JEE Main January 2023

Question Paper With Text Solution

01 February | Shift-2

MATHEMATICS

JEE MAIN FEBRUARY 2023 | 1ST FEBRUARY SHIFT-2

SECTION – A

Question ID : 7155051231

1. Let P(S) denote the power set of S = {1,2,3,......,10}. Define the relations R_1 and R_2 on P(S) as AR_1B if $\left(A\cap B^C\right)\cup\left(B\cap A^C\right)=\phi$ and AR_2B if $A\cup B^C=B\cup A^C, \forall A,B\in P(S)$. Then :

(1) only R_1 is an equivalence relation

(2) both R_1 and R_2 are equivalence relations

(3) only R_2 is an equivalence relation

(4) both R_1 and R_2 are not equivalence relations

माना P(S), S = {1,2,3,......,10} के घात समुच्चय को दर्शाता है। P(S) पर संबंध R_1 तथा R_2 इस तरह परिभाषित है कि सभी AR_1B यदि $\left(A\cap B^C\right)\cup\left(B\cap A^C\right)=\phi$ है, तथा AR_2B यदि $A\cup B^C=B\cup A^C, \forall A,B\in P(S)$ है तो :

(1) केवल R_1 एक तुल्यता संबंध है।

(2) R_1 तथा R_2 दोनों तुल्यता संबंध नहीं है।

(3) केवल R_2 एक तुल्यता संबंध है।

(4) R_1 तथा R_2 दोनों तुल्यता संबंध नहीं है।

Ans. Official Answer NTA (2)

Sol. S = {1,2,3,.....10}

P(S) = power set of S

$AR_1B \Rightarrow \left(A\cap\vec{B}\right)\cup\left(\vec{A}\cap B\right)=\phi$

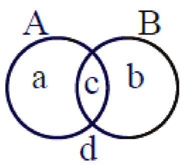
R1 is reflexive, symmetric

For transitive

$\left(A\cap\vec{B}\right)\cup\left(\vec{A}\cap B\right)=\phi;\{a\}=\phi=\{b\}A=B$

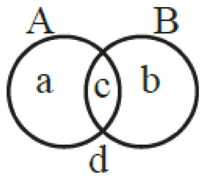
$\left(B\cap\vec{C}\right)\cup\left(\vec{B}\cap C\right)=\phi\;\therefore B=C$

$\therefore$ A = C equivalence



$R_2 \equiv A \cup \vec{B} = \vec{A} \cup B$

$R_2 \rightarrow$ Reflexive, symmetric
for transitive



$A \cup \vec{B} = \vec{A} \cup B \Rightarrow \{a, c, d\} = \{b, c, d\}$

$\{a\} = \{b\} \therefore A = B$

$B \cup \vec{C} = \vec{B} \cup C \Rightarrow B = C$

$\therefore A = C$ $\qquad \therefore A \cup \vec{C} = \vec{A} \cup C$ $\therefore$ Equivalence

Question ID : 7155051246

2. Two dice are thrown independently. Let A be the event that the number appeared on the 1st die is less than the number appeared on the 2nd die, B be the event that the number appeared on the 1st die is even and that on the second die is odd, and C be the event that the number appeared on the 1st die is odd and that on the 2nd is even. Then :

(1) the number of favourable cases of the event $(A \cup B) \cap C$ is 6

(2) A and B are mutually exclusive

(3) the number of favourable cases of the events A, B and C are 15, 6 and 6 respectively

(4) B and C are independent

दो पासे स्वतंत्रा रूप से फेंके जाते हैं। माना पहले पासे पर प्रकट होने वाली संख्या के दूसरे पासे पर प्रकट होने वाली संख्या से कम होने की घटना A है, पहले पासे पर सम संख्या तथा दूसरे पासे पर विषम संख्या के प्रकट होने की घटना B है और पहले पासे पर विषम संख्या तथा दूसरे पासे पर सम संख्या के प्रकट होने की घटना C है। तो :

(1) घटना $(A \cup B) \cap C$ के अनुकूल परिणामों की संख्या 6 है

(2) A तथा B परस्पर अपवर्जी है

(3) घटनाओं A, B तथा C के अनुकूल परिणामों की संख्या क्रमशः 15, 6 तथा 6 है

(4) B तथा C स्वतंत्रा है

Ans. Official Answer NTA (1)

Sol. A (I < II) $\qquad$ B (EO) $\qquad$ C (OE)

$n(A) = 15$ $n(B) = 9$ $n(C) = 9$

$n(A \cap B) = 3$ $n(A \cap C) = 6$ $n(B \cap C) = 0$

$n(A \cap B \cap C) = 0$

$n((A \cup B) \cap C) = n(A \cap C) + n(B \cap C) - n(A \cap B \cap C) = 6$

Question ID : 7155051250

3. Which of the following statements is a tautology?

निम्न में से कौन सा कथन पुनरूक्ति है ?

(1) $p \rightarrow (p \wedge (p \rightarrow q))$ (2) $p \vee (p \wedge q)$

(3) $(p \wedge q) \rightarrow (\sim (p) \rightarrow q)$

(4) $(p \wedge (p \rightarrow q)) \rightarrow \sim q$

Ans. Official Answer NTA (3)

Sol. (i) $p \rightarrow (p \wedge (p \rightarrow q))$

$(\sim p) \vee (p \wedge (\sim p \vee q))$

$(\sim p) \vee (f \vee (p \wedge q))$

$\sim p \vee (p \wedge q) = (\sim p \vee p) \wedge (\sim p \vee q)$

$= \sim p \vee q$

(ii) $p \vee (p \wedge q) = p$

Not tautology

(iii) $(p \wedge q) \rightarrow (\sim p \rightarrow q)$

$\sim (p \wedge q) \vee (p \vee q) = t$

$\{a,b,d\} \vee \{a,b,c\} = V$

Tautology

(iv) $(p \wedge (p \rightarrow q)) \rightarrow \sim q$

$\sim (p \wedge (\sim p \vee q)) \vee \sim q = \sim (p \wedge q) \vee \sim q = \sim p \vee \sim q$

Not tautology

Question ID : 7155051248

4. Let $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}$. Then which one of the following statements is TRUE?

(1) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is same as of $\vec{b}$.

(2) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$.

(3) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$.

(4) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is same as of $\vec{b}$.

माना दो सदिश $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ तथा $\vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}$ है। तो निम्न में से कौन सा कथन सही है ?

(1) $\vec{a}$ का $\vec{b}$ पर प्रक्षेप $\frac{-17}{\sqrt{35}}$ है तथा प्रक्षेप सदिश की दिशा $\vec{b}$ की दिशा में समान है

(2) $\vec{a}$ का $\vec{b}$ पर प्रक्षेप $\frac{17}{\sqrt{35}}$ है तथा प्रक्षेप सदिश की दिशा $\vec{b}$ की दिशा के विपरीत है

(3) $\vec{a}$ का $\vec{b}$ पर प्रक्षेप $\frac{-17}{\sqrt{35}}$ है तथा प्रक्षेप सदिश की दिशा $\vec{b}$ की दिशा में विपरीत है

(4) $\vec{a}$ का $\vec{b}$ पर प्रक्षेप $\frac{17}{\sqrt{35}}$ है तथा प्रक्षेप सदिश की दिशा $\vec{b}$ की दिशा के समान है

Ans. Official Answer NTA (Drop)

Sol. $\vec{a} = 5i - j - 3k$, $\vec{b} = i + 3j + 5k$

(1) Projection of $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a}.\vec{b}}{|\vec{b}|} = \frac{(5-3-15)}{\sqrt{1+9+25}} = \frac{-13}{\sqrt{35}}$

Question ID : 7155051242

5. Let $\alpha x = \exp\left(x^{\beta}y^{\gamma}\right)$ be the solution of the differential equation $2x^2y\,dy - (1 - xy^2)dx = 0$, $x > 0$, $y(2) = \sqrt{\log_e 2}$. Then $\alpha + \beta - \gamma$ equals :

माना अवकल समीकरण $2x^2y\,dy - (1 - xy^2)dx = 0$, $x > 0$, $y(2) = \sqrt{\log_e 2}$ का हल $\alpha x = \exp\left(x^{\beta}y^{\gamma}\right)$ है। तो $\alpha + \beta - \gamma$:

(1) -1 (2) 1 (3) 0 (4) 3

Ans. Official Answer NTA (2)

Sol. $\alpha x = e^{x^{\beta}.y^{\gamma}}$

$2x^2y\frac{dy}{dx} = 1 - x.y^2 \qquad y^2 = t$

$x^2\frac{dt}{dx} = 1 - xt$

$\frac{dt}{dx} + \frac{t}{x} = \frac{1}{x^2} \qquad$ I.F. $= e^{\ln x} = x$

$t(x) = \int \frac{1}{x^2}.xdx$

$y^2 . x = \ln x + C$

$\therefore$ 2.ln2 = ln2 + C

$\therefore$ C = ln2

Hence, $xy^2 = \ln 2x$

$\therefore 2x = e^{x.y^2}$

Hence $\alpha = 2, \beta = 1, \gamma = 2$

Question ID : 7155051233

6. The number of integral values of k, for which one root of the equation $2x^2 - 8x + k = 0$ lies in the interval (1,2) and its other roots lies in the interval (2,3), is :

k के पूर्णांक मानों, जिनके लिए समीकरण $2x^2 - 8x + k = 0$ का एक मूल अंतराल (1,2) में है, तथा दूसरा मूल अंतराल (2,3) में, की संख्या है :

(1) 0 (2) 3 (3) 1 (4) 2

Ans. Official Answer NTA (3)

Sol. Let $f(x) = 2x^2 - 8x + k = 0$

$f(1) > 0$

$f(2) < 0$

$f(3) > 0$

$f(1) > 0$ Þ $2 - 8 + k > 0 \Rightarrow k\rangle 6$ (1)

$f(2)\langle 0 \Rightarrow 8 - 16 + k\langle 0 \Rightarrow k\langle 8$ (2)

$f(3)\rangle 0 \Rightarrow 18 - 24 + k\rangle 0 \Rightarrow k\rangle 6$ (3)

$k \in (6, 8) \therefore$ integral k = 7

Question ID : 7155051249

7. Let $S=\left\{x \in R : 0<x<1 \text{ and } 2\tan^{-1}\left(\frac{1-x}{1+x}\right)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}$. If n(S) denotes the number of elements in S then :

(1) $n(S)=2$ and only one element in S is less than $\frac{1}{2}$.

(2) $n(S)=0$

(3) $n(S)=1$ and the element in S is less than $\frac{1}{2}$.

(4) $n(S)=1$ and the elements in S is more than $\frac{1}{2}$.

माना $S=\left\{x \in R : 0<x<1 \text{ and } 2\tan^{-1}\left(\frac{1-x}{1+x}\right)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}$ है। यदि n(S), S में अवयवों की संख्या को दर्शाता है, तो :

(1) $n(S)=2$ है तथा S का एक अवयव $\frac{1}{2}$ से कम है

(2) $n(S)=0$

(3) $n(S)=1$ है तथा S का अवयव $\frac{1}{2}$ से कम है

(4) $n(S)=1$ है तथा S का एक अवयव $\frac{1}{2}$ से अधिक है

Ans. Official Answer NTA (3)

Sol. $0<x<1$

$$2\tan^{-1}\left(\frac{1-x}{1+x}\right)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$\tan^{-1}x=\theta \in \left(0,\frac{\pi}{4}\right) \quad \therefore x=\tan\theta$$

$$2\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\theta\right)\right)=\cos^{-1}(\cos 2\theta)$$

$$2\left(\frac{\pi}{4}-\theta\right)=2\theta \therefore 4\theta=\frac{\pi}{2} \therefore \theta=\frac{\pi}{8}$$

$x = \tan\frac{\pi}{8} \therefore x = \sqrt{2} - 1 \simeq 0.414$

Question ID : 7155051237

8. The sum $\sum_{n=1}^{\infty} \frac{2n^2 + 3n + 4}{(2n)!}$ is equal to :

योगफल $\sum_{n=1}^{\infty} \frac{2n^2 + 3n + 4}{(2n)!}$ बराबर है :

(1) $\frac{13e}{4} + \frac{5}{4e}$

(2) $\frac{13e}{4} + \frac{5}{4e} - 4$

(3) $\frac{11e}{2} + \frac{7}{2e}$

(4) $\frac{11e}{2} + \frac{7}{2e} - 4$

Ans. Official Answer NTA (2)

Sol. $\sum_{n=1}^{\infty} \frac{2n^2 + 3n + 4}{2n!}$

$= \frac{1}{2}\sum_{n=1}^{\infty} \frac{2n(2n-1) + 8n + 8}{2n!}$

$= \frac{1}{2}\sum_{n=1}^{\infty} \left\{\frac{1}{(2n-2)!} + \frac{4}{(2n-1)!} + \frac{8}{2n!}\right\}$

$= \frac{1}{2}\left\{\left(\frac{e+e^{-1}}{2}\right) + 4\left(\frac{e-e^{-1}}{2}\right) + 8\left(\frac{e+e^{-1}}{2} - 1\right)\right\}$

$= \frac{1}{4}\left(13e + 5e^{-1} - 16\right)$

$= \frac{13}{4}e + \frac{5}{4e} - 4$

Question ID : 7155051243

9. Let $P(x_0, y_0)$ be the point on the hyperbola $3x^2 - 4y^2 = 36$, which is nearest to the line $3x + 2y = 1$. Then $\sqrt{2}\left(y_0 - x_0\right)$ is equal to :

माना अतिपरवलय $3x^2 - 4y^2 = 36$ पर बिंदु $P(x_0, y_0)$ रेखा $3x + 2y = 1$ के निकटतम है। तो $\sqrt{2}(y_0 - x_0)$ बराबर है :

(1) 3 (2) –9 (3) –3 (4) 9

Ans. Official Answer NTA (2)

Sol. $3x^2 - 4y^2 = 36$ $3x + 2y = 1$

$$m = -\frac{3}{2}$$

$$m = +\frac{3\sec\theta}{\sqrt{12}.\tan\theta}$$

$$\Rightarrow \frac{3}{\sqrt{12}} \times \frac{1}{\sin\theta} = \frac{-3}{2}$$

$$\sin\theta = -\frac{1}{\sqrt{3}}$$

$$\left(\sqrt{12}.\sec\theta, 3\tan\theta\right)$$

$$\left(\sqrt{12}.\frac{\sqrt{3}}{\sqrt{2}}, -3\times\frac{1}{\sqrt{2}}\right) \Rightarrow \left(\frac{6}{\sqrt{2}}, \frac{-3}{\sqrt{2}}\right)$$

$$\Rightarrow \sqrt{2}(y_0 - x_0) = \sqrt{2}\left(\frac{-3}{\sqrt{2}} - \frac{6}{\sqrt{2}}\right) = \frac{\sqrt{2}}{\sqrt{2}}(-9) = -9$$

Question ID : 7155051236

10. Let $f : R - \{0,1\} \rightarrow R$ be a function such that $f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$. Then $f(2)$ is equal to :

माना फलन $f : R - \{0,1\} \rightarrow R$ इस प्रकार है कि $f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$ है। तो $f(2)$ बराबर है :

(1) $\frac{9}{4}$ (2) $\frac{7}{4}$ (3) $\frac{9}{2}$ (4) $\frac{7}{3}$

Ans. Official Answer NTA (1)

Sol. , $x = 2 \Rightarrow f(2) + f(-1) = 3$ ...(1)

$, x = \frac{1}{2} \Rightarrow f\left(\frac{1}{2}\right) + f(2) = \frac{3}{2}$ ...(2)

$, x = -1 \Rightarrow f(-1) + f\left(\frac{1}{2}\right) = 0$ ...(3)

$(2) .... (3) \Rightarrow f(2) - f(-1) = \frac{3}{2}$ ...(4)

$(1) + (4) \Rightarrow 2f(2) = \frac{9}{2} \Rightarrow f(2) = \frac{9}{4}$

Question ID : 7155051240

11. The value of the integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x + \frac{\pi}{4}}{2 - \cos 2x} dx$ is :

समाकलन $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x + \frac{\pi}{4}}{2 - \cos 2x} dx$ का मान है :

(1) $\frac{\pi^2}{6\sqrt{3}}$ (2) $\frac{\pi^2}{6}$ (3) $\frac{\pi^2}{3\sqrt{3}}$ (4) $\frac{\pi^2}{12\sqrt{3}}$

Ans. Official Answer NTA (1)

Sol. $I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x + \frac{\pi}{4}}{2 - \cos 2x} dx$ (1)

$x \to -x$

$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{-x + \frac{\pi}{4}}{2 - \cos 2x} dx$ (2)

(1) + (2)

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\frac{\pi}{2}}{2-\cos 2x} dx$$

$$I = \frac{\pi}{4} \cdot 2\int_{0}^{\frac{\pi}{4}} \frac{dx}{2-\cos 2x} dx$$

$$I = \frac{\pi}{4} \cdot 2\int_{0}^{\frac{\pi}{4}} \frac{\left(1+\tan^2 x\right)dx}{2\left(1+\tan^2 x\right)-\left(1-\tan^2 x\right)}$$

$$I = \frac{\pi}{2}\int_{0}^{1} \frac{dt}{3t^2+1}$$

$$\Rightarrow I = \frac{\pi}{2\sqrt{3}} \tan^{-1}\sqrt{3}$$

$$I = \frac{\pi^2}{6\sqrt{3}}$$

Question ID : 7155051244

12. Let the plane P pass through the intersection of the planes $2x+3y-z$ and $x+2y+3z=6$, and be perpendicular to the plane $2x+y-z+1=0$. If d is the distance of P from the point (–7,1,1), then d^2 is equal to :

माना समतल P समतलों $2x + 3y - z = 2$ तथा $x + 2y + 3z = 6$ के प्रतिच्छेदन से होकर जाता है तथा समतल $2x + y - z + 1 = 0$ के लंबवत है। यदि P की बिंदु (–7,1,1) से दूरी d है, तो d^2 बराबर है :

(1) $\frac{25}{83}$ (2) $\frac{250}{82}$ (3) $\frac{250}{83}$ (4) $\frac{15}{53}$

Ans. Official Answer NTA (3)

Sol. A plane through intersection of $2x+3y-z=2$ & $x+2y+3z=6$ is

$2x + 3y - z - 2 + \lambda (x + 2y + 3z - 6) = 0$

$\Rightarrow (2 + \lambda)x + (3 + 2\lambda)y + (3\lambda - 1) z - (2 + 6\lambda) = 0$

Plane is $\perp$ to $2x + y - z + 1 = 0$

$\therefore$ 2(2 + λ) + 1 (3 + 2λ) – (3λ – 1) = 0

4 + 2λ + 3 + 2λ – 3λ + 1 = 0

λ = – 8

$\therefore$ plane is 6x + 13y + 25z – 46 = 0

$$\therefore d = \frac{|-42+13+25-46|}{\sqrt{36+169+625}} = \frac{50}{\sqrt{830}}$$

$$\therefore d^2 = \frac{50\times 50}{830} = \frac{250}{83}$$

Question ID : 7155051241

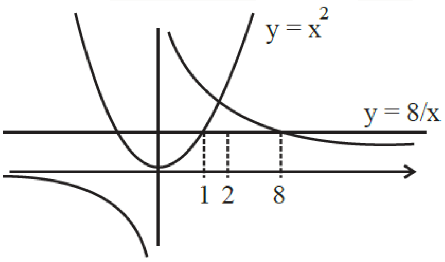
13. The area of the region given by $\{(x, y) : xy \le 8, 1 \le y \le x^2\}$ is :

$\{(x, y) : xy \le 8, 1 \le y \le x^2\}$ द्वारा दिए क्षेत्रा का क्षेत्राफल है :

(1) $8\log_e 2+\frac{7}{6}$ (2) $16\log_e 2-\frac{14}{3}$ (3) $8\log_e 2-\frac{13}{3}$ (4) $16\log_e 2+\frac{7}{3}$

Ans. Official Answer NTA (2)

Sol. $xy \le 8$ $1 \le y \le x^2$



$$\text{Area} = \int_1^2 \left(x^2 - 1\right)dx + \int_2^8 \left(\frac{8}{x} - 1\right)dx$$

$$= \left(\frac{x^3}{3}\right)_1^2 + 8\left(\ell nx\right)_2^8 - \left(x\right)_1^8$$

$$= \frac{7}{3} + 8\left(2\ell n2\right) - 7$$

$$=16\ell n2-\frac{14}{3}$$

Question ID : 7155051234

14. If $A=\frac{1}{2}\begin{bmatrix}1 & \sqrt{3}\\ -\sqrt{3} & 1\end{bmatrix}$, then :

यदि $A=\frac{1}{2}\begin{bmatrix}1 & \sqrt{3}\\ -\sqrt{3} & 1\end{bmatrix}$ है, तो :

(1) $A^{30}-A^{25}=2I$ (2) $A^{30}+A^{25}+A=I$

(3) $A^{30}+A^{25}-A=I$ (4) $A^{30}=A^{25}$

Ans. Official Answer NTA (3)

Sol. $$A=\begin{bmatrix}\frac{1}{2} & \frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & \frac{1}{2}\end{bmatrix}=\begin{bmatrix}\cos 60^{\circ} & \sin 60^{\circ}\\ -\sin 60^{\circ} & \cos 60^{\circ}\end{bmatrix} \quad ...(1)$$

$\because A(\alpha).A(\beta)=A(\alpha+\beta)$

So by above

$$A^{30}\left(\frac{\pi}{3}\right)=\begin{bmatrix}\cos 30\left(\frac{\pi}{3}\right) & \sin 30\left(\frac{\pi}{3}\right)\\ -\sin 30\left(\frac{\pi}{3}\right) & \cos 30\left(\frac{\pi}{3}\right)\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

$$A^{25}\left(\frac{\pi}{3}\right)=\begin{bmatrix}\cos 25\left(\frac{\pi}{3}\right) & \sin 25\left(\frac{\pi}{3}\right)\\ -\sin 25\left(\frac{\pi}{3}\right) & \cos 25\left(\frac{\pi}{3}\right)\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & +\frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & \frac{1}{2}\end{bmatrix}=A$$

$A^{30}+A^{25}-A=I$

Question ID : 7155051238

15. If $y(x) = x^x, x > 0$, then y"(2) – 2y'(2) is equal to :

यदि $y(x) = x^x, x > 0$ है, तो y"(2) – 2y'(2) बराबर है :

(1) $4 (\log_e 2)^2 + 2$ (2) $4 \log_e 2 + 2$

(3) $8 \log_e 2 - 2$ (4) $4 (\log_e 2)^2 - 2$

Ans. Official Answer NTA (4)

Sol. $y' = x^x$

$y' = x^x + (1 + \ln x)$

$y'' = x^x (1 + \ln x)^2 + x^x . \frac{1}{x}$

$y''(2) = 4(1 + \ln 2)^2 + 2$

$y'(2) = 4(1 + \ln 2)$

$y''(2) - 2y'(2) = 4(1 + \ln 2)^2 + 2 - 8(1 + \ln 2)$

$= 4(1 + \ln 2) [1 + \ln 2 - 2] + 2$

$= 4((\ln 2)^2 - 1) + 2$

$= 4(\ln 2)^2 - 2$

Question ID : 7155051245

16. Let $\vec{a} = 2\hat{i} - 7\hat{j} + 5\hat{k}$, $\vec{b} = \hat{i} + \hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r} \cdot \vec{b} = 0$, then $|\vec{r}|$ is equal to :

माना तीन सदिश $\vec{a} = 2\hat{i} - 7\hat{j} + 5\hat{k}$, $\vec{b} = \hat{i} + \hat{k}$ तथा $\vec{c} = \hat{i} + 2\hat{j} - 3\hat{k}$ है। यदि एक सदिश $\vec{r}$ के लिए $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ तथा $\vec{r} \cdot \vec{b} = 0$ है, तो $|\vec{r}|$ बराबर है :

(1) $\frac{11}{7}\sqrt{2}$ (2) $\frac{\sqrt{914}}{7}$ (3) $\frac{11}{5}\sqrt{2}$ (4) $\frac{11}{7}$

Ans. Official Answer NTA (1)

Sol. $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$

$\Rightarrow (\vec{r} - \vec{c}) \times \vec{a} = 0$

$\Rightarrow (\vec{r} - \vec{c}) = \lambda \vec{a}$

$\Rightarrow \vec{r} = \vec{c} + \lambda \vec{a}$

$\Rightarrow \vec{r}.\vec{b} = \vec{c}.\vec{b} + \lambda \vec{a}.\vec{b}$

$\Rightarrow 0=(1-3)+\lambda(2+5)$

$2=7\lambda \Rightarrow \lambda=\frac{2}{7}$

$\vec{r}=\vec{c}+\frac{2}{7}\vec{a}$

$=\hat{i}+2\hat{j}-3\hat{k}+\frac{2}{7}\left(2\hat{i}-7\hat{j}+5\hat{k}\right)$

$\vec{r}=\frac{11\hat{i}-11\hat{k}}{7}$

$|\vec{r}|=\frac{\sqrt{121+121}}{7}$

$|\vec{r}|=\frac{11\sqrt{2}}{7}$

Question ID : 7155051232

17. Let a, b be two real numbers such that $ab<0$. If the complex number $\frac{1+ai}{b+i}$ is of unit modulus and $a+ib$ lies on the circle $|z-1|=|2z|$, then a possible value of $\frac{1+[a]}{4b}$, where $[t]$ is greatest integer

माना a, b दो वास्तविक संख्याऐं है, जिन के लिए $ab<0$ है। यदि सम्मिश्र संख्या $\frac{1+ai}{b+i}$ इकाई मापक की है तथा $a+ib$ वृत्त, $|z-1|=|2z|$ पर है, तो $\frac{1+[a]}{4b}$ जहाँ $[t]$ महत्तम पूर्णांक फलन है, का एक संभव मान है :

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) 1 (4) –1

Ans. Official Answer NTA (Drop)

Sol. $ab<0, \left|\frac{1+ai}{b+i}\right|=1$

$|1+ai|=|b+i|$

$a^2+1=b^2+1 \Rightarrow a=\pm b \Rightarrow b=-a$ as $ab<0$

(a, b) lies on $|z-1|=|2z|$

$|a+ib-1|=2|a+ib|$

$(a-1)^2+b^2=4(a^2+b^2)$

$(a-1)^2+a^2=4(2a^2)$

$1-2a=6a^2 \Rightarrow 6a^2+2a-1=0$

$a = \frac{-2 \pm \sqrt{28}}{12} = \frac{-1 \pm \sqrt{7}}{6}$

$a = \frac{\sqrt{7} - 1}{6}$ & $b = \frac{1 - \sqrt{7}}{6}$

$[a] = 0$

$\therefore \frac{1 + [a]}{4b} = \frac{6}{4(1 - \sqrt{7})} = -\left(\frac{1 + \sqrt{7}}{4}\right)$

or $[a] = 0$

Similarly it is not matching with $a = \frac{-1 - \sqrt{7}}{6}$

No answer is matching.

Question ID : 7155051235

18. For the system of linear equations $\alpha x + y + z = 1$, $x + \alpha y + z = 1$, $x + y + \alpha z = \beta$, which one of the following statements is NOT true?

(1) It has infinitely many solutions if $\alpha = 1$ and $\beta = 1$

(2) $x + y + z = \frac{3}{4}$ if $\alpha = 2$ and $\beta = 1$

(3) It has no solution if $\alpha = -2$ and $\beta = 1$

(4) It has infinitely many solutions if $\alpha = 2$ and $\beta = -1$

रैखिक समीकरण निकाय $\alpha x + y + z = 1$, $x + \alpha y + z = 1$, $x + y + \alpha z = \beta$ के लिए निम्न में से कौन सा कथन सही नहीं है ?

(1) इसके अनंत हल है यदि $\alpha = 1$ तथा $\beta = 1$ है

(2) $x + y + z = \frac{3}{4}$ यदि $\alpha = 2$ तथा $\beta = 1$ हैं

(3) इसका कोई हल नहीं है यदि $\alpha = -2$ तथा $\beta = 1$ हैं

(4) इसके अनंत हल है यदि $\alpha = 2$ तथा $\beta = -1$ हैं

Ans. Official Answer NTA (4)

Sol. $\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = \alpha\left(\alpha^2-1\right)-\left(\alpha-1\right)+\left(1-\alpha\right) = \alpha^3-3\alpha+2 = \left(\alpha-1\right)\left(\alpha^2+\alpha-2\right)$

$(\alpha-1)(\alpha-1)(\alpha+2)$

for $\alpha = 1$ system of equations is

$x+y+z=1$

$x+y+z=1$

$x+y+z=\beta \Rightarrow \beta = 1$ then infinitely many solutions

θ.f $\alpha = 2, \beta = -1$ then

$\theta x+y+z=1$

$x+2y+z=1$

$x+y+2z=-1$ No solution

Question ID : 7155051239

19. The sum of the absolute maximum and minimum values of the function $f(x) = |x^2-5x+6|-3x+2$ in the interval [-1,3] is equal to :

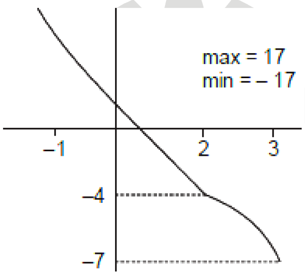
अंतराल [-1,3] में फलन $f(x) = |x^2-5x+6|-3x+2$ के निरपेक्ष उच्चतम और निरपेक्ष निम्नतम मानों का योग बराबर है:

(1) 12 (2) 24 (3) 10 (4) 13

Ans. Official Answer NTA (3)

Sol. $f(x) = |x^2-5x+6|-3x+2$

$$f(x) = \begin{cases} x^2-8x+8 & ; \quad x[-1,2] \\ -x^2+2x-4 & ; \quad x \in [2,3] \end{cases}$$



$$f'(x) = \begin{Bmatrix} 2x-8; & x \in [-1,2] \\ -2x+2; & x \in [2,3] \end{Bmatrix}$$

$f(2) = 4-16+8 = -4 = 4$

$f(-1) = 17$ (max)

$f(3) = -9+6-4 = -7$ (min)

Abs. Sum = 10

Question ID : 7155051247

20. Let $9 = x_1 < x_2 < ...... < x_7$ be in an A.P. with common difference d. If the standard deviation of $x_1, x_2, ..., x_7$ is 4 and the mean is $\overline{x}$, then $\overline{x} + x_6$ is equal to :

माना $9 = x_1 < x_2 < ...... < x_7$ समान्तर श्रेणी में हैं, जिसका सार्वान्तर d है। यदि $x_1, x_2, ..., x_7$ का मानक विचलन 4 तथा माध्य $\overline{x}$ तब $\overline{x} + x_6$ बराबर होगा :

(1) 25 (2) $2\left(9+\frac{8}{\sqrt{7}}\right)$ (3) $18\left(1+\frac{1}{\sqrt{3}}\right)$ (4) 34

Ans. Official Answer NTA (4)

Sol. Mean $\Rightarrow \overline{x} = \frac{\sum_{i=1}^{7} x_i}{7} = \frac{\frac{7}{2}[2a+6d]}{7} = a + 3d = x_4$

Variance $= \frac{\sum_{i=1}^{7}(x_i - \overline{x})^2}{7} = (4)^2 \Rightarrow \frac{\sum_{i=1}^{7}(x_i - x_4)^2}{7} = 16$

$\Rightarrow \frac{(3d)^2 + (2d)^2 + d^2 + 0 + d^2 + (2d)^2 + (3d)^2}{7} = 16$

$= 4d^2 = 16 \quad \Rightarrow \quad d = 2$

$\Rightarrow \overline{x} = 9 + 3(2) = 15$

& $x_6 = a + 5d = 9 + 5(2) = 19 \Rightarrow \overline{x} + x_6 = 34$

SECTION – B

Question ID : 7155051258

21. If the x-intercept of a focal chord of the parabola $y^2 = 8x + 4y + 4$ is 3, then the length of this chord is equal to ___________ .

यदि परवलय $y^2 = 8x + 4y + 4$ की नाभीय जीवा का x-अंतःखंड 3 है, तो इस जीवा की लंबाई बराबर है।

Ans. Official Answer NTA (16)

Sol. $y^2 = 8x + 4y + 4$

$(y-2)^2 = 8(x+1)$

$y^2 = 4ax$

$a = 2, X = x + 1, Y = y - 2$

focus (1, 2)

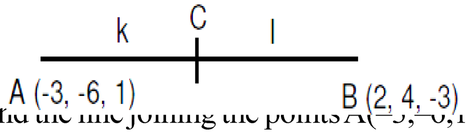
y – 2 = m (x – 1)

Put (3, 0) in the above line

m = –1

Length of focal chord = 16

Question ID : 7155051260



22. The point of intersection C of the plane 8x + y + 2z = 0 and the line joining the points A(–3,–6,1) and B(2,4,–3) divides the line segment AB internally in the ratio k : 1. If a, b, c (|a|, |b|, |c| are coprime) are the direction ratios of the perpendicular from the point C on the line $\frac{1-x}{1}=\frac{y+4}{2}=\frac{z+2}{3}$, then |a + b + c| is equal to ___________ .

समतल 8x + y + 2z = 0 तथा बिंदुओं A(–3,–6,1) और B(2,4,–3) को मिलाने वाली रेखा का प्रतिच्छेदन बिंदु C रेखाखंड AB को k : 1 के अनुपात में अंतः विभाजित करता है। यदि बिंदु C से रेखा $\frac{1-x}{1}=\frac{y+4}{2}=\frac{z+2}{3}$ पर अभिलंब के दिक् अनुपात a, b, c (|a|, |b|, |c| असहभाज्य है) हैं, तो |a + b + c| बराबर है।

Ans. Official Answer NTA (10)

Sol. $C\left(\frac{2k-3}{k+1},\frac{4k-6}{k+1},\frac{-3k+1}{k+1}\right)$

C lies on 8x + y + 2z = 0

$\therefore$ 8 (2k – 3) + (4k – 6) + 2 (–3k + 1) = 0

16k – 24 + 4k – 6 – 6k + 2 = 0

14k = 28 $\Rightarrow$ k = 2

$\therefore$ C is $\left(\frac{1}{3},\frac{2}{3},\frac{-5}{3}\right)$

Any point on line $\frac{x-1}{-1}=\frac{y+4}{2}=\frac{z+2}{2}$ is D $(1-\lambda,-4+2\lambda,-2+3\lambda)$

Drs of CD = $1-\lambda-\frac{1}{3},-4+2\lambda-\frac{2}{3},-2+3\lambda+\frac{5}{3}$

$=\frac{2-3\lambda}{3},\frac{-14+6\lambda}{3},\frac{-1+9\lambda}{3}$

CD $\perp$ to line : $-1(2-3\lambda)+2(6\lambda-14)+3(9\lambda-1)=0$

$\Rightarrow -2+3\lambda+12\lambda-28+27\lambda-3=0$

$42\lambda = 33 \Rightarrow \lambda = \frac{33}{42} = \frac{11}{14}$

$\therefore$ drs of CD $= \frac{2-\frac{33}{14}}{3}, \frac{-14+\frac{66}{14}}{3}, \frac{-1+\frac{99}{14}}{3}$

$= -5, -130, 85$

$= -1, -26, 17$

$\therefore (-1 - 26 + 17) = 10$

Question ID : 7155051255

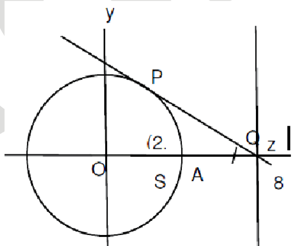
23. Number of integral solutions to the equation $x + y + z = 21$, where $x \geq 1, y \geq 3, z \geq 4$ is equal to _________ .

समीकरण $x + y + z = 21$ जहाँ $x \geq 1, y \geq 3, z \geq 4$ है, के पूर्णांकीय हलों की संख्या है।

Ans. Official Answer NTA (105)

Sol. $^{15}C_2 = \frac{15 \times 14}{2} = 105$

Question ID : 7155051259



24. The line $x = 8$ is the directrix of the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with the corresponding focus (2,0). If the tangent to E at the point P in the first quadrant passes through the point $(0, 4\sqrt{3})$ and intersects the x-axis at Q, then $(3PQ)^2$ is equal to __________ .

दीर्घवृत्त $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ की नियता $x = 8$ है तथा संगत नाभि (2,0) है। यदि प्रथम चतुर्थांश में E के बिंदु P पर स्पर्श रेखा, बिंदु $(0, 4\sqrt{3})$ से होकर जाती है तथा x-अक्ष को Q पर काटती है, तो $(3PQ)^2$ बराबर है।

Ans. Official Answer NTA (39)

Sol. $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$ae = 2$

$\frac{a}{e} = 8$

$e = \frac{1}{2}$ $\quad a^2 = 16 \quad a = 4$

$$b^2 = a^2\left(1 - e^2\right) = 16\left(1 - \frac{1}{4}\right) = 16 \times \frac{3}{4} = 12$$

$\therefore$ Ellipse is $\frac{x^2}{16} + \frac{y^2}{12} = 1$

Tangent at P is $\frac{x\cos\theta}{4} + \frac{y\sin\theta}{2\sqrt{3}} = 1$

Passing through $\left(0, 4\sqrt{3}\right)$

$$\therefore 0 + \frac{4\sqrt{3}\sin\theta}{2\sqrt{3}} = 1$$

$$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30^o$$

$\therefore$ Tangent is $\frac{x}{4}.\frac{\sqrt{13}}{2} + \frac{y}{2\sqrt{3}}.\frac{1}{2} = 1$

$$\frac{\sqrt{3}x}{8} + \frac{y}{4\sqrt{3}} = 1$$

$Q\left(\frac{8}{\sqrt{3}}, 0\right)$ $P\left(4\cos\theta, 2\sqrt{3}\sin\theta\right)$

$P\left(2\sqrt{3}, \sqrt{3}\right)$

$$PQ = \sqrt{\left(\frac{8}{\sqrt{3}} - 2\sqrt{3}\right)^2 + 3} = \sqrt{\left(\frac{2}{\sqrt{3}}\right)^2 + 3} = \sqrt{\frac{4}{3} + 3}$$

$$= \sqrt{\frac{13}{3}}$$

$$PQ^2 = \frac{13}{3} \Rightarrow (3PQ)^2 = \frac{13}{3} \times 9 = 39$$

Question ID : 7155051254

25. If the term without x in the expansion of $\left(x^{\frac{2}{3}} + \frac{\alpha}{x^3}\right)^{22}$ is 7315, then $|\alpha|$ is equal to ________ .

यदि $\left(x^{\frac{2}{3}}+\frac{\alpha}{x^{3}}\right)^{22}$ के प्रसार में x से स्वतंत्रा पद 7315 है, तो $|\alpha|$ बराबर है।

Ans. Official Answer NTA (1)

Sol. $T_{r+1} = {}^{22}C_r \cdot \left(x^{\frac{2}{3}}\right)^{22-r} .(\alpha)^r, x^{-3r}$

$= {}^{22}C_r . x^{\frac{44}{3}-\frac{2r}{3}-3r}(\alpha)^r$

$\frac{44}{3} = \frac{11r}{3}$

$r = 4$

${}^{22}C_4 . ga^4 = 7315$

$\frac{22\times21\times20\times19}{24}.\alpha^4 = 7315$

$\alpha = 1$

Question ID : 7155051256

26. If $\int_0^{\pi} \frac{5^{\cos x}\left(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x\right)dx}{1+5^{\cos x}} = \frac{k\pi}{16}$, then k is equal to ________ .

यदि $\int_0^{\pi} \frac{5^{\cos x}\left(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x\right)dx}{1+5^{\cos x}} = \frac{k\pi}{16}$ है, तो k बराबर है।

Ans. Official Answer NTA (26)

Sol. $I = \int_0^{\pi} \frac{5^{\cos x}\left(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x\right)dx}{1+5^{\cos x}}$

$= 2\int_0^{\pi/2}\left(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x\right)dx$

$= \int_0^{\pi/2} 2+\cos 4x+\cos 2x+1+\cos 2x+2\cos 3x\left(\frac{\cos 3x+3\cos x}{4}\right)dx$

$$= \int_0^{\pi/2} 3dx + 0 + 0 + 0 + \int_0^{\pi/2} \frac{1}{2}\left(\cos^2 3x + 3\cos x \cos 3x\right)dx$$

$$= \frac{3\pi}{2} + \frac{1}{4}\int_0^{\pi/2} \left(1 + \cos 6x + 3\left(\cos 4x + \cos 2x\right)\right)dx$$

$$= \frac{3\pi}{4} + \frac{1}{4}\cdot\frac{\pi}{2} = \frac{13\pi}{8} = \frac{k\pi}{16}$$

$\Rightarrow k = 26$

Question ID : 7155051252

27. Let the sixth term in the binomial expansion of $\left(\sqrt{2^{\log_2\left(10-3^x\right)}} + \sqrt[5]{2^{(x-2)\log_2 3}}\right)^m$, in the increasing powers of $2^{(x-2)\log_2 3}$, be 21. If the binomial coefficients of the second, third and fourth terms in the expansion are respectively the first, third and fifth terms of an A.P., then the sum of the squares of all possible values of x is __________ .

माना $2^{(x-2)\log_2 3}$ की बढ़ती घातों में $\left(\sqrt{2^{\log_2\left(10-3^x\right)}} + \sqrt[5]{2^{(x-2)\log_2 3}}\right)^m$ के द्विपद प्रसार में छठा पद 21 है। यदि इस प्रसार में दूसरा, तीसरा तथा चौथा द्विपद गुणांक एक A.P. के क्रमशः पहला, तीसरा तथा पाँचवा पद है, तो x के सभी संभव मानों के वर्गो का योग है।

Ans. Official Answer NTA (4)

Sol. $T_6 = {}^mC_5\left(10-3^x\right)^{\frac{m-5}{2}}\cdot\left(3^{x-2}\right) = 21$ ...(1)

${}^mC_1, {}^mC_2, {}^mC_3$ are in A.P.

$2.{}^mC_2 = {}^mC_1 + {}^mC_3$

Solving for m, we get

m = 2 (rejected), 7

Put in equation (1)

$$21.\left(10-3^x\right)\frac{3^x}{9} = 21$$

$3^x = 3^0, 3^2$

x = 0, 2

Sum of all the squares of all possible values of x = 4

Question ID : 7155051253

28. The sum of the common terms of the following three arithmetic progressions.

3,7,11,15,........,399,

2,5,8,11,.......,359 and

2,7,12,17,........,197,

is equal to _____________ .

तीन समांतर श्रेणियों

3,7,11,15,........,399,

2,5,8,11,.......,359 तथा

2,7,12,17,........,197,

के उभयनिष्ठ पदों का योग है।

Ans. Official Answer NTA (321)

Sol. $S_1 = \{2,5,8,11,14, \_\_\_\_\_, 359\}$ S_2

$S_2 = \{3,7,11,15, \_\_\_\_\_\_, 239\}$ S_1

$S_3 = \{7,12,17,22, \_\_\_\_\_, 197\}$ S_3

common AP

$S = \{47,107,167\}$

Hence sum of common AP = 321

Question ID : 7155051257

29. Let $\alpha x + \beta y + \gamma z = 1$ be the equation of a plane passing through the point (3,-2,5) and perpendicular to the line joining the points (1,2,3) and (-2,3,5). Then the value of $\alpha\beta\gamma$ is equal to __________ .

माना बिंदु (3, –2, 5) से होकर जाने वाले तथा बिंदुओं (1,2,3) और (3,-2,5) को मिलाने वाली रेखा के लंबवत समतल का समीकरण $\alpha x + \beta y + \gamma z = 1$ है। तो $\alpha\beta\gamma$ का मान बराबर है।

Ans. Official Answer NTA (6)

Sol. Given Equation is not equation of plane as γz is present. If we consider y is γ then answer would be 6.

Normal vector of plane = $3\hat{i} - \hat{j} - 2\hat{k}$

Plane : $3x - y - 2z + \lambda = 0$

Point (3 –2, 5) satisfies the plane

$\lambda = -1$

$3x - y - 2z = 1$

$\alpha\beta\gamma = 6$

Question ID : 7155051251

30. The total number of six digit numbers, formed using the digits 4, 5, 9 only and divisible by 6, is ________ .

केवल अंकों 4, 5, 9 के प्रयोग से बनायी गई छः अंको की संख्याएँ, जो 6 से विभाज्य है, की कुल संख्या है।

Ans. Official Answer NTA (81)

Sol. Unit digit must be 4 since number should be divisible by 2.
Four out of remaining five places, each has 3 options and remaining one place will have only one option so total number of six digit numbers = 3.3.3.3.1 = 81