JEE Main July 2021

Question Paper With Text Solution

20 July. | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JULY 2021 | 20TH JULY SHIFT-1

SECTION – A

1. Let $A=[a_{ij}]$ be a 3×3 matrix, where

$$a_{ij}=\begin{cases}1 & , \text{ if } i=j \\ -x & , \text{ if } |i-j|=1 \\ 2x+1, & \text{otherwise}.\end{cases}$$

Let a function $f: R\to R$ be defined as f(x) = det(A). Then the sum of maximum and minimum values of *f* on **R** is equal to :

(1) $\frac{88}{27}$

(2) $-\frac{88}{27}$

(3) $-\frac{20}{27}$

(4) $\frac{20}{27}$

Ans. Answer by Matrix (2)

Sol. $A=\begin{bmatrix}1 & -x & 2x+1 \\ -x & 1 & 2x+1 \\ 2x+1 & -x & 1\end{bmatrix}$

$\det(A)=4x^3-4x^2-4x$

$f(x)=4x^3-4x^2-4x$

$f'(x)=12x^2-8x-4$

$=4(3x^2-2x-1)$

$=4(x-1)(3x+1)$

$x=1, -1/3$ are extremum part

$f(1)=4-4-4=-4$

$f\left(\frac{-1}{3}\right)=-\frac{-4}{27}-\frac{4}{9}+\frac{4}{3}$

$=4\left(\frac{-1-3+9}{27}\right)$

$= \frac{20}{27}$

$f(1) + f(-1/3) = \frac{20}{27} - 4 = -\frac{88}{27}$

2. Let $y = y(x)$ be the solution of the differential equation

$e^{x}\sqrt{1-y^{2}}\,dx + \left(\frac{y}{x}\right)dy = 0, y(1) = -1.$

Then the value of $(y(3))^{2}$ is equal to :

(1) $1+4e^{3}$

(2) $1-4e^{3}$

(3) $1+4e^{6}$

(4) $1-4e^{6}$

Ans. Answer by Matrix (4)

Sol. $e^{x}\sqrt{1-y^{2}}\,dx + \left(\frac{y}{x}\right)dy = 0$

$\int xe^{x}dx + \int \frac{y}{\sqrt{1-y^{2}}}dy = 0$

$e^{x}(x-1) - \sqrt{1-y^{2}} = c$

Put $x = 1, \quad y = 1 \quad \Rightarrow c = 0$

$e^{x}(x-1) = \sqrt{1-y^{2}}$

Put x = 3

$e^{3}(3-1) = \sqrt{1-y^{2}}$

$2e^{3} = \sqrt{1-y^{2}}$

$4e^{6} = 1-y^{2}$

$y^{2} = 1-4e^{6}$

$\Rightarrow (y(3))^{2} = 1-4e^{6}$

3. Word with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the letter M appears at the fourth position in any such word is :

(1) $\frac{2}{11}$

(2) $\frac{1}{66}$

(3) $\frac{1}{9}$

(4) $\frac{1}{11}$

Ans. Answer by Matrix (4)

Sol. EXAMINATION

AAIINNEOXMT

$n(S) = \frac{11!}{2!2!2!}$

Event E = letter M appears at 4^{th} position.

$n(E) = \frac{10!}{2!2!2!}$

$P(E) = \frac{n(E)}{n(S)} = \frac{1}{11}$

4. Let 'a' be a real number such that the function $f(x) = ax^2 + 6x - 15, x \in R$, is increasing in $\left(-\infty, \frac{3}{4}\right)$ and decreasing in $\left(\frac{3}{4}, \infty\right)$. Then the function $g(x) = ax^2 - 6x + 15, x \in R$, has a :

(1) local minimum at $x = -\frac{3}{4}$

(2) local maximum at $x = \frac{3}{4}$

(3) local maximum at $x = -\frac{3}{4}$

(4) local minimum at $x = \frac{3}{4}$

Ans. **Answer by Matrix (3)**

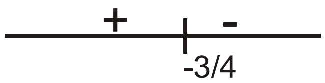
Sol. $f(x) = ax^2 + 6x + 15$

$f'(3/4) = 0$

$\Rightarrow a = -4$

$g(x) = -4x^2 - 6x + 15$

$g'(x) = -8x - 6$



$x = -3/4$ is a point of maxima

5. The probability of selecting integers $a \in [-5, 30]$ such that $x^2 + 2(a+4)x - 5a + 64 > 0$, for all $x \in R$, is :

(1) $\frac{7}{36}$

(2) $\frac{1}{6}$

(3) $\frac{1}{4}$

(4) $\frac{2}{9}$

Ans. **Answer by Matrix (4)**

Sol. $a \in [-5, 30]$

Total 36 integers

$n(S) = 36$

$x^2 + 2(a+4)x - 5a + 64 > 0 \quad \forall x - R$

$D < 0$

$4(a+4)^2 - 4(64 - 5a) < 0$

$a^2 + 13a - 48 < 0$

$(a+16)(a-3) < 0$

$a \in (-16, 3)$

$a \in \{-5, -4, -3, -2, -1, 0, 1, 2\}$

$n(E) = 8$

Prob $= \frac{n(E)}{n(S)} = \frac{8}{36} = \frac{2}{9}$

6. Let a be a positive real number such that

$\int_0^a e^{x-[x]}\,dx = 10e-9$

where $[x]$ is the greatest integer less than or equal to x. Then a is equal to :

(1) $10+\log_e(1+e)$

(2) $10+\log_e 2$

(3) $10-\log_e(1+e)$

(4) $10+\log_e 3$

Ans. Answer by Matrix (2)

Sol. $\int_0^{[a]} e^{x-[x]}dx + \int_{[a]}^{a} e^{x-[x]}dx = 10e-9$

$[a]\int_0^1 e^{x-[x]}dx + \int_0^{\{a\}} e^{x-[x]}dx = 10e-9$

$[a]\int_0^1 e^{x}dx + \int_0^{\{a\}} e^{x}dx = 10e-9$

$[a](e-1)+e^{\{a\}}-1 = 10e-9$

$[a]=10$ $\qquad$ $-[a]+e^{\{a\}}-1=-9$

$e^{\{a\}}=2$

$\{a\}=\log_e 2$

So $a=[a]+\{a\}$

$=10+\log_e 2$

7. Let $A=\begin{bmatrix}2 & 3\\ a & 0\end{bmatrix}, a\in R$ be written as P + Q where P is a symmetric matrix and Q is skew symmetric matrix. If det(Q) = 9, then the modulus of the sum of all possible values of determinant of P is equal to :

(1) 18

(2) 36

(3) 45

(4) 24

Ans. Answer by Matrix (2)

Sol. $A = \frac{A + A^T}{2} + \frac{A - A^T}{2}$

$Q = \frac{A - A^T}{2}$

$= \begin{bmatrix} 0 & \frac{3-a}{2} \\ \frac{a-3}{2} & 0 \end{bmatrix}$

det (Q) = 9

$\frac{(a-3)^2}{4} = 9$

$(a-3)^2 = 36$

$a = -3, \quad a = 9$

$P = \frac{A + A^T}{2}$

$= \begin{bmatrix} 0 & \frac{3+a}{2} \\ \frac{3+a}{2} & 0 \end{bmatrix}$

$\det(P) = -\left(\frac{3+a}{2}\right)^2$

If $a = -3 \qquad \det(P) = 0$

If $a = 9 \qquad \det(P) = -36$

sum $= 0 - 36 = -36$

$|\text{sum}| = 36$

8. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}.\vec{c} = |\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $(\vec{a} \times \vec{b})$ and $\vec{c}$ is $\frac{\pi}{6}$, then the value of $|(\vec{a} \times \vec{b}) \times \vec{c})|$ is :

(1) $\frac{3}{2}$

(2) 4

(3) 3

(4) $\frac{2}{3}$

Ans. Answer by Matrix (1)

Sol. $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k} \qquad \vec{b} = \hat{i} + \hat{j}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = 2\hat{i} - 2\hat{j} + \hat{k}$$

$|\vec{a} \times \vec{b}| = 3$

$|\vec{c} - \vec{a}| = 2\sqrt{2}$

$\vec{c}^2 + a^2 - 2\vec{a}.\vec{c} = 8$

$c^2 + a^2 - 2c = 8$

$[\because a^2 = 9]$

$c^2 - 2c + 1 = 0$

$(c-1)^2 = 0$

$c = 1$

$\Rightarrow |\vec{c}| = 1$

$|(\vec{a} \times \vec{b}) \times \vec{c}| = |\vec{a} \times \vec{b}||\vec{c}| \sin(\pi/6)$

$= (3)(1)\left(\frac{1}{2}\right)$

$= \frac{3}{2}$

9. Let [x] denote the greatest integer $\le x$. where $x \in R$. If the domain of the real valued function $f(x)=\sqrt{\frac{|[x]|-2}{|[x]|-3}}$ is $(-\infty, a)\cup[b,c)\cup[4,\infty), a<b<c$, then the value of a + b + c is :

(1) 8

(2) 1

(3) -2

(4) -3

Ans. Answer by Matrix (3)

Sol. Put $|[x]|=t \quad t\in I \quad , t\ge 0$

$$f=\sqrt{\frac{t-2}{t-3}}$$

$$\frac{t-2}{t-3}\ge 0$$

$$t\in[0,2]\cup(3,0)$$

$$\because t\in I$$

$$t\in[0,2]\cup[4,\infty)$$

$[x]\in[-2,2]$ or $[x]\in[4,\infty)$ or $[x]\in(-\infty,-4]$

$x\in[-2,3)$ or $x\in[4,\infty)$ $x\in(-\infty,-3)$

$\Rightarrow x\in(-\infty,-3)\cup[-2,3)\cup(4,\infty)$

$a=-3 \quad b=-2, \quad c=3$

$$a+b+c=-2$$

10. The Boolean expression $(p\wedge \sim q)\Rightarrow(q\vee \sim p)$ is equivalent to :

(1) $q\Rightarrow p$

(2) $p\Rightarrow q$

(3) $p\Rightarrow \sim q$

(4) $\sim q\Rightarrow p$

Ans. Answer by Matrix (2)

Sol. $(p\wedge \sim q)\rightarrow(q\vee \sim p)$

$=(\sim(p\wedge \sim q)\cup(q\vee \sim p)$

$=((\sim p)\vee q)\cup((\sim p)\vee a)$

$=((\sim p)\vee q)$

$=p\rightarrow q$

11. The mean of 6 distinct observations is 6.5 and their variance is 10.25. If 4 out of 6 observations are 2, 4, 5 and 7, then the remaining two observations are :

(1) 3, 18

(2) 1, 20

(3) 10, 11

(4) 8, 13

Ans. Answer by Matrix (3)

Sol. $\bar{x} = 6.5$

$\frac{18 + x + y}{6} = 6.5$

$\Rightarrow x + y = 21$ ....(i)

$\sigma^2 = 10.25$

$\frac{\Sigma x^2}{n} - (\bar{x})^2 = 10.25$

$\frac{94 + x^2 + y^2}{6} - (6.5)^2 = 10.25$

$x^2 + y^2 = 221$

$(x + y)^2 - 2xy = 221$

$xy = 110$ ....(ii)

solving equation (i) & (ii)

$x = 10, y = 11$

12. The coefficient of x^{256} in the expansion of $(1 - x)^{101}(x^2 + x + 1)^{100}$ **is :**

(1) $-{}^{100}C_{15}$

(2) $-{}^{100}C_{16}$

(3) ${}^{100}C_{16}$

(4) ${}^{100}C_{15}$

Ans. Answer by Matrix (4)

Sol. $(1 - x)\ \ ((1 - x)(1 + x + x^2))^{100}$

$(1 - x)\ \ (1 - x^3)^{100}$

$(1 - x^3)^{100} - x(1 - x^3)^{100}$

Coefficient of $x^{256} = 0-(-{}^{100}C_{85})$

$= {}^{100}C_{85}$

$= {}^{100}C_{15}$

13. If z and ω are two complex numbers such that $|z\omega|=1$ and $\arg(z)-\arg(\omega)=\frac{3\pi}{2}$, then

$\arg\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$ is :

(Here arg(z) denotes the principal argument of complex number z)

(1) $-\frac{\pi}{4}$

(2) $-\frac{3\pi}{4}$

(3) $\frac{\pi}{4}$

(4) $\frac{3\pi}{4}$

Ans. Answer by Matrix (2)

Sol. $|z\omega|=1=|\bar{z}\omega|$

$\arg(\omega)-\arg(z)=-3\pi/2$

$\arg(\omega\bar{z})=-3\pi/2$

$\omega\bar{z}=i \qquad (\because (\omega\bar{z})=1)$

$\arg\left(\frac{1-2i}{1+3i}\right)=\arg(1-2i)-\arg(1+3i)$

$=-\tan^{-1}(2)-\tan^{-1}(3)$

$=-3\pi/4$

14. The number of real roots of the equation

$\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{4}$ is :

(1) 1

(2) 4

(3) 0

(4) 2

Ans. Answer by Matrix (3)

Sol. Check domain of LHS

$x(x+1) \geq 0 \qquad 0 \leq x^2+x+1 \leq 1$

$\Rightarrow \; x^2+x=0$

$x = 0, -1$

Both values of x do not satisfy the given equation.

Ans = 0

15. Let the tangent to the parabola $S: y^2 = 2x$ at the point P(2, 2) meet the x-axis at Q and normal at it meet the parabola S at the point R. Then the area (in sq. units) of the triangle PQR is equal to :

(1) $\frac{15}{2}$

(2) $\frac{25}{2}$

(3) 25

(4) $\frac{35}{2}$

Ans. Answer by Matrix (2)

Sol. $y^2 = 2x \qquad a = \frac{1}{2}$

$P(2,2) \equiv P(at_1^2, 2at_1) \quad \Rightarrow t_1 = 1$

Tangent at P

$t_1 y = x + at_1^2$

$Q(-at_1^2, 0) \quad \equiv \quad Q(-2, 0)$

Normal at P intersects parabola at R

$t_2 = -t_1 - \frac{2}{t_1} = -3$

$R\left(\frac{9}{2}, -3\right)$

Area of $\Delta PQR = \frac{1}{2}\left\|\begin{matrix} -2 & 0 & 1 \\ 2 & 2 & 1 \\ \frac{9}{2} & -3 & 1 \end{matrix}\right\| = \frac{25}{2}$

16. Let a function $f : R \to R$ be defined as

$$f(x) = \begin{cases} \sin x - e^x & \text{if } x \le 0 \\ a + [-x] & \text{if } 0 < x < 1 \\ 2x - b & \text{if } x \ge 1 \end{cases}$$

where [x] is the greatest integer less than or equal to x. If *f* is continuous on R, then (a + b) is equal to :

(1) 2

(2) 5

(3) 4

(4) 3

Ans. Answer by Matrix (4)

Sol. (i) Continuous at x = 0

LHL = - 1

f(0) = -1

RHL = a - 1

a - 1 = - 1

$\Rightarrow a = 0$

(ii) Continuous at x = 1
LHL = a - 1
f(1) = 2- b
RHL = 2 - b
a - 1 = 2 - b

$\Rightarrow b = 3$

17. The value of the integral $\int_{-1}^{1} \log_e(\sqrt{1-x} + \sqrt{1+x})\,dx$ is equal to :

(1) $\log_e 2 + \frac{\pi}{2} - 1$

(2) $2\log_e 2 + \frac{\pi}{2} - \frac{1}{2}$

(3) $\frac{1}{2}\log_e 2 + \frac{\pi}{4} - \frac{3}{2}$

(4) $2\log_e 2 + \frac{\pi}{4} - 1$

Ans. Answer by Matrix (1)

Sol. $I = \int_{-1}^{1} \ln(\sqrt{1-x} + \sqrt{1+x})\,dx$

$I = 2\int_{0}^{1} \ln(\sqrt{1-x} + \sqrt{1+x})\,dx$

$= \int_{0}^{1} \ln(2 + 2\sqrt{1-x^2})\,dx$

$= \int_{0}^{1} \ln 2\,dx + \int_{0}^{1} \ln(1+\sqrt{1-x^2})\,dx$

Integrate by parts

$= \ln(2) + (x\ln(1+\sqrt{1-x^2}))_0^1 + \int_0^1 \frac{x^2\ \ dx}{(\sqrt{1-x^2})(1+\sqrt{1-x^2})}$

Put $1-x^2 = t^2$

$= \ln 2 + \int_0^1 \frac{(1-t^2)\ tdt}{t(1+t)\sqrt{1-t^2}}$

$= \ln 2 + \int_0^1 \sqrt{\frac{1-t}{1+t}}dt$

$= \ln 2 + \int_0^1 \frac{1-t}{\sqrt{1-t^2}}\,dt$

$= \ln 2 + (\sin^{-1}t)_0^1 + (\sqrt{1-t^2})_0^1$

$= \ln 2 + \left(\frac{\pi}{2} - 0\right) + (0-1)$

$= \log_e 2 + \frac{\pi}{2} - 1$

18. If α and β are the distinct roots of the equation $x^2 + (3)^{1/4}x + 3^{1/2} = 0$, then the value of $\alpha^{96}(\alpha^{12}-1) + \beta^{96}(\beta^{12}-1)$ is equal to :

(1) 56×3^{25}

(2) 56×3^{24}

(3) 28×3^{25}

(4) 52×3^{24}

Ans. **Answer by Matrix (4)**

Sol. $x^2+(3^{1/4})x+3^{1/2}=0$

$\alpha=3^{1/4}\omega \qquad \beta=3^{1/4}\omega^2$

$\alpha^{12}=3^3=27 \qquad \beta=3^3$

$\alpha^{96}=3^{24} \qquad \beta^{96}=3^{24}$

$\alpha^{96}(\alpha^{12}-1)+\beta^{96}(\beta^{12}-1)$

$=3^{24}(3^3-1)+3^{24}(3^3-1)$

$=2(26)(3^{24})$

$=52\times3^{24}$

19. If in a triangle ABC, AB = 5 units, $\angle B=\cos^{-1}\left(\frac{3}{5}\right)$ and radius of circumcircle of ΔABC is 5 units, then the area (in sq. units) of ΔABC is :

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(1) $10+6\sqrt{2}$

(2) $4+2\sqrt{3}$

(3) $6+8\sqrt{3}$

(4) $8+2\sqrt{2}$

Ans. **Answer by Matrix (3)**

Sol. ΔABC

$c=5 \quad , \quad B=\cos^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{3}{5}\right)$

R = 5

$b=2R\sin B$

b = 8

$\cos B=\frac{c^2+a^2-b^2}{2ca}$

$\frac{3}{5}=\frac{25+a^2-64}{2(5)(a)}$

$a^2-6a-39=0$

$a=3+4\sqrt{3} \quad$ or $\quad a=3-4\sqrt{3}$ (Rejected)

Area $= \frac{1}{2} ca \sin B$

$= \frac{1}{2}(5)(3+4\sqrt{3})\left(\frac{4}{5}\right)$

$= 6 + 8\sqrt{3}$

20. Let $y = y(x)$ be the solution of the differential equation

$x \tan\left(\frac{y}{x}\right)dy = \left(y \tan\left(\frac{y}{x}\right) - x\right)dx, \ -1 \le x \le 1, y\left(\frac{1}{2}\right) = \frac{\pi}{6}$.

Then the area of the region bounded by the curves $x = 0, x = \frac{1}{\sqrt{2}}$ and $y = y(x)$ in the upper half plane is :

(1) $\frac{1}{8}(\pi - 1)$

(2) $\frac{1}{12}(\pi - 3)$

(3) $\frac{1}{4}(\pi - 2)$

(4) $\frac{1}{6}(\pi - 1)$

Ans. Answer by Matrix (1)

Sol. $\tan\left(\frac{y}{x}\right)\frac{dy}{dx} = \frac{y}{x}\tan\left(\frac{y}{x}\right) - 1$

Put $y = vx$

$\frac{dy}{dx} = v + \frac{xdv}{dx}$

$(\tan v)\left(v + \frac{xdv}{dx}\right) = v \tan v - 1$

$v \tan v + \left(\frac{xdv}{dx}\right)\tan v = v \tan v - 1$

$\int \tan v \, dv = -\int \frac{dx}{v}$

$\ln(\sec v) = -\ln x + \ln c$

$\sec v = \frac{c}{x}$

$\sec\left(\frac{y}{x}\right) = \frac{c}{x}$

$x = 1/2 \Rightarrow y = \pi/6$

$\Rightarrow c = 1$

$\cos\left(\frac{y}{x}\right) = x$

$y = x\cos^{-1}(x)$

Area $= \int_0^{1/\sqrt{2}} x\cos^{-1}(x)\,dx$

$x = \cos\theta d\theta$

$dx = -\sin\theta\, d\theta$

$= \frac{1}{4}\int_{\pi/2}^{\pi/2} t\ \sin t\,dt$

$= \frac{1}{8}\left((-t\cos t)_{\pi/2} + \int_{\pi/2}^{\pi}\cos t dt\right)$

$= \frac{1}{8}\left((\pi) + (\sin\ t)_{\pi/2}^{\pi}\right)$

$= \frac{(\pi-1)}{8}$

SECTION - B

1. If the shortest distance between the lines $\vec{r_1} = \alpha\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k}), \lambda \in R$, $\alpha > 0$ and $\vec{r_2} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k}), \mu \in R$ is 9, then α is equal to ________.

Ans. Answer by Matrix (6)

Sol. $L_1\ \vec{r} = \vec{a} + \lambda\vec{p}$ $\quad \vec{a} = (\alpha, 2, 2)$

$L_2\ \vec{r} = \vec{b} + \mu\vec{q}$ $\quad \vec{b} = (-4, 0, -1)$

$\vec{p}\times\vec{q} = 8\hat{i} + 8\hat{j} + 4\hat{k}$

Shortest distance $= \left|\frac{(\vec{a}-\vec{b}).(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right| = 9$

$$\left|\frac{((\alpha+4)\hat{i}+2\hat{j}+3\hat{k}).(2\hat{i}+2\hat{j}+\hat{k})}{3}\right|=9$$

$|2\alpha+15|=27$

$2\alpha+15=27$ or $2\alpha+15=-27$

$\alpha=6$ or $\alpha=-21$ (Rejected)

2. If the value of $\lim\limits_{x\to 0}(2-\cos x\sqrt{\cos 2x})^{\left(\frac{x+2}{x^2}\right)}$ is equal e^a, then a is equal to __________.

Ans. Answer by Matrix (3)

Sol. 1^∞ Form

$$\lim_{x\to 0}(2-\cos x\sqrt{\cos 2x})^{\frac{x+2}{x^2}}=e^{\lim\limits_{x\to 0}\left(\frac{x+2}{x^2}\right)(1-\cos x\sqrt{\cos 2x})}$$

So $a=\lim\limits_{x\to 0}\dfrac{(x+2)}{x^2}(1-\cos x\sqrt{\cos 2x})$

$$=\lim_{x\to 0}\frac{2}{x^2}\frac{(1-\cos^2 x\cos 2x)}{(1+\cos x\sqrt{\cos 2x})}$$

$$=\lim_{x\to 0}\frac{2(1-\cos^2 x(2\cos^2 x-1))}{2x^2}$$

$$=\lim_{x\to 0}\frac{1+\cos^2 x-2\cos^4 x}{x^2}$$

$$=\lim_{x\to 0}\frac{(1-\cos^2 x)(1+2\cos^2 x)}{x^2}$$

$$=\lim_{x\to 0}\frac{3\sin^2 x}{x^2}=3$$

3. Let a, b, c, d be in arithmetic progression with common difference λ. If

$$\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix}=2,$$

then value of λ^2 is equal to _________.

Ans. Answer by Matrix (1)

Sol. $b-a=\lambda=c-b=d-c$

$$\begin{vmatrix} x-2\lambda & x+b & x+a \\ x-1 & x+c & x+b \\ x+2\lambda & x+d & x+c \end{vmatrix}=2$$

$R_1 \to R_1 - R_2$

$R_2 \to R_2 - R_3$

$$\begin{vmatrix} 1-2\lambda & -\lambda & -\lambda \\ -1-2\lambda & -\lambda & -\lambda \\ x+2\lambda & x+d & x+c \end{vmatrix}=2$$

$R_1 \to R_1 - R_2$

$$\begin{vmatrix} 2 & 0 & 0 \\ -1-2\lambda & -\lambda & -\lambda \\ x+2\lambda & x+d & x+c \end{vmatrix}=2$$

Expand along R_1

$2(-\lambda(x+c)+\lambda(x+d))=2$

$\lambda(x+d-x-c)=1$

$\lambda(\lambda)=1$

$\lambda^2=1$

4. The number of rational terms in the binomial expansion of $(4^{\frac{1}{4}}+5^{\frac{1}{6}})^{120}$ is _________.

Ans. Answer by Matrix (21)

Sol. $T_{r+1}={}^{120}C_r(4^{1/4})^{120-r}(5^{1/6})^r$

Powers should be integral powers

$\frac{120-r}{2}=$ integer & $\frac{r}{6}=$ Integer

$\Rightarrow r\in\{0,6,12,18......120\}$

Total 21 rational terms

5. There are 15 players in a cricket team, out of which 6 are bowlers, 7 are bstsmen and 2 are wicketkeepers. The number of ways, a team of 11 players be selected from them so as to include at least 4 bowlers, 5 batsmen and 1 wicketkeeper, is _________.

Ans. Answer by Matrix (777)

Sol. atleast 4 bowlers, atleast 5 batsmen, atleast 1wk

Bowlers(6)	Batsmen(7)	WK(2)	Number of way
5	5	1	${}^6C_5\,{}^7C_5\,{}^2C_1 = 252$
4	6	1	${}^6C_4\,{}^7C_6\,{}^2C_1 = 210$
4	5	2	${}^6C_4\,{}^7C_5\,{}^2C_2 = 315$

Total = 777

6. Let $y = mx + c, m > 0$ be the focal chord $y^2 = -64x$, which is tangent to $(x+10)^2 + y^2 = 4$. Then, the value of $4\sqrt{2}(m+c)$ is equal to _________.

Ans. Answer by Matrix (34)

Sol. $y^2 = -64x \qquad a = 16$

focus $(-16, 0)$ will lie on $y = mx + c$

$\Rightarrow c = 16m$

$y - m(x+16) = 0$

apply COT wrt $(x+10)^2 + y^2 = 4$

$$\left|\frac{(-10+16)\,m}{\sqrt{1+m^2}}\right| = 1$$

$8m^2 = 1$

$$m = \frac{1}{2\sqrt{2}}$$

$4\sqrt{2}(m+c) = 4\sqrt{2}(m+16m)$

$$= 68\sqrt{2}\left(\frac{1}{2\sqrt{2}}\right)$$

$= 34$

7. Let P be a plane passing through the pionts $(1,0,1),(1,-2,1)$ and $(0,1,-2)$. Let a vector $\vec{a}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ be such that $\vec{a}$ is parallel to the plane P, perpendicular to $(\hat{i}+2\hat{j}+3\hat{k})$ and $\vec{a}.(\hat{i}+\hat{j}+2\hat{k})=2$, then $(\alpha-\beta+\gamma)^2$ equals ________.

Ans. Answer by Matrix (81)

Sol. Plane P A(1,0,1), B(1,−2,1), C(0,1,−2)

$\vec{P}=\overrightarrow{AB}=2\hat{j}$

$\vec{q}=\overrightarrow{AC}=-\hat{i}+\hat{j}-3\hat{k}$

let $\hat{i}+2\hat{j}+3\hat{k}=\vec{r}$

$\vec{a}$ is lying in plane of $\vec{p}$ & $\vec{q}$ & perpendicular or $\vec{r}$

$\vec{a}=\lambda(\vec{p}\times\vec{q})\times\vec{r}$

$=\lambda((\vec{p}.\vec{r})\vec{q}-(\vec{q}.\vec{r})\vec{p})$

$=\lambda(4\vec{q}+8\vec{p})$

$\vec{a}=\lambda(-4\hat{i}+10\hat{j}-12\hat{k})$

$\vec{a}.(\hat{i}+\hat{j}+2\hat{k})=2$

$-4\lambda+20\lambda-24\lambda=2$

$-8\lambda=2$

$\lambda=-1/4$

$\alpha-\beta+\gamma=-4\lambda-20\lambda-12\lambda$

$=-36\lambda$

$\alpha-\beta+\gamma=9$

$(\alpha-\beta+\gamma)^2=81$

8. Let $A=\begin{pmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ 0 & 0 & 1\end{pmatrix}$ and $B=7A^{20}-20A^{7}+2I$, where I is an identity matrix of order 3×3. If $B=[b_{ij}]$, then b_{13} is equal to _________.

Ans. Answer by Matrix (910)

Sol. $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}+\begin{bmatrix}0 & -1 & 0\\ 0 & 0 & -1\\ 0 & 0 & 0\end{bmatrix}$

$A = I + C$

$$C^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; C^3 = O$$

$B = 7A^{20} - 20A^7 + 2I$

$= 7(I + C)^{20} - 20(I + C)^7 + 2I$

$= 7(I + 20C + 190C^2) - 20(I + 7C + 21C^2) + 2I$

$= -9I + 910C^2$

$$= \begin{bmatrix} -9 & 0 & 0 \\ 0 & -9 & 0 \\ 0 & 0 & -9 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 910 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & 0 & 910 \\ 0 & -9 & 0 \\ 0 & 0 & -9 \end{bmatrix}$$

9. Let $\vec{a}, \vec{b}, \vec{c}$ be three mutually perpendicular vectors of the same magnitude and equally inclined at an angle θ, with the vector $\vec{a} + \vec{b} + \vec{c}$. Then $36 \cos^2 2\theta$ is equal to ____________.

Ans. Answer by Matrix (4)

Sol. $$\cos\theta = \frac{(\vec{a} + \vec{b} + \vec{c}).\vec{a}}{|\vec{a} + \vec{b} + \vec{c}||\vec{a}|}$$

$$= \frac{r^2}{(r\sqrt{3})(r)} = \frac{1}{\sqrt{3}}$$

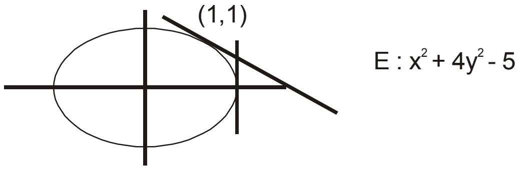
$$\cos(2\theta) = 2\cos^2\theta - 1 = \frac{-1}{3}$$

$$36\cos^2(2\theta) = 36\left(\frac{1}{9}\right) = 4$$

10. Let T be the tangent to the ellipse $E: x^2+4y^2=5$ at the point $P(1, 1)$. If the area of the region bounded by the tangent T, ellipse E, lines $x=1$ and $x=\sqrt{5}$ is $\alpha\sqrt{5}+\beta+\gamma\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$, then $|\alpha+\beta+\gamma|$ is equal to ____________.

Ans. **Answer by Matrix (1.25)**

Sol.



$T: \quad x+4y=5$

$$\text{Area}=\int_1^{\sqrt{5}}(y_{\text{LINE}}-y_{\text{ELLIPSE}})\,dx$$

$$=\int_1^{\sqrt{5}}\left(\frac{5-x}{4}-\sqrt{\frac{5-x^2}{4}}\right)dx$$

$$=\frac{1}{4}(5x)_1^{\sqrt{5}}-\left(\frac{x^2}{8}\right)_1^{\sqrt{5}}-\frac{1}{4}\left(x\sqrt{5-x^2}\right)_1^{\sqrt{5}}-\frac{5}{4}\left(\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)\right)_1^{\sqrt{5}}$$

$$=\frac{5\sqrt{5}}{4}-\frac{5}{4}-\frac{1}{2}-\frac{1}{4}(0-2)-\frac{5}{4}\left(\frac{\pi}{2}-\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)$$

$$=\frac{5\sqrt{5}}{4}-\frac{5}{4}-\frac{5}{4}\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

$$|\alpha+\beta+\gamma|=\frac{5}{4}$$