JEE Main January 2023

Question Paper With Text Solution

01 February | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911
Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in

JEE MAIN FEBRUARY 2023 | 1ST FEBRUARY SHIFT-1

SECTION – A

Question ID : 3666942554

1. If $y = y(x)$ is the solution curve of the differential equation $\frac{dy}{dx} + y\tan x = x\sec x$, $0 \le x \le \frac{\pi}{3}$, $y(0) = 1$, then $y\left(\frac{\pi}{6}\right)$ is equal to :

यदि अवकल समीकरण $\frac{dy}{dx} + y\tan x = x\sec x$, $0 \le x \le \frac{\pi}{3}$, $y(0) = 1$ का हल वक्र $y = y(x)$ है, तो $y\left(\frac{\pi}{6}\right)$ का मान है :

(1) $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$ (2) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$

(3) $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$ (4) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

Ans. Official Answer NTA (3)

Sol. Here I.F. = sec x

Then solution of D.E.

$y(\sec x) = x\tan x - \ln(\sec x) + c$

Given $y(0) = 1 \Rightarrow c = 1$

$\therefore y(\sec x) = x\tan x - \ln(\sec x) + 1$

At $x = \frac{\pi}{6}$, $y = \frac{\pi}{12} + \frac{\sqrt{3}}{2}\ln\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$

$y = \frac{\pi}{12} + \frac{\sqrt{3}}{2}\left(\log\frac{\sqrt{3}}{2} + \log e\right)$

$= \frac{\pi}{12} - \frac{\sqrt{3}}{2}\log\left(\frac{2}{e\sqrt{3}}\right)$

Question ID : 3666942558

2. The mean and variance of 5 observations are 5 and 8 respectively. If 3 observations are 1, 3, 5 then the sum of cubes of the remaining two observations is :

5 प्रेक्षणों का माध्य एवं प्रसरण क्रमशः 5 एवं 8 है। यदि तीन प्रेक्षण 1, 3, 5 हैं, तब शेष दो प्रेक्षणों के घनों का योग है :

(1) 1216 (2) 1792 (3) 1456 (4) 1072

Ans. Official Answer NTA (4)

Sol. Let remaining two obervations are α, β.

$$\text{Mean} = \frac{1+3+5+\alpha+\beta}{5} = 5$$

$\alpha + \beta = 16 \qquad ...(1)$

$$\sigma^2 = \frac{1+9+25+\alpha^2+\beta^2}{5} - 5^2$$

$$8 = \frac{35+\alpha^2+\beta^2}{5} - 25$$

$33 \times 5 - 35 = \alpha^2 + \beta^2$

$\alpha^2 + \beta^2 = 130 \qquad ...(2)$

$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$256 - 130 = 2\alpha\beta \Rightarrow \alpha\beta = 63$

$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= 4096 - 3024$

$= 1072$

Question ID : 3666942541

3. Let R be relation on R, given by $R = \{(a, b) : 3a - 3b + \sqrt{7}$ is an irrational number$\}$.

Then R is :

(1) reflexive and symmetric but not transitive

(2) reflexive but neither symmetric nor transitive

(3) an equivalence relation

(4) reflexive and transitive but not symmetric

माना IR में एक सम्बन्ध R है जो निम्न प्रकार दिया गया है $R = \{(a, b) : 3a - 3b + \sqrt{7}$ अपरिमेय संख्या है$\}$।

तब R

(1) स्वतुल्य और सममित है परन्तु संक्रामक नहीं है

(2) स्वतुल्य है परन्तु न तो सममित है और न ही संक्रामक है

(3) एक तुल्यता सम्बन्ध है

(4) स्वतुल्य और संक्रामक है परन्तु सममित नहीं है

Ans. Official Answer NTA (2)

Sol. Check for reflexivity :

As $3(a - a) + \sqrt{7} = \sqrt{7}$ which belongs to relation

so relation is reflexive
Check for symmetric

Take $a = \frac{\sqrt{7}}{3}, b = 0$

Now (a, b) $\in$ R but (b, a) $\notin$ R

As $3(b-a) + \sqrt{7} = 0$ which is rational so relation is not symmetric.

Check for transitivity :

Take (a, b) as $\left(\frac{\sqrt{7}}{3}, 1\right)$

& (b, c) as $\left(1, \frac{2\sqrt{7}}{3}\right)$

So now (a, b) $\in$ R & (b, c) $\in$ R but (a, c) $\notin$ R which means relation is not transitive.

Question ID : 3666942542

4. If the center and radius of the circle $\left|\frac{z-2}{z-3}\right| = 2$ are respectively (α, β) and γ, then $3(\alpha + \beta + \gamma)$ is equal to :

यदि वृत्त $\left|\frac{z-2}{z-3}\right| = 2$ के केन्द्र एवं त्रिज्या क्रमशः (α, β) एवं γ है, तब $3(\alpha + \beta + \gamma)$ का मान है :

(1) 9 (2) 11 (3) 10 (4) 12

Ans. Official Answer NTA (4)

Sol. $\left|\frac{z-2}{z-3}\right| = 2$

$(z-2)(\bar{z}-2) = 4(\bar{z}-3)(z-3)$

$\Rightarrow z\bar{z} - 2z - 2\bar{z} + 4 = 4\left(z\bar{z} - 3z - 3\bar{z} + 9\right)$

$3z\bar{z} - 10z - 10\bar{z} + 32 = 0$

$z\bar{z} - \frac{10}{3}z - \frac{10}{3}\bar{z} + \frac{32}{3} = 0$

$\alpha = -\frac{10}{3}$ $c = \frac{32}{3}$

centre $\left(\frac{10}{3}, 0\right)$ $\gamma = \sqrt{|a|^2 - c} = \sqrt{\frac{100}{9} - \frac{32}{3}} = \frac{2}{3}$

$\alpha = \frac{10}{3}, \beta = 0, \gamma = \frac{2}{3}$

$\therefore \alpha + \beta + \gamma = \frac{10}{3} + \frac{2}{3} = \frac{12}{3}$ $\therefore 3(\alpha + \beta + \psi) = 12$

Question ID : 3666942559

5. Let S be the set of all solutions of the equation $\cos^{-1}(2x) - 2\cos^{-1}\left(\sqrt{1-x^2}\right) = \pi$, $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$. Then $\sum_{x \in S} 2\sin^{-1}(x^2 - 1)$ is equal to :

माना समीकरण $\cos^{-1}(2x) - 2\cos^{-1}\left(\sqrt{1-x^2}\right) = \pi$, $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ के सभी हलों का समुच्चय S है। तब $\sum_{x \in S} 2\sin^{-1}(x^2 - 1)$ का मान है :

(1) 0 (2) $\pi - \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (3) $\frac{-2\pi}{3}$ (4) $\pi - 2\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

Ans. Official Answer NTA (3)

Sol. $\cos^{-1}(2x) = \pi + 2\cos^{-1}\left(\sqrt{1-x^2}\right)$

LHS = $[0, \pi]$
For equation to be meaningful

$\cos^{-1} 2x = \pi$ and $\cos^{-1}\left(\sqrt{1-x^2}\right) = 0$

$x = \frac{-1}{2}$ and $x = 0$

which is not possible
$\therefore x \in \phi$
Now $\Sigma(x) = 0$
$\therefore$ Sum over empty set is always 0

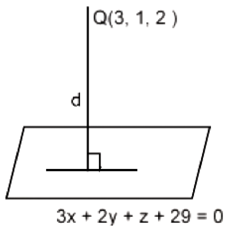
Question ID : 3666942555

6. Let the image of the point P(2, –1, 3) in the plane x + 2y – z = 0 be Q. Then the distance of the plane $3x + 2y + z + 29 = 0$ from the point Q is :

माना समतल $x + 2y - z = 0$ में बिंदु $P(2, -1, 3)$ का प्रतिबिंब Q है। तब बिंदु Q से समतल $3x + 2y + z + 29 = 0$ की दूरी

है :

(1) $2\sqrt{14}$ (2) $3\sqrt{14}$ (3) $\frac{24\sqrt{2}}{7}$



Ans. Official Answer NTA (2)

Sol. Image of point P(2, –1, 3) in palne $x + 2y - z = 0$ be

$$\frac{x-2}{1}=\frac{y+1}{2}=\frac{z-3}{-1}=\frac{-2(2-2-3)}{(1)^2+(2)^2+(-1)^2}$$

$$\frac{x-2}{1}=\frac{y+1}{2}=\frac{z-3}{-1}=\frac{6}{6}=1$$

so, $x - 2 = 1, y + 1 = 2, z - 3 =- -1$

$\therefore\ x = 3, y = 1, z = 2$

So : Q (3, 1, 2)

$$\therefore d=\left|\frac{3(3)+2(1)+2+29}{\sqrt{3^2+2^2+1^2}}\right|$$

$$d=\left|\frac{9+4+29}{\sqrt{14}}\right|$$

$$d=\frac{42}{\sqrt{14}}=3\sqrt{14}$$

Question ID : 3666942549

7. Let $f(x) = 2x + \tan^{-1}x$ and $g(x)=\log_e\left(\sqrt{1+x^2}+x\right), x \in [0, 3]$. Then :

(1) $\min f'(x) = 1 + \max g'(x)$

(2) there exist $0 < x_1 < x_2 < 3$ such that $f(x) < g(x),\ \forall\ x \in (x_1, x_2)$

(3) there exists $\hat{x} \in [0,3]$ such that $f'(\hat{x}) < g'(\hat{x})$

(4) $\max f(x) > \max g(x)$

माना $f(x) = 2x + \tan^{-1}x$ एवं $g(x)=\log_e\left(\sqrt{1+x^2}+x\right), x \in [0, 3]$ है। तब

(1) $\min f'(x) = 1 + \max g'(x)$

(2) $0 < x_1 < x_2 < 3$ का अस्तित्व है, जिनके लिए $f(x) < g(x),\ \forall\ x \in (x_1, x_2)$ है।

(3) $\hat{x} \in [0,3]$ का अस्तित्व है, जिसके लिए $f'(\hat{x}) < g'(\hat{x})$ है।

(4) max f(x) > max g(x)

Ans. Official Answer NTA (4)

Sol. $f(x)=2x+\tan^{-1}x$ and $g(x)=\ln\left(\sqrt{1+x^2}+x\right)$

and $x \in [0, 3]$

$$g'(x)=\frac{1}{\sqrt{1+x^2}}$$

Now, $0 \le x \le 3$

$0 \le x^2 \le 9$

$1 \le 1 + x^2 \le 10$

So, $2+\frac{1}{10} \le f'(x) \le 3$

$\frac{21}{10} \le f'(x) \le 3$ and $\frac{1}{\sqrt{10}} \le g'(x) \le 1$

option (4) is icorrect

From above, $g'(x) < f'(x)\ \forall\ x \in [0, 3]$

Option (1) is incorrect

f'(x) & g'(x) both positive so f(x) & g(x) both are increasing

So, max (f(x) at x = 3 is $6 + \tan^{-1} 3$)

Max (g(x) at x = 3 is $\ln\left(3+\sqrt{10}\right)$

And $6 + \tan^{-1} 3 > \ln\left(3+\sqrt{10}\right)$

Option (4) is correct.

Question ID : 3666942543

8. Let $S = \{x : x \in R$ and $\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}+\left(\sqrt{3}-\sqrt{2}\right)^{x^2-4}=10\}$. Then n(S) is equal to :

माना $S = \{x : x \in R$ एवं $\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}+\left(\sqrt{3}-\sqrt{2}\right)^{x^2-4}=10\}$ है। तब n(S) बराबर है :

(1) 0 (2) 2 (3) 6 (4) 4

Ans. Official Answer NTA (4)

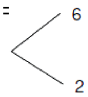
Sol. $\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}+\left(\sqrt{3}-\sqrt{2}\right)^{x^2-4}=10$

$$\left(\left(\sqrt{3}+\sqrt{2}\right)^2\right)^{\frac{x^2-4}{2}}+\left(\left(\sqrt{3}-\sqrt{2}\right)^2\right)^{\frac{x^2-4}{2}}=10$$

$$\Rightarrow\left(5+2\sqrt{6}\right)^{\frac{x^2-4}{2}}+\left(5-2\sqrt{6}\right)^{\frac{x^2-4}{2}}=10$$

Now $\left(5+2\sqrt{6}\right)\left(5-2\sqrt{6}\right)=25-24=$

& $\left(5+2\sqrt{6}\right)+\left(5-2\sqrt{6}\right)=10$



$\therefore\ \frac{x^2-4}{2}=\pm1 \qquad \Rightarrow x^2=4\pm2$

$\Rightarrow x=\pm\sqrt{6},\pm\sqrt{2}$

$\therefore\ n(S)=4$

Question ID : 3666942556

9. The shortest distance between the lines $\frac{x-5}{1}=\frac{y-2}{2}=\frac{z-4}{-3}$ and $\frac{x+3}{1}=\frac{y+5}{4}=\frac{z-1}{-5}$ is :

रेखाओं $\frac{x-5}{1}=\frac{y-2}{2}=\frac{z-4}{-3}$ एवं $\frac{x+3}{1}=\frac{y+5}{4}=\frac{z-1}{-5}$ के बीच न्यूनतम दूरी है :

(1) $5\sqrt{3}$ (2) $7\sqrt{3}$ (3) $4\sqrt{3}$ (4) $6\sqrt{3}$

Ans. Official Answer NTA (D)

Sol. Shortest distance between two lines

$\frac{x-x_1}{a_1}=\frac{y-y_1}{a_2}=\frac{z-z_1}{a_3}$ &

$\frac{x-x_2}{b_1}=\frac{y-y_2}{b_2}=\frac{z-z_2}{b_3}$ is given as

$$\frac{\begin{vmatrix} x_1-x_2 & y_1-y_2 & z_1-z_2 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}}{\sqrt{\left(a_1b_3-a_3b_2\right)^2+\left(a_1b_3-a_3b_1\right)^2+\left(a_1b_2-a_2b_1\right)^2}}$$

$$\frac{\begin{vmatrix} 5-(3) & 2-(-5) & 4-1 \\ 1 & 2 & -3 \\ 1 & 4 & -5 \end{vmatrix}}{\sqrt{(-10+12)^2+(-5+3)^2+(4-2)^2}}$$

$$\frac{\begin{vmatrix} 8 & 7 & 3 \\ 1 & 2 & -3 \\ 1 & 4 & -5 \end{vmatrix}}{\sqrt{(2)^2+(2)^2+(2)^2}}$$

$$=\frac{|8(-10+12)-7(-5+3)+3(4-2)|}{\sqrt{4+4+4}}$$

$$=\frac{16+14+6}{\sqrt{12}}=\frac{36}{\sqrt{12}}=\frac{36}{2\sqrt{3}}$$

$$=\frac{18}{\sqrt{3}}=6\sqrt{3}$$

Question ID : 3666942550

10. $\lim_{n\to\infty}\left[\frac{1}{1+n}+\frac{1}{2+n}+\frac{1}{3+n}+.....+\frac{1}{2n}\right]$ is equal to :

$\lim_{n\to\infty}\left[\frac{1}{1+n}+\frac{1}{2+n}+\frac{1}{3+n}+.....+\frac{1}{2n}\right]$ का मान है :

(1) $\log_e 2$ (2) $\log_e\left(\frac{2}{3}\right)$ (3) 0 (4) $\log_e\left(\frac{3}{2}\right)$

Ans. Official Answer NTA (1)

Sol. $\lim_{n\to\infty}\left(\frac{1}{1+n}+\frac{1}{2+n}+....+\frac{1}{2n}\right)=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{r+n}$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{\left(\frac{r}{n}+1\right)}\cdot\frac{1}{n}$$

$$\int_0^1 \frac{dx}{1+x} = \left[\ln(1+x)\right]_0^1$$

$\ln 2 - \ln 1 = \ln 2$

Question ID : 3666942547

11. The value of $\frac{1}{1!50!}+\frac{1}{3!48!}+\frac{1}{5!46!}+.......+\frac{1}{49!2!}+\frac{1}{51!1!}$ is :

$\frac{1}{1!50!}+\frac{1}{3!48!}+\frac{1}{5!46!}+.......+\frac{1}{49!2!}+\frac{1}{51!1!}$ का मान है :

(1) $\frac{2^{51}}{51!}$ (2) $\frac{2^{50}}{51!}$ (3) $\frac{2^{51}}{50!}$ (4) $\frac{2^{50}}{50!}$

Ans. Official Answer NTA (2)

Sol. $$\sum_{r=1}^{26}\frac{1}{(2r-1)!(51-(2r-1))!} = \sum_{r=1}^{26} {}^{51}C_{(2r-1)}\frac{1}{51!}$$

$$= \frac{1}{51!}\left\{{}^{51}C_1 + {}^{51}C_3 + ... + {}^{51}C_{51}\right\} = \frac{1}{51!}\left(2^{50}\right)$$

Question ID : 3666942548

12. The sum to 10 terms of the series $\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}.......$ is :

श्रेणी $\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}.......$ के दस पदों का योग :

(1) $\frac{55}{111}$ (2) $\frac{56}{111}$ (3) $\frac{59}{111}$ (4) $\frac{58}{111}$

Ans. Official Answer NTA (1)

Sol. $$t_n = \frac{n}{1+n^2+n^4} = \frac{n}{\left(n^2+n+1\right)\left(n^2-n+1\right)}$$

$$= \frac{1}{2}\left(\frac{1}{\left(n^2-n+1\right)} - \frac{1}{\left(n^2+n+1\right)}\right)$$

$t_1 = \frac{1}{2}\left(1 - \frac{1}{3}\right)$

$t_2 = \frac{1}{2}\left(\frac{1}{3} - \frac{1}{7}\right)$

$t_3 = \frac{1}{2}\left(\frac{1}{7} - \frac{1}{13}\right)$

|
|
|

$t_{10} = \frac{1}{2}\left(\frac{1}{91} - \frac{1}{111}\right)$

$\therefore S_{10} = t_1 + t_2 + t_3 + \ldots + t_{10}$

$= \frac{1}{2}\left(1 - \frac{1}{111}\right) = \frac{55}{111}$

Question ID : 3666942557

13. For a triangle ABC, the value of $\cos 2A + \cos 2B + \cos 2C$ is least. If its inradius is 3 and incentre is M, then which of the following is NOT correct? :

(1) $\overrightarrow{MA} \cdot \overrightarrow{MB} = -18$

(2) perimeter of ΔABC is $18\sqrt{3}$

(3) $\sin 2A + \sin 2B + \sin 2C = \sin A + \sin B + \sin C$

(4) area of ΔABC is $\frac{27\sqrt{3}}{2}$

एक त्रिभुज ABC के लिए $\cos 2A + \cos 2B + \cos 2C$ का मान न्यूनतम है। यदि इसके अंतःवृत्त की त्रिज्या 3 तथा अन्तःवृत्त का केन्द्र M है, तब निम्न में से कौन सा सही नहीं है ?

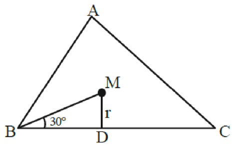
(1) $\overrightarrow{MA} \cdot \overrightarrow{MB} = -18$

(2) ΔABC का परिमाण $18\sqrt{3}$

(3) $\sin 2A + \sin 2B + \sin 2C = \sin A + \sin B + \sin C$

(4) ΔABC का क्षेत्रफल $\frac{27\sqrt{3}}{2}$

Ans. Official Answer NTA (4)

Sol. 

If cos 2A + cos 2B + cos 2C is minimun then A = B = C = 60°

So ΔABC is equilateral

Now in–radias r = 3

So in ΔMBD we have

$$\tan 30^o = \frac{MD}{BD} = \frac{r}{a/2} = \frac{6}{a}$$

$$\frac{1}{\sqrt{3}} = \frac{1}{a} = a = 6\sqrt{3}$$

Perimeter of $\Delta ABC = 18\sqrt{3}$

Area of $\Delta ABC = \frac{\sqrt{3}}{4}a^2 = 27\sqrt{3}$

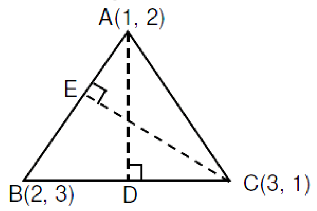
Question ID : 3666942553

14. If the orthocenter of the triangle, whose vertices are (1, 2), (2, 3) and (3, 1) is (α, β), then the quadratic equation whose roots are α + 4β and 4α + β, is :

एक त्रिभुज के शीर्ष (1, 2), (2, 3) और (3, 1) है। यदि इसका लंबकेन्द्र (α, β) है, तब α + 4β और 4α + β किस समीकरण के मूल है :

(1) $x^2 - 19x + 90 = 0$ (2) $x^2 - 20x + 99 = 0$ (3) $x^2 - 18x + 80 = 0$ (4) $x^2 - 22x + 120 = 0$

Ans. Official Answer NTA (2)

Sol. 

$$\text{Slope (AB)} = m_{AB} = \frac{2-3}{1-2} = 1$$

∴ Slope (CE) = –1

∴ Equation of CE is : y – 1 = –1(x – 3)

⇒ y – 1 = –x + 3

⇒ x + y = 4 ...(i)

Similarly $m_{BC} = \frac{2}{-1} = -2$

Equation of AD is : $y - 2 = \frac{1}{2}(x - 1)$

$2y - 4 = x - 1$

$\therefore\ x - 2y + 3 = 0$ ...(ii)

Solving equation (i) & (ii)

We have $x = \frac{5}{3},\ y = \frac{7}{3}$

$\therefore \alpha = \frac{5}{3}, \beta = \frac{7}{3}$

$\therefore \alpha + 4\beta = \frac{5}{3} + \frac{28}{3} = \frac{33}{3} = 11$

$4\alpha + \beta = 4 \times \frac{5}{3} + \frac{7}{3} = \frac{27}{3} = 9$

$\therefore$ Quadratic Equation is

$x^2 -$ (sum of roots)$x +$ (products of roots) $= 0$

$x^2 - 20x + 99 = 0$

Question ID : 3666942560

15. The negation of the expression $q \vee ((\sim q) \wedge p)$ is equivalent to :

$q \vee ((\sim q) \wedge p)$ का निषेधन किस के तुल्य है ?

(1) $p \wedge (\sim q)$ (2) $(\sim p) \wedge (\sim q)$ (3) $(\sim p) \vee (\sim q)$ (4) $(\sim p) \times q$

Ans. Official Answer NTA (2)

Sol. $\sim (q \vee ((\sim q) \wedge p))$

$= \sim q \wedge \sim ((\sim q) \wedge p)$

$= \sim q \wedge (q \vee \sim p)$

$= (\sim q \wedge q) \vee (\sim q \wedge \sim p)$

$= (\sim q \wedge \sim p)$

Question ID : 3666942552

16. The combined equation of the two lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ can be written as $(ax + by + c)(a'x + b'y + c') = 0$. The equation of the angle bisectors of the lines represented by the equation $2x^2 + xy - 3y^2 = 0$ is :

दो रेखाओं $ax + by + c = 0$ तथा $a'x + b'y + c' = 0$ का संयुक्त समीकरण $(ax + by + c)(a'x + b'y + c') = 0$ लिखा जा सकता है। समीकरण $2x^2 + xy - 3y^2 = 0$ से प्राप्त दो रेखाओं के कोण समद्विभाजकों का समीकरण है।

(1) $x^2 - y^2 - 10xy = 0$ (2) $3x^2 + 5xy + 2y^2 = 0$

(3) $x^2 - y^2 + 10xy = 0$ (4) $3x^2 + xy - 2y^2 = 0$

Ans. Official Answer NTA (1)

Sol. Equation of angle bisectors is $\frac{x^2 - y^2}{2+3} = \frac{xy}{\frac{1}{2}}$

$\Rightarrow \frac{x^2 - y^2}{5} = 2xy$

$\Rightarrow x^2 - y^2 = 10xy$

$\Rightarrow x^2 - y^2 - 10xy = 0$

Question ID : 3666942551

17. The area enclosed by the closed curve C given by the differential equation $\frac{dy}{dx} + \frac{x+a}{y-2} = 0$, $y(1) = 0$ is 4π. Let P and Q be the points of intersection of the curve C and the y-axis. If normals at P and Q on the curve C intersect x-axis at points R and S respectively, then the length of the line segment RS is :

अवकल समीकरण $\frac{dy}{dx} + \frac{x+a}{y-2} = 0$, $y(1) = 0$ से प्राप्त संवृत्त वक्र C से घिरे क्षेत्रा का क्षेत्राफल 4π है। माना वक्र C व y अक्ष के प्रतिच्छेदन बिंदु P व Q है। यदि वक्र C के बिंदु P व Q पर अभिलम्ब x अक्ष को क्रमशः बिंदु R व S पर मिलते है, तब रेखा खण्ड RS की लम्बाई है :

(1) $\frac{2\sqrt{3}}{3}$ (2) 2 (3) $2\sqrt{3}$ (4) $\frac{4\sqrt{3}}{3}$

Ans. Official Answer NTA (4)

Sol. $\frac{dy}{dx} + \frac{x+a}{y-2} = 0$

$\frac{dy}{dx} = \frac{x+a}{2-y}$

$(2 - y)\,dy = (x + a)\,dx$

$2y - \frac{y^2}{2} = \frac{x^2}{2} + ax + c$

$a + c = -\frac{1}{2}$ as $y(1) = 0$

$X^2 + Y^2 + 2ax - 4y - 1 - 2a = 0$

$\pi r^2 = 4\pi$

$r^2 = 4$

$4 = a^2 + 4 + 1 + 2a$

$(a + 1)^2 = 0$

$P \text{ or } Q = \left(0, 2 \pm \sqrt{3}\right)$

Equation of normal at P, Q are $y - 2 = \sqrt{3}(x - 1)$

$y - 2 = -\sqrt{3}(x - 1)$

$R = \left(1 - \frac{2}{\sqrt{3}}, 0\right)$

$S = \left(1 + \frac{2}{\sqrt{3}}, 0\right)$

$RS = \frac{4}{\sqrt{3}} = 4\frac{\sqrt{3}}{3}$

Question ID : 3666942546

18. Let S denote the set of all real values of λ such that the system of equations

$\lambda x + y + z = 1$
$x + \lambda y + z = 1$
$x + y + \lambda z = 1$

is inconsistent, then $\sum_{\lambda \in S}\left(|\lambda|^2 + |\lambda|\right)$ is equal to :

माना λ के सभी वास्तविक मानों, जिसके लिए समीकरण निकाय

$\lambda x + y + z = 1$
$x + \lambda y + z = 1$
$x + y + \lambda z = 1$

असंगत है, का समुच्चय S है, तब $\sum_{\lambda \in S}\left(|\lambda|^2 + |\lambda|\right)$ का मान है :

(1) 4 (2) 6 (3) 2 (4) 12

Ans. Official Answer NTA (2)

Sol. $D=\begin{vmatrix}\lambda & 1 & 1\\ 1 & \lambda & 1\\ 1 & 1 & \lambda\end{vmatrix}=(\lambda-1)^2(\lambda+2)$ If $D = 0 \Rightarrow \lambda = 1, -2$

$D_1=\begin{vmatrix}1 & 1 & 1\\ 1 & \lambda & 1\\ 1 & 1 & \lambda\end{vmatrix}=(\lambda-1)^2$ When $\lambda = -2, D \neq 0$

when $\lambda = 1$, all equations are identical so number of solutions are infinite.
So for inconsistent system, λ can be equal to –2 only.

$\therefore \sum_{\lambda\to 5}\left(|\lambda|^2+|\lambda|\right) = |-2|^2 + |-2| = 4 + 2 = 6$

Question ID : 3666942545

19. In a binomial distribution B(n, p), the sum and the product of the mean and the variance are 5 and 6 respectively, then 6(n + p – q) is equal to :

एक द्विपद बंटन B(n, p) में माध्य तथा प्रसरण के योग एवं गुणनफल क्रमशः 5 व 6 हैं, तब 6(n + p – q) का मान है :

(1) 52 (2) 53 (3) 50 (4) 51

Ans. Official Answer NTA (1)

Sol. $np + npq = 5,\ np \cdot npq = 6$

$np(1 + q) = 5,\ n^2p^2q = 6$

$n^2p^2(1 + q)^2 = 25,\ n^2p^2q = 6$

$\frac{6}{q}(1+q)^2 = 25$

$6q^2 + 12q + 6 = 25q$

$6q^2 - 13q + 6 = 0$

$6q^2 - 9q - 4q + 6 = 0$

$(3q - 2)(2q - 3) = 0$

$q=\frac{2}{3},\frac{3}{2}, q=\frac{2}{3}$ is accepted

$p=\frac{1}{3}\Rightarrow n\cdot\frac{1}{3}+n\cdot\frac{1}{3}\cdot\frac{2}{3}=5$

$\frac{3n+2n}{9}=45$

$n = 9$

So $6(n+p-q) = 6\left(9+\frac{1}{3}-\frac{2}{3}\right) = 52$

Question ID : 3666942544

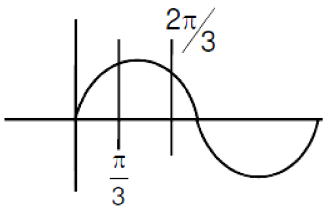
20. Let $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$, $x \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$. If α and β respectively are the maximum and the minimum values of f, then :

माना $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$, $x \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ हैं यदि f के अधिकतम व न्यूनतम मान क्रमशः α व β हैं, तब :

(1) $\beta^2 + 2\sqrt{\alpha} = \frac{19}{4}$ (2) $\alpha^2 - \beta^2 = 4\sqrt{3}$ (3) $\beta^2 - 2\sqrt{\alpha} = \frac{19}{4}$ (4) $\alpha^2 + \beta^2 = \frac{9}{2}$

Ans. Official Answer NTA (3)

Sol. $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin(2x) \\ \sin^2 x & \cos^2 x & 1+\sin(2x) \end{vmatrix}, x \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$



$R_1 \to R_1 - R_2$
$R_2 \to R_2 - R_3$

$f(x) = \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \sin^2 x & \cos^2 x & 1+\sin(2x) \end{vmatrix}$

$= 1\,(1 + \sin(2x)) + \cos^2 x + \sin^2 x$

$f(x) = 2 + \sin 2x$ $\left[\begin{matrix} x \in \left(\frac{\pi}{6}, \frac{\pi}{3}\right) \\ 2x \in \left(\frac{\pi}{3}, \frac{2\pi}{3}\right) \end{matrix}\right]$

$\max(f(x)) = 3 = \alpha$

$\text{Min}(f(x)) = 2 + \frac{\sqrt{3}}{2} = \beta$

$\therefore \beta^2 = \left(2+\frac{\sqrt{3}}{2}\right)^2 = 4+\frac{3}{4}+2\sqrt{3} = \frac{19}{4}+2\sqrt{3} \quad \therefore \beta^2 - 2\sqrt{\alpha} = \frac{19}{4}+2\sqrt{3}-2\sqrt{3} = \frac{19}{4}$

SECTION – B

Question ID : 3666942565

21. If $\int_0^1 \left(x^{21}+x^{14}+x^7\right)\left(2x^{14}+3x^7+6\right)^{\frac{1}{7}} dx = \frac{1}{\ell}(11)^{\frac{m}{n}}$ where l, m, n $\in$ N, m and n are coprime then $l+m+n$ is equal to ________.

यदि $\int_0^1 \left(x^{21}+x^{14}+x^7\right)\left(2x^{14}+3x^7+6\right)^{\frac{1}{7}} dx = \frac{1}{\ell}(11)^{\frac{m}{n}}$ है, जहाँ l, m, n $\in$ N, m एवं n सहअभाज्य हैं, तब $l+m+n$ का मान है।

Ans. Official Answer NTA (63)

Sol. $\int \left(x^{20}+x^{13}+x^6\right)\left(2x^{21}+3x^{14}+6x^7\right)^{1/7} dx$

$2x^{21} + 3x^{14} + 6x^7 = t$

$42(x^{20} + x^{13} + x^6)\, dx = dt$

$$\frac{1}{42}\int_0^{11} t^{\frac{1}{7}} dt = \left(\frac{t^{\frac{8}{7}}}{\frac{8}{7}} \times \frac{1}{42}\right)_0^{11}$$

$$\frac{1}{48}\left(t^{\frac{8}{7}}\right)_0^{11} = \frac{1}{48}(11)^{8/7}$$

$l = 48$, $m = 8$, $n = 7$

$l + m + n = 63$

Question ID : 3666942561

22. If $f(x) = x^2 + g'(1)x + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f''(x)$, then the value of $f(4) - g(4)$ is equal to ________.

यदि $f(x) = x^2 + g'(1)x + g''(2)$ एवं $g(x) = f(1)x^2 + xf'(x) + f''(x)$ है, तो $f(4) - g(4)$ का मान है।

Ans. Official Answer NTA (14)

Sol. $f(x) = x^2 + xg'(1) + g''(2)$ ...(1)

$\therefore f'(x) = 2x + g'(1)$

$f''(x) = 2$

$\therefore g(x) = f(1)x^2 + x(2x + g'(1)) + 2 = f(1)x^2 + 2x^2 + xg'(1) + 2$

$g'(x) = 2f(1)x + 4x + g'(1)$ ...(2)

$g''(x) = 2(1) + 4$ ...(3)

from (2) put x = 1

$g'(1) + 2f(1) + 4 + g'(1)$

$\Rightarrow 2f(1) + 4 = 0$

$\Rightarrow f(1) = -2 \Rightarrow g''(2) = -4 + 4 = 0$ from (3)

from (1)

$f(1) = 1 + g'(1) + 0$

$\Rightarrow g'(1) = -2 - 1 = -3$

$\because f'(x) = 2x - 3$

$f(x) = x^2 - 3x + c$

put x = 1, f(1) = – 2 $\quad \therefore c = 0$

$\therefore f(x) = x^2 - 3x$

$\therefore g(x) = -3x + 2$

$\Rightarrow f(4) - g(4) = 4 - (-10) = 14$

Question ID : 3666942567

23. Let A be the area bounded by the curve $y = x|x-3|$, the x-axis and the ordinates $x = -1$ and $x = 2$. Then 12A is equal to ________.

माना वक्र $y = x|x-3|$, x-अक्ष तथा कोटियों $x = -1$ व $x = 2$ से घिरे क्षेत्रा का क्षेत्राफल A है। तब 12A का मान है।

Ans. Official Answer NTA (62)

Sol. $$A = \int_{-1}^{0} (x^2 - 3x)dx + \int_{0}^{2} (3x - x^2)dx$$

$$\Rightarrow A = \left.\frac{x^3}{3} - \frac{3x^2}{2}\right|_{-1}^{0} + \left.\frac{3x^2}{2} - \frac{x^3}{3}\right|_{0}^{2}$$

$$\Rightarrow A = \frac{11}{6} + \frac{10}{3} = \frac{31}{6}$$

$\therefore 12A = 62$

Question ID : 3666942570

24. Let $\vec{v}=\alpha\hat{i}+2\hat{j}-3\hat{k}$, $\vec{w}=2\alpha\hat{i}+\hat{j}-\hat{k}$ and $\vec{u}$ be a vector such that $|\vec{u}|=\alpha>0$. If the minimum value of the scalar triple product $[\vec{u}\ \vec{v}\ \vec{w}]$ is $-\alpha\sqrt{3401}$, and $|\vec{u}\cdot\hat{i}|^2=\frac{m}{n}$ where m and n are coprime natural numbers, then m + n is equal to ________.

माना $\vec{v}=\alpha\hat{i}+2\hat{j}-3\hat{k}$, $\vec{w}=2\alpha\hat{i}+\hat{j}-\hat{k}$ है तथा एक सदिश $\vec{u}$ के लिए $|\vec{u}|=\alpha>0$ है। यदि अदिश त्रिक गुणनफल $[\vec{u}\ \vec{v}\ \vec{w}]$ का न्यूनतम मान $-\alpha\sqrt{3401}$ है, और $|\vec{u}\cdot\hat{i}|^2=\frac{m}{n}$ है जहाँ m एवं n सहअभाज्य धन पूर्णांक हैं, तब m + n का मान है।

Ans. Official Answer NTA (3501)

Sol. $[\vec{u}\vec{v}\vec{w}]=\vec{u}.(\vec{v}\times\vec{w})$

$\left(|u||\vec{v}\times\vec{w}|\cos\theta\right)=-\alpha\sqrt{3401}$

$\Rightarrow \cos\theta=-1$

$|u|=\alpha$ (Given)

$|\vec{v}\times\vec{w}|=\sqrt{3401}$

$$\vec{v}\times\vec{w}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \alpha & 2 & -3\\ 2\alpha & 1 & -1\end{vmatrix}$$

$\vec{v}\times\vec{w}=\hat{i}-5\alpha j-3\alpha k$

$|\vec{v}\times\vec{w}|=\sqrt{1+25\alpha^2+9\alpha^2}=\sqrt{3401}$

$34\alpha^2=3400$

$\alpha^2=100$

$\alpha=10$ (as $\alpha>0$)

So $\vec{u}=\lambda\left(\hat{i}-5\alpha j-3\alpha k\right)$

$\vec{u}=\sqrt{\lambda^2+25\alpha^2\lambda^2+9\alpha^2\lambda}$

$\alpha^2=\lambda^2\left(1+25\alpha^2+9\alpha^2\right)$

$100=\lambda^2(1+34\times100)$

$\lambda^2=\frac{100}{3401}=\frac{m}{n}$

Question ID : 3666942564

25. The remainder, when $19^{200}+23^{200}$ is divided by 49, is ________.

$19^{200}+23^{200}$ को 49 से विभाजित करने पर शेषफल है।

Ans. Official Answer NTA (29)

Sol. $(21-2)^{200}+(21+2)^{200}=49\lambda+2^{201}$

$2^{201}=8^{67}=(7+1)^{67}=49\lambda+7\times 67+1$

$=49\lambda+470$

$=49(\lambda+9)+29$

So, remainder = 29

Question ID : 3666942569

26. A(2, 6, 2), B(–4, 0, λ), C(2, 3, –1) and D(4, 5, 0), $|\lambda|\le 5$ are the vertices of a quadrilateral ABCD. If its area is 18 square units, then $5-6\lambda$ is equal to ________.

A(2, 6, 2), B(–4, 0, λ), C(2, 3, –1) तथा D(4, 5, 0), $|\lambda|\le 5$ एक चतुर्भुज ABCD के शीर्ष है। यदि इसका क्षेत्रफल 18 वर्ग इकाई है, तो $5-6\lambda$ है।

Ans. Official Answer NTA (11)

Sol. A(2, 6, 2), B(–4, 0, λ), C(2, 3, –1), D(4, 5, 0)

$$\text{Area}=\frac{1}{2}\left|\overrightarrow{BD}\times\overrightarrow{AC}\right|=18$$

$$\overrightarrow{AC}\times\overrightarrow{BD}=\begin{vmatrix}\hat{i} & j & k\\ 0 & -3 & -3\\ 8 & 5 & -\lambda\end{vmatrix}$$

$$=(3\lambda+15)\hat{i}-j(-24)+k(-24)$$

$$\overrightarrow{AC}\times\overrightarrow{BD}=(3\lambda+15)\hat{i}+24j-24k$$

$$=\sqrt{(3\lambda+15)^2+(24)^2+(24)^2}=36$$

$=\lambda^2+10\lambda+9=0$

$=\lambda=-1,-9$

$|\lambda|\le 5\Rightarrow\lambda=-1$

$5-6\lambda=5-6(-1)=11$

Question ID : 3666942566

27. Let $a_1, = 8, a_2, a_3, ...a_n$, be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170, then the product of its middle two terms is ________.

माना $a_1, = 8, a_2, a_3, ...a_n$, एक A.P. है। यदि इसके प्रथम चार पदों का योग 50 है तथा इसके अन्तिम चार पदों का योग 170 है, तब इसके मध्य दो पदों का गुणनफल है।

Ans. Official Answer NTA (754)

Sol. $S_4 = 50$

$2(16 + 3d) = 50$

$d = 3$

$4a + d(4n - 10) = 170$

$32 + 3(4n - 10) = 170$

$4n - 10 = 46$

$n = 14$

Middle terms are T_7, T_8

$T_7T_8 = (8 + 6 \times 3)(8 + 7 \times 3) = 26 \times 29$

$= 754$

Question ID : 3666942568

28. Let $f : R \to R$ be differentiable function such that $f'(x) + f(x) = \int_0^2 f(t)dt$. If $f(0) = e^{-2}$, then $2f(0) - f(2)$ is equal to ________.

माना $f : R \to R$ एक अवकलनीय फलन है, जिसके लिए ताकि $f'(x) + f(x) = \int_0^2 f(t)dt$ है। यदि $f(0) = e^{-2}$ है, तो $2f(0) - f(2)$ का मान है।

Ans. Official Answer NTA (1)

Sol. $\frac{dy}{dx} + y = k$

$y \cdot e^x = k \cdot e^x + c$

$f(0) = e^{-2}$

$\Rightarrow c = e^{-2} - k$

$\therefore\ y = k + (e^{-2} - k)e^{-x}$

now $k = \int_0^2 \left(k + \left(e^{-2} - k\right)e^{-x}\right)dx$

$\Rightarrow k = e^{-2} - 1$

$\therefore\ y = (e^{-2} - 1) + e^{-x}$

$f(2) = 2e^{-2} - 1,\ f(0) = e^{-2}$

$2f(0) - f(2) = 1$

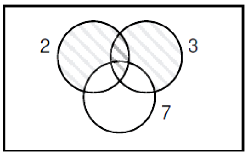
Question ID : 3666942563

29. The number of 3-digit numbers, that are divisible by either 2 or 3 but not divisible by 7, is ________.

तीन अंकों की संख्याओं, जो या तो 2 या 3 से विभाज्य हैं परन्तु 7 से विभाज्य नहीं हे, की संख्या है।

Ans. Official Answer NTA (514)

Sol.



Divisible by 2 100, 102, 104, ..., 998

$t_n = a + (n-1)d \Rightarrow 998 = 100 + (n-1)2$

$\Rightarrow \frac{898}{2} = n - 1$

$\Rightarrow n = 449 + 1 = 450$

Divisible by 3 102, 105, 108, ..., 999

$t_n = a + (n-1)d \Rightarrow 999 - 102 = (n-1)3$

$\Rightarrow \frac{897}{3} = n - 1$

$\Rightarrow n = 299 + 1 = 300$

Divisiblle by 2 & 3 both 102, 108, 114, ..., 996

$996 - 102 = (n-1)6$

$n = 1 + \frac{894}{6} = 1 + 149 = 150$

$\therefore$ No. divisible by 2 or 3 = 450 + 300 – 150 = 600

No. divisible by 2 and 7 both 112,126,140,154, ..., 994

$994 - 112 = (n-1)14$

$882 = (n-1)14$

$\Rightarrow n = 63 + 1 = 64$

No. divisible by 3 and 7 both 105, 126, 147, ..., 987

$987 - 105 = (n-1)\ 21$

$\Rightarrow n = 1 + 42 = 43$

No. divisible by 2, 3 and 7 126, 168, 252, 294, ..., 966

$966 - 126 = (n - 1)42$

$\Rightarrow 840 = (n - 1)\ 42$

$\Rightarrow n = 1 + 20 = 21$

$\therefore$ Total required numbers $= 600 - (64 + 43 - 21) = 600 + 21 - 107$

$= 621 - 107 = 514$

Question ID : 3666942562

30. The number of words, with or without meaning, that can be formed using all the letters of the word ASSASSINATION so that the vowels occur together, is ________.

ASSASSINATION शब्द के सभी अक्षरों के प्रयोग से, अर्थपूर्ण या अर्थहीन बनाए जा सकने वाले शब्दों, जिनमें सभी स्वर एक साथ हों, की संख्या है।

Ans. Official Answer NTA (50400)

Sol. Vowels : A, A, A, I, I, O

Consonants : S, S, S, S, N, N, T

$\because$ Total number of ways in which vowels come together

$$= \frac{\lfloor 8}{\lfloor 4 \lfloor 2} \times \frac{\lfloor 6}{\lfloor 3 \lfloor 2} = 50400$$