JEE Main January 2023

Question Paper With Text Solution

24 January | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JANUARY 2023 | 24TH JANUARY SHIFT-2

SECTION – A

Question ID : 7155051604

1. The locus of the mid points of the chords of the circle $C_1 : (x-4)^2 + (y-5)^2 = 4$ which subtend an angle θ_i at the centre of the circle C_i, is a circle of radius r_i. If $\theta_1 = \frac{\pi}{3}$, $\theta_3 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$, the θ_2 is equal to :

वृत्त $C_1 : (x-4)^2 + (y-5)^2 = 4$ की उन जीवाओं, जो वृत्त C_i (जो r_i त्रिज्या का वृत्त है) के केन्द्र पर θ_i कोण बनाती है, के मध्य बिंदुओं का बिन्दुपथ होगा। यदि $\theta_1 = \frac{\pi}{3}$, $\theta_3 = \frac{2\pi}{3}$ और $r_1^2 = r_2^2 + r_3^2$ है, तब θ_2 बराबर है :

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{3\pi}{4}$ (4) $\frac{\pi}{2}$

Ans. Official Answer NTA (4)

Sol. If a chord of circle of radius R subtends angle θ_i at the centre then locus of the midpoint of this chord is a circle of radius $r_i = R \cdot \cos\left(\frac{\theta_i}{2}\right)$

Given

$$r_1^2 = r_2^2 + r_3^2$$

$$\Rightarrow \cos^2\frac{\theta_1}{2} = \cos^2\frac{\theta_2}{2} + \cos^2\frac{\theta_2}{2}$$

$$\Rightarrow \cos^2\frac{\pi}{6} = \cos^2\frac{\pi}{3} + \cos^2\frac{\theta_2}{2}$$

$$\Rightarrow \frac{3}{4} = \frac{1}{4} + \cos^2\frac{\theta_2}{2}$$

$$\Rightarrow \cos^2\frac{\theta_2}{2} = \frac{1}{2}$$

$$\Rightarrow \cos^{\frac{\theta_2}{2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{\theta_2}{2} = \frac{\pi}{4}$$

$$\therefore \theta_3 = \frac{\pi}{2}$$

Question ID : 7155051598

2. If $({}^{30}C_1)^2 + 2({}^{30}C_2)^2 + 3({}^{30}C_3)^2 + \ldots\ldots + 30({}^{30}C_{30})^2 = \frac{\alpha 60!}{(30!)^2}$ then α is equal to :

यदि $({}^{30}C_1)^2 + 2({}^{30}C_2)^2 + 3({}^{30}C_3)^2 + \ldots\ldots + 30({}^{30}C_{30})^2 = \frac{\alpha 60!}{(30!)^2}$ है, तो α बराबर है :

(1) 15 (2) 60 (3) 10 (4) 30

Ans. Official Answer NTA (1)

Sol. $S = 0\cdot\left({}^{30}C_0\right)^2 + 1\cdot\left({}^{30}C_1\right)^2 + 2\cdot\left({}^{30}C_2\right)^2 + \ldots\ldots + 30\cdot\left({}^{30}C_{30}\right)^2$

$S = 30\cdot\left({}^{30}C_0\right)^2 + 29\cdot\left({}^{30}C_1\right)^2 + 28\cdot\left({}^{30}C_2\right)^2 + \ldots + 0\cdot\left({}^{30}C_0\right)^2$

$2S = 30\cdot\left({}^{30}C_0{}^2 + {}^{30}C_1{}^2 + \ldots\ldots + {}^{30}C_{30}{}^2\right)$

$S = 15\cdot{}^{60}C_{30} = 15\cdot\frac{60!}{(30!)^2}$

$\frac{15\cdot 10!}{(30!)^2} = \frac{\alpha\cdot 60!}{(30!)^2}$

$\Rightarrow \alpha = 15$

Question ID : 7155051603

3. Let $y = y(x)$ be the solution of the differential equation $(x^2 - 3y^2)\,dx + 3xy\,dy = 0$, $y(1) = 1$. Then $6\,y^2(e)$ is equal to :

माना अवकल समीकरण $(x^2 - 3y^2)\,dx + 3xy\,dy = 0$, $y(1) = 1$ का हल $y = y(x)$ है। तब $6\,y^2(e)$ बराबर है :

(1) $3e^2$ (2) e^2 (3) $\frac{3}{2}e^2$ (4) $2e^2$

Ans. Official Answer NTA (4)

Sol. D.E. $\rightarrow \frac{2ydy}{dx} - \frac{2y^2}{x} = \frac{-2x}{3}$

Let $y^2 = t \Rightarrow 2y\frac{dy}{dx} = \frac{dt}{dx}$

D.E. $\rightarrow \frac{dt}{dx} - \frac{2t}{x} = -\frac{2x}{3}$

I.F. $= e^{J -\frac{2}{x}dx} = e^{-2\,en|x|} = e^{tn\frac{1}{x^2}} = \frac{1}{x^2}$

Solution $\Rightarrow t\cdot\frac{1}{x^2}=\int\frac{1}{x^2}\left(\frac{-2x}{3}\right)dx+c\Rightarrow\frac{y^2}{x^2}=\frac{-2}{3}\ell n|x|+c$

$\therefore x=1, y=1\Rightarrow 1=0+c\Rightarrow c=1$

$\Rightarrow\frac{y^2}{x^2}=-\frac{2}{3}\ell n|x|+1$

$x=e\Rightarrow\frac{y^2(e)}{e^2}=\frac{-2}{3}+1\Rightarrow y^2(e)=\frac{e^2}{3}\Rightarrow 6y^2(e)=2e^2$

Question ID : 7155051607

4. If the foot of the perpendicular drawn from (1, 9, 7) to the line passing through the point (3, 2, 1) and parallel to the planes $x + 2y + z = 0$ and $3y – z = 3$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to :

यदि बिंदु (3, 2, 1) से होकर जाने वाली तथा समतलों $x + 2y + z = 0$ और $3y – z = 3$ के समान्तर रेखा पर बिंदु (1, 9, 7) से डाले गए लम्ब का पाद (α, β, γ) है, तब $\alpha + \beta + \gamma$ बराबर है :

(1) 1 (2) 3 (3) –1 (4) 5

Ans. Official Answer NTA (4)

Sol. D.R's of line be

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 1\\ 0 & 3 & -1\end{vmatrix}$$

$$i(-2-3)-j(-1)+\hat{k}(3)$$

$$=-5i+j+3\hat{k}$$

Equation of line

$$\frac{x-3}{-5}=\frac{y-2}{1}=\frac{z-1}{3}=\lambda$$

$N(-5\lambda+3,\lambda+2,3\lambda+1)$

D.R's of AN, : $-5\lambda+2,\lambda-7,3\lambda-6$

Now, $(-5\lambda+2)(-5)+(\lambda-7)(1)+(3\lambda-6)3=0$

$25\lambda-10+\lambda-7+9\lambda-18=0$

$35\lambda=35\Rightarrow\lambda=1$

N (–2, 3, 4)

$\alpha+\beta+\gamma=-2+3+4=5$

Question ID : 7155051594

5. If the system of equations

$x + 2y + 3z = 3$
$4x + 3y - 4z = 4$
$8x + 4y - \lambda z = 9 + \mu$

has infinitely many solutions, then the ordered pair (λ, μ) is equal to :

यदि समीकरणों

$x + 2y + 3z = 3$
$4x + 3y - 4z = 4$
$8x + 4y - \lambda z = 9 + \mu$

के अनंत हल है, तब क्रमित युग्म (λ, μ) बराबर है :

(1) $\left(\frac{72}{5}, \frac{21}{5}\right)$ (2) $\left(-\frac{72}{5}, -\frac{21}{5}\right)$ (3) $\left(\frac{72}{5}, -\frac{21}{5}\right)$ (4) $\left(-\frac{72}{5}, \frac{21}{5}\right)$

Ans. Official Answer NTA (3)

Sol.

$x + 2y + 3z = 3$ ________(i)

$4x + 3y - 4z = 4$ ________(ii)

$8x + 4y - \lambda z = 9 + \mu$ ________(iii)

$(i) \times 4 - (ii) \Rightarrow 5y + 16z = 8$ ________(iv)

$(ii) \times 2 - (iii) \Rightarrow 2y + (\lambda - 8)z = -1 - \mu$ ________(v)

$(iv) \times 2 - (iii) \times 5 \Rightarrow (32 - 5(\lambda - 8))z = 16 - 5(-1 - \mu)$

For infinite solutions $\Rightarrow 72 - 5\lambda = 0 \Rightarrow \lambda = \frac{72}{5}$

$21 + 5\mu = 0 \Rightarrow \mu = \frac{-21}{5}$

$\Rightarrow (\lambda, \mu) \equiv \left(\frac{72}{5}, \frac{-21}{5}\right)$

Question ID : 7155051602

6. $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9 - 4x^2}} dx$ is equal to :

$\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9 - 4x^2}} dx$ बराबर है :

(1) $\frac{\pi}{3}$ (2) 2π (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{6}$

Ans. Official Answer NTA (2)

Sol. $\frac{48}{2}\left(\sin^{-1}\left(\frac{2x}{3}\right)\right)_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} = 24\left(\sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}\frac{1}{\sqrt{2}}\right)$

$= 24\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = 24\left(\frac{\pi}{12}\right) = 2\pi$

Question ID : 7155051597

7. The number of integers, greater than 7000 that can be formed, using the digits 3, 5, 6, 7, 8 without repetition, is :

बिना पुनरावृति के अंकों 3, 5, 6, 7, 8 के प्रयोग से बनने वाली 7000 से बड़ी पूर्ण संख्याओं की संख्या है :

(1) 220 (2) 168 (3) 48 (4) 120

Ans. Official Answer NTA (2)

Sol. Four digit numbers greater than 7000

$= 2 \times 4 \times 3 \times 2 = 48$

Five digit number = 5! = 120

Total number greater than 7000

$= 120 + 48 = 168$

Question ID : 7155051609

8. Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$ be in A.P. and $a_1 + a_3 = 10$. If the mean of these six numbers is $\frac{19}{2}$ and their variance is σ^2, then $8\sigma^2$ is equal to :

माना छः संख्यायें $a_1, a_2, a_3, a_4, a_5, a_6$ समान्तर श्रेणी में है और $a_1 + a_3 = 10$ है। यदि इन छः संख्याओं का माध्य $\frac{19}{2}$ है और प्रसरण σ^2 है, तब $8\sigma^2$ का मान है :

(1) 220 (2) 105 (3) 200 (4) 210

Ans. Official Answer NTA (4)

Sol. $\dfrac{\frac{6}{2}[2a+5d]}{6}=\dfrac{19}{2}$

2a + 5d = 19 ________(i)

$a_1 + a_3 = 10$

a + a + 2d = 10

2a + 2d = 10 ________(ii)

from (i) and (ii)

d = 3

a = 2

Numbers are 2, 5, 8, 11, 14, 17

$\sigma^2=\dfrac{2^2+5^2+8^2+11^3+14^2+17^2}{6}-\left(\dfrac{19}{2}\right)^2$

$=\dfrac{699}{6}-\left(\dfrac{19}{2}\right)^2=\dfrac{233}{2}-\dfrac{361}{4}$

$\sigma^2=\dfrac{466-361}{4}=\dfrac{105}{4}$

$\Rightarrow 8\sigma^2=210$

Question ID : 7155051599

9. Let f(x) be a function such that $f(x+y)=f(x)\cdot f(y)$ for all $x, y \in N$. If $f(1)=3$ and $\sum_{k=1}^{n} f(k)=3279$, then the value of n is :

माना एक फलन f(x) इस प्रकार है कि सभी $x, y \in N$ के लिए $f(x+y)=f(x)\cdot f(y)$ है। यदि $f(1)=3$ और $\sum_{k=1}^{n} f(k)=3279$ है, तब n का मान है :

(1) 9 (2) 8 (3) 7 (4) 6

Ans. Official Answer NTA (3)

Sol. $f(x+y)=f(x)\cdot f(y)\forall x,y\in N, f(1)=3$

$f(2)=f^2(1)=3^2$

$f(3)=f(1)f(2)=3^3$

$f(4)=3^4$

$f(k)=3^k$

$$\sum_{k=1}^{n} f(k) = 3279$$

$$f(1)+f(2)+f(3)+\ldots\ldots\ldots+f(k)=3279$$

$$3+3^2+3^3+\ldots\ldots\ldots 3^k=3279$$

$$\frac{3\left(3^k-1\right)}{3-1}=3279$$

$$\frac{3^k-1}{2}=1093$$

$$3^k-1=2186$$

$$3^k=2187$$

$$\boxed{k=7}$$

Question ID : 7155051595

10. Let A be a 3 × 3 matrix such that $|\text{adj}(\text{adj}(\text{adj } A))| = 12^4$. Then $|A^{-1}\text{ adj } A|$ is equal to :

माना A एक 3 × 3 का आव्यूह है तथा $|\text{adj}(\text{adj}(\text{adj } A))| = 12^4$ है तब $|A^{-1}\text{ adj } A|$ बराबर है :

(1) $\sqrt{6}$ (2) 1 (3) 12 (4) $2\sqrt{3}$

Ans. Official Answer NTA (4)

Sol. $|\text{Adj}(\text{Adj}(\text{Adj } A))|$

$=|\text{Adj}(\text{Adj } A)|^2$

$=|\text{Adj } A|^4$

$\Rightarrow |A|^3=12^4$

$\Rightarrow |A|=12^{1/2}$

Now, $|A^{-1}\text{ Adj } A|=|A^{-1}||\text{Adj} A|$

$$=\frac{1}{|A|^2}|A|^2=|A|=12^{1/2}=2\sqrt{3}$$

Question ID : 7155051596

11. The number of square matrices of order 5 with entries from the set {0, 1}, such that the sum of all the elements

in each row is 1 and the sum of all the elements in each column is also 1, is :

कोटि 5 के वर्ग आव्यूहों, जिनके अवयव समुच्चय $\{0, 1\}$ से है, प्रत्येक पंक्ति के सभी अवयवों का योग 1 है तथा प्रत्येक स्तम्भ के सभी अवयवों का योग भी 1 है, की संख्या है :

(1) 225 (2) 125 (3) 150 (4) 120

Ans. Official Answer NTA (4)

Sol.

$$\begin{bmatrix} \text{-------------} \\ \text{-------------} \\ \text{-------------} \\ \text{-------------} \\ \text{-------------} \end{bmatrix}$$

In each row and each column exactly one is to be placed :

$\therefore$ No. of such matrices $= 5 \times 4 \times 3 \times 2 \times 1 = 120$

Alternate :

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \to 5\text{ways} \\ \to 4\text{ways} \\ \to 3\text{ways} \\ \to 4\text{ways} \\ \to 1\text{ways} \end{matrix}$$

Step-1 : Select any 1 place for 1's in row 1. Automatically some column will get filled with 0's.

Step-2 : From next now select 1 place for 1's. Automatically some column will get filled with 0's.

$\Rightarrow$ Each time one less place will be available for putting 1's.

Repeat step-2 till last row.

Req. ways $= 5 \times 4 \times 3 \times 2 \times 1 = 120$

Question ID : 7155051600

12. The set of all values of a for which $\lim_{x \to a}([x-5]-[2x+2]) = 0$, where $[\propto]$ denotes the greatest integer less than or equal to $\propto$ is equal to :

a के सभी मानों, जिनके लिए $\lim_{x \to a}([x-5]-[2x+2]) = 0$ है, जहाँ $[\propto]$ महत्तम पूर्णांक $\leq \propto$ है, का समुच्चय है :

(1) (–7.5, –6.5) (2) (–7.5, –6.5] (3) [–7.5, –6.5] (4) [–7.5, –6.5)

Ans. Official Answer NTA (1)

Sol. $\lim_{x \to a}([x-5]-[2x+2]) = 0$

$\lim_{x \to a}([x]-5-[2x]-2) = 0$

$\lim_{x\to a}([x]-[2x])=7$

Let $a\in\left[n,n+\frac{1}{2}\right)$ then n-2 n=7

$n=-7$

Let $a\in\left[n+\frac{1}{2},n+1\right)$ then

$n-(2n+1)=7$
$-n=8$
$n=-8$

$a\in\left[-7\frac{1}{2},-7\right)$

but limit does not exists at $a=-7.5$

Hence $a\in(-7.5,-6.5)$

Question ID : 7155051608

13. Let $\vec{\alpha}=4\hat{i}+3\hat{j}+5\hat{k}$ and $\beta=\hat{i}+2\hat{j}-4\hat{k}$. Let $\vec{\beta}_1$ be parallel to $\vec{\alpha}$ and $\vec{\beta}$ be perpendicular to $\vec{\alpha}$. If $\vec{\beta}=\vec{\beta}_1+\vec{\beta}_2$, then the value of $5\vec{\beta}_2\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)$ is :

माना $\vec{\alpha}=4\hat{i}+3\hat{j}+5\hat{k}$ एवं $\beta=\hat{i}+2\hat{j}-4\hat{k}$ है। माना $\vec{\beta}_1$, $\vec{\alpha}$ के समान्तर कोण तथा $\vec{\beta}$, $\vec{\alpha}$ के लम्बवत है यदि $\vec{\beta}=\vec{\beta}_1+\vec{\beta}_2$ है, तब $5\vec{\beta}_2\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)$ का मान है :

(1) 7 (2) 6 (3) 9 (4) 11

Ans. Official Answer NTA (1)

Sol. Let $\vec{\beta}_1=\lambda\vec{\alpha}$

Now $\vec{\beta}_2=\vec{\beta}-\vec{\beta}_1$

$=(\hat{i}+2\hat{j}-4\hat{k})-\lambda(4\hat{i}+3\hat{j}+5\hat{k})$

$=(1-4\lambda)\hat{i}+(2-3\lambda)\hat{j}-(5\lambda+4)\hat{k}$

$\vec{\beta}_2\cdot\vec{\alpha}=0$

$\Rightarrow 4(1-4\lambda)+3(2-3\lambda)-5(5\lambda+4)=0$

$\Rightarrow 4-16\alpha+6-9\lambda-25\lambda-20=0$

$\Rightarrow 50\lambda=-10$

$\Rightarrow \boxed{\lambda=\frac{-1}{5}}$

$\vec{\beta}_2 = \left(1+\frac{4}{5}\right)\hat{i} + \left(2+\frac{3}{5}\right)\hat{j} - (-1+4)\hat{k}$

$\vec{\beta}_2 = \frac{9}{5}\hat{i} + \frac{13}{5}\hat{j} - 3\hat{k}$

$5\vec{\beta}_2 = 9\hat{i} + 13\hat{j} - 15\hat{k}$

$5\vec{\beta}_2 \cdot (\hat{i}+\hat{j}+\hat{k}) = 9+13-15 = 7$

Question ID : 7155051601

14. If $f(x) = x^3 - x^2 f'(1) + xf''(2) - f'''(3)$, $x \in R$, then :

यदि $f(x) = x^3 - x^2 f'(1) + xf''(2) - f'''(3)$, $x \in R$ है, तब :

(1) $2f(0) - f(1) + f(3) = f(2)$
(2) $f(1) + f(2) + f(3) = f(0)$
(3) $3f(1) + f(2) = f(3)$
(4) $f(3) - f(2) = f(1)$

Ans. Official Answer NTA (1)

Sol. $f(x) = x^3 - x^2 f'(1) + xf''(2) - f'''(3)$

$f'(x) = 3x^2 - 2xf'(1) + f''(2)$

$f''(x) = 6x - 2f'(1)$

$f'''(x) = 6 \Rightarrow f'''(3) = 6$

from (3) $\rightarrow f''(2) = 12 - 2f'(1)$

from(2) $\rightarrow f'(1) = 3(1)^2 - 2f'(1) + f''(2)$

$\Rightarrow f''(2) = 3f'(1) - 3$

$\Rightarrow 12 - 2f'(1) = 3f'(1) - 3 \Rightarrow f'(1) = 3$

$f''(2) = 12 - 6 = 6$

$f(x) = x^3 - 3x^2 + 6x - 6, \quad f(0) = -6$

$f(1) = -2, \qquad f(2) = 2, \qquad f(3) = 12$

Question ID : 7155051591

15. If $f(x) = \frac{2^{2x}}{2^{2x}+2}$, $x \in R$, then $f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + ... + f\left(\frac{2022}{2023}\right)$ is equal to :

यदि $f(x)=\frac{2^{2x}}{2^{2x}+2}$, $x \in R$ है, तब $f\left(\frac{1}{2023}\right)+f\left(\frac{2}{2023}\right)+......+f\left(\frac{2022}{2023}\right)$ बराबर है :

(1) 2010 (2) 1010 (3) 2011 (4) 1011

Ans. Official Answer NTA (4)

Sol. $f(x)=\frac{4^x}{4^x+2}$

$f(x)+f(1-x)=\frac{4^x}{4^x+2}+\frac{4^{1-x}}{4^{1-x}+2}$

$=\frac{4^x}{4^x+2}+\frac{4}{4+2\left(4^x\right)}$

$=\frac{4^x}{4^x+2}+\frac{2}{2+4^x}$

$=1$

$\Rightarrow f(x)+f(1-x)=1$

Now $f\left(\frac{1}{2023}\right)+f\left(\frac{2}{2023}\right)+f\left(\frac{3}{2023}\right)+........+f\left(1-\frac{3}{2023}\right)+f\left(1-\frac{2}{2023}\right)+f\left(1-\frac{1}{2023}\right)$

Now sum of terms equidistant from beginning and end is 1

Sum = 1 + 1 + 1 + ...+ 1 (1011 times)

= 1011

Question ID : 7155051593

16. The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is :

$\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ का मान है :

(1) $-\frac{1}{2}\left(\sqrt{3}-i\right)$ (2) $-\frac{1}{2}\left(1-i\sqrt{3}\right)$ (3) $\frac{1}{2}\left(1-i\sqrt{3}\right)$ (4) $\frac{1}{2}\left(\sqrt{3}+i\right)$

Ans. Official Answer NTA (1)

Sol. Let $\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}=z$

$$\left(\frac{1+z}{1+\bar{z}}\right)^3 = \left(\frac{1+z}{1+\frac{1}{z}}\right)^3 = z^3$$

$$\Rightarrow \left(i\left(\cos\frac{2\pi}{9} - i\sin\frac{2\pi}{9}\right)\right)^3$$

$$= -i\left(\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}\right) = -i\left(\frac{-1}{2} - i\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \frac{-1}{2}(\sqrt{3} - i)$$

Question ID : 7155051592

17. The number of real solutions of the equation $3\left(x^2+\frac{1}{x^2}\right)-2\left(x+\frac{1}{x}\right)+5=0$, is :

समीकरण $3\left(x^2+\frac{1}{x^2}\right)-2\left(x+\frac{1}{x}\right)+5=0$, के वास्तविक हलों की संख्या है :

(1) 4 (2) 0 (3) 2 (4) 3

Ans. Official Answer NTA (2)

Sol. $3\left(x^2+\frac{1}{x^2}\right)-2\left(x+\frac{1}{x}\right)+5=0$

$$3\left[\left(x+\frac{1}{x}\right)^2-2\right]-2\left(x+\frac{1}{x}\right)+5=0$$

Let $x+\frac{1}{x}=t; t \in (-\infty-2]\cup[2,\infty)$

$3t^2-6-2t+5=0$

$3t^2-2t-1=0$

$t=1, t=\frac{-1}{3}$ But $t \in (-\infty,-2]\cup[2,\infty)$

$\Rightarrow$ No. real solution

Question ID : 7155051605

18. The equations of the sides AB and AC of a triangle ABC are $(\lambda + 1)x + \lambda y = 4$ and $\lambda x + (1 - \lambda)y + \lambda = 0$ respectively. Its vertex A is on the y-axis and its orthocentre is (1, 2). The length of the tangent from the point C to the part of the parabola $y^2 = 6x$ in the first quadrant is :

एक त्रिभुज ABC की भुजाओं AB और AC के समीकरण क्रमशः $(\lambda + 1)x + \lambda y = 4$ और $\lambda x + (1 - \lambda)y + \lambda = 0$ है। इसका शीर्ष A, y-अक्ष पर है और इसका लंबकेन्द्र (1, 2) है। बिंदु C से परवलय $y^2 = 6x$ के प्रथम चतुर्थांश के भाग पर खींची गई स्पर्श रेखा की लंबाई है :

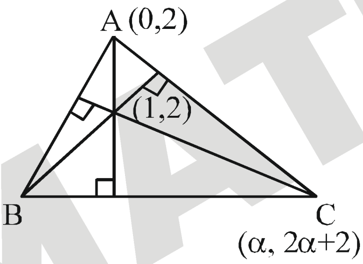
(1) $2\sqrt{2}$ (2) 2 (3) 4 (4) $\sqrt{6}$

Ans. Official Answer NTA (1)

Sol. $AB : (\lambda + 1)x + \lambda y = 4$

$AC : \lambda x + (1 - \lambda)y + \lambda = 0$

Vertex A is on y-axis



$\Rightarrow x = 0$

$y = \frac{4}{\lambda}, y = \frac{\lambda}{\lambda - 1}$

$\Rightarrow \frac{4}{\lambda} = \frac{\lambda}{\lambda - 1}$

$\Rightarrow \lambda = 2$

$AB : 3x + 2y = 4$

$AC : 2x - y + 2 = 0$

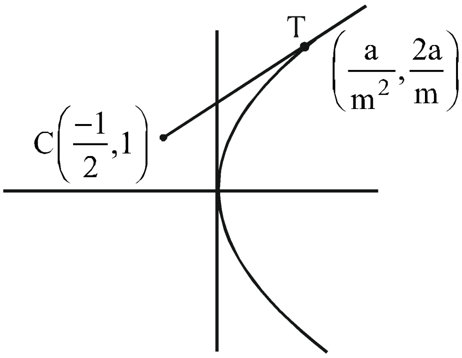
$\Rightarrow A(0, 2)$

Let $C(\alpha, 2\alpha + 2)$

Now (Slope of Altitude through C) $\left(-\frac{3}{2}\right) = -1$

$\left(\frac{2\alpha}{\alpha - 1}\right)\left(-\frac{3}{2}\right) = -1 \Rightarrow \alpha = -\frac{1}{2}$

So $C\left(-\frac{1}{2}, 1\right)$



Let Equation of tangent be $y = mx + \frac{3}{2m}$

$m^2 + 2m - 3 = 0$

$\Rightarrow m = 1, -3$

So tangent which touches in first quadrant at T is

$T \equiv \left(\frac{a}{m^2}, \frac{2a}{m}\right)$

$\equiv \left(\frac{3}{2}, 3\right)$

$\Rightarrow CT = \sqrt{4+4} = 2\sqrt{2}$

Question ID : 7155051606

19. Let the plane containing the line of intersection of the planes $P_1 : x + (\lambda + 4)y + z = 1$ and $P_2 : 2x + y + z = 2$ pass through the points (0, 1, 0) and (1, 0, 1). Then the distance of the point $(2\lambda, \lambda, -\lambda)$ from the plane P2 is :

माना समतल, जिसमें समतलों $P_1 : x + (\lambda + 4)y + z = 1$ तथा $P_2 : 2x + y + z = 2$ की प्रतिच्छेदन रेखा स्थित है, बिंदुओं (0, 1, 0) तथा (1, 0, 1) से होकर जाता है। तो बिंदु $(2\lambda, \lambda, -\lambda)$ की समतल P2 से दूरी है :

(1) $5\sqrt{6}$ (2) $2\sqrt{6}$ (3) $4\sqrt{6}$ (4) $3\sqrt{6}$

Ans. Official Answer NTA (4)

Sol. Eq. of plane containing the line of intersection of planes is given by

$p_1 + k\,p_2 = 0$

$X + (\lambda + 4)\,y + z - 1 + k\,(2x + y + z - 2) = 0$

It passes through (0, 1, 0) & (1, 0, 1)

$\lambda + 3 + k\,(1 - 2) = 0 \Rightarrow \lambda - k = -3$ ....(1)

And $(1 + 1 - 1) + k\,(2 + 1 - 2) = 0 \Rightarrow 1 + k = 0$

$\Rightarrow k = -1, \lambda = -4$

Now given point (–8, –4, 4)

Its distance from plane $p_2 = \left|\frac{-16-4+4-2}{\sqrt{6}}\right| = 3\sqrt{6}$

Question ID : 7155051610

20. Let p and q be two statements. Then $\sim(p \wedge (p \Rightarrow \sim q))$ is equivalent to :

माना p व q दो कथन है। तब $\sim(p \wedge (p \Rightarrow \sim q))$ के तुल्य कथन है :

(1) $p \vee (p \wedge (\sim q))$ (2) $p \vee (p \wedge q)$ (3) $p \vee ((\sim p) \wedge q)$ (4) $(\sim p) \vee q$

Ans. Official Answer NTA (4)

Sol. $\sim (p \wedge (p \rightarrow \sim q))$

$\equiv \sim p \vee \sim (\sim p \vee \sim q)$

$\equiv \sim p \vee (p \wedge q)$

$\equiv (\sim p \vee p) \wedge (\sim p \vee q)$

$\equiv t \wedge (\sim p \vee q)$

$\equiv \sim p \vee q$

SECTION – B

Question ID : 7155051615

21. Let f be a differentiable function defined on $\left[0, \frac{\pi}{2}\right]$. such that f(x) > 0 and $f(x) + \int_0^x f(t)\sqrt{1-(\log_e f(t))^2}\,dt = e$, $\forall x \in \left[0, \frac{\pi}{2}\right]$. Then $\left(6\log_e f\left(\frac{\pi}{6}\right)\right)^2$ is equal to ________.

माना $\left[0, \frac{\pi}{2}\right]$ पर परिभाषित एक अवकलनीय फलन f है जिसके लिए f(x) > 0 और $f(x) + \int_0^x f(t)\sqrt{1-(\log_e f(t))^2}\,dt = e$, $\forall x \in \left[0, \frac{\pi}{2}\right]$ है। तब $\left(6\log_e f\left(\frac{\pi}{6}\right)\right)^2$ बराबर है ________

Ans. Official Answer NTA (27)

Sol. $t(x) = \int_0^x f(t)\sqrt{1-\ell nt(x)^2}\,dt = e : \nabla x \in \left[0, \frac{\pi}{2}\right]$

Differentiate on both side

$f\cdot(x)+f(x)\sqrt{1-knf\left(xm^2\right.}=0$

$\int\frac{f(x)dx}{f(x)\sqrt{1-tnf(x)m^2}}=-\int dx$

Let $(n+\alpha)=U$

$\frac{1}{f(x)}\cdot f'(x)dx=du$

$\int\frac{du}{\sqrt{1-u^2}}=-\int dx$

$\sin^{-1}\left(\ell n\left(f\left(x\right)\right)=-x+c\right.$

put $x=0$

$\sin^{-1}\left(\ell n\left(f\left(0\right)\right)=0+c\right.$

$\sin^{-1}(1)=c$

$c=\frac{\pi}{2}$

$\sin^{-1}\left(\ell n\left(f\left(x\right)\right)=-x+\frac{\pi}{2}\right.$

$\ell n\left(f\left(x\right)\right)=\sin\left(\frac{\pi}{2}-x\right)=\cos x$

$f\left(x\right)=e^{\cos x}$

$f\left(\frac{\pi}{6}\right)=e^{\frac{\sqrt{3}}{2}}$

Now $\left(6\ell ne^{\frac{\sqrt{3}}{2}}\right)^2=\left(6\times\frac{\sqrt{3}}{2}\right)^2=27$

Question ID : 7155051620

22. If the shortest distance between the lines $\frac{x+\sqrt{6}}{2}=\frac{y-\sqrt{6}}{3}=\frac{z-\sqrt{6}}{4}$ and $\frac{x-\lambda}{3}=\frac{y-2\sqrt{6}}{4}=\frac{z+2\sqrt{6}}{5}$ is 6, then the square of sum of all possible values of λ is ________.

यदि $\frac{x+\sqrt{6}}{2}=\frac{y-\sqrt{6}}{3}=\frac{z-\sqrt{6}}{4}$ और $\frac{x-\lambda}{3}=\frac{y-2\sqrt{6}}{4}=\frac{z+2\sqrt{6}}{5}$ के बीच न्यूनतम दूरी 6 है, तब λ के सभी संभव मानों का योग का वर्ग है _________

Ans. Official Answer NTA (384)

Sol. In vector form

$$\vec{r}=\left(-\sqrt{6}\hat{i}+\sqrt{6}\hat{j}+\sqrt{6}\hat{k}\right)+A\left(2\hat{i}+3\hat{j}+4\hat{k}\right)$$

$$\vec{r}=\left(\lambda\hat{i}+2\sqrt{6}\hat{j}-2\sqrt{6}\hat{k}\right)+B\left(3\hat{i}+4\hat{j}+5\hat{k}\right)$$

$$\overrightarrow{a_2}-\overrightarrow{a_1}=\left(\lambda+\sqrt{6}\right)\hat{i}+\sqrt{6}\hat{j}-3\sqrt{6}\hat{k}$$

$$\vec{b}_1\times\vec{b}_2=-\hat{i}+2\hat{j}-\hat{k}$$

$$\left|\vec{b}_1\times\vec{b}_2\right|=\sqrt{6}$$

$$\left(\vec{a}_2-\vec{a}_1\right)\left(\vec{b}_1\times\vec{b}_2\right)=\left(4\sqrt{6}-\lambda\right)$$

$$\text{S.D.}=\left|\frac{\left(\vec{a}_2-\vec{a}_1\right)\left(\vec{b}_1\times\vec{b}_2\right)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right|=6$$

$$\frac{4\sqrt{6}-\lambda}{\sqrt{6}}=\pm 6$$

$$\lambda_1=-2\sqrt{6},\lambda_2=10\sqrt{6}$$

$$\lambda_1+\lambda_2=8\sqrt{6}$$

$$\left(\lambda_1+\lambda_2\right)^2=384$$

$= 384$

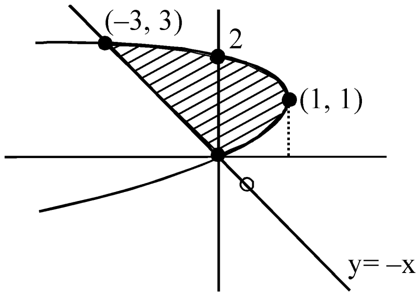
Question ID : 7155051614

23. If the area of the region bounded by the curves $y^2-2y=-x, x+y=0$ is A, then 8 A is equal to _________.

यदि वक्रों $y^2-2y=-x, x+y=0$ से घिरे क्षेत्र का क्षेत्रफल A है, तो 8 A बराबर है _________

Ans. Official Answer NTA (36)

Sol.



$$A = \int_0^3 \left[\left(2y - y^2\right) - (-y)\right] dy = \int_0^3 \left(3y - y^2\right) dy = \left[\frac{3y^2}{2} - \frac{y^3}{3}\right]_0^3$$

$$= \frac{3\times 9}{2} - \frac{27}{3} = \frac{9}{2} \qquad \text{So, } 8A = 36$$

Question ID : 7155051611

24. The minimum number of elements that must be added to the relation R = {(a, b), (b, c), (b, d) on the set{a, b, c, d} so that it is an equivalence relation, is ________.

समुच्चय {a, b, c, d} पर संबंध R = {(a, b), (b, c), (b, d) में न्यूनतम कितने अवयव जोड़े जाए जिससे कि यह एक तुल्यता संबंध हो ________

Ans. Official Answer NTA (13)

Sol. Given R = {(a, b), (b, c), (b, d)}
In order to make it equivalence relation as per given set, R must be
{(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (b, c), (c, b),
(b, d), (d, b), (a, c), (a, d), (c, d), (d, c), (c, a), (d, a)}
There already given so 13 more to be added.

Question ID : 7155051618

25. Let $S = \{\theta \in [0, 2\pi) : \tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0\}$. Then $\sum_{\theta\in S} \sin^2\left(\theta + \frac{\pi}{4}\right)$ is equal to ________.

माना $S = \{\theta \in [0, 2\pi) : \tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0\}$ है। तब $\sum_{\theta\in S} \sin^2\left(\theta + \frac{\pi}{4}\right)$ बराबर है ________

Ans. Official Answer NTA (2)

Sol. $\tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0$

$\tan(\pi\cos\theta) = -\tan(\pi\sin\theta)$

$\tan(\pi\cos\theta) = \tan(-\pi\sin\theta)$

$\pi\cos\theta = n\pi - \pi\sin\theta, n \in I$

$\sin\theta + \cos\theta = n$

$\therefore \sin\theta + \cos\theta \in \left[-\sqrt{2}, \sqrt{2}\right]$

Then possible value of $n = -1, 0, 1$

Now $\sin\theta + \cos\theta = 1$

$\theta = 0, \frac{\pi}{2},$

$$\left|\begin{array}{c} \sin\theta + \cos\theta = 0 \\ \tan\theta - 1 \\ \theta = \frac{3\pi}{4}, \frac{7\pi}{4}, \end{array}\right|$$

$\sin\theta + \cos\theta = -1$

$\theta = \frac{3\pi}{2}, \pi$

So, $\sum_{\theta\in S} \sin^2\left(\theta + \frac{\pi}{4}\right) = 4\left(\frac{1}{2}\right) + 2(0) = 2$

Question ID : 7155051613

26. If $\frac{1^3 + 2^3 + 3^3 + \text{....up to n term}}{1.3 + 2.5 + 3.7 + \text{....up to n term}} = \frac{9}{5}$, then the value of n is _________.

यदि $\frac{1^3 + 2^3 + 3^3 + \text{.... n पदों तक}}{1.3 + 2.5 + 3.7 + \text{.... n पदों तक}} = \frac{9}{5}$, है तो n का मान है।

Ans. Official Answer NTA (5)

Sol. $\frac{1^3 + 2^3 + 3^3 + ...upto\ n\ terms}{1\cdot3 + 2\cdot5 + 3\cdot7 + ...upto\ n\ terms}$

$$\Rightarrow \frac{\frac{n^2(n+1)^2}{4}}{\Sigma n(2n+1)} = \frac{9}{5}$$

$$\frac{\frac{n^2(n+1)^2}{4}}{2\cdot\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}}=\frac{9}{5}$$

$$\frac{\frac{n(n+1)}{4}\cdot 6}{2(2n+1)+3}=\frac{9}{5}$$

$5n^2 - 19n - 30 = 0$

$n = 5, -\frac{6}{5}$

$n = 5$

Question ID : 7155051619

27. The equations of the sides AB, BC and CA of a triangle ABC are : $2x + y = 0$, $x + py = 21a$, $(a \neq 0)$ and $x - y = 3$ respectively. Let P(2, a) be the centroid of ΔABC. Then $(BC)^2$ is equal to ________.

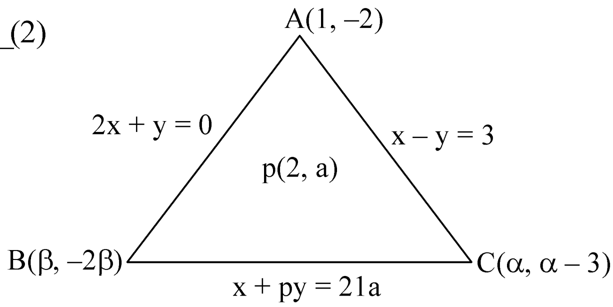
एक त्रिभुज ABC की भुजाओं AB, BC तथा CA के समीकरण क्रमशः $2x + y = 0$, $x + py = 21a$, $(a \neq 0)$ तथा $x - y = 3$ है। माना ΔABC का केन्द्रक P(2, a) है। तब $(BC)^2$ बराबर है।

Ans. Official Answer NTA (122)

Sol. Now $\frac{1+\alpha+\beta}{3}=2$

$\alpha+\beta=5\ldots(1)$

and $\frac{-2-2\beta+\alpha-3}{3}=a \Rightarrow \alpha-2\beta=3a+5$ ______(2)



from (1) and (2) $\alpha = a+5, \beta = -a$

$B(-a, 2a)$, $C(a + 5, a + 2)$

B and C lies on $x + py = 21a$

$-a + p(2a) = 21a \Rightarrow p = 11$

and $a + 5 + 11(a + 2) = 21a$

$\Rightarrow a = 3$

$B(-3, 6)$, $C(8, 5)$, $BC^2 = 122$

Question ID : 7155051616

28. Let $\vec{a}=\hat{i}+2\hat{j}+\lambda\hat{k}$, $\vec{b}=3\hat{i}-5\hat{j}-\lambda\hat{k}$, $\vec{a}\cdot\vec{c}=7, 2\vec{b}\cdot\vec{c}+43=0$, $\vec{a}\times\vec{c}=\vec{b}\times\vec{c}$. Then $|\vec{a}\cdot\vec{b}|$ is equal to ________.

माना $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}$, $\vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}$, $\vec{a}\cdot\vec{c} = 7, 2\vec{b}\cdot\vec{c} + 43 = 0$, $\vec{a}\times\vec{c} = \vec{b}\times\vec{c}$ है। तब $\left|\vec{a}\cdot\vec{b}\right|$ बराबर है।

Ans. Official Answer NTA (8)

Sol. $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}, \quad \vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}, \vec{a}\cdot\vec{c} = 7$

$\vec{a}\times\vec{c} - \vec{b}\times\vec{c} = \vec{0}$

$(\vec{a} - \vec{b})\times\vec{c} = 0 \Rightarrow (\vec{a} - \vec{b})$ is paralleled to $\vec{c}$

$\vec{a} - \vec{b} = \mu\vec{c}$, where μ is a scalar

$-2\hat{i} + 7\hat{j} + 2\lambda\hat{k} = \mu\cdot\vec{c}$

Now $\vec{a}\cdot\vec{c} = 7$ gives $2\lambda^2 + 12 = 7\mu$

And $\vec{b}\cdot\vec{c} = -\dfrac{43}{2}$ gives $4\lambda^2 + 82 = 43\mu$

$\mu = 2$ and $\lambda^2 = 1$

$|\vec{a}\cdot\vec{b}| = 8$

Question ID : 7155051612

29. Let the sum of the coefficients of the first three terms in the expansion of $\left(x - \dfrac{3}{x^2}\right)^n$, $x \neq 0, n \in N$, be 376.

Then the coefficients of x^4 is ________.

माना $\left(x - \dfrac{3}{x^2}\right)^n$, $x \neq 0, n \in N$ के प्रसार में प्रथम तीन पदों के गुणांकों का योग 376 है। तो x^4 का गुणांक ________ है।

Ans. Official Answer NTA (405)

Sol. ${}^nC_0 - {}^nC_1\cdot 3 + {}^nC_2\cdot 3^2 = 376$

$\Rightarrow n = 10$

Now $T_{r+1} = {}^{10}C_r\cdot\left(\dfrac{-3}{x^2}\right)^r\cdot(x)^{10-r}$

$= {}^{10}C_r\cdot(-3)^r\cdot x^{10-3r}$

For $x^4 \Rightarrow 10 - 3r = 4 \Rightarrow r = 2$

Coff. of $x^4 = {}^{10}C_2\cdot(-3)^2 = 405$

Question ID : 7155051617

30. Three urns A, B and C contain 4 red, 6 black; 5 red, 5 black; and and λ red, 4 black balls respectively. One of the urns is selected at random and a ball is drawn. If the ball drawn is red and the probability that it is drawn from urn C is 0.4 then the square of the length of the side of the largest equilateral triangle, inscribed in the parabola $y^2 = \lambda x$ with one vertex at the vertex of the parabola, is ________.

तीन कलश A, B एवं C में क्रमशः 4 लाल, 6 काली; 5 लाल, 5 काली एवं λ लाल, 4 काली गेंद है। एक कलश यादृच्छया चुना जाता है तथा इसमें से एक गेंद निकाली जाती है। यदि निकाली गई गेंद लाल है तथा इसके कलश C से निकाले जाने की प्रायिकता 0.4 है, तो परवलय $y^2 = \lambda x$ के अंतर्गत सबसे बड़े समबाहु त्रिभुज, जिसका एक शीर्ष परवलय की शीर्ष पर है, की भुजा की लंबाई का वर्ग है।

Ans. Official Answer NTA (432)

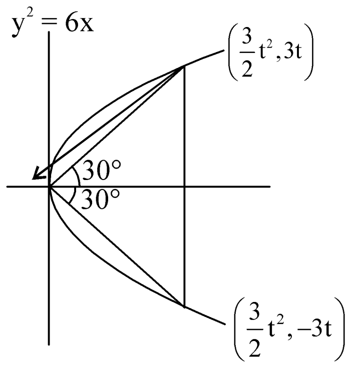
Sol.

Urn	A	Urn	B	Urn	C
Red	Black	Red	Black	Red	Black
4	6	5	5	λ	4

$$P\left(\frac{C}{R}\right) = \frac{P(C)P\left(\frac{R}{C}\right)}{P(A)P\left(\frac{R}{A}\right)+P(B)P\left(\frac{R}{B}\right)+P(C)P\left(\frac{R}{C}\right)}$$

$$0.4 = \frac{\frac{1}{3}\times\frac{\lambda}{(\lambda+4)}}{\frac{1}{3}\times\frac{4}{10}+\frac{1}{3}\times\frac{5}{10}+\frac{1}{3}\frac{\lambda}{(\lambda+4)}}$$

$$\Rightarrow \lambda = 6$$



$$\tan 30° = 3t = \frac{3}{2}t^2$$

$$\frac{1}{\sqrt{3}} = \frac{2}{t}$$

$$t = 2\sqrt{3}$$

$$\left(\frac{3}{2}t^2, 3t\right) = \left(18, 6\sqrt{3}\right)$$

$$\ell^2 = 18^2 + \left(6\sqrt{3}\right)^2$$

$= 324 + 108$

$= 432$