JEE Main March 2021

Question Paper With Text Solution

18 March. | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN MARCH 2021 | 18TH MARCH SHIFT-1

SECTION – A

1. Let $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + ... + a_{40}x^{40}$. Then, $a_1 + a_3 + a_5 + ... + a_{37}$ is equal to :

(1) $2^{19}(2^{20} + 21)$ (2) $2^{19}(2^{20} - 21)$ (3) $2^{20}(2^{20} - 21)$ (4) $2^{20}(2^{20} + 21)$

Ans. Official Answer NTA (2)

Sol. $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^{2+} ..... + a_{40}x^{40}$

Put x = 1

$4^{20} = a_0 + a_1 + a_2 + a_3 + ... a_{40}$ ....(1)

Put x = –1

$2^{20} = a_0 - a_1 + a_2 - a_3 + ... a_{40}$ ....(2)

(1) – (2)

$\Rightarrow 4^{20} - 2^{20} = [a_1 + a_3 + a_5 + ... a_{37} + a_{39}]$

$\Rightarrow a_1 + a_3 + a_5 + ... a_{37} + a_{39} = \frac{1}{2}[4^{20} - 2^{20}]$

a_{39} = Coefficient of $x^{39} = {}^{20}C_{19} . 2^{19} \times 1$
$= 5.2^{21}$

So $a_1 + a_3 + ... a_{37} = 2^{39} - 2^{19} - 5.2^{21}$
$= 2^{19} [2^{20} - 21]$

2. $\frac{1}{3^2-1}+\frac{1}{5^2-1}+\frac{1}{7^2-1}+......+\frac{1}{(201)^2-1}$ is equal to :

(1) $\frac{101}{404}$ (2) $\frac{101}{408}$ (3) $\frac{99}{400}$ (4) $\frac{25}{101}$

Ans. Official Answer NTA (4)

Sol. General term

$$T_r = \frac{1}{(2r+1)^2-1} = \frac{1}{2r(2r+2)}$$

$$T_r = \frac{1}{4}\left[\frac{1}{r}-\frac{1}{r+1}\right]$$

Number of terms = 100

$$\Rightarrow S_n = \sum_{r=1}^{100} T_r = \frac{1}{4}\left[\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+....\left(\frac{1}{100}-\frac{1}{101}\right)\right]$$

$= \frac{1}{4} \times \frac{100}{101} = \frac{25}{101}$

3. The equation of one of the straight lines which passes through the point (1, 3) and makes an angle $\tan^{-1}\left(\sqrt{2}\right)$ with the straight line, $y+1=3\sqrt{2}\,x$ is :

(1) $4\sqrt{2}\,x-5y-\left(5+4\sqrt{2}\right)=0$
(2) $5\sqrt{2}\,x+4y-\left(15+4\sqrt{2}\right)=0$
(3) $4\sqrt{2}\,x+5y-4\sqrt{2}=0$
(4) $4\sqrt{2}\,x+5y-\left(15+4\sqrt{2}\right)=0$

Ans. Official Answer NTA (4)

Sol. $\tan\theta = \sqrt{2}$

$\Rightarrow \sqrt{2} = \left|\frac{m-3\sqrt{2}}{1+3\sqrt{2}m}\right|$

$\Rightarrow \sqrt{2} + 6m = \pm\,(m-3\sqrt{2})$

$\Rightarrow m = \frac{2\sqrt{2}}{7}$ or $\frac{-4\sqrt{2}}{5}$

Equation of line :

$(y-3) = \frac{-4\sqrt{2}}{5}(x-1)$

$\Rightarrow 4\sqrt{2}\,x+5y-\left(15+4\sqrt{2}\right)=0$

4. For the four circles M, N, O and P, following four equations are given :

Circle M : $x^2 + y^2 = 1$

Circle N : $x^2 + y^2 - 2x = 0$

Circle O : $x^2 + y^2 - 2x - 2y + 1 = 0$

Circle P : $x^2 + y^2 - 2y = 0$

If the centre of circle M is joined with centre of the circle N, further centre of circle N is joined with centre of the circle O, centre of circle O is joined with the centre of circle P and lastly, centre of circle P is joined with centre of circle M, then these lines from the sides of a :

(1) Square (2) Rectangle (3) Parallelogram (4) Rhombus

Ans. Official Answer NTA (1)

Sol. Center of the four circles are :

$C_1 : (0, 0)$
$C_2 : (1, 0)$
$C_3 : (1, 1)$
$C_4 : (0, 1)$
Which are vertices of a square.

5. If the functions are defined as $f(x)=\sqrt{x}$ and $g(x)=\sqrt{1-x}$, then what is the common domain of the following functions : f + g, f – g, f/g, g/f, g – f, where

$$(f \pm g)(x) = f(x) \pm g(x), (f/g)(x) = \frac{f(x)}{g(x)}$$

(1) $0 < x \le 1$ (2) $0 < x < 1$ (3) $0 \le x \le 1$ (4) $0 \le x < 1$

Ans. Official Answer NTA (2)

Sol. $f(x) = \sqrt{x}$
Domain $D_1 : [0, \infty)$
$g(x) = \sqrt{1-x}$
Domain $D_2 : (-\infty, 1]$.
Common domain of

$f + g, f - g, \frac{f}{g}, \frac{g}{f}, g - f$ will be

a subset of $D_1 \cap D_2 = [0, 1]$.

as $x \neq 0$; For $\frac{g}{f}$ and $x \neq 1$ for $\frac{f}{g}$

$x \in (0,1)$.

6. If $f(x)=\begin{cases} \frac{1}{|x|} & ;|x| \ge 1 \\ ax^2+b & ;|x|<1 \end{cases}$ is differentiable at every point of the domain, then the values of a and b are respectively :

(1) $\frac{1}{2}, -\frac{3}{2}$ (2) $\frac{5}{2}, -\frac{3}{2}$ (3) $-\frac{1}{2}, \frac{3}{2}$ (4) $\frac{1}{2}, \frac{1}{2}$

Ans. Official Answer NTA (3)

Sol. f (x) is continuous at x = 1

$\Rightarrow 1 = a + b$ ...... (1).

f (x) is differentiable at x = 1

$\Rightarrow f'(1^-) = f'(1^+)$

$\Rightarrow 2a = -1 \Rightarrow a = \frac{-1}{2}$,

$b = 1 + \frac{1}{2} = \frac{3}{2}$.

7. The number of integral values of m so that the abscissa of point of intersection of lines 3x + 4y = 9 and y = mx + 1 is also an integer, is :

(1) 1 (2) 0 (3) 3 (4) 2

Ans. Official Answer NTA (4)

Sol. $3x + 4y = 9$; $y = mx + 1$

$\Rightarrow 3x + 4(mx + 1) = 9$

$\Rightarrow x = \frac{9-4}{3+4m} = \frac{5}{3+4m}$.

For x to be integer;

3 +4m = 1, –1, 5 or –5

$\Rightarrow m = \frac{-1}{2}, -1, \frac{1}{2}$ or –2

Two of which are integers

8. The differential equation satisfied by the system of parabolas $y^2 = 4a(x + a)$ is :

(1) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) + y = 0$ (2) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) - y = 0$

(3) $y\left(\frac{dy}{dx}\right) + 2x\left(\frac{dy}{dx}\right) - y = 0$ (4) $y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$

Ans. Official Answer NTA (4)

Sol. $y^2 = 4\,ax + 4\,a^2$ ..... (1)

Differentiating;

$2y\frac{dy}{dx} = 4\,a \Rightarrow a = \frac{y}{2}\frac{dy}{dx}$

Substituting in (1)

$$y^2 = 4\left(\frac{y}{2}\frac{dy}{dx}\right)x + 4\left(\frac{y}{2}\frac{dy}{dx}\right)^2$$

$$\Rightarrow y = 2x\ \frac{dy}{dx} + y\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow\ y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y = 0$$

9. The solutions of the equation

$$\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0,\ (0 < x < \pi),\text{ are :}$$

(1) $\frac{\pi}{12},\frac{\pi}{6}$ (2) $\frac{7\pi}{12},\frac{11\pi}{12}$ (3) $\frac{5\pi}{12},\frac{7\pi}{12}$ (4) $\frac{\pi}{6},\frac{5\pi}{6}$

Ans. Official Answer NTA (2)

Sol. $R_1 \to R_1 + R_2$

$$\Rightarrow \Delta = \begin{vmatrix} 2 & 2 & 1 \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin^2 x & 4\sin^2 x & 1+4\sin^2 x \end{vmatrix}$$

$C_2 \to C_2 - C_1\ :\ C_3 \to C_3 - C_1$

$$\Rightarrow \Delta \begin{vmatrix} 2 & 2 & -1 \\ \cos^2 x & 1 & 0 \\ 4\sin^2 x & 0 & 1 \end{vmatrix} = 0$$

$\Rightarrow\ 2 + 4\sin 2x = 0$

$$\Rightarrow \sin 2x = -\frac{1}{2}$$

$$\Rightarrow 2x = \frac{7\pi}{6}.\frac{11\pi}{6} \Rightarrow x = \frac{7\pi}{12},\frac{11\pi}{12}$$

10. The integral $\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{4x^2-4x+6}}\,dx$ is equal to :

(where c is a constant of integration)

(1) $\frac{1}{2}\sin\sqrt{(2x-1)^2+5}+c$ (2) $\frac{1}{2}\cos\sqrt{(2x+1)^2+5}+c$

(3) $\frac{1}{2}\cos\sqrt{(2x-1)^2+5}+c$ (4) $\frac{1}{2}\sin\sqrt{(2x-1)^2+5}+c$

Ans. Official Answer NTA (1)

Sol. $(2x-1) = t$; $2dx = dt$

$$I=\int\frac{t\cos\sqrt{t^2+5}}{t^2+5}\frac{dt}{2}$$

$t^2 + 5 = u^2$

$\Rightarrow 2tdt = 2udu$

$$\Rightarrow I=\int\frac{\cos u.u\,du}{2u}$$

$$=\frac{1}{2}\sin u+c=\frac{1}{2}\sin\sqrt{t^2+5}+C$$

$$=\frac{1}{2}\sin\sqrt{(2x-1)^2+5}+C$$

11. If $\lim\limits_{x\to 0}\frac{\sin^{-1}x-\tan^{-1}x}{3x^3}$ is equal to L, then the value of $(6L + 1)$ is :

(1) $\frac{1}{2}$ (2) 6 (3) $\frac{1}{6}$ (4) 2

Ans. Official Answer NTA (4)

Sol. $\lim\limits_{x\to 0}\frac{\sin^{-1}x-\tan^{-1}x}{3x^3}$

This is $\frac{0}{0}$ Indeterminate form

Applying L'Hopital Rule

$$L=\lim_{x\to 0}\frac{\frac{1}{\sqrt{1-x^2}}-\frac{1}{1+x^2}}{9x^2}$$

$$=\lim_{x\to 0}\frac{\left(1+x^2-\sqrt{1-x^2}\right)\left(1+x^2\sqrt{1-x^2}\right)}{\left(\sqrt{1+x^2}\right)\left(1+x^2\right)9x^2\left(1+x^2+\sqrt{1-x^2}\right)}$$

$$=\lim_{x\to 0}\frac{\left(1+x^2\right)^2-\left(1-x^2\right)}{9x^2\times 2}$$

$$=\lim_{x\to 0}\frac{x^4+3x^2}{18x^2}=\frac{1}{6}$$

$\Rightarrow 6L+1=2$

12. The sum of all 4-digit distinct numbers that can be formed with the digits 1, 2, 2 and 3 is :

(1) 26664 (2) 22264 (3) 122664 (4) 122234

Ans. Official Answer NTA (1)

Sol. Total no. of such numbers $\frac{4!}{2!} = 12$

At every place (Unit's, Ten's, hundred's and thousand's)
1 will occur 3 times, 3 will occur 3 times and 2 will occur 6 times.
So sum = $(1 + 10 + 100 + 1000) \times (1 \times 3 + 2 \times 6 + 3 \times 3)$
$= 1111 \times 24$
$= 26664$

13. If α, β are natural numbers such that $100^{\alpha} - 199\beta = (100)(100) + (99)(101) + (98)(102) + ... + (1)(199)$, then the slope of the line passing through (α , β) and origin is :

(1) 530 (2) 550 (3) 510 (4) 540

Ans. Official Answer NTA (2)

Sol. Sum S = $\sum_{r=0}^{99}(100-r)(100+r)$

$= \sum_{r=0}^{99}(100^2 - r^2)$

$= 100^2\sum_{r=0}^{99}1 - \sum_{r=0}^{99}r^2$

$= 1000000 - \frac{99 \times 100 \times 199}{6}$

$= 100^3 - 199 \times 1650$

$= 100^{\alpha} - 199\beta \Rightarrow \alpha = 3;\ \beta = 1650$

slope = $\frac{\beta}{\alpha} = \frac{1650}{3} = 550$

14. A vector $\vec{a}$ has components 3p and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to new system, $\vec{a}$ has components p + 1 and $\sqrt{10}$, then a value of p is equal to :

(1) 1 (2) $\frac{4}{5}$ (3) $-\frac{5}{4}$ (4) –1

Ans. Official Answer NTA (4)

Sol. Since lengths do not change

$\Rightarrow \sqrt{(3p)^2 + (1)^2} = \sqrt{(p+1)^2 + \left(\sqrt{10}\right)^2}$

$\Rightarrow 9p^2 + 1 = p^2 + 2p + 1 + 10$

$\Rightarrow 8p^2 - 2p - 10 = 0 \Rightarrow 4p^2 - p - 5 = 0$

$\Rightarrow 4p^2 - 5p + 4p - 5 = 0$

$\Rightarrow p = -1 \text{ or } p = \frac{5}{4}$.

15. The real valued function $f(x) = \frac{\operatorname{cosec}^{-1} x}{\sqrt{x - [x]}}$, where [x] denotes the greatest integer less than or equal to x, is defined for all x belonging to :

(1) All reals except integers

(2) All non-integers except the interval [–1, 1]

(3) All integers except 0, –1, 1

(4) All reals except the interval [–1, 1]

Ans. Official Answer NTA (2)

Sol. $f(x) = \frac{\operatorname{cosec}^{-1} x}{\sqrt{x - [x]}}$

Domain of $\operatorname{cosec}^{-1} x$ is $x \in (-\infty, -1] \cup [1, \infty)$

for $\sqrt{x - [x]}$ or $\sqrt{\{x\}}$

$\{x\} \neq 0 \Rightarrow x \neq$ Integer

So, final Domain is all non-integers execpet interval [–1, 1]

16. Let $A + 2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$ and $2A - B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}$. If Tr(A) denotes the sum of all diagonal elements of the matrix A, then Tr(A) – Tr(B) has value equal to :

(1) 2 (2) 1 (3) 3 (4) 0

Ans. Official Answer NTA (1)

Sol. $Tr(A+2B) = Tr(A) + 2\,Tr(B) = 1 - 3 + 1 = -1$ ......(1)

$Tr(2A - B) = 2Tr(A) - Tr(B) = 2 - 1 + 2 = 3$ ....(2)

Solving (1) & (2) ;

$Tr(A) = ;\ Tr(B) = -1$

$Tr(A) - Tr(B) = 2.$

17. Let α, β, γ be the real roots of the equation $x^3 + ax^2 + bx + c = 0$, ($a, b, c \in R$ and $a, b \neq 0$). If the system of equations (in u, v, w) given by $\alpha u + \beta v + \gamma w = 0;\ \beta u + \gamma v + \alpha w = 0;\ \gamma u + \alpha v + \beta w = 0$ has non - trivial solution, then the value of $\frac{a^2}{b}$ is :

(1) 5 (2) 3 (3) 0 (4) 1

Ans. Official Answer NTA (2)

Sol. For the system of equations to have a non-trivial solution.

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$$

$\Rightarrow \alpha^3 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma$

$\Rightarrow \alpha = \beta = \gamma$ or $\alpha + \beta + \gamma = 0$

But since $\alpha + \beta + \gamma = -a$ and $a \neq 0$

$\Rightarrow \alpha + \beta + \gamma \neq 0$

So, $\alpha = \beta = \gamma$

So $3\alpha = -a$; $3\alpha^2 = b$

$$\frac{a^2}{b} = \frac{9\alpha^2}{3\alpha^2} = 3$$

18. The value of $3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{3 + \ldots\ldots\infty}}}}$ is equal to :

(1) $3 + 2\sqrt{3}$ (2) $2 + \sqrt{3}$ (3) $4 + \sqrt{3}$ (4) $1.5 + \sqrt{3}$

Ans. Official Answer NTA (4)

Sol. Let $x = 3 + \cfrac{1}{4 + \cfrac{1}{x}}$

$\Rightarrow x = 3 + \frac{x}{4x+1}$

$\Rightarrow x = \frac{13x+3}{4x+1}$

$\Rightarrow 4x^2 - 12x - 3 = 0$

$\Rightarrow x = \frac{12 \pm \sqrt{144+48}}{B} = \frac{3 \pm 2\sqrt{3}}{2}$

Since $x > 0$

so $x = 1.5 + \sqrt{3}$

19. Choose the correct statement about two circles whose equations are given below :

$x^2 + y^2 - 10x - 10y + 41 = 0$

$x^2 + y^2 - 22x - 10y + 137 = 0$

(1) Circles have only one meeting point

(2) Circles have no meeting point

(3) Circles have two meeting points

(4) Circles have same centre

Ans. Official Answer NTA (1)

Sol. $S_1 = x^2 + y^2 - 10x - 10y + 41 = 0$

Centre $C_1 = (5, 5)$

Radius $R_1 = 3$.

$S_2 = x^2 + y^2 - 22x - 10y + 137 = 0$

Centre $C_2 = (11, 5)$

Radias $R_2 = 3$

Distance between centres, $d = 6 = r_1 + r_2$

$\Rightarrow$ Circles have only one meeting point

20. If the equation $a|z|^2 + \overline{\overline{\alpha}z + \alpha\overline{z}} + d = 0$ represents a circle where a, d are real constants, then which of the following condition is correct ?

(1) $\alpha = 0$, a, d $\in R^+$

(2) $|\alpha|^2 - ad \neq 0$

(3) $|\alpha|^2 - ad > 0$ and $a \in R - \{0\}$

(4) $|\alpha|^2 - ad \geq 0$ and $a \in R$

Ans. Official Answer NTA (3)

Sol. $a|z|^2 + \overline{\overline{\alpha}z + \alpha\overline{z}} + d = 0$

$\Rightarrow a|z|^2 + \alpha\overline{z} + \overline{\alpha}\,z + d = 0$

Let $z = x + i\,y$; $\alpha = p + i\,q$

$\Rightarrow a\,(x^2 + y^2) + (p + i\,q)\,(x - i\,y) + (p - i\,q)\,(x + i\,y) + d = 0$

$\Rightarrow a\,(x^2 + y^2) + 2\,px + 2qy + d = 0$

$\Rightarrow x^2 + y^2 + \frac{2p}{a}x + \frac{2q}{a}y + \frac{d}{a} = 0$

Represents a real circle if

$g^2 + f^2 - c > 0 \qquad \Rightarrow \frac{p^2}{a^2} + \frac{q^2}{a^2} - \frac{d}{a} > 0$

$\Rightarrow p^2 + q^2 > a\,d$

$\Rightarrow |\alpha|^2 - ad > 0\ \&\ a \neq 0.$

SECTION – B

1. Let the plane $ax + by + cz + d = 0$ bisect the line joining the points $(4, -3, 1)$ and $(2, 3, -5)$ at the right angles. If a, b, c, d are integers, then the minimum value of $(a^2 + b^2 + c^2 + d^2)$ is :

Ans. Official Answer NTA (28)

Sol. Direction ratios of normal to the plane are proportional to :

$(2 - 4, 3 - (-3), (-5 - 1)) = (-2, 6, -6)$

or $(1, -3, 3)$

Also midpoint : $(3, 0, -2)$ will lie on plane

$ax + by + cz + d = 0$

$\Rightarrow x - 3y + 3z + d = 0$

$\Rightarrow 3 - 0 + 3 \times (-2) + d = 0 \Rightarrow d = 3$

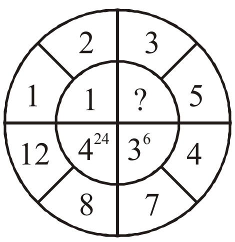
So equaton of plane is

$x - 3y + 3z + 3 = 0$

$a^2 + b^2 + c^2 + d^2 = k^2\,(1^2 + 3^2 + 3^2 + 3^2)$

minimum $= 28$

2. The missing value in the following figure is :



Ans. Official Answer NTA (4)

Sol. All the terms in the inner circle

are difference of pair of numbers in the outer circle; raised to the power factorial of difference

e.g. 1, 2 $\rightarrow (2-1)^{(2-)!}$

3, 5 $\rightarrow (5-3)^{(5-3)!} = 2^2 = 4$

4, 7 $\rightarrow (7,-4)^{(7-4)!} = 3^6$

12, 8 $\rightarrow (12-8)^{(12-8)!} = 4^{24}$

3. The number of solutions of the equation |cotx| = $\cot x + \frac{1}{\sin x}$ in the interval [0, 2 π] is :

Ans. Official Answer NTA (1)

Sol. Case-1 $\cot x > 0 \Rightarrow \in (0, \pi/2) \cup (\pi, 3\pi/2)$

$\Rightarrow \cot x = \cot x + \frac{1}{\sin x}$

$\Rightarrow \frac{1}{\sin x} = 0$ $\Rightarrow$ No. solution

Case-2 $\cot x \le 0 \Rightarrow x \in [\pi/2, \pi) \cup [3\pi/2, 2\pi)$

$\Rightarrow -\cot x = \cot x + \frac{1}{\sin x}$

$\Rightarrow -2\frac{\cot x}{\sin x} = \frac{1}{\sin x} \Rightarrow \cos x = \frac{-1}{2}$

$\Rightarrow x : \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3} = \frac{2\pi}{3}, \frac{4\pi}{3}$

But only one of these lies in desired

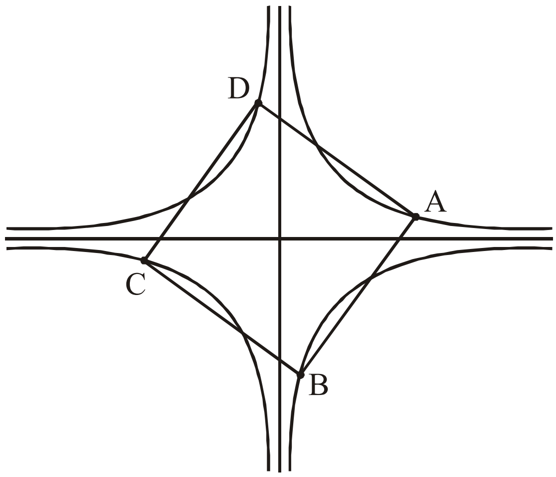
interval i.e $\frac{2\pi}{3}$

So, only one solution

4. A square ABCD has all its vertices on the curve $x^2y^2 = 1$. The midpoints of its sides also lie on the same curve. Then, the square of area of ABCD is :

Ans. Official Answer NTA (80)

Sol. The curves are $xy = 1$ or $xy = -1$



As per the diagram

Let $A\left(t, \frac{1}{t}\right)$ $C\left(-t, \frac{-1}{t}\right)$

$B\left(\frac{1}{t}, -t\right)$ $D\left(-\frac{1}{t}, t\right)$

Mid-point of AB lies on $xy = 1$

$$\Rightarrow \left(\frac{t+\frac{1}{t}}{2}\right) \times \left(\frac{\frac{1}{t}-t}{2}\right) = 1$$

$$\Rightarrow \frac{1}{t^2} - t^2 = 4 \Rightarrow \left(t^2 + \frac{1}{t^2}\right)^2 = \left(t^2 - \frac{1}{t^2}\right)^2 + 4 = 20$$

$\Rightarrow t^4 + 4t^2 - 1 = 0.$

Side of square $AB = \sqrt{\left(t - \frac{1}{t}\right)^2 + \left(\frac{1}{t} + t\right)^2}$

$$= \sqrt{2\left(t^2 + \frac{1}{t^2}\right)}$$

Area of square $2 \times \left(t^2 + \frac{1}{t^2}\right) = 2 \times \sqrt{20}$

Square of Area $4 \times 20 = 80$

5. The mean age of 25 teachers in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. If the mean age of the teachers in this school now is 39 years, then the age (in years) of the newly appointed teacher is :

Ans. Official Answer NTA (35)

Sol. Sum of ages = 25 × 40 = 1000

Let the age of new teacher = x years

$$\frac{1000-60+x}{25} = 39$$

$\Rightarrow x = 35$

6. Let f(x) and g(x) be two functions satisfying $f(x^2) + g(4-x) = 4x^3$ and $g(4-x) = 0$.

Then the value of $\int_{-4}^{4} f(x^2)\,dx$ is :

Ans. Official Answer NTA (512)

Sol. $f(x^2) = 4x^3 - g(4-x)$

and $g(4-x) = 0$

$\Rightarrow f(x^2) = 4x^3$

$$I = \int_{-4}^{4} f(x^2)\,dx$$

Even funciton

$$= 2\int_{0}^{4} 4x^3 dx$$

$$= 2 \times 4 \times \left[\frac{x^4}{4}\right]_0^4 = 512$$

7. Let z_1, z_2 be the roots of the equation $z^2 + az + 12 = 0$ and z_1, z_2 form an equilateral triangle with origin. Then, the value of $|a|$ is :

Ans. Official Answer NTA (6)

Sol. $z_1 z_2 = 12$; $z_1 + z_2 = -a$

............(1)

Since 0, z_1, z_2 form an equilateral triangel, $|z_1| = |z_2| = |z_1 - z_2|$

Also from rotation theorem

$\frac{z_2}{z_1} = e^{i\frac{\pi}{3}}$ ....(2)

From (1) & (2);

$z_2^2 = 12\,e^{i\frac{\pi}{3}}$ $|z_2| = \sqrt{12} = |z_1|$

$|z_1 + z_2| = \left|\left(z_1 + z_1 e^{i\frac{\pi}{3}}\right)\right|$

$= |z_1| \left|1 + e^{i\frac{\pi}{3}}\right|$

$= \sqrt{12} \times \left|1 + \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right|$

$= \sqrt{12} \times \left|\frac{3}{2} + i\frac{\sqrt{3}}{2}\right| = \sqrt{12} \times \frac{\sqrt{9+3}}{2}$

$= 6$

8. The number of times the digit 3 will be written when listing the integers from 1 to 1000 is :

Ans. Official Answer NTA (300)

Sol. Number of times; digit 3 will be written = 10 × 10 + 10 × 10 + 10 × 10

$\underline{3}\ \ \underline{10} \times \underline{10}\qquad \underline{10}\ \ \underline{3}\ \times\ \underline{10}\qquad \underline{10} \times \underline{10}\ \ \underline{3}$

9. If $f(x) = \int \frac{5x^8 + 7x^6}{(x^2+1+2x^7)^2}dx$, $(x \geq 0)$, f(0) = 0 and $f(1) = \frac{1}{K}$, then the value of K is :

Ans. Official Answer NTA (4)

Sol. $f(x) = \int \frac{5x^8 + 7x^6}{(x^2+1+2x^7)^2}\ dx$

$= \int \frac{\frac{5}{x^6} + \frac{7}{x^8}}{\left(\frac{1}{x^5} + \frac{1}{x^7} + 2\right)^2}\ dx$

Let $\frac{1}{x^5} + \frac{1}{x^7} + 2 = t$

$\Rightarrow \left(-\frac{5}{x^6} - \frac{7}{x^8}\right)dx = dt$

$\Rightarrow f(x) = -\int \frac{dt}{t^2} = \frac{1}{t} + c$

$f(x) = \frac{1}{\frac{1}{x^5} + \frac{1}{x^7} + 2} + c$

$\Rightarrow f(x) = \frac{x^7}{x^2 + 1 + 2x^7} + c$

$f(0) = 0 \Rightarrow c = 0$

$f(1) = \frac{1}{4} = \frac{1}{k}$

$\Rightarrow k = 4$

10. The equation of the planes parallel to the plane x – 2y + 2z – 3 = 0 which are at unit distance from the point (1, 2, 3) is ax + by + cz + d = 0. If (b – d) = K(c – a), then the positive value of K is :

Ans. Official Answer NTA (4)

Sol. Let the equation of plane be

$x - 2y + 2z + \lambda = 0.$

Distance of (1, 2, 3) from plane =1

$\Rightarrow \left|\frac{1-4+6+\lambda}{\sqrt{1^2+2^2+2^2}}\right| = 1$

$\Rightarrow |\lambda + 3| = 3$

$\Rightarrow \lambda = 0$ or –6

$x - 2y + 3z + 0 = 0$

or $x - 2y + 2z - 6 = 0$

$(b - d) = k(c - a)$

$\Rightarrow (-2-(-6)) = k(2-1)$

$\Rightarrow 4 = k$

or $(-2 - 0) = k(2 - 1) \Rightarrow k = -2$

So positive value of k = 4