JEE Main June 2022

Question Paper With Text Solution

24 June | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JUNE 2022 | 24TH JUNE SHIFT-1

SECTION – A

Question ID : 101661

Complex number

1. Let $A = \{z \in C : 1 \le |z - (1+i)| \le 2\}$ and $B = \{z \in A : |z - (1-i)| = 1\}$. Then, B :

(1) is an empty set

(2) contains exactly two elements

(3) contains exactly three elements

(4) is an infinite set

माना $A = \{z \in C : 1 \le |z - (1+i)| \le 2\}$ तथा $B = \{z \in A : |z - (1-i)| = 1\}$ है, तब, B :

(1) एक रिक्त समुच्चय है

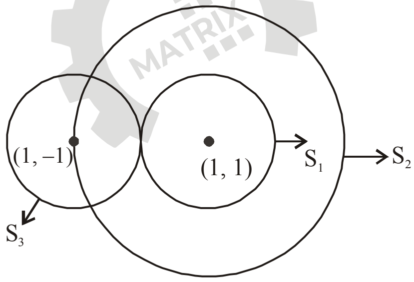
(2) में ठीक दो अवयव हैं

(3) में ठीक तीन अवयव हैं

(4) एक अनन्त समुच्चय है

Ans. Official Answer NTA (4)

Sol. Set A ≡ region between two concentric circle,
centre is (1, 1)



(1, –1) will lie on onto circle
B ≡ Portion of circle S_3 lies between concentric circle S_1 and S_2
≡ infinite points in B

Question ID : 101662

Binomial Theorem

2. The remainder when 3^{2022} is divided by 5 is :

3^{2022} को 5 से विभाजित करने पर शेषफल है :

(1) 1 (2) 2 (3) 3 (4) 4

Ans. Official Answer NTA (4)

Sol. $3^{2022} = 3^2 (3^{2020}) = 9(81)^{505}$

$= 9(80 + 1)^{505}$

$= 9(80k_1 + 1)$

$= 9(5k_2 + 1)$

$= 5k_3 + 9 = 5k_3 + 5 + 4 = 5k_4 + 4$

remainder = 4

Question ID : 101663

Tangent and normal

3. The surface area of a balloon of spherical shape being inflated, increases at a constant rate. If initially, the radius of balloon is 3 units and after 5 seconds, it becomes 7 units, then its radius after 9 seconds is:

गोले के आकार के एक गुब्बारे में हवा भरने के दौरान उसके पृष्ठ का क्षेत्रफल एक स्थिर दर से बढ़ता है। यदि आरंभ में गुब्बारे की त्रिज्या 3 इकाई है तथा 5 सैकण्ड पश्चात् यह 7 इकाई हो जाती है, तो 9 सैकण्ड पश्चात् इसकी त्रिज्या होगी :

(1) 9 unit (2) 10 unit (3) 11 unit (4) 12 unit

Ans. Official Answer NTA (1)

Sol. $S = 4\pi r^2$

$$\frac{ds}{dt} = k$$

$$8\pi r\frac{dr}{dt} = k$$

$$\int r\,dr = \int \frac{k}{8\pi}dt$$

$$\frac{r^2}{2} = \frac{kt}{8\pi} + c$$

at $t = 0, r = 3 \Rightarrow c = \frac{9}{2}$

$\Rightarrow r^2 = \frac{kt}{4\pi} + 9$

at $t = 5, r = 7 \Rightarrow k = 32\pi$

$\Rightarrow r^2 = 8t + 9$

at $t = 9 \Rightarrow r^2 = 81$

$\Rightarrow r = 9$

Question ID : 101664

Probability

4. Bag A contains 2 white, 1 black and 3 red balls and bag B contains 3 black, 2 red and n white balls. One bag is chosen at random and 2 balls drawn from it at random, are found to be 1 red and 1 black. If the probability that both balls come from Bag A is $\frac{6}{11}$, then n is equal to _______.

थैले A में 2 सफेद, 1 काली तथा 3 लाल गेंद है तथा थैले B में 3 काली, 2 लाल तथा n सफेद गेंद हैं। एक थैला यादृच्छया चुना जाता है तथा इसमें से यादृच्छया निकाली गई दो गेंदों में एक लाल तथा एक काली पायी जाती है। यदि दोनों गेंदों के थैले A में से निकलने की प्रायिकता $\frac{6}{11}$ है, तो n बराबर है ________

(1) 13 (2) 6 (3) 4 (4) 3

Ans. Official Answer NTA (3)

Sol. A ≡ Bag A is chosen

B ≡ Bag B is chosen

E ≡ 1 red and 1 black ball are drawn

$$P\left(\frac{A}{E}\right) = \frac{6}{11}$$

$$\frac{P(A \cap E)}{P(E)} = \frac{6}{11}$$

$$\frac{P(A \cap E)}{P(A \cap E) + P(B \cap E)} = \frac{6}{11}$$

$$\frac{P(A)P\left(\frac{E}{A}\right)}{P(A)P\left(\frac{E}{A}\right)+P(B)P\left(\frac{E}{B}\right)}=\frac{6}{11}$$

$$\frac{\frac{1}{2}\times\frac{3}{15}}{\frac{1}{2}\times\frac{3}{15}+\frac{1}{2}\times\frac{6\times 2}{(n+5)(n+4)}}=\frac{6}{11}$$

$\Rightarrow (n + 5)(n + 4) = 72$

$\Rightarrow n = 4$ or $n = -13$ (rejected)

Question ID : 101665

Parabola

5. Let $x^2 + y^2 + Ax + By + C = 0$ be a circle passing through (0, 6) and touching the parabola $y = x^2$ at (2, 4). Then A + C is equal to ________.

माना बिन्दु (0, 6) से होकर जाने वाला तथा परवलय $y = x^2$ को (2, 4) पर स्पर्श करने वाला एक वृत्त $x^2 + y^2 + Ax + By + C = 0$ है, तो A + C बराबर है ________

(1) 16 (2) $\frac{88}{5}$ (3) 72 (4) –8

Ans. Official Answer NTA (1)

Sol. Tangent to $y = x^2$ at (2, 4)

$$\frac{dy}{dx}=2x$$

$$\left(\frac{dy}{dx}\right)_{x=2}=4$$

$y - 4 = 4(x - 2)$

$4x - y - 4 = 0$

equation of circle touching $y = x^2$ at (2, 4)

$(x - 2)^2 + (y - 4)^2 + \lambda(4x - y - 4) = 0$

(0, 6) lies on it $\Rightarrow \lambda = \frac{4}{5}$

$(x-2)^2+(y-4)^2+\frac{4}{5}(4x-y-4)=0$

$A=-4+\frac{16}{5}$ $\quad$ $c=20-\frac{16}{5}$

$\Rightarrow A+C=16$

Question ID : 101666

Determinant

6. The number of values of α for which the system of equation :

$x+y+z=\alpha$

$\alpha x+2\alpha y+3z=-1$

$x+3\alpha y+5z=4$

is inconsistent, is :

α के मानों, जिनके लिए समीकरण निकाय :

$x+y+z=\alpha$

$\alpha x+2\alpha y+3z=-1$

$x+3\alpha y+5z=4$

असंगत है, की संख्या है :

(1) 0 (2) 1 (3) 2 (4) 3

Ans. Official Answer NTA (2)

Sol. $D=0$

$$\begin{vmatrix}1 & 1 & 1\\ \alpha & 2\alpha & 3\\ 1 & 3\alpha & 5\end{vmatrix}=0$$

$3(\alpha-1)^2=0$

$\alpha=1$

$D_x \neq 0$

So system is incensistent $\Rightarrow \alpha=1$

Question ID : 101667

Quadratic Equation

7. If the sum of the squares of the reciprocals of the roots α and β of the equation $3x^2 + \lambda x - 1 = 0$ is 15, then $6(\alpha^3 + \beta^3)^2$ is equal to :

यदि समीकरण $3x^2 + \lambda x - 1 = 0$ के मूल α तथा β के व्युत्क्रमों के वर्गों का योगफल 15 है, तो $6(\alpha^3 + \beta^3)^2$ बराबर है :

(1) 18 (2) 24 (3) 36 (4) 96

Ans. Official Answer NTA (2)

Sol. $3x^2 + \lambda x - 1 = 0$

$\alpha+\beta = \frac{-\lambda}{3}$, $\alpha\beta = \frac{-1}{3}$

$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=15$

$\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}=15$

$\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}=15$

$\Rightarrow \lambda^2 = 9$ $\Rightarrow \lambda = \pm 3$

$6(\alpha^3+\beta^3)^2 = 6\left((\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)\right)^2$

$= 6(4) = 24$

Question ID : 101668

ITF

8. The set of all values of k for which $(\tan^{-1}x)^3 + (\cot^{-1}x)^3 = k\pi^3, x \in R$, is the interval :

k के सभी मानों, जिनके लिए $(\tan^{-1}x)^3 + (\cot^{-1}x)^3 = k\pi^3, x \in R$ है, का समुच्चय कौन–सा अंतराल है :

(1) $\left[\frac{1}{32},\frac{7}{8}\right)$ (2) $\left(\frac{1}{24},\frac{13}{16}\right)$ (3) $\left[\frac{1}{48},\frac{13}{16}\right]$ (4) $\left[\frac{1}{32},\frac{9}{8}\right)$

Ans. Official Answer NTA (1)

Sol. Note : Language is incomplete

$(\tan^{-1}(x))^3 + (\cot^{-1}(x))^3 = k\pi^3$ has a solution

$\Rightarrow k\pi^3$ will lie in range of LHS

$y = (\tan^{-1}(x))^3 + (\cot^{-1}(x))^3$

put $\tan^{-1}(x) = \alpha$, $\cot^{-1}(x) = \beta$

$$\alpha + \beta = \frac{\pi}{2}$$

$$\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$y = \alpha^3 + \beta^3$

$$= (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$$

$$y = \frac{\pi^3}{8} - \frac{3\pi}{2}\alpha\left(\frac{\pi}{2} - \alpha\right),\ \alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$y)_{\min} = \frac{\pi^3}{32} \qquad \text{when } \alpha = \frac{\pi}{4}$$

$$y)_{\max} \to \frac{7\pi^3}{8} \qquad \text{when } \alpha \to -\frac{\pi}{2}$$

$$y \in \left[\frac{\pi^3}{32}, \frac{7\pi^3}{8}\right)$$

$$\Rightarrow k \in \left[\frac{1}{32}, \frac{7}{8}\right)$$

Question ID : 101669

Matrices

9. Let $S = \left\{\sqrt{n} : 1 \le n \le 50 \text{ and } n \text{ is odd}\right\}$. Let $a \in S$ and $A = \begin{bmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}$. If $\sum_{a \in S} \det(\operatorname{adj} A) = 100\lambda$ then λ is equal to :

माना $S = \{\sqrt{n} : 1 \le n \le 50$ तथा n विषम है} माना $a \in S$ तथा $A = \begin{bmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}$ हैं। यदि $\sum_{a \in S} \det(\operatorname{adj} A) = 100\lambda$ है, तो λ बराबर है :

(1) 218 (2) 221 (3) 663 (4) 1717

Ans. Official Answer NTA (2)

Sol. $S = \left\{1, \sqrt{3}, \sqrt{5}, \sqrt{7}, ...\sqrt{49}\right\} \equiv \left\{\sqrt{2r-1} : 1 \le r \le 25, r \in N\right\}$

$$A = \begin{bmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}$$

$|A| = 1 + a^2$

$|\text{adj}(A)| = |A|^2 = (1 + a^2)^2$

$$\sum_{a \in S} \det(\text{adj}(A)) = \sum_{r=1}^{2}(1+2r-1)^2 = \sum_{r=1}^{25} 4r^2$$

$$= 4\sum_{r=1}^{25}(r^2)$$

$$100\lambda = \frac{4(25)(26)(51)}{6}$$

$\Rightarrow \lambda = 221$

Question ID : 101670

Monotonocity

10. For the function $f(x) = 4\log_e(x-1) - 2x^2 + 4x + 5,\ x > 1$, which one of the following is NOT correct?

(1) f is increasing in (1, 2) and decreasing in $(2, \infty)$

(2) $f(x) = -1$ has exactly two solutions

(3) $f'(e) - f''(2) < 0$

(4) $f(x) = 0$ has a root in the interval $(e, e+1)$

फलन $f(x) = 4\log_e(x-1) - 2x^2 + 4x + 5,\ x > 1$, के लिए निम्न में से कौनसा एक सही नहीं है?

(1) f अन्तराल (1, 2) में वर्धमान तथा $(2, \infty)$ में ह्रासमान है

(2) $f(x) = -1$ के ठीक दो हल हैं

(3) $f'(e) - f''(2) < 0$ है

(4) $f(x) = 0$ का एक मूल अंतराल $(e, e+1)$ में है

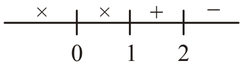
Ans. Official Answer NTA (3)

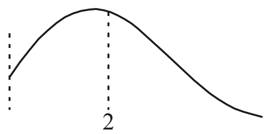
Sol. $f(x) = 4\log_e(x-1) - 2x^2 + 4x + 5$

$$f'(x) = \frac{4}{x-1} - 4x + 4$$

$$= -\frac{4(x)(x-2)}{(x-1)}$$

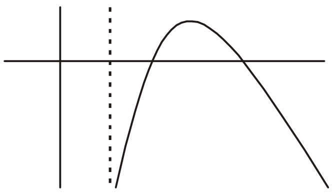
$$f''(x) = \frac{-4}{(x-1)^2} - 4$$





$\Rightarrow$ Option A is true

graph of f(x)



$\Rightarrow$ Option B is also true

$$f'(e) = \frac{4}{e-1} - 4(e-1)$$

$$f''(2) = -8$$

$$f'(e) - f''(2) > 0$$

$\Rightarrow$ Option C is false

Question ID : 101671

Tangent and normal

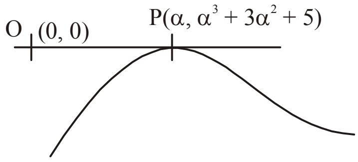
11. If the tangent at the point(x_1, y_1) on the curve $y = x^3 + 3x^2 + 5$ passes through the origin, then (x_1, y_1) does NOT lie on the curve :

यदि वक्र $y = x^3 + 3x^2 + 5$ के बिन्दु(x_1, y_1) पर स्पर्श रेखा मूल बिन्दु से होकर जाती है, तो (x_1, y_1) निम्न में से किस वक्र पर स्थित नहीं है :

(1) $x^2 + \frac{y^2}{81} = 2$ (2) $\frac{y^2}{9} - x^2 = 8$ (3) $y = 4x^2 + 5$ (4) $\frac{x}{3} - y^2 = 2$

Ans. Official Answer NTA (4)

Sol.



equate slope of OP

$$3\alpha^2 + 6\alpha = \frac{\alpha^3 + 3\alpha^2 + 5}{\alpha}$$

$2\alpha^3 + 3\alpha^2 = 5 \Rightarrow \alpha = 1$

P(1, 9)

P does not lie on $\frac{x}{3} - y^2 = 2$

Question ID : 101672

Maxima and Minima

12. The sum of absolute maximum and absolute minimum values of the function $f(x) = |2x^2 + 3x - 2| + \sin x \cos x$ in the interval [0, 1] is :

अंतराल [0, 1] में फलन $f(x) = |2x^2 + 3x - 2| + \sin x \cos x$ के निरपेक्ष उच्चतम तथा निरपेक्ष निम्नतम मानों का योगफल है :

(1) $3 + \frac{\sin(1)\cos^2\left(\frac{1}{2}\right)}{2}$

(2) $3 + \frac{1}{2}(1 + 2\cos(1))\sin(1)$

(3) $5 + \frac{1}{2}(\sin(1) + \sin(2))$

(4) $2 + \sin\left(\frac{1}{2}\right)\cos\left(\frac{1}{2}\right)$

Ans. Official Answer NTA (2)

Sol.

$$f(x) = \begin{cases} 2x^2 + 3x - 2 + \frac{\sin 2x}{2} & \frac{1}{2} \le x \le 1 \\ -(2x^2 + 3x - 2) + \frac{\sin 2x}{2} & 0 \le x < \frac{1}{2} \end{cases}$$

$$f'(x) = \begin{cases} 4x + 3 + \cos(2x) & \frac{1}{2} < x \le 1 \\ -4x - 3 + \cos 2x & 0 \le x < \frac{1}{2} \end{cases}$$

$$f'(x) < 0 \quad \forall x \in \left[0, \frac{1}{2}\right)$$

$$f'(x) > 0 \quad \forall x \in \left(\frac{1}{2}, 1\right]$$

$$f(x)_{\min} = f\left(\frac{1}{2}\right) = \frac{\sin(1)}{2}$$

$$f(x)_{\max} = \max\{f(0), f(1)\}$$

$$= f(1) = 3 + \sin 1 \cos 1$$

$$\text{sum} = 3 + \sin 1 \cos 1 + \frac{\sin 1}{2}$$

Question ID : 101673

Sequence and progression

13. If $\{a_i\}_{i=1}^{n}$, where n is an even integer, is an arithmetic progression with common difference 1, and $\sum_{i=1}^{n} a_i = 192, \sum_{i=1}^{\frac{n}{2}} a_{2i} = 120$, then n is equal to :

यदि $\{a_i\}_{i=1}^{n}$, जहाँ n एक सम पूर्णांक है, एक समांतर श्रेढ़ी है, जिसका सार्व अंतर 1 है, तथा $\sum_{i=1}^{n} a_i = 192, \sum_{i=1}^{\frac{n}{2}} a_{2i} = 120$ हैं, तो n बराबर है :

(1) 48 (2) 96 (3) 92 (4) 104

Ans. Official Answer NTA (2)

Sol. $a_1 + a_2 + a_3 + ... + a_n = 192$ ——(1)

$a_2 + a_4 + a_6 + ... + a_n = 120$ ——(2)

(1) – (2)

$a_1 + a_3 + a_5 + ... + a_{n-1} = 72$ ——(3)

(2) – (3)

1 + 1 + ... + 1 = 48, Total terms = n, number of odd terms = n/2

$$\frac{n}{2} = 48$$

$$n = 96$$

Question ID : 101674

Differential Equation

14. If x = x(y) is the solution of the differential equation $y\frac{dx}{dy} = 2x + y^3(y+1)e^y, x(1) = 0$; then x(e) is equal to :

यदि अवकल समीकरण $y\frac{dx}{dy} = 2x + y^3(y+1)e^y, x(1) = 0$ का हल x = x(y) है, तो x(e) बराबर है :

(1) $e^3(e^e - 1)$ (2) $e^e(e^3 - 1)$ (3) $e^2(e^e + 1)$ (4) $e^e(e^2 - 1)$

Ans. Official Answer NTA (1)

Sol. Linear differential equation

$$y\frac{dx}{dy} - 2x = y^3(y+1)e^y$$

$$\frac{dx}{dy} - \frac{2}{y}x = y^2(y+1)e^y$$

$$\text{I.f} = e^{\int -\frac{2}{y}dy} = \frac{1}{y^2}$$

$$\int d\left(\frac{x}{y^2}\right) = \int (y+1)e^y dy$$

$$\frac{x}{y^2} = ye^y + c$$

$x(1) = 0 \Rightarrow c = -e$

$$\frac{x}{y^2} = ye^y - e$$

put y = e

$$\frac{x}{e^2} = ee^e - e = e\left(e^e - 1\right)$$

$$x = e^3\left(e^e - 1\right)$$

Question ID : 101675

Hyperbola

15. Let $\lambda x - 2y = \mu$ be a tangent to the hyperbola $a^2x^2 - y^2 = b^2$. then $\left(\frac{\lambda}{a}\right)^2 - \left(\frac{\mu}{b}\right)^2$ is equal to :

माना अतिपरवलय $a^2x^2 - y^2 = b^2$ की एक स्पर्श रेखा $\lambda x - 2y = \mu$ है। तो $\left(\frac{\lambda}{a}\right)^2 - \left(\frac{\mu}{b}\right)^2$ बराबर है :

(1) – 2 (2) – 4 (3) 2 (4) 4

Ans. Official Answer NTA (4)

Sol. Hyperbola $a^2x^2 - y^2 = b^2$

$$\frac{x^2}{\left(\frac{b^2}{a^2}\right)} - \frac{y^2}{b^2} = 1$$

Apply cot

$c^2 = a^2m^2 - b^2$

$$\left(-\frac{\mu}{2}\right)^2 = \left(\frac{b^2}{a^2}\right)\left(\frac{\lambda}{2}\right)^2 - b^2$$

$$\frac{\lambda^2}{a^2} - \frac{\mu^2}{b^2} = 4$$

Question ID:101676

Vectors

16. Let $\hat{a}$, $\hat{b}$ be unit vectors. If $\vec{c}$ be a vectors such that the angle between $\hat{a}$ and $\hat{c}$ is $\frac{\pi}{12}$, and $\hat{b} = \vec{c} + 2(\vec{c} \times \hat{a})$, then $|6\vec{c}|^2$ is equal to :

माना Let $\hat{a}$, $\hat{b}$ इकाई सदिश हैं। यदि एक सदिश $\vec{c}$ इस प्रकार है कि $\hat{a}$ तथा $\hat{c}$ के बीच का कोण $\frac{\pi}{12}$ है तथा $\hat{b} = \vec{c} + 2(\vec{c} \times \hat{a})$ है, तो $|6\vec{c}|^2$ बराबर है :

(1) $6(3-\sqrt{3})$ (2) $3 + \sqrt{3}$ (3) $6(3+\sqrt{3})$ (4) $6(\sqrt{3}+1)$

Ans. Official Answer NTA (3)

Sol. Let $|\vec{c}| = x$

$\hat{b} = \vec{c} + (\vec{c} \times \hat{a})$

$|\hat{b}| = 1$

$|\vec{c} + 2(\vec{c} \times \hat{a})| = 1$

$|\vec{c}|^2 + 4(\vec{c} \times \hat{a})^2 + 4\vec{c}.(\vec{c} \times \hat{a}) = 1$

$x^2 + 4(x^2)(1)\sin^2(15°) + 0 = 1$

$x^2 + 2x^2\left(1 - \frac{\sqrt{3}}{2}\right) = 1$

$x^2 + x^2(2 - \sqrt{3}) = 1$

$x^2(3 - \sqrt{3}) = 1$

$x^2 = \frac{3 + \sqrt{3}}{6}$

Ans = $(6x)^2 = 36x^2 = 6(3 + \sqrt{3})$

Question ID:101677

Probability

17. If a random variable X follows the Binomial distribution B(33, p) such that 3P (X = 0) = P(X = 1), then the value of $\frac{P(X=15)}{P(X=18)} - \frac{P(X=16)}{P(X=17)}$ is equal to :

यदि एक यादृच्छया चर X, द्विपद बंटन B(33, p) का अनुसरण करता है तथा 3P(X = 0) = P(X = 1) है, तो $\frac{P(X=15)}{P(X=18)} - \frac{P(X=16)}{P(X=17)}$ का मान बराबर है :

(1) 1320 (2) 1088 (3) $\frac{120}{1331}$ (4) $\frac{1088}{1089}$

Ans. Official Answer NTA (1)

Sol. $B(n_1 p) \equiv B(33_1 p)$

$3p(X = 0) = p(X = 1)$

$3({}^{33}C_0 (1 - p)^{33}) = {}^{33}C_1\, p(1 - p)^{32}$

$\Rightarrow p = \frac{1}{12} \qquad \Rightarrow \frac{1-p}{p} = \frac{1}{p} - 1 = 11$

Req Value = $\frac{p(X=15)}{p(X=18)} - \frac{p(X=16)}{p(X=17)}$

$= \frac{{}^{33}C_{15}\, p^{15}(1-p)^{18}}{{}^{33}C_{18}\, p^{18}(1-p)^{15}} - \frac{{}^{33}C_{16} p^{16}(1-p)^{17}}{{}^{33}C_{16}\, p^{17}(1-p)^{16}}$

$= \left(\frac{1-p}{p}\right)^3 - \left(\frac{1-p}{p}\right)$

$= (11)^3 - (11) = 1320$

Question ID:101678
ITF

18. The domain of the function $f(x) = \frac{\cos^{-1}\left(\frac{x^2-5x+6}{x^2-9}\right)}{\log_e\left(x^2-3x+2\right)}$ is :

फलन $f(x) = \frac{\cos^{-1}\left(\frac{x^2-5x+6}{x^2-9}\right)}{\log_e\left(x^2-3x+2\right)}$ का प्रांत है :

(1) $(-\infty,1)\cup(2,\infty)$

(2) $(2, \infty)$

(3) $\left[-\frac{1}{2},1\right)\cup(2,\infty)$

(4) $\left[-\frac{1}{2},1\right)\cup(2,\infty)-\left\{\frac{3+\sqrt{5}}{2},\frac{3-\sqrt{5}}{2}\right\}$

Ans. Official Answer NTA (Bonus)

Sol. $-1 \le \frac{x^2-5x+6}{x^2-9} \le 1$

$x^2 - 3x + 2 > 0$

$x^2 - 3x + 2 \ne 1$

$\Rightarrow x \in \left[-\frac{1}{2},1\right)\cup(2,\infty) - \left\{\frac{3+\sqrt{5}}{2},\frac{3-\sqrt{5}}{2},3\right\}$

Bonus

Question ID:101679

Trigonometric Equation

19. Let $S = \left\{\theta, \in [-\pi, \pi] - \left\{\pm\frac{\pi}{2}\right\} : \sin\theta\tan\theta + \tan\theta = \sin 2\theta\right\}$. If $T = \sum_{\theta\in S}\cos 2\theta$, then T + n (S) is equal to :

माना $S = \left\{\theta, \in [-\pi, \pi] - \left\{\pm\frac{\pi}{2}\right\} : \sin\theta\tan\theta + \tan\theta = \sin 2\theta\right\}$ है। यदि $T = \sum_{\theta\in S}\cos 2\theta$ है, तो T + n (S) बराबर है :

(1) $7 + \sqrt{3}$ (2) 9 (3) $8 + \sqrt{3}$ (4) 10

Ans. Official Answer NTA (2)

Sol. $\sin\theta\tan\theta + \tan\theta = \sin 2\theta$

$s\left(\frac{s}{c}\right) + \frac{s}{c} = 2sc$

$\sin\theta = 0$ $\frac{s+1}{c} = 2c$

$\theta = -\pi, 0, \pi$ $s + 1 = 2c^2$

$s + 1 = 2 - 2s^2$

$2s^2 + s - 1 = 0$

$(s + 1)(2s - 1) = 0$

$\sin\theta = -1$ $\sin\theta = \frac{1}{2}$

(Rejected) $\Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$S = \left\{-\pi, 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}\right\}$

$n(S) = 5$

$T = \cos(-2\pi) + \cos 0 + \cos 2\pi + \cos\frac{\pi}{3} + \cos\frac{5\pi}{3}$

$= 4$

$T + n(S) = 9$

Question ID:101680

Mathematical Reasoning

20. The number of choices for $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$, such that $(p \Delta q) \Rightarrow ((p \Delta \sim q) \vee ((\sim p) \Delta q))$ is a tautology is :

कितने $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$ के लिए $(p \Delta q) \Rightarrow ((p \Delta \sim q) \vee ((\sim p) \Delta q))$ एक पुनरुक्ति :

(1) 1 (2) 2 (3) 3 (4) 4

Ans. Official Answer NTA (2)

Sol. For tautology $((p\Delta \sim q) \vee ((\sim p)\Delta q))$ must be true.

This is possible only when $\Delta = \vee \& \Rightarrow$

SECTION – B

Question ID:101681

Function

21. The number of one-one functions $f : \{a, b, c, d\} \to \{0, 1, 2, ..., 10\}$ such that $2f(a) - f(b) + 3f(c) + f(d) = 0$ is ________.

एकैकी फलनों $f : \{a, b, c, d\} \to \{0, 1, 2, ..., 10\}$, जिनके लिए $2f(a) - f(b) + 3f(c) + f(d) = 0$ है, की संख्या है ________

Ans. Official Answer NTA (31)

Sol. $\because 3f(c) + 2f(a) + f(d) = f(b)$

Value of f(c)	Value of f(a)	Number of functions
0	1	7
	2	5
	3	3
	4	2
1	0	6
	2	2
	3	1
2	0	3
	1	1
3	0	1
	Total Number of functions =	**31**

Question ID:101682

P & C

22. In an examination, there are 5 multiple choice questions with 3 choices, out of which exactly one is correct. There are 3 marks for each correct answer, –2 marks for each wrong answer and 0 mark if the equation is not attempted. Then, the number of ways a students appearing in the examination gets 5 marks is ______.

एक परीक्षा में 5 बहुविकल्पी प्रश्न हैं। इनमें उत्तर के लिए 3 विकल्प हैं, जिनमें से ठीक एक सही है। प्रत्येक सही उत्तर के लिए 3 अंक है, प्रत्येक गलत उत्तर के लिए –2 अंक हैं तथा कोई भी उत्तर न देने पर 0 अंक है। तो उन तरीकों, जिनमें परीक्षा देने वाले एक छात्र को 5 अंक मिलते हैं, की संख्या है _________

Ans. Official Answer NTA (40)

Sol. $x_1 + x_2 + x_3 + x_4 + x_5 = 5$

Only one possibilities 3, 3, 3, –2, –2

Number of ways is $= \frac{5!}{3!2!} \times 2 \times 2 = 40$

Question ID:101683

Straight Line

23. Let $A\left(\frac{3}{\sqrt{a}}, \sqrt{a}\right)$, a > 0, be a fixed point in the xy-plane. The image of A in y-axis be B and the image of B in x-axis be C. If $D(3\cos\theta, a\sin\theta)$ is a point in the fourth quadrant such that the maximum area of ΔACD is 12 square units, then a is equal to ________.

माना xy-समतल में एक निश्चित बिन्दु $A\left(\frac{3}{\sqrt{a}}, \sqrt{a}\right)$, a > 0 है। y-अक्ष में A का प्रतिबिंब B तथा x-axis में B का प्रतिबिंब C है। यदि चतुर्थ चतुर्थांश में एक बिंदु $D(3\cos\theta, a\sin\theta)$ के लिए ΔACD का अधिकतम क्षेत्रफल 12 वर्ग इकाई है, तो a बराबर है ________

Ans. Official Answer NTA (8)

Sol. Clearly B is $\left(-\frac{3}{\sqrt{a}}, +\sqrt{a}\right)$ and C is $\left(-\frac{3}{\sqrt{a}}, -\sqrt{a}\right)$

$$\text{Area of } \Delta ACD = \frac{1}{2}\begin{vmatrix} \frac{3}{\sqrt{a}} & \sqrt{a} & 1 \\ -\frac{3}{\sqrt{a}} & -\sqrt{a} & 1 \\ 3\cos\theta & a\sin\theta & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 0 & 0 & 1 \\ -\frac{3}{\sqrt{a}} & -\sqrt{a} & 1 \\ 3\cos\theta & a\sin\theta & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \left|-3\sqrt{a}\sin\theta + 3\sqrt{a}\cos\theta\right| = 3\sqrt{a}\left|\cos\theta - \sin\theta\right|$$

$$\Rightarrow \Delta_{max} = 3\sqrt{a}.\sqrt{2} = 12 \Rightarrow a = \left(2\sqrt{2}\right)^2 = 8$$

Question ID:101684

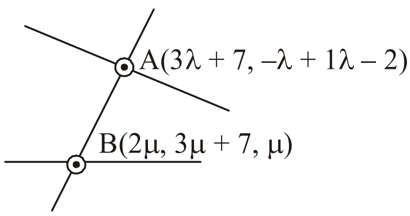
3D Geometry

24. Let a line having direction ratios 1, –4, 2 intersect the lines $\frac{x-7}{3} = \frac{y-1}{-1} = \frac{z+2}{1}$ and $\frac{x}{2} = \frac{y-7}{3} = \frac{z}{1}$ at the points. A and B. Then $(AB)^2$ is equal to ______.

माना एक रेखा जिसके दिक अनुपात 1, –4, 2 हैं रेखाओं $\frac{x-7}{3} = \frac{y-1}{-1} = \frac{z+2}{1}$ तथा $\frac{x}{2} = \frac{y-7}{3} = \frac{z}{1}$ को बिन्दुओं A तथा B पर काटती है। तो $(AB)^2$ बराबर है __________

Ans. Official Answer NTA (84)

Sol.



DR's of AB

$(3\lambda - 2\mu + 7, -\lambda - 3\mu - 6, \lambda - \mu - 2)$

$$\frac{3\lambda - 2\mu + 7}{1} = \frac{-\lambda - 3\mu - 6}{-4} = \frac{\lambda - \mu - 2}{2}$$

Taking first (2) $-12\lambda + 8\mu - 28 = -\lambda - 3\mu - 6$

$\lambda - \mu + 2 = 0$

Taking second & third $-2\lambda - 6\mu - 12 = -4\lambda + 4\mu + 8$

$\lambda - 5\mu - 10 = 0$

After solving above two equation $\lambda = -5$, $\mu = -3$

A = (–8, 6, – 7)

B = (–6, –2, –3)

$(AB)^2 = 4 + 64 + 16 = 84$

Question ID:101685

Continuity & Differentiability

25. The number of points where the function $f(x) = \begin{cases} |2x^2 - 3x - 7| & \text{if } x \le -1 \\ [4x^2 - 1] & \text{if } -1 < x < 1 \\ |x+1| + |x-2| & \text{if } x \ge 1, \end{cases}$

[t] denotes the greatest integer ≤ t, is discontinuous is ____.

उन बिन्दुओं, जहाँ फलन $f(x) = \begin{cases} |2x^2 - 3x - 7| & \text{यदि } x \le -1 \text{ है} \\ [4x^2 - 1] & \text{यदि } -1 < x < 1 \text{ है} \\ |x+1| + |x-2| & \text{यदि } x \ge 1 \text{ है,} \end{cases}$

[t] महत्तम पूर्णांक ≤ t है, असंतत है, की संख्या है _________

Ans. Official Answer NTA (7)

Sol. $\because$ f (–1) = 2 and f (1) = 3

For $x \in (-1, 1)$, $(4x^2 - 1) \in [-1, 3)$

hence f(x) will be discontinuous at x = 1 and also

whenever $4x^2 - 1 = 0$, 1 or 2

$\Rightarrow x = \pm\frac{1}{2}, \pm\frac{1}{\sqrt{2}}$ and $\pm\frac{\sqrt{3}}{2}$

So there are total 7 points of discontinuity

Question ID:101686

Definite Integration

26. Let $f(\theta) = \sin\theta + \int_{-\pi/2}^{\pi/2} (\sin\theta + t\cos\theta) f(t)dt$. Then value of $\left|\int_0^{\pi/2} f(\theta)d\theta\right|$ is _______.

माना $f(\theta) = \sin\theta + \int_{-\pi/2}^{\pi/2} (\sin\theta + t\cos\theta) f(t)dt$ है। तो $\left|\int_0^{\pi/2} f(\theta)d\theta\right|$ का मान है _________

Ans. Official Answer NTA (1)

Sol. $f(\theta)=\sin\theta+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\sin\theta+t\cos\theta)f(t)dt$

$f(\theta)=\sin\theta+\sin\theta+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}f(t)dt+\cos\theta\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}tf(t)dt$

Let $A=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}f(t)dt, B=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}tf(t)dt$

$f(\theta)=\sin\theta+A\sin\theta+B\cos\theta$

$f(\theta)=(A+1)\sin\theta+B\cos\theta$

$A=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(A+1)\sin t+B\cos t\,dt$

A = 2B ——— (1)

$B=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}t((A+1)\sin t+B\cos t)$

$B=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}t(A+1)\sin t$

$B=(A+1)2\int_{0}^{\frac{\pi}{2}}t\sin t\,dt$

B = (A + 1) 2.1

2A + 2 – B = 0 ——— (2)

After solving

$B=-\frac{2}{3}, A=-\frac{4}{3}$

$$\left|\int_0^{\frac{\pi}{2}} f(\theta)\,d\theta\right| = \left|\int_0^{\frac{\pi}{2}} -\frac{1}{3}\sin\theta - \frac{2}{3}\cos\theta\right|$$

$= 1$

Question ID:101687

Definite Integration

27. Let $\underset{0\le x\le 2}{\text{Max}}\left\{\frac{9-x^2}{5-x}\right\} = \alpha$ and $\underset{0\le x\le 2}{\text{Min}}\left\{\frac{9-x^2}{5-x}\right\} = \beta$. If $\int_{\beta-\frac{8}{3}}^{2\alpha-1} \text{Max}\left\{\frac{9-x^2}{5-x}, x\right\}dx = \alpha_1 + \alpha_2 \log_e\left(\frac{8}{15}\right)$ then $\alpha_1 + \alpha_2$ is equal to _________.

माना $\underset{0\le x\le 2}{\text{Max}}\left\{\frac{9-x^2}{5-x}\right\} = \alpha$ तथा $\underset{0\le x\le 2}{\text{Min}}\left\{\frac{9-x^2}{5-x}\right\} = \beta$ हैं। यदि $\int_{\beta-\frac{8}{3}}^{2\alpha-1} \text{Max}\left\{\frac{9-x^2}{5-x}, x\right\}dx = \alpha_1 + \alpha_2 \log_e\left(\frac{8}{15}\right)$ है, तो $\alpha_1 + \alpha_2$ बराबर है _________

Ans. Official Answer NTA (34)

Sol. Let f(x) = $\frac{x^2-9}{x-5} \Rightarrow f'(x) = \frac{(x-1)(x-9)}{(x-5)^2}$

So, $\alpha = f(1) = 2$ and $\beta = \min(f(0), f(2)) = \frac{5}{3}$

Now, $\int_{-1}^{3} \max\left\{\frac{x^2-9}{x-5}, x\right\}dx = \int_{-1}^{\frac{9}{5}} \frac{x^2-9}{x-5}dx + \int_{\frac{9}{5}}^{3} x\,dx$

$$= \int_{-1}^{\frac{9}{5}}\left(x+5+\frac{16}{x-5}\right)dx + \left.\frac{x^2}{2}\right|_{\frac{9}{5}}^{3}$$

$$= \frac{28}{25} + 14 + 16\ln\left(\frac{8}{15}\right) + \frac{72}{25} = 18 + 16\ln\left(\frac{8}{15}\right)$$

Clearly $\alpha_1 = 18$ and $\alpha_2 = 16$, so $\alpha_1 + \alpha_2 = 34$

Question ID : 101688

Ellipse

28. If two tangents drawn from a point (α, β) lying on the ellipse $25x^2 + 4y^2 = 1$ to the parabola $y^2 = 4x$ are such that the slope of one tangent is four times the other, then the value of $(10\alpha + 5)^2 + (16\beta^2 + 50)^2$ equals _________.

दीर्घवृत्त $25x^2 + 4y^2 = 1$ पर एक बिन्दु (α, β) से परवलय $y^2 = 4x$ पर दो स्पर्श रेखाएँ खीची जाती हैं। यदि एक स्पर्श रेखा की प्रवणता दूसरे की चार गुना है, तो $(10\alpha + 5)^2 + (16\beta^2 + 50)^2$ का मान बराबर है ________

Ans. Official Answer NTA (2929)

Sol. $\alpha = \frac{1}{5}\cos\theta, \beta = \frac{1}{2}\sin\theta$

Equation of tangent to $y^2 = 4x$

$$y = mx + \frac{1}{m}$$

It passes through (α, β)

$$\frac{1}{2}\sin\theta = m\frac{1}{5}\cos\theta + \frac{1}{m}$$

$$m^2\left(\frac{\cos\theta}{5}\right) - m\left(\frac{1}{2}\sin\theta\right) + 1 = 0$$

It has two rootss m_1 and m_2 where $m_1 = 4m_2$

$$m_1 + m_2 = \frac{\frac{1}{2}\sin\theta}{\frac{\cos\theta}{5}}$$

$$m_1 m_2 = \frac{5}{\cos\theta}$$

After eliminating m_1 and m_2

$$\cos\theta = \frac{-5 \pm \sqrt{29}}{2}$$

$$\alpha = \frac{-5 \pm \sqrt{29}}{10} \Rightarrow 10\alpha + 5 = \pm\sqrt{29}$$

$$\beta^2 = \frac{1}{4}\sin^2\theta \Rightarrow 16\beta^2 = -50 \pm 10\sqrt{29}$$

$$(10\alpha + 5)^2 + (16\beta^2 + 50)^2 = 2929$$

Question ID : 101689

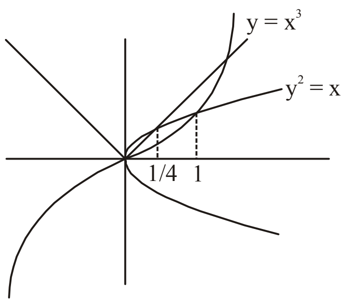
Area Under Curve

29. Let S be the region bounded by the curves $y = x^3$ and $y^2 = x$. The curve $y = 2|x|$ divides S into two regions of areas R_1 and R_2. If max $\{R_1, R_2\} = R_2$, then $\frac{R_2}{R_1}$ is equal to ______.

माना वक्रों $y = x^3$ तथा $y^2 = x$ द्वारा घिरा क्षेत्र S है। वक्र $y = 2|x|$, क्षेत्र S को R_1 तथा R_2 क्षेत्रफल के दो क्षेत्रों में बाँटता है। यदि max $\{R_1, R_2\} = R_2$ है, तो $\frac{R_2}{R_1}$ बराबर है _________

Ans. Official Answer NTA (19)

Sol.



$C_1 : y = x^3$
$C_2 : y^2 = x$ and
C_1 and C_2 intersect at (1, 1)
C_2 and C_3 intersect at $\left(\frac{1}{4}, \frac{1}{2}\right)$

Clearly $R_1 = \int_0^{1/4}\left(\sqrt{x} - 2x\right)dx = \frac{2}{3}\left(\frac{1}{8}\right) - \frac{1}{16} = \frac{1}{48}$ and $R_1 + R_2 = \int_0^1\left(\sqrt{x} - x^3\right)dx = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$

so, $\frac{R_1 + R_2}{R_1} = \frac{5/12}{1/48} \Rightarrow 1 + \frac{R_2}{R_1} = 20$

$\Rightarrow \frac{R_2}{R_1} = 19$

Question ID : 101690

3D Geometry

30. If the shortest distance between the lines $\vec{r}=\left(-\hat{i}+3\hat{k}\right)+\lambda\left(\hat{i}-a\hat{j}\right)$ and $\vec{r}=\left(-\hat{j}+2\hat{k}\right)+\mu\left(\hat{i}-\hat{j}+\hat{k}\right)$ is $\sqrt{\frac{2}{3}}$, then the integral value of a is equal to _____.

यदि रेखाओं $\vec{r}=\left(-\hat{i}+3\hat{k}\right)+\lambda\left(\hat{i}-a\hat{j}\right)$ तथा $\vec{r}=\left(-\hat{j}+2\hat{k}\right)+\mu\left(\hat{i}-\hat{j}+\hat{k}\right)$ के बीच न्यूनतम दूरी $\sqrt{\frac{2}{3}}$ है, तो a का पूर्णांकीय मान बराबर है __________

Ans. Official Answer NTA (2)

Sol. $a_1 = (-1, 0, 3)$

$a_2 = (0, -1, 2)$

$b_1 = (1, -a, 0)$ dr's of line (1)

$b_2 = (1, -1, 1)$ dr's of line (2)

$\bar{a}_2 - \bar{a}_1 = (1, -1, -1)$

$$\bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & -a & 0 \\ 1 & -1 & 1 \end{vmatrix}$$

$\bar{b}_1 \times \bar{b}_2 = \hat{i}(-a) - \hat{j} + k(a-1)$

$\left|\bar{b}_1 \times \bar{b}_2\right| = \sqrt{a^2 + 1 + (a-1)^2}$

$a_2 - a_1 . \bar{b}_1 \times \bar{b}_2 = 2 - 2a$

$$\frac{2(1-a)}{\sqrt{a^2+1+(a-1)^2}} = \sqrt{\frac{2}{3}}$$

Squaring an both the side

After solving a = $2, \frac{1}{2}$