \left(x^2 + 6 - \frac{5x^2}{2}\right)dx = \int_0^{\frac{2\alpha}{5}} \left(\alpha x - \frac{5x^2}{2}\right)dx$$

$$\Rightarrow \int_0^{\frac{2\alpha}{5}} \left(\alpha x - \frac{5x^2}{2}\right)dx = 16$$

$$\Rightarrow \alpha^3 = 600$$

Question ID : 3666941257

30. Let x and y be distinct integers where $1 \le x \le 25$ and $1 \le y \le 25$. Then, the number of ways of choosing x and y, such that x + y is divisible by 5, is ________.

माना x तथा y भिन्न पूर्णांक हैं जहाँ $1 \le x \le 25$ तथा $1 \le y \le 25$। तब x तथा y के मध्य चुने जाने के तरीकों की संख्या, जबकि x + y, 5 से विभाज्य हो।

Ans. Official Answer NTA (120 Eng.)
112 (Hindi)

Sol. we divides numbers $1 \le x \le 25$, $1 \le y \le 25$, $x \in N$ in to following five groups

Ist	IInd	IIIrd	IVth	Vth
1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

x + y is divisible by 5 if $x \in I^{st}$ and $y \in IV^{th} \Rightarrow {}^5C_1 \times {}^5C_1 = 25$
$x \in II^{nd}$ and $y \in III^{rd} \Rightarrow {}^5C_1 \times {}^5C_1 = 25$

$x \in III^{rd}$ and $y \in II^{nd} \Rightarrow {}^5C_1 \times {}^5C_1 = 25$

$x \in IV^{th}$ and $y \in I^{st} \Rightarrow {}^5C_1 \times {}^5C_1 = 25$

$x \in V^{th}$ and $y \in V^{th} \Rightarrow {}^5C_2 \times 2 = 20$

Total = 120