JEE Main March 2021

Question Paper With Text Solution

17 March. | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN MARCH 2021 | 17TH MARCH SHIFT-1

SECTION – A

1. The value of $\lim\limits_{x\to 0^+} \dfrac{\cos^{-1}\left(x-[x]^2\right)\cdot\sin^{-1}\left(x-[x]^2\right)}{x-x^3}$, where [x] denotes the greatest integer $\leq$ x is :

(1) π (2) $\frac{\pi}{2}$ (3) 0 (4) $\frac{\pi}{4}$

Ans. Official Answer NTA (2)

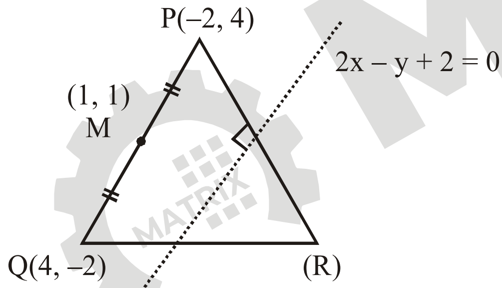
Sol. $\lim\limits_{x\to 0^+} \dfrac{\cos^{-1}x\ \sin^{-1}x}{x\left(1-x^2\right)} = \dfrac{\pi}{2}$

2. In a triangle PQR, the co-ordinates of the points P and Q are (–2, 4) and (4, –2) respectively. If the equation of the perpendicular bisector of PR is 2x – y + 2 = 0, then the centre of the circumcircle of the ΔPQR is :

(1) (–1, 0) (2) (–2, –2) (3) (1, 4) (4) (0, 2)

Ans. Official Answer NTA (2)

Sol.



Slope of PQ = – 1
slope of perpendicular bisector of PQ = 1
equation of perpendicular bisector of PQ
y – 1 = 1 (x –1)
y = x
circumcentre is P.O. I. of
2x – y + z = 0 and y = x
(–2, – 2)

3. The system of equations ky + y + z = 1, x + ky + z = k and x + y + zk = k² has no solution if k is equal to :

(1) –2 (2) 0 (3) –1 (4) 1

Ans. Official Answer NTA (1)

Sol. $\Delta = \begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} = (k-1)^2 (k+2)$

$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ k & k & 1 \\ k^2 & 1 & k \end{vmatrix} = -(k-1)^2 (k+1)$

for $k = -2$, $D = 0$ and $D_x \neq 0$

4. Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = 7\hat{i} + \hat{j} - 6\hat{k}$.

If $\vec{r} \times \vec{a} = \vec{r} \times \vec{b}$, $\vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = -3$, then $\vec{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k})$ is equal to :

(1) 12 (2) 13 (3) 8 (4) 10

Ans. Official Answer NTA (1)

Sol. $\vec{r} \times \vec{a} - \vec{r} \times \vec{b} = 0$

$\vec{r} \times (\vec{a} - \vec{b}) = 0$

$\vec{r} \,//\, \vec{a} - \vec{b}$

$\vec{r} = \lambda (\vec{a} - \vec{b})$

$\vec{r} = \lambda (-5\hat{i} - 4\hat{j} + 10\hat{k})$

$\vec{r} \cdot (\hat{i} - 4\hat{j} + \hat{k}) = -3$

$\lambda = 3$

$\vec{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k}) = (-5\hat{i} - 4\hat{j} + 10\hat{k}) \cdot (2\hat{i} - 3\hat{j} + \hat{k})$

$= 12$

5. Which of the following is true for y(x) that satisfies the differential equation $\frac{dy}{dx} = xy - 1 + x - y$; y (0) = 0;

(1) $y(1) = e^{\frac{1}{2}} - 1$ (2) $y(1) = 1$ (3) $y(1) = e^{\frac{1}{2}} - e^{-\frac{1}{2}}$ (4) $y(1) = e^{-\frac{1}{2}} - 1$

Ans. Official Answer NTA (4)

Sol. $\frac{dy}{dx} = y(x-1) + (x-1)$

$= (x-1)(y+1)$

$\int \frac{dy}{y+1} = \int (x-1)dx$

$\ln(y+1) = \frac{x^2}{2} - x + C$

$y(0) = 0 \Rightarrow c = 0$

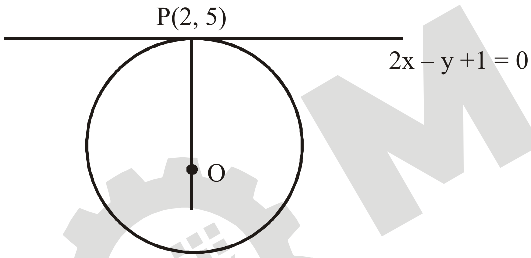
$y = e^{\frac{x^2}{2} - x} - 1$

6. The line $2x - y + 1 = 0$ is a tangent to the circle at the point (2, 5) and the center of the circle lies on $x - 2y = 4$. Then, the radius of the circle is :

(1) $4\sqrt{5}$ (2) $5\sqrt{3}$ (3) $3\sqrt{5}$ (4) $5\sqrt{4}$

Ans. Official Answer NTA (3)

Sol.



Equation of OP

$y - 5 = \frac{-1}{2}(x-2)$

$2y - 10 = x + 2$

$x + 2y = 12$

Centre of circle is P.O.I of lines $x + 2y = 12$ ans $x - 2y = 4$

0(8, 2)

radius = OP = $\sqrt{36+9} = 3\sqrt{5}$

7. The sum of possible values of x for $\tan^{-1}(x+1)+\cot^{-1}\left(\frac{1}{x-1}\right)=\tan^{-1}\left(\frac{8}{31}\right)$ is :

(1) $-\frac{32}{4}$ (2) $-\frac{31}{4}$ (3) $-\frac{33}{4}$ (4) $-\frac{30}{4}$

Ans. Official Answer NTA (1)

Sol. $\tan^{-1}(x+1)+\cot^{-1}\left(\frac{1}{x-1}\right)=\tan^{-1}\left(\frac{8}{31}\right)$

$\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\left(\frac{8}{31}\right)$

$\frac{(x+1)+(x-1)}{1-(x^2-1)}=\frac{8}{31}$

$31(2x)=8(2-x^2)$

$31x=8-4x^2$

$4x^2+31x-8=0$

$4x^2+32x-x-8=0$

$4x(x+8)-1(x+8)=0$

$x=-8 \quad x=\frac{1}{4}$

put in original equation

$\tan^{-1}(-7)+\text{cost}^{-1}\left(-\frac{1}{9}\right)$

$-\tan^{-1}(7)+\pi-\cot^{-1}\left(\frac{1}{9}\right)$

$\pi-(\tan^{-1} 7+\tan^{-1}(9))$

$\pi+\tan^{-1}\left(\frac{8}{31}\right)-\pi=\text{RHS}$

but at $x=\frac{1}{4}$

LHS $>\frac{\pi}{2}$ and RHS $<\frac{\pi}{2}$

So, only solution is

x = –8

8. The value of $4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{4+......\infty}}}}$ is :

(1) $4+\frac{4}{\sqrt{5}}\sqrt{30}$ (2) $2+\frac{4}{\sqrt{5}}\sqrt{30}$ (3) $5+\frac{2}{5}\sqrt{30}$ (4) $2+\frac{2}{5}\sqrt{30}$

Ans. Official Answer NTA (4)

Sol. $y = 4+\cfrac{1}{5+\cfrac{1}{y}}$, $y > 0$

$y = 4+\frac{y}{5y+1}$

$5y^2 + y = 20y + 4 + y$

$5y^2 – 20y – 4 = 0$

$y = \frac{20\pm\sqrt{400+80}}{10}$

$y = \frac{20\pm\sqrt{480}}{10}$

$y = 2+\frac{2}{5}\sqrt{30}$ or $y = 2-\frac{2}{5}\sqrt{30}$ (Rejected)

9. Choose the incorrect statement about the two circles whose equations are given below :

$x^2 + y^2 – 10x – 10y + 41 = 0$ and

$x^2 + y^2 – 16x – 10y + 80 = 0$

(1) Circles have two intersection points.

(2) Both circles' centres lie inside region of one another.

(3) Distance between two centres is the average of radii of both the circles.

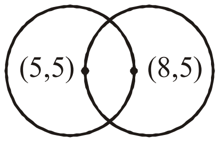
(4) Both circles pass through the centre of each other.

Ans. Official Answer NTA (2)

Sol. $C_1(5, 5)$ $r_1 = \sqrt{25+25-41} = 3$

$C_2(8, 5)$ $r_2 = \sqrt{64+25-80} = 3$

$d = C_1C_2 = 3$



10. Which of the following statements is incorrect for the function g(α) for $\alpha \in R$ such that

$$g(\alpha) = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin^{\alpha} x}{\cos^{\alpha} x + \sin^{\alpha} x} dx$$

(1) g(α) is an even fuction

(2) g(α) is a strictly increasing function

(3) g(α) has an inflection point at $\alpha = -\frac{1}{2}$

(4) g(α) is a strictly decreasing function

Ans. Official Answer NTA (1)

Sol. $$g(\alpha) = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(\sin x)^{\alpha}}{(\sin x)^{\alpha} + (\cos x)^{\alpha}} dx \qquad ....(1)$$

apply king

$$g(\alpha) = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(\cos x)^{\alpha}}{(\cos x)^{\alpha} + (\sin x)^{\alpha}} dx \qquad ....(2)$$

add (1) + (2)

$$2g(\alpha) = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dx$$

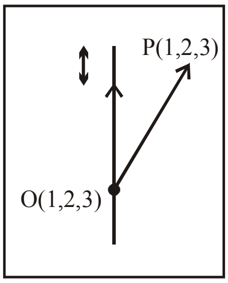
$$g(\alpha) = \frac{\pi}{12}$$

11. The equation of the plane which contains the y - axis and passes through the point (1, 2, 3) is :

(1) x + 3z = 0 (2) 3x + z = 6 (3) x + 3z = 10 (4) 3x – z = 0

Ans. Official Answer NTA (4)

Sol.



$$\vec{n} = \overrightarrow{OP} \times \hat{j} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 1 & 0 \end{vmatrix} = -3\hat{i} + \hat{k}$$

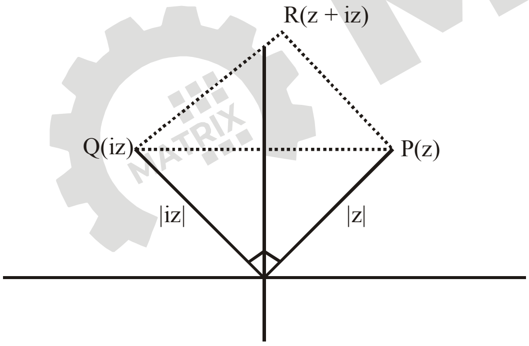
$-3x + z = 0$
$3x + z = 0$

12. The area of the triangle with vertices A(z), B(iz) and C(z + iz) is :

(1) $\frac{1}{2}$ (2) $\frac{1}{2}[z]^2$ (3) 1 (4) $\frac{1}{2}|z+iz|^2$

Ans. Official Answer NTA (2)

Sol.



Required Area = Area of ΔOPR

$\frac{1}{2}(OP)(PR) = \frac{1}{2}|z||iz|$

$= \frac{|z|^2}{2}$

13. If $A = \begin{pmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{pmatrix}$ and $\det\left(A^2 - \frac{1}{2}I\right) = 0$, then a possible value of α is :

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

Ans. Official Answer NTA (3)

Sol. $A^2 = \begin{bmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{bmatrix}\begin{bmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} \sin^2\alpha & 0 \\ 0 & \sin^2\alpha \end{bmatrix}$

$A^2 - \frac{I}{2} = \begin{bmatrix} \sin^2\alpha - \frac{1}{2} & 0 \\ 0 & \sin^2\alpha - \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\left(\sin^2\alpha - \frac{1}{2}\right)^2 = 0$

$\sin\alpha = \frac{1}{\sqrt{2}}$, $\sin\alpha = \frac{-1}{\sqrt{2}}$ Rejected

$\alpha = \frac{\pi}{4}$ $\alpha \in \left[0 \quad \frac{\pi}{2}\right]$

14. If the fourth term in the expression of $\left(x + x^{\log_2^x}\right)^7$ is 4480, then the value of x where $x \in N$ is equal to :

(1) 3 (2) 2 (3) 4 (4) 1

Ans. Official Answer NTA (2)

Sol. $T_{r+1} = {}^7C_r \quad x^{7-r}\left(x^{\log_2^x}\right)^r$

$T_4 = {}^7C_4 \quad x^4\left(x^{\log_2^x}\right)^3 = 4480$

$x^4 . x^{3\log_2^x} = 2^4 \times 2^3$

$x = 2$

15. If the Boolean expression $(p \Rightarrow q) \Leftrightarrow (q * (\square p))$ is a tautology, then the Boolean expression $p * (\square q)$ is equivalent to :

(1) $\sim q \Rightarrow p$ (2) $p \Rightarrow q$ (3) $q \Rightarrow p$ (4) $p \Rightarrow \sim q$

Ans. Official Answer NTA (3)

Sol.

P	a	P→a	a*(~p)	qv(~p)
T	T	T	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

q* (~p) ≡ qv(~p) ⇒ *is equivalent to v

p* ~q = pv~q = ~qvp = q → p

16. If $\cot^{-1}(\alpha) = \cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + \cot^{-1} 32 + ...$ upto 100 terms, then α is :

(1) 1.03 (2) 1.01 (3) 1.02 (4) 1.00

Ans. Official Answer NTA (2)

Sol. $T_r = \cos^{-1}\left(2r^2\right) = \tan^{-1}\left(\frac{2}{4r^2}\right)$

$T_r = \tan^{-1}\left(\frac{(2r+1)-(2r-1)}{1+(2r+1)(2r-1)}\right)$

$T_r = \tan^{-1}(2r+1) - \tan^{-1}(2r-1)$

$T_1 = \tan^{-1}(3) - \tan^{-1}(1)$

$T_2 = \tan^{-1}(5) - \tan^{-1}(3)$

| | |

| | |

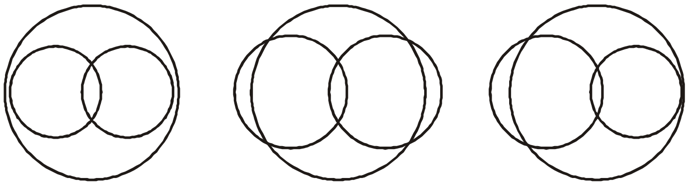
$T_{100} = \tan^{-1}(201) - \tan^{-1}(199)$

$\cot^{-1}(\alpha) = \tan^{-1}(201) - \tan^{-1}(1)$

$\cot^{-1}(\alpha) = \tan^{-1}\left(\frac{200}{202}\right)$

$\alpha = 1.01$

17. In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagrams can justify the above statement ?



(1) P and Q (2) Q and R (3) None of these (4) P and R

Ans. Official Answer NTA (3)

Sol. None of P, Q, R represent the exact condition given in the problem. So option (3) is right answer

18. Team 'A' consists of 7 boys and n girls and Team 'B' has 4 boys and 6 girls. If a total of 52 single matches can be arranged between these two teams when a boy plays against a boy and a girl plays against a girl, then n is equal to :

(1) 6 (2) 4 (3) 5 (4) 2

Ans. Official Answer NTA (2)

Sol. $^7C_1\,{}^4C_1 + {}^nC_1\,{}^6C_1 = 52$
$28 + 6n = 52$
$n = 4$

19. Two dices are rolled. If both dices have six faces numbered 1, 2, 3, 5, 7 and 11, then the probability that the sum of the numbers on the top faces is less than or equal to 8 is :

(1) $\frac{1}{2}$ (2) $\frac{4}{9}$ (3) $\frac{17}{36}$ (4) $\frac{5}{12}$

Ans. Official Answer NTA (3)

Sol. n(s) = 36
possible ordered pair
(1, 1) (1, 2) (1, 3) (1, 5) (1, 7)
(2, 1) (2, 2) (2, 3) (2, 5)
(3, 1) (3, 2) (3, 3) (3, 5)
(5, 1) (5, 2) (5, 3)
(7, 1)

Probalbility = $\frac{17}{36}$

20. The inverse of $y = 5^{\log x}$ is :

(1) $x = y^{\log 5}$ (2) $x = y^{\frac{1}{\log 5}}$ (3) $x = 5^{\frac{1}{\log y}}$ (4) $x = 5^{\log y}$

Ans. Official Answer NTA (1 and 4)

Sol. $y = 5^{\log x}$

for inverse

$x \leftrightarrow y$

$x = 5^{\log y}$

$x = y^{\log 5}$

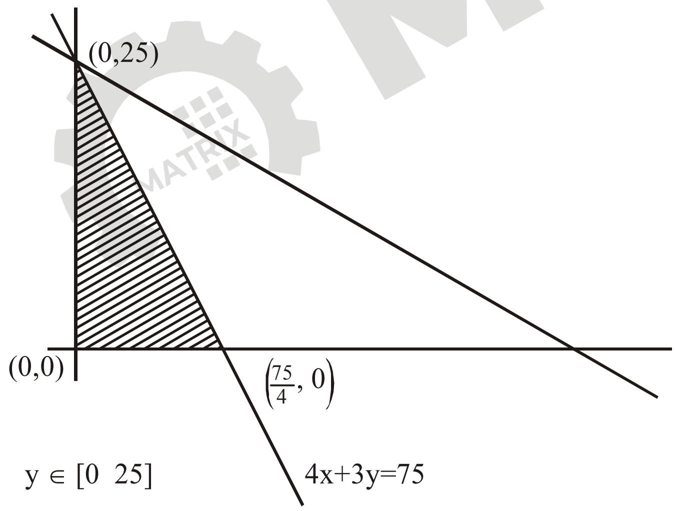
Ans. 1 and 4

SECTION – B

1. The maximum value of z in the following equation $z = 6xy + y^2$, where $3x + 4y \leq 100$ and $4x + 3y \leq 75$ for $x \geq 0$ and $y \geq 0$ is :

Ans. Official Answer NTA (904)

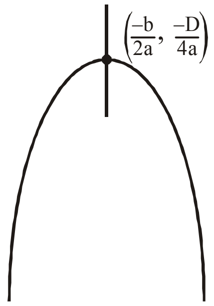
Sol.



$z = y(6x + y)$ and $4x + 3y = 75$

$$z = \left(\frac{75-3y}{4}\right)6y + y^2$$

$$z = \frac{225}{2}y - \frac{7}{2}y^2 \qquad y \in [0 \ \ 25]$$



$$Z_{max} = \frac{-D}{4a} = \frac{225 \times 225}{4 \times 2 \times 7} = 904.017857$$

Ans. 904

2. The minimum distance between any two points P_1 and P_2 while considering point P_1 on one circle and point P_2 on the other circle for the given circles' equations

$x^2 + y^2 - 10x - 10y + 41 = 0$

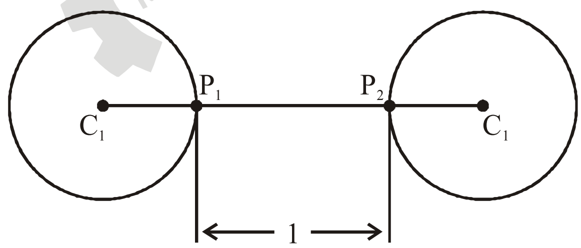
$x^2 + y^2 - 24x - 10y + 160 = 0$ is ________ .

Ans. Official Answer NTA (1)

Sol. $(x-5)^2 + (y-5)^2 = 9 \Rightarrow C_1(5, 5), r_1 = 3$

$(x-12)^2 + (y-5)^2 = 9 \Rightarrow C_2(12, 5), r_2 = 3$

$C_1C_2 = 7$



$(P_1P_2)_{min} = 1$

3. Let there be three independent events E_1, E_2 and E_3. The probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let 'p' denote the probability of none of events occurs that satisfies the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in

the interval (0, 1).

Then, $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$ is equal to _____.

Ans. Official Answer NTA (6)

Sol. Let x, y, z be the probability of E_1, E_2, E_3 respectively

$\alpha = x(1-y)(1-z)$
$\beta = y(1-x)(1-z)$
$\gamma = z(1-x)(1-y)$
$p = (1-x)(1-y)(1-z)$
put α, β, γ and p in
$(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3r)p = 2\beta r$
We get, $x = 2y$
$y = 3z$
$\Rightarrow x = 6z$

$\frac{x}{z} = 6$

4. If $f(x) = \sin\left(\cos^{-1}\left(\frac{1-2^{2x}}{1+2^{2x}}\right)\right)$ and its first derivative with respect to x is $-\frac{b}{a}\log_e 2$ when x = 1, where a and b are integers, then the minimum value of $|a^2 - b^2|$ is _____ .

Ans. Official Answer NTA (481)

Sol. $\cos^{-1}\left(\frac{1-4^x}{1+4^x}\right)$

Let $2^x = t$
$t > 0$

$\cos^{-1}\left(\frac{1-t^2}{1+t^2}\right)$ put $t = \tan\theta$

$= \cos^{-1}(\cos 2\theta)$ $\theta \in \left(0, \frac{\pi}{2}\right)$

$= \cos^{-1}(\cos 2\theta)$ $2\theta \in (0, \pi)$
$= 2\theta$

$\therefore \sin\left(\cos^{-1}\left(\frac{1-4^x}{1+4^x}\right)\right) = \sin 2\theta$

$y = \frac{2\tan\theta}{1+\tan^2\theta} = \frac{2t}{1+t^2} = \frac{2.2^x}{1+4^x}$

$\frac{dy}{dx} = \frac{\left(1+4^x\right)\cdot 2.2^x \ln 2 - 2.2^x\left(4^x\right)\ln 4}{\left(1+4^x\right)^2}$

$\frac{dy}{dx} = \frac{-12\ln 2}{25}$

a = 25 and b = 12

$|a^2 - b^2|_{min} = 481$

5. If $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$, then the value of det($A^4$) + det($A^{10}$–(Adj(2A))^{10}) is equal to :

Ans. Official Answer NTA (16)

Sol. $|A| = -2$

$|A|^4 = 16$

$A^{10} = \begin{bmatrix} 2^{10} & 2^{10}-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1024 & 1023 \\ 0 & 1 \end{bmatrix}$

$2A = \begin{bmatrix} 4 & 6 \\ 0 & -2 \end{bmatrix}$

$\text{adj}(2A) = \begin{bmatrix} -2 & -6 \\ 0 & 4 \end{bmatrix}$

$(\text{adj}(2A))^{10} = 2^{10}\begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix}^{10}$

$= 2^{10}\begin{bmatrix} 1 & -\left(2^{10}-1\right) \\ 0 & 2^{10} \end{bmatrix}$

$A^{10} - (\text{adj}(2A))^{10} = \begin{bmatrix} 0 & 2^{11}\times 1023 \\ 0 & 1-\left(1024\right)^2 \end{bmatrix}$

$\left|A^{10} - \left(\text{adj}(2a)\right)^{10}\right| = 0$

6. If the function $f(x) = \frac{\cos(\sin x) - \cos x}{x^4}$ is continuous at each point in its domain and $f(0) = \frac{1}{k}$, then k is :

Ans. Official Answer NTA (6)

Sol. $\frac{1}{k} = \lim_{x\to 0} \frac{2\sin\left(\frac{\sin x + x}{2}\right)\sin\left(\frac{x - \sin x}{2}\right)}{x^4}$

$= \lim_{x\to 0}\left(\frac{\sin x + x}{2}\right)\left(\lim_{x\to 0}\frac{x - \sin x}{2}\right)$

$= 1 \times \frac{1}{6} = \frac{1}{6}$

$k = 6$

7. If $\vec{a} = \alpha\hat{i} + \beta\hat{j} + 3\hat{k}$

$\vec{b} = -\beta\hat{i} - \alpha\hat{j} - \hat{k}$ and

$\vec{c} = \hat{i} - 2\hat{j} - \hat{k}$

such that $\vec{a}\cdot\vec{b} = 1$ and $\vec{b}\cdot\vec{c} = -3$, then $\frac{1}{3}\left(\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}\right)$ is equal to :

Ans. Official Answer NTA (2)

Sol. $\vec{a} = (\alpha, \beta, 3)$, $\vec{b} = (-\beta, -\alpha, -)$ $\vec{c} = (1, -2, -1)$

$\vec{a}.\vec{b} = 1$

$\alpha\beta = -2$

$\vec{b}.\vec{c} = -3$

$2\alpha - \beta = -4$

$\therefore \alpha = -1$

$\beta = 2$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 3 \\ -2 & 1 & -1 \end{vmatrix} = -5\hat{i} - 7\hat{j} + 3\hat{k}$$

$\frac{1}{3}(\vec{a}\times\vec{b})\cdot\vec{C} = 2$

8. If the equation of the plane passing through the line of intersection of the planes $2x - 7y + 4z - 3 = 0$, $3x - 5y + 4z + 11 = 0$ and the point $(-2, 1, 3)$ is $ax + by + cz - 7 = 0$, then the value of $2a + b + c - 7$ is :

Ans. Official Answer NTA (4)

Sol. Equation of plane

$P_1 + \lambda P_2 = 0$

$(2x - 7y + 4z - 3) + \lambda (3x - 5y + 47 + 11) = 0$

It passes through (–2, 1, 3)

$\lambda = \frac{1}{6}$

$15x - 47y + 28z - 7 = 0$

$a = 15, \; b = -47, \; c = 28$

$2a + b + c - 7 = 4$

9. If $[\cdot]$ represents the greatest integer function, then the value of

$$\left| \int_0^{\sqrt{\frac{\pi}{2}}} \left[\left[x^2 \right] - \cos x \right] dx \right| \text{ is ______.}$$

Ans. Official Answer NTA (1)

Sol. $$\left| \int_0^{\sqrt{\frac{\pi}{2}}} \left[x^2 \right] dx + \int_0^{\sqrt{\frac{\pi}{2}}} \left[-\cos x \right] dx \right|$$

$$= \left| \int_0^1 0 \; dx + \int_1^{\sqrt{\frac{\pi}{2}}} 1 \; dx + \int_1^{\sqrt{\frac{\pi}{2}}} (-1) \; dx \right|$$

$$= \left| \sqrt{\frac{\pi}{2}} - 1 - \sqrt{\frac{\pi}{2}} \right| = |-1| = 1$$

10. If $(2021)^{3762}$ is divided by 17, then the remainder is ________.

Ans. Official Answer NTA (4)

Sol. $(2021)^{3768} = (2023\text{-}z)^{3762}$

$= 17\,k + 2^{3762} \quad \{k \in I\}$

$= 17\,k + 2^2 \; (2^4)^{940}$

$= 17\,k + 4\,(17 - 1)^{940}$

$= 17\,k + 4\,(17\lambda + 1) \quad \{\lambda \in I\}$

$= 17k + 4\,(17\lambda) + 4$

Remainder = 4