JEE Main February 2021

Question Paper With Text Solution

24 Feb.| Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN FEB 2021 | 24TH FEB SHIFT-1

SECTION – A

1. The value of

$-{}^{15}C_1 + 2 \cdot {}^{15}C_2 - 3 \cdot {}^{15}C_3 + ... - 15 \cdot {}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + ... + {}^{14}C_{11}$ is

(1) $2^{13} - 14$ (2) 2^{14} (3) $2^{16} - 1$ (4) $2^{13} - 13$

Ans. Official Answer NTA : (1)

Sol. $(1+x)^{15} = {}^{15}C_0 + {}^{15}C_1 x + {}^{15}C_2 x^2 + ... + {}^{15}C_{15} x^{15}$

D w r to x

$15(1+x)^{14} = {}^{15}C_1 + 2 \cdot {}^{15}C_2 x + ... + 15 \cdot {}^{15}C_{15} x^{14}$

Put $x = -1$

$-{}^{15}C_1 + 2 \cdot {}^{15}C_2 + ... - 15\ {}^{15}C_{15} = 0$

${}^{14}C_1 + {}^{14}C_3 + ... + {}^{14}C_{11} + {}^{14}C_{13} = 2^{13}$

${}^{14}C_1 + {}^{14}C_3 + ... + {}^{14}C_{11} = 2^{13} - 14$

$S = 2^{13} - 14$

2. $\lim\limits_{x \to 0} \dfrac{\int_0^{x^2} \left(\sin\sqrt{t}\right) dt}{x^3}$ is equal to :

(1) 0 (2) $\frac{1}{15}$ (3) $\frac{2}{3}$ (4) $\frac{3}{2}$

Ans. Official Answer NTA : (3)

Sol. Use L' Hospital

$$\lim_{x \to 0} \frac{\sin|x|.2x}{3x^2} \Rightarrow \lim_{x \to 0^+} \frac{2\sin x}{3x}$$

Best possible answer $= \frac{2}{3}$

3. The locus of the mid–point of the line segment joining the focus of the parabola $y^2 = 4ax$ to a moving point of the parabola is another parabola whose directrix is :

(1) $x = -\frac{a}{2}$ (2) $x = 0$ (3) $x = \frac{a}{2}$ (4) $x = a$

Ans. Official Answer NTA : (2)

Sol. $P(at^2, 2at)$

$S(a, 0)$

$2h = a(1+t^2)$

$2k = 2at$

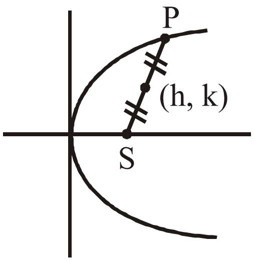
$k^2 = 2a\left(h - \frac{a}{2}\right)$

$y^2 = 2a\left(x - \frac{a}{2}\right)$

Directrix

$x - \frac{a}{2} = -\frac{a}{2}$

$x = 0$



4. If the tangent to the curve $y = x^3$ at the point $P(t, t^3)$ meets the curve again at Q, then the ordinate of the point which divides PQ internally in the ratio 1 : 2 is :

(1) $-t^3$ (2) $2t^3$ (3) 0 (4) $-2t^3$

Ans. Official Answer NTA : (4)

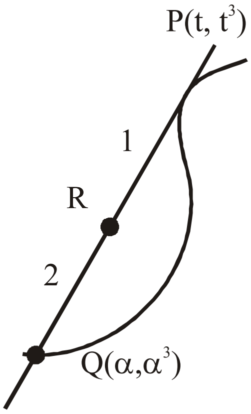
Sol. $(m_T)_P = m_{PQ}$

$3t^2 = \frac{t^3 - \alpha^3}{t - \alpha}$

$\alpha = 1$ or $\alpha = -2t$; $Q = (-2t, -8t^3)$

$y_R = \frac{1\cdot(-8t^3) + 2\cdot(t^3)}{1+2}$

$y_R = -2t^3$

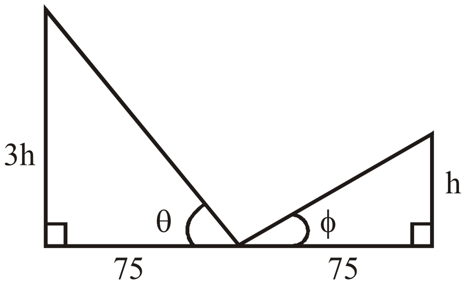


5. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is :

(1) $20\sqrt{3}$ (2) 25 (3) $25\sqrt{3}$ (4) 30

Ans. Official Answer NTA : (3)

Sol.



$$\theta + \phi = \frac{\pi}{2}$$

$$\tan\theta = \cot\phi$$

$$\frac{3h}{75} = \frac{75}{h}$$

$$h = 25\sqrt{3}$$

6. If $f: R \to R$ is a function defined by $f(x) = [x-1]\cos\left(\frac{2x-1}{2}\right)\pi$, where $[\cdot]$ denotes the greatest integer function then f is :

(1) discontinuous only at x = 1

(2) continuous for every real x

(3) discontinuous at all integral values of x except at x = 1

(4) continuous only at x = 1

Ans. Official Answer NTA : (2)

Sol. $f(x) = \underbrace{[x-1]}_{\substack{\text{Discontinuous} \\ \text{at } x \in I}} \underbrace{\cos(2x-1)\frac{\pi}{2}}_{\text{always continuous}}$

For $x \in I$ $\quad \cos(2x-1)\frac{\pi}{2} = 0$

So f (x) is always continous.

7. $f: R \to R$ be defined as $f(x) = 2x - 1$ and $g: R - \{1\} \to R$ be defined as $g(x) = \frac{x - \frac{1}{2}}{x - 1}$. Then the composition function f(g(x)) is :

(1) neither one-oen nor onto

(2) both one-one and onto

(3) onto but not one-one

(4) one-one but not onto

Ans. Official Answer NTA : (4)

Sol. $f(x) = 2x - 1$

$$g(x) = \frac{x - \frac{1}{2}}{x - 1}$$

$$f(g(x)) : R - \{1\} \to R$$

$$f(g(x)) = 2\left(\frac{x - \frac{1}{2}}{x - 1}\right) - 1$$

$$f(g(x)) = \frac{x}{x-1} \quad (x \neq 1) \qquad \left(\frac{L}{L}\right) \quad \text{Always one-one}$$

$$R_{f(g(x))} = R - \{1\} \qquad \text{not onto}$$

8. An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is :

(1) $\frac{1}{2}$ (2) $\frac{5}{16}$ (3) $\frac{1}{32}$ (4) $\frac{3}{16}$

Ans. Official Answer NTA : (1)

Sol. P(odd two times) = P(Even three times)

$${}^nC_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{n-2} = {}^nC_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{n-3}$$

$$n = 5$$

P(1O) + P(3O) + P(5O)

$$= \left({}^5C_1 + {}^5C_3 + {}^5C_5\right) \times \left(\frac{1}{2}\right)^5$$

$$= \frac{1}{2}$$

9. The system of linear equations

$3x - 2y - kz = 10$

$2x - 4y - 2z = 6$

$x + 2y - z = 5m$

(1) $k = 3, m \neq \frac{4}{5}$ (2) $k \neq 3, m \neq \frac{4}{5}$ (3) $k \neq 3, m \in R$ (4) $k = 3, m = \frac{4}{5}$

Ans. Official Answer NTA : (1)

Sol. $3x - 2y - kz = 10$

$2x - 4y - 2z = 6$

$x + 2y - z = 5m$

eliminate y

$2x + (1 - k)z = 7$

$4x + (-k - 1)z = 10 + 5m$

For inconsistant $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\frac{2}{4} = \frac{1-k}{-k-1} \neq \frac{7}{10+5}$

k = 3

$m \neq \frac{4}{5}$

10. A scientific committee is to be formed from 6 Indians and 8 foreigners, which includs at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed, is :

(1) 575 (2) 560 (3) 1050 (4) 1625

Ans. Official Answer NTA : (4)

Sol. No. of ways $= 2I\,4F + 3I\,6F + 4I\,8F$

$= {}^6C_2 \cdot {}^8C_4 + {}^6C_3\ {}^8C_6 + {}^6C_4 \cdot {}^8C_8$

$= 1625$

11. The function $f(x)=\frac{4x^3-3x^2}{6}-2\sin x+(2x-1)\cos x$:

(1) Decreases in $\left[\frac{1}{2},\infty\right)$

(2) Decreases in $\left(-\infty,\frac{1}{2}\right]$

(3) Increases in $\left(-\infty,\frac{1}{2}\right]$

(4) Increases in $\left[\frac{1}{2},\infty\right)$

Ans. Official Answer NTA : (4)

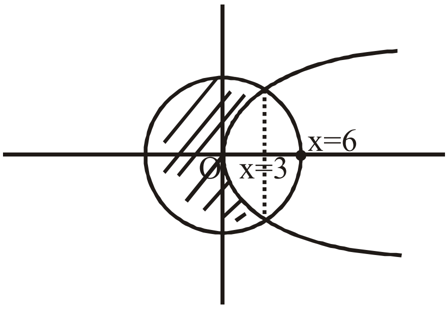
Sol. $f'(x)=(2x-1)(x-\sin x)\geq 0$ for $x\in\left[\frac{1}{2},\infty\right)$

$f(x)$ is increasing for $x\in\left[\frac{1}{2},\infty\right)$

12. The area (in sq. units) of the part of the circle $x^2+y^2=36$, which is outside the parabola $y^2=9x$, is :

(1) $24\pi+3\sqrt{3}$ (2) $12\pi-3\sqrt{3}$ (3) $24\pi-3\sqrt{3}$ (4) $12\pi+3\sqrt{3}$

Ans. Official Answer NTA : (3)



Sol. $\left.\begin{array}{l}C: \quad x^2+y^2=36 \\ P: \quad y^2=9x\end{array}\right\}$ Solve

$x = 3$ or $x = -12$

$$A=\pi.36-3\left[\int_0^3 3\sqrt{x}dx+\int_3^6\sqrt{36-x^2}dx\right]$$

$$A=24\pi-3\sqrt{3}$$

13. The statement among the following that is a tautology is :

(1) $A\vee(A\wedge B)$

(2) $B\rightarrow[A\wedge(A\rightarrow B)]$

(3) $A\wedge(A\vee B)$

(4) $[A\wedge(A\rightarrow B)]\rightarrow B$

Ans. Official Answer NTA : (4)

Sol.

A	B	$A\wedge B$	$A\vee B$	$A\vee(A\wedge B)$	$A\wedge(A\vee B)$	$A\to B$	$A\wedge(A\to B)$	$A\wedge(A\to B)\to B$	$B\to(A\wedge(A\to B)$
T	T	T	T	T	T	T	T	T	T
T	F	F	T	T	T	F	F	T	T
F	T	F	T	F	F	T	F	T	F
F	F	F	F	F	F	T	F	T	T

14. If $e^{\left(\cos^2 x+\cos^4 x+\cos^6 x+....\infty\right)\log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of

$$\frac{2\sin x}{\sin x+\sqrt{3}\cos x}\left(0<x<\frac{\pi}{2}\right) \text{ is :}$$

(1) $\sqrt{3}$ (2) $\frac{1}{2}$ (3) $2\sqrt{3}$ (4) $\frac{3}{2}$

Ans. Official Answer NTA : (2)

Sol. $e^{\left(\frac{\cos^2 x}{1-\cos^2 x}\right)\log_e{}^2} = 2^{\cot^2 x}$

$t^2 - 9t + 8 = 0$

$t = 1, 8$

$2^{\cot^2 x} = 1, 8$

$\cot^2 x = 0, 3$

$\cot x = 0, \sqrt{3}, -\sqrt{3}$ $\quad x \in \left(0, \frac{\pi}{2}\right)$

$\cot x = \sqrt{3} \Rightarrow x = \pi/6$

$$\frac{2\sin x}{\sin x+\sqrt{3}\cos x} = \frac{2\cdot\frac{1}{2}}{\frac{1}{2}+\sqrt{3}\cdot\frac{\sqrt{3}}{2}} = \frac{1}{4}$$

15. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C are placed at the points (1, 1), (2, 2) and (4, 4) respectively. Then which of these stones is/are on the path of the man ?

(1) C only (2) B only (3) All the three (4) A only

Ans. Official Answer NTA : (2)

Sol. $\left\{\begin{matrix}\text{Let} & x-\text{intereept}=a \\ & y-\text{intereept}=b\end{matrix}\right\}$

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\frac{\frac{1}{a}+\frac{1}{b}}{2}=\frac{1}{4} \Rightarrow \frac{1}{a}+\frac{1}{b}=\frac{1}{2}$$

$$\frac{2}{a}+\frac{2}{b}=1$$

Point (2, 2) lies on the line

16. Let p and q be two positive numbers such that $p + q = 2$ and $p^4 + q^4 = 272$. Then p and q are roots of the equation :

(1) $x^2 - 2x + 8 = 0$
(2) $x^2 - 2x + 136 = 0$
(3) $x^2 - 2x + 16 = 0$
(4) $x^2 - 2x + 2 = 0$

Ans. Official Answer NTA : (3)

Sol. $p + q = 2$

$p^4 + q^4 = 272$

$p^2 + q^2 + 2pq = 4$

$p^2 + q^2 = 4 - 2pq$

$p^4 + q^4 + 2p^2q^2 = 16 + 4p^2q^2 - 16pq$

$pq = 16$

Equation of parabola

$x^2 - 2x + 16 = 0$ (but $D < 0$ so roots are imaginary) **(bonus)**

17. The population $P = P(t)$ at time 't' of a certain species follows the differential equation $\frac{dP}{dt} = 0.5P - 450$. If $P(0) = 850$, then the time at which population becomes zero is :

(1) $\frac{1}{2}\log_e 18$
(2) $\log_e 9$
(3) $2\log_e 18$
(4) $\log_e 18$

Ans. Official Answer NTA : (3)

Sol. $\frac{dP}{dt}=\frac{P-900}{2}$

$$\int_{850}^{0}\frac{dP}{P-900}=\int_{0}^{t}\frac{dt}{2}\Rightarrow \ln|P-900|\Big|_{850}^{0}=\frac{t}{2}$$

$t = 2\log_e 18$

18. If $\int\frac{\cos x-\sin x}{\sqrt{8-\sin 2x}}dx = a\sin^{-1}\left(\frac{\sin x+\cos x}{b}\right)+c$, where c is a constant of integration, then the ordered pair (a, b) is equal to :

(1) (3, 1) (2) (1, –3) (3) (1, 3) (4) (–1, 3)

Ans. Official Answer NTA : (3)

Sol. $\sin x + \cos x = t$

$1 + \sin 2x = t^2$

$$I=\int\frac{dt}{\sqrt{9-t^2}}=\sin^{-1}\left(\frac{t}{3}\right)+C$$

$$I=\sin^{-1}\left(\frac{\sin x+\cos x}{3}\right)+C$$

$a = 1$
$b = 3$

$(a, b) = (1, 3)$

19. The equation of the plane passing through the point (1, 2, –3) and perpendicular to the planes $3x + y - 2z = 5$ and $2x - 5y - z = 7$, is :

(1) $11x + y + 17z + 38 = 0$ (2) $6z - 5y + 2z + 10 = 0$

(3) $3x - 10y - 2z + 11 = 0$ (4) $6x - 5y - 2z - 2 =0$

Ans. Official Answer NTA : (1)

Sol. $$\vec{n}=\vec{n}_1\times\vec{n}_2=\begin{vmatrix} i & j & k\\ 3 & 1 & -2\\ 2 & -5 & -1\end{vmatrix}$$

$= -11i - j - 17k$

DR'S of normal = 11, 1, 17

Equation of plane $11(x-1) + 1, (y-2) + 17(z+3) = 0$

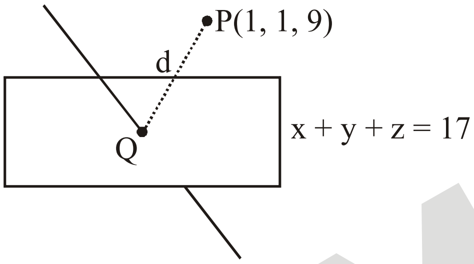
$11x + y + 17z + 38 = 0$

20. The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and the plane $x + y + z = 17$ is :

(1) $\sqrt{38}$ (2) 38 (3) $2\sqrt{19}$ (4) $19\sqrt{2}$

Ans. Official Answer NTA : (1)

Sol. $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2} = \lambda$



$Q = (3 + \lambda, 4 + 2\lambda, 5 + 2\lambda)$

Lies on plane

$12 + 5\lambda = 17$

$\lambda = 1$

$Q = (4, 6, 7)$

$PQ = \sqrt{38}$

SECTION – B

1. Let three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ be such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$, $\vec{a}\cdot\vec{c} = 7$ and $\vec{b}$ is perpendicular to $\vec{c}$, where $\vec{a} = -\hat{i}+\hat{j}+\hat{k}$ and $\vec{b} = 2\hat{i}+\hat{k}$, then the value of $2\left|\vec{a}+\vec{b}+\vec{c}\right|^2$ is ______.

Ans. Official Answer NTA : (75)

Sol. $\vec{c} = \lambda\left(\left(\vec{a}\times\vec{b}\right)\times\vec{b}\right)$

$\vec{c} = \lambda\left(\left(\vec{a}\cdot\vec{b}\right)\vec{b}\right) - \left(\vec{b}\cdot\vec{b}\right)\vec{a}\Big)$

$\vec{c} = \lambda\left(-\vec{b}-5\vec{a}\right)$

$\vec{a}\cdot\vec{c} = \lambda\left(-\vec{a}\cdot\vec{b}-5\vec{a}\cdot\vec{a}\right) = 7$

$\lambda = \frac{-1}{2}$

$\vec{c} = \frac{5\vec{a}+\vec{b}}{2} = \frac{-3i+5i+6k}{2}$

$\vec{a}+\vec{b}+\vec{c} = \frac{-i+7j+10k}{2}$

$\left|\vec{a}+\vec{b}+\vec{c}\right| = \frac{1}{2}\times\sqrt{150}$

$2\left|\vec{a}+\vec{b}+\vec{c}\right|^2 = 75$

2. Let B_i (i = 1, 2, 3) be three independent events in a sample space. The probability that only B_1 occur is α, only B_2 occurs is β and only B_3 occurs is γ. Let p be the probability that none of the events B_i occurs and these 4 probabilities stisfy the equations $(\alpha - 2\beta)\,p = \alpha\beta$ and $(\beta - 3\gamma)\,p = 2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0, 1)). Then $\frac{P(B_1)}{P(B_3)}$ is equal to _______.

Ans. Official Answer NTA : (6)

Sol. Let $P(B_1) = a$

$P(B_2) = b$

$P(B_3) = c$

$a(1-b)(1-c) = \alpha$

$$(1-a)\,b(1-c)=\beta$$

$$(1-a)(1-b)\,c=\gamma$$

$$(1-a)(1-b)(1-c)=p$$

$$(\alpha-2\beta)p=\alpha\beta \text{ and } (\beta-3\gamma)p=2\beta\gamma$$

$$\frac{a}{b}=2 \quad \& \quad \frac{b}{c}=3$$

$$\frac{a}{c}=6=\frac{P(B_1)}{P(B_3)}$$

3. If $\int\limits_{-a}^{a}\left(|x|+|x-2|\right)dx=22$ $(a>2)$ and $[x]$ denotes the greatest integer $\le x$, then $\int\limits_{a}^{-a}\left(x+[x]\right)$ dx is equal to ___________.

Ans. Official Answer NTA : (3)

Sol. $$I=\int\limits_{-a}^{a}|x|\,dx+\int\limits_{-a}^{a}|x-2|\,dx=22$$

$$2\int\limits_{0}^{a}x\,dx+\int\limits_{-a}^{2}(2-x)\,dx+\int\limits_{2}^{a}(x-2)\,dx=22$$

$$a=3$$

$$\int\limits_{a}^{-a}\left(x+[x]\right)dx=\int\limits_{3}^{-3}x\,dx+\int\limits_{3}^{-3}[x]\,dx$$

$$=0+3=3$$

4. The minimum value of α for which the equation $\frac{4}{\sin x}+\frac{1}{1-\sin x}=\alpha$ has at least one solution in $\left(0,\frac{\pi}{2}\right)$ is ________ .

Ans. Official Answer NTA : (9)

Sol. $$f(x)=\frac{4}{\sin x}+\frac{1}{1-\sin x}=\frac{4-3\sin x}{\sin x(1-\sin x)}$$

$$f'(x)=\frac{(2-\sin x)(3\sin x-2)}{\sin^2 x(1-\sin x)^2}$$

$$f'(x)\Rightarrow \underset{\sin x\,=\,2/3}{\overset{(-)\text{ve}\quad O\quad (+)\text{ve}}{\longleftarrow\!\bullet\!\longrightarrow}}$$

$$\alpha_{\min}=\frac{4}{2/3}+\frac{1}{1-2/3}=9$$

5. Let $P=\begin{bmatrix}3 & -1 & -2\\ 2 & 0 & \alpha\\ 3 & -5 & 0\end{bmatrix}$, where $\alpha\in R$. Suppose $Q=[q_{ij}]$ is a matrix satisfying $PQ=kI_3$ for some non–zero $k\in R$. If $q_{23}=-\frac{k}{8}$ and $|Q|=\frac{k^2}{2}$ then α^2+k^2 is equal to ___________ .

Ans. Official Answer NTA : (17)

Sol. $P=\begin{bmatrix}3 & -1 & -2\\ 2 & 0 & \alpha\\ 3 & -5 & 0\end{bmatrix}$

$|P|=4(3\alpha+5)$

$PQ=kI_3$

$|P|\cdot|Q|=k^3$

$4(3\alpha+5)\cdot\frac{k^2}{2}=k^3$

$k=2(3\alpha+5)$ ......(1)

$Q=k\,P^{-1}$

$Q=\frac{k\,\text{adj.}P}{|P|}$

$q_{23}=\frac{k}{4(3\alpha+5)}.C_{32}$ $(C_{ij}=\text{Cofacor of }a_{ij})$

$\frac{-k}{8}=\frac{k}{4(3\alpha+5)}\times(-1)\times(3\alpha+6)$

$\alpha=-1$

$\alpha^2+k^2=17$

6. Let M be any 3 × 3 matrix with entries from the set {0, 1, 2}. The maximum number of such matrices, for which the sum of diagonal elements of M^TM is seven, is _______ .

Ans. Official Answer NTA : (540)

Sol. $M=\begin{bmatrix}a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}$

$\text{tr}(M^TM) = a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 7$

Case – 1 2, 1, 1, 1, 0, 0, 0, 0, 0

Case – 2 1, 1, 1, 1, 1, 1, 1, 0, 0

$$\text{Total} = \frac{9!}{3!5!} + \frac{9!}{7!2!}$$

= 540

7. $\lim\limits_{n\to\infty} \tan\left\{\sum\limits_{r=1}^{n} \tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right\}$ is equal to __________ .

Ans. Official Answer NTA : (1)

Sol. $\tan^{-1}\left(\frac{1}{1+r(r+1)}\right) = \tan^{-1}(r+1) - \tan^{-1} r$

$$S_n = \sum_{r=1}^{n} \tan^{-1}(r+1) - \tan^{-1} r$$

$S_n = \tan^{-1}(n+1) - \tan^{-1} 1$

$$S_\infty = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$\lim_{n\to\infty} \tan\left(\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right) = \tan\left(\frac{\pi}{4}\right) = 1$$

8. Let A = {n∈N : n is a 3–digit number}

B = {9k + 2 : k∈N}

and C = {9k + *l* : k∈N} for some *l* (0 < *l* < 9)

If the sum of all the elements of the set $A \cap (B \cup C)$ is 274 × 400, then *l* is equal to ________ .

Ans. Official Answer NTA : (5)

Sol. If $B \cup C = B$

$$\sum_{K=11}^{110} (9k+2) = 9\left(\sum_{K=1}^{110} K - \sum_{K=1}^{10} K\right) + 200$$

$= 54650 \neq 274 \times 400$

If $B \cup C \neq B$

$$\sum_{K=11}^{110}(9k+2)+\sum_{K=11}^{110}(9k+\ell)=274\times 400$$

$$\ell=5$$

9. If the least and the largest real values of α, for which the equation $z + \alpha\,|z - 1| + 2i = 0$ ($z\in C$ and $i = \sqrt{-1}$) has a solution, are p and q respectively; then $4(p^2 + q^2)$ is equal to ________.

Ans. Official Answer NTA : (10)

Sol. $z=x+iy$

$$x+\alpha\sqrt{(x-1)^2+y^2}+i(y+2)=0+0i$$

$$y=-2$$

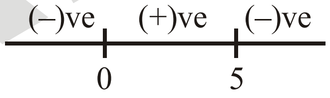
$$x+\alpha\sqrt{(x-1)^2+y^2}=0$$

$$x^2=\alpha^2\left((x-1)^2+4\right)$$

$$\alpha^2=\frac{x^2}{x^2-2x+5}=f(x)$$

$$f'(x)=\frac{-2x(x-5)}{(x^2-2x+5)^2}$$

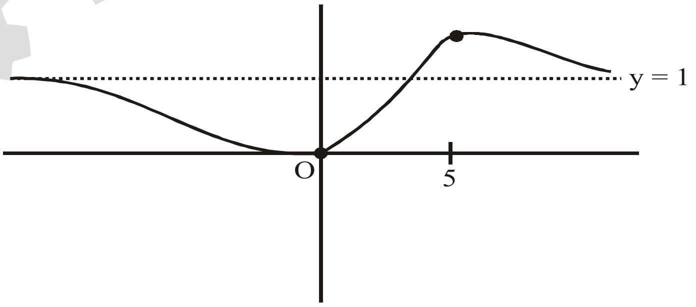
f '(x) =



$$\lim_{a\to\pm\infty} f(x)=1$$

$$f(0)=0$$

$$f(5)=\frac{5}{4}$$



$$\alpha^2\in\left[0,\frac{5}{4}\right]$$

$$\alpha\in\left[\frac{-\sqrt{5}}{2},\frac{\sqrt{5}}{2}\right]$$

$$P=\frac{-\sqrt{5}}{2}$$

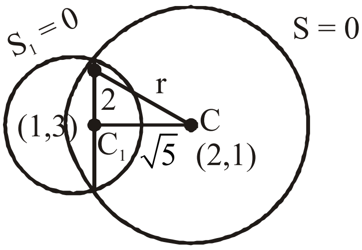
$$q=\frac{\sqrt{5}}{2}$$

$$4(P^2+q^2)=10$$

10. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle 'C', whose center is at (2, 1), then its radius is ________.

Ans. Official Answer NTA : (3)

Sol.



$S_1 : x^2 + y^2 - 2x - 6y + 6 = 0$

$C_1 = (1,3)$

$r_1 = 2$

$r = \sqrt{r_1^{\,2} + CC_1^{\,2}}$

$r = 3$