JEE Main January 2023

Question Paper With Text Solution

24 January | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911
Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in

JEE MAIN JANUARY 2023 | 24TH JANUARY SHIFT-1

SECTION – A

Question ID : 7155051507

1. $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right)+\sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$ is equal to :

$\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right)+\sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$ बराबर है :

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{3}$

Ans. Official Answer NTA (4)

Sol. $E=\tan^{-1}\left(\frac{1+\sqrt{3}}{\sqrt{3}(\sqrt{3}+1)}\right)+\sec^{-1}\left(\frac{16+8\sqrt{3}}{12+6\sqrt{3}}\right)^{1/2}$

$\because \frac{16+8\sqrt{3}}{12+6\sqrt{3}}=\frac{4(\sqrt{3}+1)^2}{(3+\sqrt{3})^2}$

$\therefore \left(\frac{16+8\sqrt{3}}{12+6\sqrt{3}}\right)^{1/2}=\frac{2(\sqrt{3}+1)}{3+\sqrt{3}}=\frac{2}{\sqrt{3}}$

$E=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)+\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)+\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$E=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)+\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$=\frac{\pi}{6}+\frac{\pi}{6}=\frac{\pi}{3}$

Question ID : 7155051505

2. If A and B are two non-zero n × n matrics such that $A^2+B=A^2B$, then :

यदि A तथा B, n × n के दो शून्येत्तर आव्यूह इस प्रकार हैं कि $A^2+B=A^2B$ है, तो :

(1) $A^2B=BA^2$ (2) $A^2=I$ or $B=I$ (3) $AB=I$ (4) $A^2B=I$

Ans. Official Answer NTA (1)

Sol. $A^2 + B = A^2 B$

$(A^2 - I)(B - I)$ .....(1)

$A^2 + B = A^2 B$

$A^2 (B - I) = B$

$A^2 B (B - I)^{-1}$

$A^2 = B (A^2 - I)$

$A^2 = BA^2 - B$

$A^2 + B = BA^2$

$A^2B = BA|^2$

Question ID : 7155051515

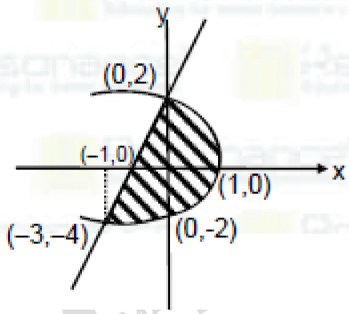
3. The area enclosed by the curves $y^2 + 4x = 4$ and $y - 2x = 2$ is :

वक्रो $y^2 + 4x = 4$ तथा $y - 2x = 2$ से घिरे क्षेत्र का क्षेत्रफल है :

(1) $\frac{25}{3}$ (2) 9 (3) $\frac{23}{3}$ (4) $\frac{22}{3}$

Ans. Official Answer NTA (2)

Sol.



Required area $= \int_{-3}^{0} \sqrt{4-4x}dx - \frac{1}{2}(2)(4) + \frac{1}{2}(1)(2) + 2\int_{0}^{1} \sqrt{4-4x}dx$

$= 2\int_{-3}^{0} \sqrt{1-x}dx + 4\int_{0}^{1} \sqrt{1-x}dx - 3$

$= -2 \times \frac{2}{3}\left[(1-x)^{3/2}\right]_{-3}^{0} - 4\frac{2}{3}\left[(1-x)^{3/2}\right] - 3$

$= -\frac{4}{3}(1-8) - \frac{8}{3}(0-1) - 3$

$= \frac{28}{3} + \frac{8}{3} - 3 = 9$

Question ID : 7155051503

4. Let $p, q \in R$ and $\left(1-\sqrt{3}i\right)^{200} = 2^{199}\left(p+iq\right)$, $i = \sqrt{-1}$. Then $p+q+q^2$ and $p-q+q^2$ are roots of the equation :

माना $p, q \in R$ तथा $\left(1-\sqrt{3}i\right)^{200} = 2^{199}\left(p+iq\right)$, $i = \sqrt{-1}$ तो $p+q+q^2$ तथा $p-q+q^2$ किस समीकरण के मूल है :

(1) $x^2-4x-1=0$ (2) $x^2-4x+1=0$ (3) $x^2+4x+1=0$ (4) $x^2+4x-1=0$

Ans. Official Answer NTA (2)

Sol. $(1+\sqrt{3}i)^{200} = 2^{199}(p+iq)$

$$\Rightarrow 2^{200}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^{200} = 2^{199}(p+iq)$$

$$\Rightarrow 2^{200}\left(\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)\right)^{200} = 2^{199}(p+iq)$$

$$\Rightarrow 2^{200}\left(e^{i\frac{\pi}{2}}\right)^{200} = 2^{199}(p+iq)$$

$$\Rightarrow 2^{200}\left(\cos\left(\frac{200\pi}{3}\right)+i\sin\left(\frac{200\pi}{3}\right)\right) = 2^{199}(p+iq)$$

$$\Rightarrow 2^{200}\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right) = 2^{199}(p+iq)$$

$$\Rightarrow 2^{199}(-1+\sqrt{3}i) = 2^{199}(p+iq)$$

$$\Rightarrow p=-1, q=\sqrt{3}$$

$$p+q+q^2 = 2+\sqrt{3}$$

$$p-q+q^2 = 2-\sqrt{3}$$

Question ID : 7155051511

5. The distance of the point(7, –3, –4) from the plane passing through the points (2, –3, 1), (–1, 1, –2) and (3, –4, 2) is :

बिंदु (7, –3, –4) की बिंदुओं (2, –3, 1), (–1, 1, –2) और (3, –4, 2) से होकर जाने वाले समतल से दूरी है :

(1) 5 (2) $5\sqrt{2}$ (3) 4 (4) $4\sqrt{2}$

Ans. Official Answer NTA (2)

Sol. Equation of Plane is

$$= \begin{vmatrix} x-2 & y+3 & z-1 \\ -3 & 4 & -3 \\ 4 & -5 & 4 \end{vmatrix}$$

$x - z - 1 = 0$

Distance of P(7, –3, –4) from Plane is

$$d = \left|\frac{7+4-1}{\sqrt{2}}\right| = 5\sqrt{2}$$

Question ID : 7155051510

6. Let a tangent to the curve $y^2 = 24x$ meet the curve $xy = 2$ at the points A and B. Then the mid points of such line segments AB lie on a parabola with the :

(1) length of latus rectum $\frac{3}{2}$ (2) directrix $4x = -3$

(3) directrix $4x = 3$ (4) length of latus rectum 2

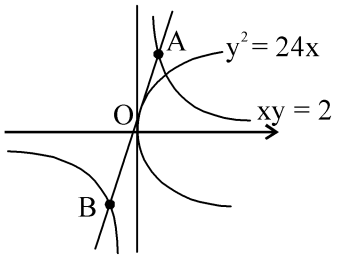
माना वक्र $y^2 = 24x$ की एक स्पर्श रेखा, वक्र $xy = 2$ को बिंदुओं A तथा B पर मिलती है। तो ऐसे रेखाखंडों AB का मध्यबिंदु एक परवलय पर है, जिसकी

(1) नाभिलंब जीवा की लंबाई $\frac{3}{2}$ है (2) नियता $4x = -3$ है

(3) नियता $4x = 3$ है (4) नाभिलंब जीवा की लंबाई 2 है

Ans. Official Answer NTA (3)

Sol.



$y^2 = 24x$ ..... (1)

$xy = 2$ ..... (2)

let mid-point of chord AB of $xy = 2$ be m (x_1, y_1)

$\therefore$ equation of AB be $T = S_1$

$$\frac{x(y_1)+y(x_1)}{2}-2=x_1y_1-2$$

$$\Rightarrow x(y_1)+y(x_1)=2x_1y_1 \Rightarrow y(x_1)=-x(y_1)+2x_1y_1$$

$$\Rightarrow y=x\left(\frac{-y_1}{x_1}\right)+2y_1 \quad .....(3)$$

$\because$ is tanget to (1)

$$\therefore c=\frac{a}{m} \Rightarrow 2y_1=-\frac{6x_1}{y_1}$$

$$\therefore y_1^2=-3x$$

$\therefore$ locus of mid-point m (x_1, y_1) is

$y^2=-3x$

length of latus rectum = 3

Directrix $\Rightarrow x=a$

$$x=\frac{3}{4} \Rightarrow x=a$$

Question ID : 7155051509

7. The value of $\sum_{r=0}^{22} {}^{22}C_r\,{}^{23}C_r$ is :

$\sum_{r=0}^{22} {}^{22}C_r\,{}^{23}C_r$ का मान है :

(1) ${}^{44}C_{22}$ (2) ${}^{45}C_{24}$ (3) ${}^{45}C_{23}$ (4) ${}^{44}C_{23}$

Ans. Official Answer NTA (3)

Sol. $$\sum_{r=0}^{22} {}^{22}C_r\,{}^{23}\sum_{r=0}^{22} {}^{22}C_r\,{}^{23}C_r=\sum_{r=0}^{22} {}^{22}C_r\,{}^{23}\sum_{r=0}^{22} {}^{22}C_r\,{}^{23}C_{23-r}$$

$$={}^{22}C_0\,{}^{23}C_{23}+{}^{22}C_1\,{}^{23}C_{22}+\cdots+{}^{22}C_{21}\,{}^{23}C_2+{}^{22}C_{22}\,{}^{23}C_1$$

$$(1+x)^{22}={}^{22}C_0+{}^{22}C_1x+\cdots+{}^{22}C_{21}x^{21}+{}^{22}C_{22}x^{22}$$

$$(1+x)^{23}={}^{23}C_0+{}^{23}C_1x+\cdots+{}^{23}C_{22}x^{22}+{}^{23}C_{23}x^{23}$$

coefficient of x^{23} in $(1+x)^{22}(1+x)^{23}=\sum_{r=0}^{22} {}^{22}C_r\,{}^{23}\sum_{r=0}^{22} {}^{22}C_r\,{}^{23}C_{23-r}$

$={}^{45}C_{23}$

Question ID : 7155051504

8. Let N denote the number that turns up when a fair die is rolled. If the probability that the system of equations

$x + y + z = 1$
$2x + Ny + 2z = 2$
$3x + 3y + Nz = 3$

has unique solution is $\frac{k}{6}$, then the sum of value of k and all possible values of N is :

माना एक न्याय पासे को फेंकने पर प्राप्त संख्या N है यदि समीकरण निकाय
$x + y + z = 1$
$2x + Ny + 2z = 2$
$3x + 3y + Nz = 3$

के अद्वितीय हल होने की प्रायिकता $\frac{k}{6}$ है, तो k तथा N के सभी संभव मानों का योग है :

(1) 21 (2) 18 (3) 20 (4) 19

Ans. Official Answer NTA (3)

Sol. $x + y + z = 1$
$2x + Ny + 2z = 2$
$3x + 3y + Nz = 3$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & N & 2 \\ 3 & 3 & N \end{vmatrix}$$

$= (N - 2)(N - 3)$
For unique solution $\Delta \neq 0$
So $N \neq 2, 3$

$\Rightarrow$ P (system has unique solution) $= \frac{4}{6}$

So k = 4
Therefore sum = 4 + 1 + 4 + 5 + 6 = 20

Question ID : 7155051502

9. The relation $R = \{(a, b): \gcd(a, b) = 1, 2a \neq b, a, b \in Z\}$ is :

(1) symmetric but not transitive (2) transitive but not reflexive

(3) reflexive but not symmetric (4) neither symmetric nor transitive

संबंध $R = \{(a, b): \gcd(a, b) = 1, 2a \neq b, a, b \in Z\}$:

(1) सममित है परन्तु संक्रामक नहीं है (2) संक्रामक है परन्तु स्वतुल्य नहीं है

(3) स्वतुल्य है परन्तु सममित नहीं है (4) न तो सममित है न ही संक्रामक है

Ans. Official Answer NTA (4)

Sol. (1) Reflexive

let $a \in z$

$aRa \Rightarrow \gcd(a, a) = a$

$\Rightarrow$ R is not reflexive

(2) Let $a, b \in z$ and aRb

So, $aRb \Rightarrow \gcd(a,b) = 1$

$\Rightarrow \gcd(b,a) = 1, \ \forall\, a, b \in z$ $\Rightarrow b\,Ra, \ \forall\, a, b \in z$ $\Rightarrow$ R is symmetric relation on z

Question ID : 7155051517

10. Let $\vec{u} = \hat{i} - \hat{j} - 2\hat{k}$, $\vec{v} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{v}.\vec{w} = 2$ and $\vec{v} \times \vec{w} = \vec{u} + \lambda\vec{v}$. Then $\vec{u}.\vec{w}$ is equal to :

माना $\vec{u} = \hat{i} - \hat{j} - 2\hat{k}$, $\vec{v} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{v}.\vec{w} = 2$ तथा $\vec{v} \times \vec{w} = \vec{u} + \lambda\vec{v}$. तो $\vec{u}.\vec{w}$ बराबर है :

(1) $-\frac{2}{3}$ (2) $\frac{3}{2}$ (3) 2 (4) 1

Ans. Official Answer NTA (4)

Sol. $\vec{v} \times \vec{w} = \vec{u} + \lambda\vec{v}$

$(\vec{v} \times \vec{w})\vec{v} = \vec{u} \cdot \vec{v} + \lambda\vec{v} \cdot \vec{v}$

$0 = \vec{u} \cdot \vec{v} + \lambda |v|^2$

$$\lambda = -\left(\frac{\vec{u} \cdot \vec{v}}{|v|^2}\right) = -\left(\frac{2 - 1 + 2}{6}\right) = -\frac{1}{2}$$

Now, $(\vec{v} \times \vec{w})\vec{w} = \vec{u} \cdot \vec{w} + \lambda\vec{v} \cdot \vec{w}$

$\Rightarrow 0 = \vec{u} \cdot \vec{w} + 2\lambda \Rightarrow \vec{u} \cdot \vec{w} = -2\lambda$

$$\Rightarrow \vec{u} \cdot \vec{w} = -2\left(-\frac{1}{2}\right) = 1$$

Question ID : 7155051508

11. Let $y = y(x)$ be the solution of the differential equation $x^3\,dy + (xy-1)dx = 0, x > 0, y\left(\frac{1}{2}\right) = 3 - e$. Then $y(1)$ is equal to :

माना अवकल समीकरण $x^3\,dy + (xy-1)dx = 0, x > 0, y\left(\frac{1}{2}\right) = 3 - e$ का हल $y = y(x)$ है। तो $y(1)$ बराबर है :

(1) $2 - e$ (2) e (3) 1 (4) 3

Ans. Official Answer NTA (3)

Sol. $\frac{dy}{dx} = \frac{1-xy}{x^3} = \frac{1}{x^3} - \frac{y}{x^2}$

$\frac{dy}{dx} + \frac{y}{x^2} = \frac{1}{x^3}$

If $= e^{\int \frac{1}{x^2}dx} = e^{-\frac{1}{x}}$

$y \cdot e^{-\frac{1}{x}} = \int e^{-\frac{1}{x}} \cdot \frac{1}{x^3}dx$ (put $-\frac{1}{x} = t$)

$y = \frac{1}{x} + 1 + Ce^{\frac{1}{x}}$

Where C is constant

Put $x = \frac{1}{2}$

$3 - e = 2 + 1 + Ce^2$

$C = -\frac{1}{e}$

$y(1) = 1$

Question ID : 7155051512

12. For three positive integers p, q, r, $x^{pq^2} = y^{qr} = z^{p^2r}$ and $r = pq + 1$ such that $3, 3\log_y x, 3\log_z y, 7\log_x z$ are in A.P. with common difference $\frac{1}{2}$. Then r-p-q is equal to :

तीन घनात्मक पूर्णांकों p, q, r के लिए $x^{pq^2} = y^{qr} = z^{p^2r}$, $r = pq + 1$ है तथा $3, 3\log_y x, 3\log_z y, 7\log_x z$ एक A.P. में है, जिसका सार्व अंतर $\frac{1}{2}$ हो तो r-p-q बराबर है :

(1) 2 (2) –6 (3) 6 (4) 12

Ans. Official Answer NTA (1)

Sol. Here

$$\frac{3\log x}{\log y}-3=\frac{1}{2}\Rightarrow\frac{\log x}{\log y}=\frac{7}{6} \quad \text{_____(1)}$$

$$\Rightarrow\frac{3\log y}{\log z}-\frac{3\log y}{\log z}=\frac{1}{2}\Rightarrow\frac{\log y}{\log z}=\frac{4}{3} \quad \text{_____(2)}$$

$$\Rightarrow\frac{7\log z}{\log x}-\frac{3\log y}{\log 2}=\frac{1}{2}\Rightarrow\frac{\log z}{\log x}=\frac{9}{14} \quad \text{_____(3)}$$

Now by $x^{pq^2}=y^{qr}=z^{p^2r}$

$pq^2\log x=qr\log y=p^2r\log z$ _____(4)

by equation (1), (2) , (3) and (4)

$$\frac{\log x}{\log y}=\frac{r}{pq}=\frac{7}{6} \quad \text{_____(5)}$$

$$\frac{\log y}{\log z}=\frac{p^2}{q}=\frac{4}{3} \quad \text{_____(6)}$$

$$\frac{\log z}{\log x}=\frac{q^2}{pr}=\frac{9}{14} \quad \text{_____(7)}$$

$r=pq+1$ (given) _____(8)

By solving equation (5), (6), (7) & (8) we get

$r=7, p=2$ and $q=3$

So, $r-p-q=7-2-3=2$

Question ID : 7155051501

13. The equation $x^2-4x+[x]+3=x[x]$, where [x] denotes the greatest integer function, has :

(1) no solution (2) a unique solution in $(-\infty,\infty)$

(3) a unique solution in $(-\infty,1)$ (4) exactly two solution in $(-\infty,\infty)$

समीकरण $x^2-4x+[x]+3=x[x]$, जहाँ [x] महत्तम पूर्णांक फलन है :

(1) का कोई हल नहीं है (2) का $(-\infty,\infty)$ में ठीक एक हल है

(3) का $(-\infty,1)$ में ठीक एक हल है (4) का $(-\infty,\infty)$ में ठीक दो हल है

Ans. Official Answer NTA (2)

Sol. $x^2-4x+[x]+3=x[x]$

$\Rightarrow x^2 - 4x + 3 = x[x] - [x]$

$\Rightarrow (x-1)(x-3) = [x] . (x-1)$

$\Rightarrow x = 1$ or $x - 3 = [x]$

$\Rightarrow x - [x] = 3$

$\Rightarrow \{x\} = 3$ (Not Possible)

Only one solution $x = 1$ in $(-\infty, \infty)$

Question ID : 7155051519

14. Let Ω be the sample space and $A \subseteq \Omega$ be an event. Given below are two statements :

(S1) : If $P(A) = 0$, then $A = \emptyset$

(S2) : If $P(A) = 1$, then $A = \Omega$

Then :

(1) Only (S1) is true

(2) both (S1) and (S2) are true

(3) only (S2) is true

(4) both (S1) and (S2) are false

माना Ω एक प्रतिदर्श समष्टि है तथा $A \subseteq \Omega$ एक घटना है। नीचे दो कथन दिए गए है :

(S1) : यदि $P(A) = 0$ है, तो $A = \emptyset$ है

(S2) : यदि $P(A) = 1$ है, तो $A = \Omega$ है

तो :

(1) केवल (S1) सत्य है

(2) (S1) तथा (S2) दोनों सत्य है

(3) केवल (S2) सत्य है

(4) (S1) तथा (S2) दोनों असत्य है

Ans. Official Answer NTA (4)

Sol. Ω = sample space

A = be an event

$A = \left\{\frac{1}{2}\right\}, \Omega = [0, 1]$

If $P(A) = 0 \Rightarrow A \neq \emptyset$

If $P\left(\overline{A}\right) = 1 \Rightarrow \overline{A} \neq \Omega$

Then both statement are false

Question ID : 7155051518

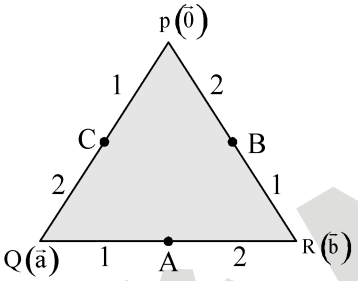
15. Let PQR be triangle. The point A, B and C are on the sides QR, RP and PQ respectively such that $\frac{QA}{AR}=\frac{RB}{BP}=\frac{PC}{CQ}=\frac{1}{2}$. Then $\frac{\text{Area}(\Delta PQR)}{\text{Area}(\Delta ABC)}$ is equal to :

माना PQR एक त्रिभुज है। भुजाओं QR, RP तथा PQ पर क्रमशः बिंदु A, B तथा C इस प्रकार है कि $\frac{QA}{AR}=\frac{RB}{BP}=\frac{PC}{CQ}=\frac{1}{2}$ है, तो $\frac{\text{Area}(\Delta PQR)}{\text{Area}(\Delta ABC)}$ बराबर है :

(1) 2 (2) 3 (3) 4 (4) $\frac{5}{2}$

Ans. Official Answer NTA (2)

Sol.



Let position vector of P, Q, R be $\bar{0}, \bar{a}$ and $\bar{b}$ respectively

$\Rightarrow$ P.V. of $C=\frac{\bar{a}}{3}$, P.V of $A=\frac{2\bar{a}+\bar{b}}{3}$ and P.V of $B=\frac{2\bar{b}}{3}$

$\therefore \overline{CA}=\frac{\vec{a}+\vec{b}}{3}$ and $\overline{AB}=\frac{\bar{b}-2\bar{a}}{3}$

$\Delta PQR=\frac{1}{2}|\overline{PQ}\times\overline{PR}|=\frac{1}{2}|\bar{a}\times\bar{b}|$

$\Delta ABC=\frac{1}{2}|\overline{CA}\times\overline{AB}|=\frac{1}{2}\left|\left(\frac{(\bar{a}+\bar{b})}{3}\times\frac{(\bar{b}-2\bar{a})}{3}\right)\right|=\frac{1}{2}\left|\frac{\bar{a}\times\bar{b}}{3}\right|$

$=\frac{\triangle PQR}{\triangle ABC}=3$

Question ID : 7155051520

16. The compound statement $(\sim(P \wedge Q)) \vee ((\sim P) \wedge Q) \Rightarrow ((\sim P) \wedge (\sim Q))$ is equivalent to :

मिश्र कथन $(\sim(P \wedge Q)) \vee ((\sim P) \wedge Q) \Rightarrow ((\sim P) \wedge (\sim Q))$ किस के तुल्य है ?

(1) $(\sim P) \vee Q$ (2) $((\sim P) \vee Q) \wedge (\sim Q)$

(3) $(\sim Q) \vee P$ (4) $((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$

Ans. Official Answer NTA (4)

Sol. Let $r = (\sim (P \wedge Q)) \vee ((\sim P) \wedge Q)$; $s = ((\sim P) \wedge (\sim Q))$

P	Q	~(P∧Q)	(~P)∧Q	r	s	r→s
T	T	F	F	F	F	T
T	F	T	F	T	F	F
F	T	T	T	T	F	F
F	F	T	F	T	T	T

Option (A) : $((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$
is equivalent to (not of only P) $\wedge$ (not of only Q)
= (Both P, Q) and (neither P nor Q)

Question ID : 7155051514

17. $\lim\limits_{t\to 0}\left(1^{\frac{1}{\sin^2 t}} + 2^{\frac{1}{\sin^2 t}} + \ldots\ldots + n^{\frac{1}{\sin^2 t}}\right)^{\sin^2 t}$ is equal to :

$\lim\limits_{t\to 0}\left(1^{\frac{1}{\sin^2 t}} + 2^{\frac{1}{\sin^2 t}} + \ldots\ldots + n^{\frac{1}{\sin^2 t}}\right)^{\sin^2 t}$ बराबर है :

(1) $\frac{(n+1)}{2}$ (2) n^2 (3) $n^2 + n$ (4) n

Ans. Official Answer NTA (4)

Sol. $\lim\limits_{t\to 0}\left(\frac{1}{n^{\sin^2 t}}\right)^{\sin^2 t}\left[1+\left(\frac{1}{n}\right)^{\text{cosec}^2 t}+\left(\frac{2}{n}\right)^{\text{cosec}^2 t}+\ldots\ldots\left(\frac{n-1}{n}\right)^{\text{cosec}^2 t}\right]^{\sin^2 t}$

as $0 < \frac{1}{n} < 1$, So $\left(\frac{1}{n}\right)^{\infty} = 0$

So, $n\,[1+0]^0 = n$

Question ID : 7155051506

18. Let α be a root of the equation $(a-c)x^2+(b-a)x+(c-b)=0$ where a, b, c are distinct real numbers such that the matrix $\begin{bmatrix}\alpha^2 & \alpha & 1\\ 1 & 1 & 1\\ a & b & c\end{bmatrix}$ is singular. Then, the value of $\frac{(a-c)^2}{(b-a)(c-b)}+\frac{(b-a)^2}{(a-c)(c-b)}+\frac{(c-b)^2}{(a-c)(b-a)}$ is :

माना समीकरण $(a-c)x^2+(b-a)x+(c-b)=0$, का मूल α है तथा आव्यूह जहाँ a, b, c भिन्न वास्तविक संख्याऐं है जबकि आव्यूह $\begin{bmatrix}\alpha^2 & \alpha & 1\\ 1 & 1 & 1\\ a & b & c\end{bmatrix}$ अव्युत्क्रमणीय है। तो $\frac{(a-c)^2}{(b-a)(c-b)}+\frac{(b-a)^2}{(a-c)(c-b)}+\frac{(c-b)^2}{(a-c)(b-a)}$ का मान है :

(1) 3 (2) 12 (3) 6 (4) 9

Ans. Official Answer NTA (1)

Sol. $\Delta=0=\begin{vmatrix}\alpha^2 & \alpha & 1\\ 1 & 1 & 1\\ a & b & c\end{vmatrix}$

$\Rightarrow \alpha^2(c-b)-\alpha(c-a)+(b-a)=0$

It is singular when $\alpha=1$

$$\frac{(a-c)^2}{(b-a)(c-b)}+\frac{(b-a)^2}{(a-c)(c-b)}+\frac{(c-b)^2}{(a-c)(b-a)}$$

$$\frac{(a-b)^3+(b-c)^3+(c-a)^3}{(a-b)(b-c)(c-a)}$$

$$=3\frac{(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)}=3$$

Question ID : 7155051513

19. Let $f(x)=\begin{cases} x^2\sin\left(\frac{1}{x}\right), & x\neq 0 \\ 0, & x=0 \end{cases}$

Then at $x=0$:

(1) f is continuous but f' is not continuous

(2) f and f' both are continuous

(3) f is continuous but not differentiable

(4) f' is continuous but not differentiable

माना $f(x)=\begin{cases} x^2\sin\left(\frac{1}{x}\right), & x\neq 0 \\ 0, & x=0 \end{cases}$ है।

तो $x=0$ पर :

(1) f संतत है परन्तु f' संतत नहीं है

(2) f तथा f' दोनों संतत है

(3) f संतत है परन्तु अवकलनीय है

(4) f' संतत है परन्तु अवकलनीय है

Ans. Official Answer NTA (1)

Sol. at x= 0

$$\text{LHD}=\lim_{h\to 0^+}\frac{f(0-h)-f(0)}{-h}=\lim_{h\to 0^+}\frac{-h^2\sin(1/h)}{-h}=\lim_{h\to 0}\frac{\sin(1/h)}{(1\setminus h)}=0$$

$$\text{RHD}=\lim_{h\to 0^+}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0^+}\frac{h^2\sin(1/h)}{h}=\lim_{h\to 0^+}\frac{\sin(1/h)}{1/h}=0$$

$\Rightarrow$ f(x) = is continuous and differential at $x=0$

$$\text{Now } f'(x)=\begin{cases} 2x\sin(1/x)-\cos(1/x), & x\neq 0 \\ 0, & x=0 \end{cases}$$

clearly f'(x) is discontinuous at $x=0$

Question ID : 7155051516

20. The distance of the point (–1, 9, –16) from the plane $2x + 3y – z = 5$ measured parallel to the line $\frac{x+4}{3} = \frac{2-y}{4} = \frac{z-3}{12}$ is :

बिंदु (–1, 9, –16) की समतल $2x + 3y – z = 5$ से रेखा $\frac{x+4}{3} = \frac{2-y}{4} = \frac{z-3}{12}$ के समांतर मापी दूरी है :

(1) $20\sqrt{2}$ (2) $13\sqrt{2}$ (3) 31 (4) 26

Ans. Official Answer NTA (4)

Sol. Equation of line

$$\frac{x+1}{3} = \frac{y-9}{-4} = \frac{z+16}{12}$$

G.P on line $(3\lambda-1, -4\lambda+9, 12\lambda-16)$

point of intersection of line and plane

$6\lambda – 2 – 12\lambda + 27 – 12\lambda + 16 = 5$

$\lambda = 2$

Point (5, 1, 8)

Distance = $\sqrt{36+64+576} = 26$

SECTION – B

Question ID : 7155051522

21. The number of 9 digit numbers, that can be formed using all the digits of the number 123412341 so that the even digits occupy only even places, is _____.

संख्या 123412341 के सभी अंकों के प्रयोग से बनाई जा सकने वाली 9 अंकों की ऐसी संख्याओं, कि सम अंक केवल सम स्थानों पर हो, की संख्या है।

Ans. Official Answer NTA (60)

Sol. Four even digits at four places in $\frac{4!}{2!2!} = 6$ ways

Five odd digits at odd places = $\frac{5!}{3!2!} = 10$ ways

Required number of numbers = 60 ways.

Question ID : 7155051525

22. A boy needs to select five courses from 12 available course, out of which 5 courses are language courses. If he

can choose at most two language courses, then the number of ways he can choose five courses is _____.

12 उपलब्ध पाठक्रमों, जिनके 5 भाषा के पाठयक्रम है, में से एक लड़के को पाँच पाठयक्रम लेने है। यदि वह अधिकतम दो भाषा के पाठयक्रम ले सकता है, तो उसके द्वारा पाँच पाठयक्रम लेने के तरीकों की संख्या है।

Ans. Official Answer NTA (546)

Sol. For at most two language courses

$= {}^5C_2 \times {}^7C_3 + {}^5C_1 \times {}^7C_4 + {}^7C_5 = 546$

Question ID : 7155051523

23. The 4^{th} term of GP is 500 and its common ratio is $\frac{1}{m}$, $m \in N$. Let S_n denote the sum of the first n terms of this GP. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$, then the number of possible values of m is _____.

एक GP का चौथा पद 500 है तथा इसका सार्वअनुपात $\frac{1}{m}$, $m \in N$ है। माना इस GP के प्रथम n पदों का योग S_n है। यदि $S_6 > S_5 + 1$ तथा $S_7 < S_6 + \frac{1}{2}$ हो, तो m के संभव मानों की संख्या है।

Ans. Official Answer NTA (12)

Sol. Given 4^{th} term of GP = 500

$$C.R. = \frac{1}{m}$$

$$\frac{a}{m^3} = 500$$

$$S_n = \frac{a\left[\left(\frac{1}{m}\right)^n - 1\right]}{\frac{1}{m} - 1}$$

$$S_n = \frac{a\left(1 - m^n\right)}{m^n \frac{(1-m)}{m}}$$

$$S_n = \frac{a\left(1 - m^n\right)}{m^{n-1}(1-m)}$$

$S_6 > S_5 + 1$

$$\frac{a\left(1-m^6\right)}{m^5(1-m)}>\frac{a\left(1-m^5\right)}{m^4(1-m)}+1$$

$$a-am^6>m^5-m^6+am-am^6$$

$$a(1-m)>m^5(1-m)$$

$$a>m^5 \quad \ldots(2)$$

$$S_7<S_6+\frac{1}{2}$$

$$\frac{a\left(1-m^7\right)}{m^6(1-m)}<\frac{a\left(1-m^6\right)}{m^5(1-m)}+\frac{1}{2}$$

$$a-am^7<am-am^7+\frac{1}{2}m^6-\frac{m^7}{2}$$

$$a(1-m)<\frac{1}{2}m^6(1-m)$$

$$a<\frac{m^6}{2}\cdots(3)$$

So $m^5<a<\frac{m^6}{2}$

$$m^2<\frac{a}{m^3}<\frac{m^3}{2}$$

$$m^2<500<\frac{m^3}{2}$$

$m \in [11,22]$

so, no. of possible value of m is 12

Question ID : 7155051528

24. The value of $\frac{8}{\pi}\int_0^{\frac{\pi}{2}}\frac{(\cos x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}}dx$ is _____.

$\frac{8}{\pi}\int_0^{\frac{\pi}{2}}\frac{(\cos x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}}dx$ का मान है _____.

Ans. Official Answer NTA (2)

Sol. $I=\frac{8}{\pi}\int_0^{\frac{\pi}{2}}\frac{(\cos x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}}dx$ _____(1)

Using $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

$I=\frac{8}{\pi}\int_0^{\frac{\pi}{2}}\frac{(\sin x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}}dx$ _____(2)

Adding (1) and (2)

$2I=\frac{8}{\pi}\int_0^{\frac{\pi}{2}}1dx$

$I=2$

Question ID : 7155051521

25. Let a tangent to the curve $9x^2+16y^2=144$ intersects the coordinate axes at the points A and B. Then, the minimum length of the line segment AB is _____.

माना वक्र $9x^2+16y^2=144$ की एक स्पर्श रेखा निर्देशांक अक्षों को बिंदुओं A तथा B पर मिलती है। तो रेखाखंड AB की न्यूनतम लंबाई है _____

Ans. Official Answer NTA (7)

Sol. $\frac{x^2}{16}+\frac{y^2}{9}=1$

Equation of tangent $\frac{x}{4}\cos\theta+\frac{y}{3}\sin\theta=1$

A (4sec θ, 0) B(0, 3(cosec θ)

$AB=\sqrt{16\sec^2\theta+9\operatorname{cosec}^2\theta}=\sqrt{25+\left(4\tan\theta-3\cot\theta\right)^2+24}\geq\sqrt{49}\geq 7$

$AB_{min}=7$

Question ID : 7155051526

26. Let $\lambda\in R$ and let the equation E be $|x|^2-2|x|+|\lambda-3|=0$. Then the largest element in the set $S=\{x+\lambda : x \text{ is an integer solution of E}\}$ is _____.

माना $\lambda\in R$ है तथा माना समीकरण $E: |x|^2-2|x|+|\lambda-3|=0$ है। तो समुच्चय $S=\{x+\lambda : x, E \text{ का एक पूर्णांक हल है}\}$ में सबसे बड़ा अवयव है _____

Ans. Official Answer NTA (5)

Sol. $|x|^2 - 2|x| + |\lambda - 3| = 0$

$|x|^2 - 2|x| + |\lambda - 3| - 1 = 0$

$(|x| - 1)^2 + |\lambda - 3| = 1$

At $\lambda = 3$, $x = 0$ and 2,
at $\lambda = 4$ or 2, then
$x = 1$ or -1
So maximum value of $x + \lambda = 5$

Question ID : 7155051524

27. Suppose $\sum_{r=0}^{2023} r^2 \, {}^{2023}C_r = 2023 \times \alpha \times 2^{2022}$. Then the value of α is _____.

माना $\sum_{r=0}^{2023} r^2 \, {}^{2023}C_r = 2023 \times \alpha \times 2^{2022}$ है। तो α का मान है _____

Ans. Official Answer NTA (1012)

Sol. $\sum r \cdot {}^{2023}C_r = \sum r \cdot (2023)^{2022} C_{r-1}$

$2023 \sum r^{2022} C_{r-1}$

$2023 \left(\sum_{1}^{2023} (r-1)^{2022} C_{r-1} + {}^{2022}C_{r-1} \right)$

$= 2023 \left(\left(\sum_{r=2}^{2023} (2022)^{2021} C_{r-2} \right) + \sum_{r=1}^{2023} {}^{2022}C_{r-1} \right) = 2023 \left(2022 \cdot 2^{2021} + 2^{2022} \right)$

$= 2^{2022} \cdot 2023(1011+1) = 2^{2022}(1012) \cdot (2023)$

$\because \ \alpha = 1012$

Question ID : 7155051530

28. The shortest distance between the lines $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-6}{2}$ and $\frac{x-6}{3} = \frac{1-y}{2} = \frac{z+8}{0}$ is equal to _____.

रेखाओं $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-6}{2}$ तथा $\frac{x-6}{3} = \frac{1-y}{2} = \frac{z+8}{0}$ के बीच न्यूनतम दूरी है _____

Ans. Official Answer NTA (14)

Sol. Shortest distance between the lines

$$= \frac{\begin{vmatrix} 4 & 2 & -14 \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & k \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix}}$$

$$= \frac{16+12+168}{|-4\hat{i}+6\hat{j}-12k|} = \frac{196}{14} = 14$$

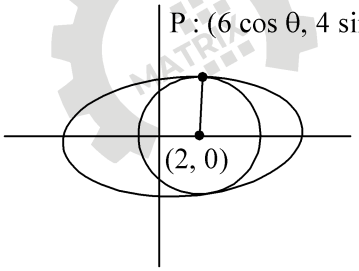
Question ID : 7155051529

29. Let C be the largest circle centred at (2, 0) and inscribed in the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$. If $(1, \alpha)$ lies on C, then 10 α^2 is equal to _____.

माना C सबसे बड़ा वृत्त हैरू जिसका केन्द्र (2, 0) पर है तथा जो दीर्घवृत $\frac{x^2}{36}+\frac{y^2}{16}=1$ के अन्तर्गत है। यदि बिंदु $(1, \alpha)$ वृत्त C पर है, तो 10 α^2 बराबर है _____

Ans. Official Answer NTA (118)

Sol.



equation of ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$

$$\frac{x}{18}+\frac{yy'}{8}=0$$

$$y' = \frac{-x}{18}\cdot\frac{8}{y}$$

$$Y_N = \frac{9y}{4x} = 9\cdot\frac{4\sin\theta}{4\cdot 6\cos\theta} = \frac{3}{2}\frac{\sin\theta}{\cos\theta}$$

$$Y_N(\text{ Slope }) = \frac{4\sin\theta}{6\cos\theta - 2} = \frac{2\sin\theta}{3\cos\theta - 1}$$

$$\frac{3}{2}\frac{\sin\theta}{\cos\theta} = \frac{2\sin\theta}{3\cos\theta - 1}$$

$$\cos\theta = \frac{3}{5}$$

$$P : \left(\frac{18}{5}, \frac{16}{5}\right)$$

$$S = \sqrt{\left(\frac{8}{5}\right)^2 + \left(\frac{16}{5}\right)^2}$$

Now $(x-2)^2 + y^2 = r^2$

$$(1, \alpha) \to 1 + \alpha^2 = \frac{8^2}{5^2} + \frac{16^2}{5^2} = 12.8$$

$\alpha^2 = 11.8$

$10\alpha^2 = 118$

Question ID : 7155051527

30. The value of $12\int_0^3 \left|x^2 - 3x + 2\right| dx$ is _____.

$12\int_0^3 \left|x^2 - 3x + 2\right| dx$ का मान है _____

Ans. Official Answer NTA (22)

Sol. $12\int_0^3 \left|x^2 - 3x + 2\right| dx$

$$= 12\int_0^3 \left|\left(x - \frac{3}{2}\right)^2 - \frac{1}{4}\right| dx$$

If $x - \frac{3}{2} = t$

$dx = dt$

$$= 24\int_0^{3/2} \left|t^2 - \frac{1}{4}\right| dt$$

$$= 24\left[-\int_0^{1/2}\left(t^2 - \frac{1}{4}\right)dt + \int_{1/2}^{3/2}\left(t^2 - \frac{1}{4}\right)dt\right] = 22$$