JEE Main February 2021

Question Paper With Text Solution

24 Feb.| Shift-2

MATHEMATICS

JEE MAIN FEB 2021 | 24TH FEB SHIFT-2

SECTION – A

1. Let A and B be 3 × 3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $(A^2B^2 - B^2A^2)X = O$, where X is a 3 × 1 column matrix of unknown variables and O is a 3 × 1 null matrix, has :

(1) no solution
(2) exactly two solutions
(3) infinitely many solutions
(4) a unique solution

Ans. Official Answer NTA : (3)

Sol. $(A^2B^2 - B^2A^2)^T = (A^2B^2)^T - (B^2A^2)^T$

$= B^2A^2 - A^2B^2$

$\Rightarrow A^2B^2 - B^2A^2$ is skew – symmetic

$\Rightarrow |A^2B^2 - B^2A^2| = 0$

$\Rightarrow$ option (3)

2. If $n \geq 2$ is a positive integer, then the sum of the series ${}^{n+1}C_2 + 2\left({}^2C_2 + {}^3C_2 + {}^4C_2 + \ldots + {}^nC_2\right)$ is :

(1) $\dfrac{n(n+1)^2(n+2)}{12}$
(2) $\dfrac{n(n+1)(2n+1)}{6}$
(3) $\dfrac{n(2n+1)(3n+1)}{6}$
(4) $\dfrac{n(n-1)(2n+1)}{6}$

Ans. Official Answer NTA : (2)

Sol. $E = {}^{n+1}C_2 + 2\left({}^3C_3 + {}^3C_2 + {}^4C_2 + \ldots + {}^nC_2\right)$

$= {}^{n+1}C_2 + 2\,({}^{n+1}C_3)$

$= \dfrac{(n+1)n}{2} + \dfrac{2(n+1)n(n-1)}{6}$

$= \dfrac{n(n+1)(3+2n-2)}{6}$

$= \dfrac{n(n+1)(2n+1)}{6}$

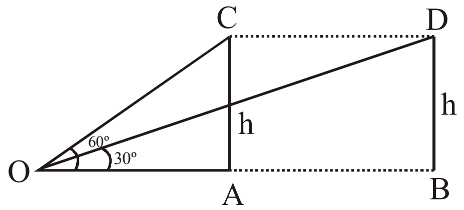
$\Rightarrow$ option (2)

3. The angle of elevation of a jet plane from a point A on the ground is 60º. After a flight of 20 seconds at the speed of 432 km/hour, the angle of elevation changes to 30º. If the jet plane is flying at a constant height, then its height is :

(1) $1800\sqrt{3}$m (2) $1200\sqrt{3}$m (3) $3600\sqrt{3}$m (4) $2400\sqrt{3}$m

Ans. Official Answer NTA : (2)

Sol.



$AB = CD = \frac{432000}{3600} \times 20$

$= 2400$

AB = h cot 30º – h cot 60º = 2400

$\frac{2h}{\sqrt{3}} = 2400 \Rightarrow h = 1200\sqrt{3}$

$\Rightarrow$ option (2)

4. Let f(x) be a differentiable function defined on [0, 2] such that f'(x) = f' (2 – x) for all $x \in (0, 2)$, f(0) = 1 and f(2) = e^2. Then the value of $\int_0^2 f(x)\,dx$ is :

(1) $1 - e^2$ (2) $1 + e^2$ (3) $2(1 - e^2)$ (4) $2(1 + e^2)$

Ans. Official Answer NTA : (2)

Sol. f'(x) = f'(2 – x)

f'(x) – f' (2 – x) = 0

Integrate w.r.t. x

f(x) + f(2 – x) = C

$\because$ f(0) = 1, f(2) = $e^2 \Rightarrow C = e^2 + 1$

$I = \int_0^2 f(x)\,dx$

$I=\int_0^2 f(2-x)\,dx$ $\left(\because \int_a^b f(x)\,dx=\int_a^b f(a+b-x)\,dx\right)$

Add

$2I=\int_0^2 (f(x)+f(2-x))dx$

$2I=\int_0^2 (e^2+1)\,dx$

$\Rightarrow I = e^2 + 1$

option (2)

5. If a curve $y = f(x)$ passes through the point (1, 2) and satisfies $x\frac{dy}{dx}+y=bx^4$, then for what value of b, $\int_1^2 f(x)\,dx=\frac{62}{5}$?

(1) 10 (2) 5 (3) $\frac{31}{5}$ (4) $\frac{62}{5}$

Ans. Official Answer NTA : (1)

Sol. $\frac{xdy}{dx}+y=bx^4$

$\int d(xy)=\int bx^4dx$

$xy=\frac{bx^5}{5}+c$

$y=\frac{bx^4}{5}+\frac{c}{x}$

Passes though (1, 2) $\Rightarrow \frac{b}{5}+c=2$ .....(1)

$\int_1^2 f(x)\,dx=\frac{62}{5}$

$\left(\frac{bx^5}{25}+c\ln x\right)_1^2=\frac{62}{5}$

$\frac{31b}{25}+c\ln 2=\frac{62}{5}$ ......(2)

$\Rightarrow$ solving equation (1) and (2)

$\Rightarrow$ b = 10 and c = 0

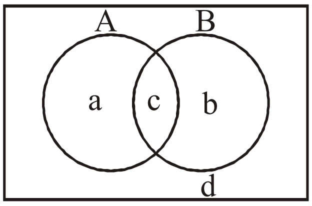
$\Rightarrow$ option (1)

6. The probability that two randomly selected subsets of the set {1, 2, 3, 4, 5} have exactly two elements in their intersection, is :

(1) $\frac{35}{2^7}$ (2) $\frac{65}{2^7}$ (3) $\frac{135}{2^9}$ (4) $\frac{65}{2^8}$

Ans. Official Answer NTA : (3)

Sol.



n $(A \cap B) = 2$

for sample space

4 position for each element

$\Rightarrow n(S) = 4^5 = 2^{10}$

for event

n $(A \cap B) = 2$

n (Event) $= {}^5C_2 \times (3)^3$

p(Event) $= \frac{10 \times 27}{2^{10}} = \frac{135}{2^9}$

Option (3)

7. Let f : R $\rightarrow$ R be defined as

$$f(x) = \begin{cases} -55x, & \text{if } x < -5 \\ 2x^3 - 3x^2 - 120x, & \text{if } -5 \le x \le 4 \\ 2x^3 - 3x^2 - 36x - 336, & \text{if } x > 4, \end{cases}$$

Let A = {$x \in R$: f is increasing}. Then A is equal to :

(1) $(-5, \infty)$ (2) $(-\infty, -5) \cup (4, \infty)$

(3) $(-5, -4) \cup (4, \infty)$ (4) $(-\infty, -5) \cup (-4, \infty)$

Ans. Official Answer NTA : (3)

Sol.

$$f(x) = \begin{cases} -55x & x < -5 \\ 2x^3 - 3x^2 - 120x & -5 \le x \le 4 \\ 2x^3 - 3x^2 - 36x - 336 & x > 4 \end{cases}$$

$$f'(x) = \begin{cases} -55 & x < -5 \\ 6x^2 - 6x - 120 & -5 < x < +4 \\ 6x^2 - 6x - 36 & x > 4 \end{cases}$$

C – I $\quad x < -5$

$f'(x) < 0$

$\Rightarrow$ f is decreasing

C – II $\quad -5 < x < 4$

$f'(x) > 0 \Rightarrow x \in (-\infty, -4) \cup (5, \infty)$

$\Rightarrow x \in (-5, -4) \Rightarrow$ f is increasing

C – III $\quad x > 4$

$f'(x) > 0$

$6(x^2 - x - 6) > 0 \Rightarrow x \in (-\infty, -2) \cup (3, \infty)$

$\Rightarrow x \in (4, \infty) \Rightarrow$ f is increasing

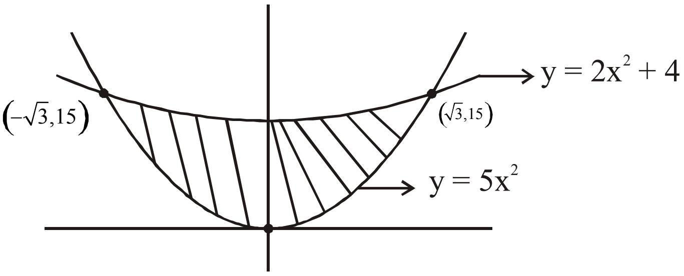
f is increasing $\forall\ x \in (-5, -4) \cup (4, \infty)$ $\quad \Rightarrow$ option (3)

8. The area of the region : $R = \{(x, y) : 5x^2 \le y \le 2x^2 + 9\}$ is :

(1) $9\sqrt{3}$ square units
(2) $12\sqrt{3}$ square units
(3) $6\sqrt{3}$ square units
(4) $11\sqrt{3}$ square units

Ans. Official Answer NTA : (2)

Sol.



Required Area = $\int_{-\sqrt{3}}^{\sqrt{3}} (2x^2 + 9 - 5x^2)\,dx$

$= 12\sqrt{3}$

$\Rightarrow$ option (2)

9. Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices (a, c), (2, b) and (a, b) be $\left(\frac{10}{3}, \frac{7}{3}\right)$. If α, β are the roots of the equations $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is :

(1) $\frac{71}{256}$ (2) $-\frac{71}{256}$ (3) $\frac{69}{256}$ (4) $-\frac{69}{256}$

Ans. Official Answer NTA : (2)

Sol. $G\left(\frac{2a+2}{3}, \frac{2b+c}{3}\right) \equiv \left(\frac{10}{3}, \frac{7}{3}\right)$

$\frac{2a+2}{3} = \frac{10}{3}$ $\qquad$ $\frac{2b+c}{3} = \frac{7}{3}$

$a = 4$ $\qquad$ $2b + c = 7$

$2(a + d) + a + 2d = 7$

$3a + 4d = 7$

$12 + 4d = 7$

$4d = -5 \Rightarrow d = \frac{5}{4}$

$\Rightarrow b = 4 - \frac{5}{4} = \frac{11}{4}$

Quadratic equation

$ax^2 + bx + 1 = 0$

$4x^2 + \frac{11x}{4} + 1 = 0$

$16x^2 + 11x + 4 = 0 < \begin{matrix}\alpha \\ \beta\end{matrix}$

$E = \alpha^2 + \beta^2 - \alpha\beta$

$= (\alpha + \beta)^2 - 3\alpha\beta$

$= \frac{121}{256} - 3\left(\frac{1}{4}\right)$

$= \frac{121-192}{256} = \frac{-71}{256}$

$\Rightarrow$ option (2)

10. For which of the following curves, the line $x + \sqrt{3}y = 2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$?

(1) $2x^2 - 18y^2 = 9$

(2) $x^2 + y^2 = 7$

(3) $y^2 = \frac{1}{6\sqrt{3}}x$

(4) $x^2 + 9y^2 = 9$

Ans. Official Answer NTA : (4)

Sol. Tangent $x + \sqrt{3}y = 2\sqrt{3}$ at $(x_1, y_1) \equiv \left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$

Check options

(1) $2xx_1 - 18yy_1 = 9$

$2x\left(\frac{3\sqrt{3}}{2}\right) - 18y\left(\frac{1}{2}\right) = 9$

$2\sqrt{3}\,x - 9y = 9$ No

(2) $xx_1 + yy_1 = 7$

$x\frac{3\sqrt{3}}{2} + y\left(\frac{1}{2}\right) = 7$ No

(3) $yy_1 = \frac{1}{12\sqrt{3}}(x + x_1)$

$$\frac{y}{2}=\frac{1}{12\sqrt{3}}\left(x+\frac{3\sqrt{3}}{2}\right)$$ No

(4) $xx_1 + 9yy_1 = 9$

$$x\frac{3\sqrt{3}}{2}+9y\left(\frac{1}{2}\right)=9$$

$$x+\sqrt{3}y=2\sqrt{3}$$ Yes

$\Rightarrow$ option (4)

11. Let f be twice differentiable function defined on R such that f(0) = 1, f'(0) = 2 and f' (x) $\neq$ 0 for all x $\in$ R. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$, for all x $\in$ R, then value of f(1) lies in the interval :

(1) (3, 6) (2) (6, 9) (3) (0, 3) (4) (9, 12)

Ans. Official Answer NTA : (2)

Sol. $f(x)\, f''(x) - (f'(x))^2 = 0 \;\forall x \in R$

$$\frac{d}{dx}\left(\frac{f'(x)}{f(x)}\right)=0$$

$$\frac{f'(x)}{f(x)}=c$$

$\because f(0)=1, f'(0)=2 \Rightarrow c=2$

$$\frac{f'(x)}{f(x)}=2$$

$$\int\frac{dy}{y}=\int 2dx$$

$\ln y = 2x + \ln k \Rightarrow y = k\,e^{2x}$

$\because\; f(0) = 1 \Rightarrow k = 1$

$f(x) = e^{2x}$

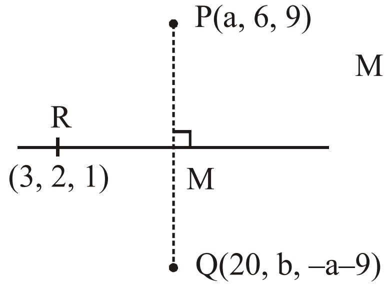
$f(1) = e^2$

12. Let a, b $\in$ R. If the mirror image of the point P (a, 6, 9) with respect of the line $\frac{x-3}{7}=\frac{y-2}{5}=\frac{z-1}{-9}$ is (20, b, –a –9), then |a + b| is equal to :

(1) 86 (2) 90 (3) 88 (4) 84

Ans. Official Answer NTA : (3)

Sol.



M $\equiv$ mid point of pcs

$M\left(\frac{20+a}{2},\frac{2+b}{2},\frac{-a}{2}\right)$

$\overrightarrow{MR}$ || line

$\overrightarrow{MR}=\left(\frac{a+14}{2},\frac{b+2}{2},\frac{-a-2}{2}\right)$

$$\frac{\frac{a+14}{2}}{7}=\frac{\frac{b+2}{2}}{5}=\frac{\frac{-a-2}{2}}{-9}$$

$\Rightarrow a=-56 \qquad b=-32$

$\Rightarrow |a+b|=88$

option (3)

13. The negation of the statement $\sim p \wedge (p \vee q)$ is :

(1) $p \wedge \sim q$ (2) $\sim p \vee q$ (3) $\sim p \wedge q$ (4) $p \vee \sim q$

Ans. Official Answer NTA : (4)

Sol. $\sim(\sim p \wedge (p \vee q))$

$p \vee ((\sim p) \wedge (\sim q))$

$(p \vee (\sim p)) \wedge (p \vee (\sim q))$

$t \vee (p \vee (\sim q))$

$\equiv p \vee (\sim q)$ $\Rightarrow$ option (4)

14. A possible value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is :

(1) $\frac{1}{2\sqrt{2}}$ (2) $2\sqrt{2}-1$ (3) $\frac{1}{\sqrt{7}}$ (4) $\sqrt{7}-1$

Ans. Official Answer NTA : (3)

Sol. Let $\sin^{-1}\left(\frac{\sqrt{63}}{8}\right)=\theta$ $\sin\theta=\frac{\sqrt{63}}{8}\Rightarrow\cos\theta=\frac{1}{8}$

$\cos\frac{\theta}{2}=\sqrt{\frac{1+\cos\theta}{2}}$; $\sin\frac{\theta}{2}=\sqrt{\frac{1-\cos\theta}{2}}$

$=\frac{3}{4}$ $=\frac{\sqrt{7}}{4}$

$$\tan\frac{\theta}{4}=\frac{\sin\frac{\theta}{2}}{1+\cos\frac{\theta}{2}}$$

$$=\frac{\frac{\sqrt{7}}{4}}{1+\frac{3}{4}}=\frac{\sqrt{7}}{7}=\frac{1}{\sqrt{7}}$$

$\Rightarrow$ option (3)

15. For the statements p and q, consider the following compond statements :

(a) $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$

(b) $((p \vee q) \wedge \sim p) \rightarrow q$

Then which of the following statements is correct ?

(1) (a) is a tautology but not (b). (2) (b) is a tautology but not (a).

(3) (a) and (b) both are not tautologies. (4) (a) and (b) both are tautologies.

Ans. Official Answer NTA : (4)

Sol. (a) $((\sim q) \wedge (p \rightarrow q)) \rightarrow (\sim p)$

$\sim ((\sim q) \wedge (p \rightarrow q)) \vee (\sim p)$

$(q \vee (\sim (p \rightarrow q))) \vee (\sim p)$

$(q \vee (p \wedge \sim q)) \vee (\sim p)$

$(q \vee p) \wedge (q \vee \sim q) \vee (\sim p)$

$(q \vee p) \vee (\sim p) \equiv t$

(b) $((p \vee q) \wedge (\sim p)) \rightarrow q$

$\sim ((p \vee q) \wedge (\sim p)) \vee q$

$((\sim (p \vee q)) \vee p) \vee q$

$((\sim p) \wedge (\sim q) \vee p) \vee q$

$(p \vee \sim q) \vee q \equiv t$

$\Rightarrow$ option (4)

16. The vector equation of the plane passing through the intersection of the planes $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=1$ and $\vec{r}\cdot(\hat{i}-2\hat{j}) = -2$, and the point (1, 0, 2) is :

(1) $\vec{r}\cdot(\hat{i}-7\hat{j}+3\hat{k})=\frac{7}{3}$

(2) $\vec{r}\cdot(\hat{i}+7\hat{j}+3\hat{k})=\frac{7}{3}$

(3) $\vec{r}\cdot(3\hat{i}+7\hat{j}+3\hat{k})=7$

(4) $\vec{r}\cdot(\hat{i}+7\hat{j}+3\hat{k})=7$

Ans. Official Answer NTA : (4)

Sol. By concept of family of planes

equation of required plane is

$x + y + z - 1 + \lambda (x - 2y + 2) = 0$

(1, 0, 2) will lie on it

$1 + 2 - 1 + \lambda (1 + 2) = 0$

$\lambda = \frac{-2}{3}$

$\frac{x}{3}+\frac{7y}{3}+z-\frac{7}{3}=0$

$x + 7y + 3z = 7$

Vector equation

$\vec{r}\cdot\left(\hat{i}+7\hat{j}+3\hat{k}\right)=7$

$\Rightarrow$ option (4)

17. For the system of linear equations :

$x-2y=1,\ x-y+kz=-2,\ ky+4z=6,\ k\in R$

Consider the following statements :

(A) The system has unique solution if $k\neq 2, k\neq -2$.

(B) The system has unique solution if $k=-2$.

(C) The system has unique solution if $k=2$.

(D) The system has no–solution if $k=2$.

(E) The system has infinite number of solutions if $k\neq -2$.

Which of the following statements are correct ?

(1) (C) and (D) only
(2) (A) and (E) only
(3) (B) and (E) only
(4) (A) and (D) only

Ans. Official Answer NTA : (4)

Sol. $$D=\begin{vmatrix}1 & -2 & 0\\ 1 & -1 & k\\ 0 & k & 4\end{vmatrix}=4-k^2=(2-k)(2+k)$$

$$D_x=\begin{vmatrix}1 & -2 & 0\\ -2 & -1 & k\\ 6 & k & 4\end{vmatrix}=-(k+10)(k+2)$$

$D\neq 0\Rightarrow k\neq\pm 2$

$\Rightarrow$ system will have unique solution

$\Rightarrow$ A is true; E, B, C are false

If $k=2\Rightarrow D=0\ D_x\neq 0$

$\Rightarrow$ System will have no solution $\Rightarrow$ D is true

$\Rightarrow$ A & D are true.

18. If the curve $y = ax^2 + bx + c$, $x \in R$, passes through the point (1, 2) and the tangent line to this curve at origin is $y = x$, then the possible values of a, b, c are :

(1) $a = 1, b = 0, c = 1$ (2) $a = -1, b = 1, c = 1$

(3) $a = 1, b = 1, c = 0$ (4) $a = \frac{1}{2}, b = \frac{1}{2}, c = 1$

Ans. Official Answer NTA : (3)

Sol. Curve passes through (0, 0) $\Rightarrow c = 0$

$y = ax^2 + bx$

Tangent at (0, 0)

$y = bx \Rightarrow b = 1$ ($\because$ comparing with $y = x$)

$y = ax^2 + x$

Curve passes through (1, 2)

$2 = a + 1 \Rightarrow a = 1$

$(a, b, c) = (1, 1, 0)$

$\Rightarrow$ option (3)

19. The valur of the integral, $\int_1^3 [x^2 - 2x - 2]\,dx$, where [x] denotes the greatest integer less than or equal to x, is :

(1) –5 (2) $-\sqrt{2}-\sqrt{3}+1$ (3) $-\sqrt{2}-\sqrt{3}-1$ (4) –4

Ans. Official Answer NTA : (3)

Sol. $I = \int_1^3 [x^2 - 2x - 2]\,dx = \int_1^3 [(x-1)^2 - 3]\,dx$

Put $x - 1 = t$

$I = \int_0^2 [t^2 - 3]\,dx = \int_0^2 [t^2]\,dx - 3\int_0^2 dx$

$= \int_0^1 0\,dt + \int_1^{\sqrt{2}} 1\,dt + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dt + \int_{\sqrt{3}}^{2} 3\,dt - 6$

$= -\sqrt{2} - \sqrt{3} - 1$

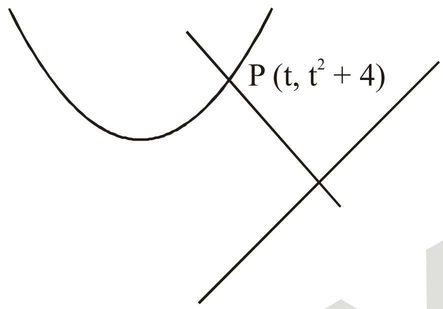
$\Rightarrow$ option (3)

20. If P is a point on the parabola $y = x^2 + 4$ which is closest to the straight line $y = 4x - 1$, then the co–ordinates of P are :

(1) (–2, 8) (2) (2, 8) (3) (1, 5) (4) (3, 13)

Ans. Official Answer NTA : (2)

Sol.



Tangent at P will be parallel to $y = 4x - 1$

$\left.\frac{dy}{dx}\right)_P = 4$

$2t = 4 \Rightarrow t = 2$

P (2, 8)

$\Rightarrow$ option (2)

SECTION – B

1. The students $S_1, S_2, ..., S_{10}$ are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is _______ .

Ans. Official Answer NTA : (31650)

Sol. Case – I Group C has 1 student

number of ways = ${}^{10}C_1 (2^9 - 2) = A_1 = 5100$

Case – II Group C has 2 student

number of ways = $^{10}C_2\,(2^8 - 2) = A_2 = 11430$

Case – III Group C has 3 student

number of ways = $^{10}C_3\,(2^7 - 2) = A_3 = 15120$

Ans. = $A_1 + A_2 + A_3 = 31650$

2. Let a point P be such that its distance from the point (5, 0) is thrice the distance of P form the point (–5, 0). If the locus of the point P is a circle of radius r, then $4r^2$ is equal to _____________.

Ans. Official Answer NTA : (56)

Sol. P(x, y) A(5, 0) B (–5, 0)

PA = 3PB

$(PA)^2 = 9(PB)^2$

$(x-5)^2 + y^2 = 9\,((x+5)^2 + y^2)$

$8x^2 + 8y^2 + 100x + 200 = 0$

$$x^2 + y^2 + \frac{25}{2}x + 25 = 0$$

$$r = \sqrt{\left(\frac{28}{4}\right)^2 - 25}$$

$$4r^2 = 4\left(\frac{625}{16} - 25\right)$$

$$= \frac{625}{4} - 100 = \frac{225}{4} = 56.25$$

3. If the variance of 10 natural numbers 1, 1, 1, ..., 1, k is less than 10, then the maximum possible value of k is ________.

Ans. Official Answer NTA : (11)

Sol. $\sigma^2 < 10$

$$\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 < 10$$

$$\frac{9+k^2}{10}-\left(\frac{9+k}{10}\right)^2<10$$

$9(k^2 - k) < 991$

$k(k-2) < \dfrac{991}{9}$

k = 11 is largest integral value satisfying given inequality.

4. If $a+\alpha=1, b+\beta=2$ and $a\,f(x)+\alpha f\left(\frac{1}{x}\right)=bx+\frac{\beta}{x}, x\neq 0$, then the value of the expression $\dfrac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}$ is ________.

Ans. Official Answer NTA : (2)

Sol. $a+\alpha=1$ $\qquad b+\beta=2$

$$af(x)+\alpha f\left(\frac{1}{x}\right)=bx+\frac{\beta}{x} \qquad ....(1)$$

$x \to \dfrac{1}{x}$

$$af\left(\frac{1}{x}\right)+\alpha f(x)=\frac{b}{x}+\beta \qquad ....(2)$$

(1) + (2)

$$(a+\alpha)\ f\left(f(x)+f\left(\frac{1}{x}\right)\right)=(b+\beta)\left(x+\frac{1}{x}\right)$$

$$f(x)+f\left(\frac{1}{x}\right)=2\left(x+\frac{1}{x}\right)$$

$$\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}=2$$

5. Let $i = \sqrt{-1}$. If $\frac{\left(-1+i\sqrt{3}\right)^{21}}{(1-i)^{24}} + \frac{\left(1+i\sqrt{3}\right)^{21}}{(1+i)^{24}} = k$, and n = [|k|] be the greatest integral part of |k|. Then $\sum_{j=0}^{n+5}(j+5)^2 - \sum_{j=0}^{n+5}(j+5)$ is equal to ______ .

Ans. Official Answer NTA : (310)

Sol. $k = \left(\frac{\left(2e^{i\frac{2\pi}{3}}\right)^{21}}{\left(\sqrt{2}e^{-i\frac{\pi}{4}}\right)^{24}}\right) + \left(\frac{\left(2e^{i\frac{\pi}{3}}\right)^{21}}{\left(\sqrt{2}e^{i\frac{\pi}{4}}\right)^{24}}\right)$

$= 2^9 - 2^9 = 0$

$\Rightarrow n = 0$

$E = \sum_{j=0}^{5}(j+5)^2 - \sum_{j=0}^{5}(j+5)$

$= (5^2 + 6^2 + ... + 10^2) - (5 + 6 + ... + 10)$

$= 310$

6. If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x-2)^2 + (y-3)^2 = 25$ at the point (5, 7) is A, then 24A is equal to _________.

Ans. Official Answer NTA : (1225)

Sol. Normal at P (5, 7)

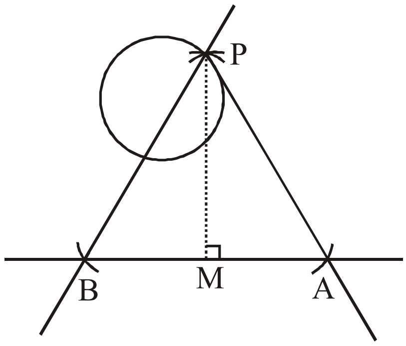
$4x - 3y + 1 = 0$

$\Rightarrow B\left(-\frac{1}{4}, 0\right)$

Tangent at P

$3x + 4y = 43$

$\Rightarrow A\left(\frac{43}{3}, 0\right)$



PB is normal at P, PA is tangent at P

Area = $\frac{1}{2}$ (AB) (PM)

$$A = \frac{1}{2}\left(\frac{43}{3}+\frac{1}{4}\right)(7) = \frac{1225}{24}$$

24A = 1225

7. The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α, then 2α is ___.

Ans. Official Answer NTA : (3)

Sol. GP a, ar, ar^2, ar^3

$a + ar + ar^2 + ar^3 = \frac{65}{12}$

$$S_4 = a\frac{(r^4-1)}{r-1} = \frac{65}{12} \quad ....(1)$$

$$\frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2}+\frac{1}{ar^3} = \frac{65}{18}$$

$$\frac{\frac{1}{ar^3}(r^4-1)}{r-1} = \frac{65}{18} \quad ....(2)$$

(1) / (2)

$a^2r^3 = \frac{3}{2}$

$a \times ar \times ar^2 = 1$

$a^3r^3 = 1$

$\Rightarrow a = \frac{2}{3}$

$r = \frac{3}{2}$

$\alpha = ar^2 = \frac{3}{2}$

$2\alpha = 3$

8. The number of the real roots of the equation $(x + 1)^2 + |x-5| = \frac{27}{4}$ is ______.

Ans. Official Answer NTA : (2)

Sol. C – I $\quad x \geq 5$

$(x + 1)^2 + x - 5 = \frac{27}{4}$

$x^2 + 3x - \frac{43}{4} = 0$ (Both roots < 5)

$\Rightarrow x \in \phi$

C – II $\quad x < 5$

$(x + 1)^2 - (x - 5) = \frac{27}{4}$

$x^2 + x - \frac{3}{4} = 0$ (as both roots < 5)

$\Rightarrow$ 2 real roots

9. Let λ be an integer. If the shortest distance between the lines $x - \lambda = 2y - 1 = -2z$ and $x = y + 2\lambda = z - \lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is _______.

Ans. Official Answer NTA : (1)

Sol. $L_1 \; \frac{x-\lambda}{2} = \frac{y-\frac{1}{2}}{1} = \frac{z}{-1}$

$L_2 \; \frac{x}{1} = \frac{y+2\lambda}{1} = \frac{z-\lambda}{1}$

$\vec{p} = (2,1,-1), \vec{q} = (1,1,1)$

$A\left(\lambda, \frac{1}{2}, 0\right)$ $\quad\quad$ B(0, –2λ, λ)

$\vec{p} \times \vec{q} = 2\hat{i} - 3\hat{j} + \hat{k}$

$$\left|\frac{\overrightarrow{AB} \cdot (\vec{p} \times \vec{q})}{(\vec{p} \times \vec{q})}\right| = \frac{\sqrt{7}}{2\sqrt{2}}$$

$$\frac{\left|-2\lambda + 3\left(2\lambda + \frac{1}{2}\right) + \lambda\right|}{\sqrt{14}} = \frac{\sqrt{7}}{2\sqrt{2}}$$

$\Rightarrow \lambda = -1$ $\quad\quad$ or $\lambda = \frac{2}{5}$

$\Rightarrow |\lambda| = 1$ $\quad\quad$ (as $\lambda \in I$)

10. For integers n and r, let $\binom{n}{r} = \begin{matrix} {}^nC_r, \text{if } n \geq r \geq 0 \\ 0, \quad \text{otherwise} \end{matrix}$

The maximum value of k for which the sum

$\sum_{i=0}^{k} \binom{10}{i}\binom{15}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i}\binom{13}{k+1-i}$ exists, is equal to ______.

Ans. Official Answer NTA : (12)

Sol. Sum = $\sum_{i=0}^{k} {}^{10}C_i \, {}^{15}C_{k-i} + \sum_{i=0}^{k} {}^{12}C_i \, {}^{12}C_{k+1-5}$

$= {}^{25}C_k + {}^{25}C_{k+1}$

$= {}^{26}C_{k+1}$

Maximum when k + 1 = 13

$\Rightarrow k = 12$

Note : Solution is provided assuming

"sum is maximum then value of k is"

for given language, maximum value of k is not defined