# JEE Main March 2021

## Question Paper With Text Solution

## 16 March. | Shift-2

# MATHEMATICS


MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation



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# JEE MAIN MARCH 2021 | 16$^{TH}$ MARCH SHIFT-2

## SECTION – A

1. Let A denote the event that a 6 digit integer formed by 0, 1, 2, 3, 4, 5, 6 without repetitions, be divisible by 3. Then probability of event A is equal to:

(1) $\frac{11}{27}$ (2) $\frac{9}{56}$ (3) $\frac{3}{7}$ (4) $\frac{4}{9}$

Ans. Offical Answer NTA (4)

Sol. Case I : 1, 2, 3, 4, 5, 6 Total = 6!

Case II : 0, 1, 2, 4, 5, 6 Total = 5 × 5!

Case III : 0, 1, 2, 3, 4, 5 $\underline{\text{Total} = 5\times 5!}$

$16\times 5!$

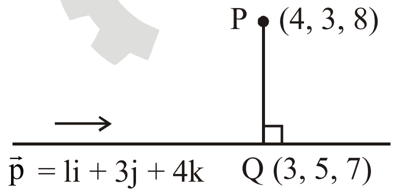
$$P(A) = \frac{16\times 5!}{6\times(6\times5\times4\times3\times2)} = \frac{4}{9}$$

2. If the foot of the perpendicular from point (4, 3, 8) on the line $L_1 : \frac{x-a}{l} = \frac{y-2}{3} = \frac{z-b}{4}$ ; $l \neq 0$ is (3, 5, 7 ), then the shortest distance between the line $L_1$ and line $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ equal to :

(1) $\sqrt{\frac{2}{3}}$ (2) $\frac{1}{\sqrt{6}}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{1}{2}$

Ans. Offical Answer NTA (2)

Sol.




$L_1 : \frac{x-a}{l} = \frac{y-2}{3} = \frac{z-b}{4}$

$\overrightarrow{PQ} = -i + 2j - k$

$\overrightarrow{PQ}\cdot\vec{p} = 0 \Rightarrow l = 2$

$\frac{3-a}{2} = \frac{5-2}{3} = \frac{7-b}{4}$

$\Rightarrow$ a = 1, b = 3

$L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ : A = (1, 2, 3) $\overrightarrow{AB} = i + 2j + 2k$

$\vec{p} = 2i + 3j + 4k$

$$\vec{p}\times\vec{q} = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ : B = (2, 4, 5)

$\vec{p}\times\vec{q} = -i + 2j - k$

$\vec{q} = 3i + 4j + 5k$

d = Projection of $\overrightarrow{AB}$ on $\vec{p}\times\vec{q}$

$$d = \left|\frac{\overrightarrow{AB}\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$$

$$d = \left|\frac{-1+4-2}{\sqrt{6}}\right| = \frac{1}{\sqrt{6}}$$

3. Consider the integral $I = \int_0^{10} \frac{[x]e^{\{x\}}}{e^{x-1}} dx$ where [x] denotes the greatest integer less than or equal to x. Then the value of I is equal to:

(1) 45(e – 1) (2) 45(e + 1) (3) 9(e + 1) (4) 9(e – 1)

Ans. Offical Answer NTA (1)

Sol. $x \frac{dv}{dx} = \frac{-1 - v2}{}$

$$I = \left[e\int_0^1 \frac{0}{e^{\{x\}}}dx + \int_1^2 \frac{1}{e^{\{x\}}}dx + \int_2^3 \frac{2}{e^{\{x\}}}dx + ..... + \int_9^{10} \frac{9}{e^{\{x\}}}dx\right]$$

$$I = e(0+1+2+....+9)\int_0^1 \frac{dx}{e^x}$$

$$I = 45e\int_0^1 e^{-x}dx$$

$$I = 45e\left[-e^{-x}\right]_0^1$$

$$I = 45e\left[-e^{-1}+1\right]$$

$$I = 45(e-1)$$

4. Let $C_1$ be the curve obtained by the solution of differential equation $2xy\frac{dy}{dx} = y^2 - x^2, x > 0$. Let the curve $C_2$ be the solution of $\frac{2xy}{x^2 - y^2} = \frac{dy}{dx}$. If both the curves pass through (1, 1), then the area enclosed by the curves $C_1$ and $C_2$ is equal to :

(1) $\frac{\pi}{2} - 1$  (2) $\frac{\pi}{4} + 1$  (3) $\pi - 1$  (4) $\pi + 1$

Ans. Offical Answer NTA (1)

Sol. $C_1 : \frac{dy}{dx} = \frac{\left(\frac{y}{x}\right)^2 - 1}{2(y/x)}$

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{v^2 - 1}{2v} \Rightarrow x\frac{dv}{dx} = \frac{-1 - v^2}{2v}$

$\int \frac{2v}{v^2 + 1} dv = -\int \frac{dx}{x}$

$\ln(v^2 + 1) = -\ln x + \ln c$

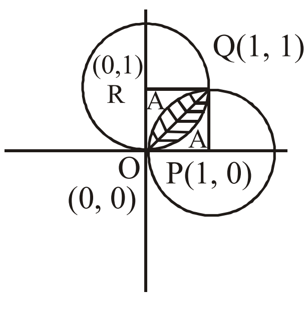
$x\left(\left(\frac{y}{x}\right)^2 + 1\right) = C$

$x^2 + y^2 = Cx$ passes through (1, 1)

$C = 2$

$C_1 : x^2 + y^2 - 2x = 0$

Similarly $C_2 : x^2 + y^2 - 2y = 0$




A = Area of square – Area of sector OQR

$A = (1\times1) - \frac{1}{4}\times\pi\times(1)^2 = 1 - \frac{\pi}{4}$

Required area = (1 × 1) – 2A

$= \frac{\pi}{2} - 1$

5. The least value of |z| where z is complex number which satisfies the inequality

$$\exp\left(\frac{(|z|+3)(|z|-1)}{||z|+1|}\log_e 2\right) \geq \log_{\sqrt{2}} |5\sqrt{7}+9i|, i = \sqrt{-1} \text{ is equal to :}$$

(1) 3 (2) 8 (3) 2 (4) $\sqrt{5}$

Ans. Offical Answer NTA (1)

Sol. $\left(e^{\log_e 2}\right)^{\frac{(|z|+3)(|z|-1)}{(|z|+1)}} \geq \log_{\sqrt{2}} 2^4$

$2^{\frac{(|z|+3)(|z|-1)}{(|z|+1)}} \geq 2^3$

$\frac{(|z|+3)(|z|-1)}{(|z|+1)} \geq 3$

$|z|^2 - |z| - 6 \geq 0$

$(|z| - 3)(|z| + 2) \geq 0 \Rightarrow$ 5 C 1 $(|z| \geq 0)$

$|z| \in [3, \infty)$

$|z|_{min} = 3$

6. Let A = {2, 3, 4, 5,...., 30} and '$\simeq$' be an equivalence relation on A × A, defined by (a, b) $\simeq$ (c, d), if and only if and only if ad = bc. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair (4, 3) is equal to :

(1) 5 (2) 8 (3) 6 (4) 7

Ans. Offical Answer NTA (4)

Sol. $(a, b) \simeq (c, d) \Leftrightarrow \frac{a}{b} = \frac{c}{d}$

$\frac{4}{3} = \frac{8}{6} = \frac{12}{9} = \frac{16}{12} = \frac{20}{15} = \frac{24}{18} = \frac{28}{21}$

Ordered pairs (4, 3), (8, 6), (12, 9), (16, 12), (20, 15), (24, 18), (28, 21)

Total = 7

7. Let $f$ be a real valued function, defined on R –{–1, 1} and given by $f(x) = 3\log_e\left|\frac{x-1}{x+1}\right| - \frac{2}{x-1}$ Then in which of the following intervals, function $f(x)$ is increasing ?

(1) $(-\infty, \infty) - \{-1, 1\}$

(2) $\left(-\infty, \frac{1}{2}\right] - \{-1\}$

(3) $\left(-1, \frac{1}{2}\right]$

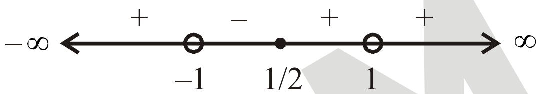
(4) $(-\infty, -1) \cup \left(\left[\frac{1}{2}, \infty\right) - \{1\}\right)$

Ans. Offical Answer NTA (4)

Sol. $f(x) = 3 \ln (x - 1) - 3 \ln (x + 1) - \frac{2}{(x-1)}$

$$f'(x) = \frac{3}{x-1} - \frac{3}{x+1} + \frac{2}{(x-1)^2}$$

$$f'(x) = \frac{4(2x-1)}{(x-1)^2(x+1)} \geq 0$$




$$x \in (-\infty, -1) \cup \left(\left[\frac{1}{2}, \infty\right) - \{1\}\right)$$

8. The maximum value of $f(x) = \begin{vmatrix} \sin^2 x & 1+\cos^2 x & \cos 2x \\ 1+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}, x \in R$ is :

(1) $\frac{3}{4}$ (2) $\sqrt{7}$ (3) $\sqrt{5}$ (4) 5

Ans. Offical Answer NTA (3)

Sol. $$f(x) = \begin{vmatrix} \sin^2 x & 1+\cos^2 x & \cos 2x \\ 1+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}$$

$C_1 \to C_1 + C_2$

$$f(x) = \begin{vmatrix} 2 & 1+\cos^2 x & \cos 2x \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

$R_1 \rightarrow R_1 - R_2$

$$f(x) = \begin{vmatrix} 0 & 1 & 0 \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

$f(x) = \cos 2x - 2\sin 2x$

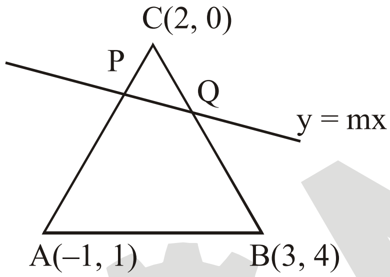
$f(x) \in \left[-\sqrt{5}, \sqrt{5}\right] \Rightarrow f(x)_{max} = \sqrt{5}$

9. Let A (–1, 1), B(3, 4) and C(2, 0) be given three points. A line $y = mx$, $m > 0$, intersects lines AC and BC at point P and Q respectively. Let $A_1$ and $A_2$ be the areas of $\Delta ABC$ and $\Delta PQC$ respectively, such that $A_1 = 3A_2$, then the value of m is equal to:

(1) $\frac{4}{15}$ (2) 1 (3) 2 (4) 3

Ans. Offical Answer NTA (2)

Sol.




Equation of AC $\Rightarrow x + 3y = 2$

Equation of BC $\Rightarrow 4x - y = 8$

$$P = \left(\frac{2}{3m+1}, \frac{2m}{3m+1}\right); Q = \left(\frac{8}{4-m}, \frac{8m}{4-m}\right)$$

$$A_1 = \frac{1}{2}\left|\begin{vmatrix} -1 & 1 & 1 \\ 3 & 4 & 1 \\ 2 & 0 & 1 \end{vmatrix}\right|$$

$$A_1 = \frac{13}{2}$$

$$A_2 = \frac{A_1}{3} = \frac{13}{6}$$

$$\frac{1}{2}\left|\begin{vmatrix} 2 & 0 & 1 \\ \frac{2}{3m+1} & \frac{2m}{3m+1} & 1 \\ \frac{8}{4-m} & \frac{8m}{4-m} & 1 \end{vmatrix}\right| = \frac{13}{6}$$

$$\frac{1}{(3m+1)(4-m)}\begin{vmatrix} 2 & 0 & 1 \\ 2 & 2m & 3m+1 \\ 8 & 8m & 4-m \end{vmatrix} = \pm\frac{13}{3}$$

$(+) \Rightarrow m^2 + 11m + 4 = 0$ (D < 0)

$(-) \Rightarrow 7m^2 - 11m - 4 = 0 \Rightarrow m = 1$ or $\frac{-4}{7}$ $(m > 0)$

$\Rightarrow m = 1$

10. Let C be the locus of the mirror image of a point on the parabola $y^2 = 4x$ with respect to the line $y = x$. Then the equation of tangent to C at P(2, 1) is

(1) $x + 2y = 4$ (2) $2x + y = 4$ (3) $x - y = 1$ (4) $x + 3y = 5$

Ans. Offical Answer NTA (3)

Sol. Image of $y^2 = 4x$ in the line $y = x$ is

$C : x^2 = 4y$

Equation of tangent at P(2, 1)

$T = 0 \Rightarrow x \cdot 2 = 4\left(\frac{y+1}{2}\right)$

$\Rightarrow x - y = 1$

11. Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 5\hat{k}$ If $\vec{r} \times \vec{a} = \vec{b} \times \vec{r}, \vec{r}.(\alpha\hat{i} + 2\hat{j} + \hat{k}) = 3$ and $\vec{r} \cdot (2\hat{i} + 5\hat{j} - \alpha\hat{k}) = -1, \alpha \in R$, then the value of $\alpha + |\vec{r}|^2$ is equal to :

(1) 15 (2) 13 (3) 11 (4) 9

Ans. Offical Answer NTA (1)

Sol. $\vec{r} \times \vec{a} = \vec{b} \times \vec{r}$

$\vec{r} \times \vec{a} + \vec{r} \times \vec{b} = \vec{0}$

$\vec{r} \times (\vec{a} + \vec{b}) = \vec{0}$

$\vec{r} = \lambda(\vec{a} + \vec{b})$

$\vec{r} = \lambda(3i - j + 2k)$

$\vec{r} \cdot (\alpha i + 2j + k) = 3$

$\lambda(3\alpha - 2 + 2) = 3 \Rightarrow \lambda(3\alpha) = 3 \Rightarrow \lambda\alpha = 1$ ... (i)

$\vec{r} \cdot (2i + 5j - \alpha k) = -1$

$\lambda(6 - 5 - 2\alpha) = -1 \Rightarrow \lambda(1 - 2\alpha) = -1$ ... (ii)

From (i) and (ii)

$\alpha = 1, \lambda = 1$

$\vec{r} = 3i - j + 2k$

$\alpha + |\vec{r}|^2 = 1 + 14 = 15$

12. Let $P(x) = x^2 + bx + c$ be a quadratic polynomial with real coefficients such that $\int_0^1 P(x)dx = 1$ and P(x) leaves remainder 5 when it is divided by (x–2). That the value of 9 (b + c) is equal to :

(1) 7 (2) 9 (3) 11 (4) 15

Ans. Offical Answer NTA (1)

Sol. $P(x) = x^2 + bx + c$

$\int_0^1 (x^2 + bx + c)dx = 1$

$\frac{1}{3} + \frac{b}{2} + c = 1$

3b + 6c = 4 .... (i)

P(2) = 5

4 + 2b + c = 5

2b + c = 1 .... (ii)

From (i) and (ii)

$b = \frac{2}{9}, c = \frac{5}{9}$

$b + c = \frac{7}{9}$

9 (b + c) = 7

13. Let $f : S \to S$ where $S = (0, \infty)$ be a twice differentiable function such that $f(x + 1) = xf(x)$. If $g: S \to R$ be defined as $g(x) = \log_e f(x)$, then the value of $|g''(5) - g''(1)|$ is equal to :

(1) 1 (2) $\frac{205}{144}$ (3) $\frac{197}{144}$ (4) $\frac{187}{144}$

Ans. Offical Answer NTA (2)

Sol. $f(x + 1) = x\, f(x)$

$\ln(x + 1) = \ln x + \ln(x)$

$g(x + 1) = \ln x + g(x)$

$g(x + 1) - g(x) = \ln x$

$g'(x + 1) - g'(x) = 1/x$

$g''(x + 1) - g''(x) = -1/x^2$

$x = 4$ $\quad g''(5) - g''(4) = \frac{-1}{16}$

$x = 3$ $\quad g''(4) - g''(3) = \frac{-1}{9}$

$x = 2$ $\quad g''(3) - g''(2) = \frac{-1}{4}$

$x = 1$ $\quad g''(2) - g''(1) = \frac{-1}{1}$

$g''(5) - g''(1) = -\left(1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16}\right)$

$|g''(5) - g''(1)| = \frac{205}{144}$

14. Let the lengths of intercepts on x-axis and y-axis made by the circle $x^2 + y^2 + ax + 2ay + c = 0$. $(a < 0)$ be $2\sqrt{2}$ be $2\sqrt{5}$, respectively. Then the shortest distance from origin to a tangent to this circle which is perpendicular to the line $x + 2y = 0$, is equal to :

(1) $\sqrt{6}$ (2) $\sqrt{11}$ (3) $\sqrt{7}$ (4) $\sqrt{10}$

Ans. Offical Answer NTA (1)

Sol. $C : x^2 + y^2 + ax + 2ay + c = 0$

X - intercept = $2\sqrt{g^2 - c} = 2\sqrt{2}$

$g^2 - c = 2$

$\frac{a^2}{4} - c = 2$ ... (i)

Y - intercept = $2\sqrt{f^2 - c} = 2\sqrt{5}$

$f^2 - c = 5$

$a^2 - c = 5$ ... (ii)

From (i) and (ii)

$a = -2$ or $2$ $(a < 0)$

$a = -2$

$c = -1$

$c : x^2 + y^2 - 2x - 4y - 1 = 0$

$c : (x - 1)^2 + (y - 2)^2 = 6$

Equation of tangent perpendicular to the line $x + 2y = 0$ is

$\Rightarrow 2x - y + \lambda = 0$

By applying condition of tangency

$\Rightarrow \left|\frac{-2 + 2 + \lambda}{\sqrt{5}}\right| = \sqrt{6}$

$\Rightarrow 2x - y \pm \sqrt{30} = 0$

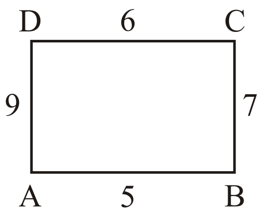
Perpendicular distance from origin = $\left|\frac{0 - 0 \pm \sqrt{30}}{\sqrt{5}}\right| = \sqrt{6}$

15. Consider a rectangle ABCD having 5, 7, 6, 9 points in the interior of the line seqments AB, CD, BC, DA respectively. Let $\alpha$ be the number of triangles having these points from different sides as vertices and $\beta$ be the number of quadrilaterals having these points from different sides as vertices. Then $(\beta - \alpha)$ is equal to :

(1) 1890 (2) 717 (3) 1173 (4) 795

Ans. Offical Answer NTA (2)

Sol.




$\alpha = {}^5C_1 \cdot {}^7C_1 \cdot {}^6C_1 + {}^7C_1 \cdot {}^6C_1 \cdot {}^9C_1 + {}^6C_1 \cdot {}^9C_1 \cdot {}^5C_1 + {}^9C_1 \cdot {}^5C_1 \cdot {}^7C_1$

$\alpha = 1173$

$\beta = {}^5C_1 \cdot {}^7C_1 \cdot {}^6C_1 \cdot {}^9C_1$

$\beta = 1890$

$\beta - \alpha = 717$

16. If (x, y, z) be an arbitrary point lying on a plane P which passes through the points (42, 0, 0), (0, 42, 0) and (0, 0, 42), then the value of the expression

$3+\frac{x-11}{(y-19)^2(z-12)^2}+\frac{y-19}{(x-11)^2(z-12)^2}+\frac{z-12}{(x-11)^2(y-19)^2}-\frac{x+y+z}{14(x-11)(y-19)(z-12)}$ is equal to :

(1) – 45 (2) 39 (3) 3 (4) 0

Ans. Offical Answer NTA (3)

Sol. P : x + y + z = 42

(x – 11) + (y – 19) + (z – 12) = 0

Let x – 11 = a

y – 19 = b

z – 12 = c

a + b + c = 0

$$3+\frac{a}{b^2c^2}+\frac{b}{a^2c^2}+\frac{c}{a^2b^2}-\frac{42}{14abc}$$

$$\Rightarrow\ 3+\frac{a^3+b^3+c^3}{a^2b^2c^2}-\frac{3}{abc}$$

$$\Rightarrow\ 3+\frac{3abc}{a^2b^2c^2}-\frac{3}{abc}$$

$\Rightarrow 3$

17. If y = y(x) is the solution of the differential equation $\frac{dy}{dx}+(\tan x)y=\sin x, 0\le x\le\frac{\pi}{3}$ with y(0), then $y\left(\frac{\pi}{4}\right)$ equal to :

(1) $\left(\frac{1}{2\sqrt{2}}\right)\log_e 2$ (2) $\frac{1}{4}\log_e 2$ (3) $\log_e 2$ (4) $\frac{1}{2}\log_e 2$

Ans. Offical Answer NTA (1)

Sol. $\frac{dy}{dx}+\left(\tan x\right)y=\sin x$ (LDE)

$IF = e^{\int \tan x\, dx} = \sec x$

$y \cdot \sec x = \int \sin x \cdot \sec x \, dx$

$y \sec x = \ln(\sec x) + C$

$y(0) = 0 \Rightarrow C = 0$

$y = \cos x \cdot \ln(\sec x)$

$$y\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \ln \sqrt{2} = \frac{1}{2\sqrt{2}} \ln 2$$

18. Let $\alpha \in R$ be such that the function $f(x) = \begin{cases} \dfrac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3} & , x \neq 0 \\ \alpha & , x = 0 \end{cases}$ is continuous at $x = 0$,

where $\{x\} = x-[x]$, $[x]$ is the greatest integer less than or equal to x. then :

(1) $\alpha = 0$ (2) $\alpha = \frac{\pi}{4}$ (3) $\alpha = \frac{\pi}{\sqrt{2}}$ (4) No such $\alpha$ exists

Ans. Offical Answer NTA (4)

Sol. $$f(0^-) = \lim_{x\to 0^-} \frac{\cos^{-1}\left(1-\{x\}^2\right)\sin^{-1}\left(1-\{x\}\right)}{\{x\}\left(1+\{x\}\right)\left(1-\{x\}\right)}$$

$$f(0^-) = \lim_{x\to 0^-} \frac{\cos^{-1}(0)}{1\cdot(1+1)} \cdot \left(\frac{\sin^{-1}\left(1-\{x\}\right)}{\left(1-\{x\}\right)}\right)$$

$$f(0^-) = \frac{\pi}{4}$$

$$f(0^+) = \lim_{x\to 0^+} \frac{\cos^{-1}\left(1-x^2\right)\sin^{-1}\left(1-x\right)}{x\left(1-x^2\right)}$$

$$= \lim_{x\to 0^+} \frac{\cos^{-1}\left(1-x^2\right)}{x} \cdot \frac{\pi}{2}$$

Let $\cos^{-1}(1-x^2) = \theta$

$1 - x^2 = \cos\theta$

$$x = \sqrt{1-\cos\theta} = \sqrt{2}\sin\frac{\theta}{2}$$

$$f(0^+) = \lim_{\theta\to 0^+} \frac{\theta}{\sqrt{2}\sin\frac{\theta}{2}} \cdot \frac{\pi}{2} = \frac{\pi}{\sqrt{2}}$$

$f(0^-) \neq f(0^+)$

No such $\alpha$ exists

19. If the points of intersections of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the circle $x^2 + y^2 = 4b$, $b > 4$ lie on the curve $y^2 = 3x^2$, then b is equal to :

(1) 10 (2) 5 (3) 12 (4) 6

Ans. Offical Answer NTA (3)

Sol. $C_1 : \frac{x^2}{16}+\frac{y^2}{b^2}=1$

$C_2 : x^2 + y^2 = 4b \ (b > 4)$

$C_3 : y^2 = 3x^2$

From $C_2$ and $C_3$

$x^2 = b$

$y^2 = 3b$, puttingb in $C_1$

$\frac{b}{16}+\frac{3b}{b^2}=1$

$b = 4$ or $12$ $(b > 4)$

$b = 12$

20. Given that the inverse trigonometric functions take principal values only. Then, the number of real values of x which satisfy $\sin^{-1}\left(\frac{3x}{5}\right)+\sin^{-1}\left(\frac{4x}{5}\right)=\sin^{-1}x$ is equal to :

(1) 0 (2) 1 (3) 3 (4) 2

Ans. Offical Answer NTA (3)

Sol. $\sin^{-1}\left(\frac{3x}{5}\right)+\sin^{-1}\left(\frac{4x}{5}\right)=\sin^{-1}x$

$\sin\left(\sin^{-1}\left(\frac{3x}{5}\right)+\sin^{-1}\left(\frac{4x}{5}\right)\right)=\sin\left(\sin^{-1}x\right)$

$\frac{3x}{5}\sqrt{1-\left(\frac{4x}{5}\right)^2}+\frac{4x}{5}\sqrt{1-\left(\frac{3x}{5}\right)^2}=x$

$x = 0$ or

$3\sqrt{25-16x^2}+4\sqrt{25-9x^2}=25$

$x^2 = 1 \Rightarrow x = \pm 1$

We can cross check by putting x = 0, 1, –1 in given equation.

Number of values = 3

**SECTION – B**

1. For real numbers $\alpha$, $\beta$, $\gamma$ and $\delta$ if

$$\int \frac{(x^2-1)+\tan^{-1}\left(\frac{x^2+1}{x}\right)}{(x^4+3x^2+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx$$

$$=\alpha\log_e\left(\tan^{-1}\left(\frac{x^2+1}{x}\right)\right)+\beta\tan^{-1}\left(\frac{\gamma(x^2-1)}{x}\right)+\delta\tan^{-1}\left(\frac{x^2+1}{x}\right)+C$$

where C is an arbitrary constant, then the value of $10(\alpha+\beta\gamma+\delta)$ is equal to _______.

Ans. Offical Answer NTA (6)

Sol. $$I=\int \frac{(x^2-1)}{(x^4+3x^2+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx+\int\frac{dx}{(x^4+3x^2+1)}$$

$$I=\int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x^2+3+\frac{1}{x^2}\right)\tan^{-1}\left(x+\frac{1}{x}\right)}+\frac{1}{2}\int\frac{(x^2+1)-(x^2-1)}{(x^4+3x^2+1)}dx$$

$$I=\int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(\left(x+\frac{1}{x}\right)^2+1\right)\tan^{-1}\left(x+\frac{1}{x}\right)}+\frac{1}{2}\int\frac{x^2+1}{(x^4+3x^2+1)}dx-\frac{1}{2}\int\frac{x^2-1}{x^4+3x^2+1}dx$$

$$I=ln\left(\tan^{-1}\left(x+\frac{1}{x}\right)\right)+\frac{1}{2}\int\frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+(\sqrt{5})^2}dx-\frac{1}{2}\int\frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2+1}dx$$

$$I=ln\left(\tan^{-1}\left(\frac{x^2+1}{x}\right)\right)+\frac{1}{2}\cdot\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{5}}\right)-\frac{1}{2}\tan^{-1}\left(x+\frac{1}{x}\right)+C$$

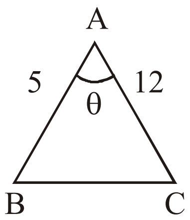
$$I=ln\left(\tan^{-1}\left(\frac{x^2+1}{x}\right)\right)+\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{\frac{1}{\sqrt{5}}(x^2-1)}{x}\right)-\frac{1}{2}\tan^{-1}\left(\frac{x^2+1}{x}\right)+C$$

$\alpha = 1, \beta = \frac{1}{2\sqrt{5}}, \gamma = \frac{1}{\sqrt{5}}, \delta = \frac{-1}{2}$ ]

10 ( α + β γ + δ) = 6

2. In Δ ABC, the lengths of sides AC and AB are 12 cm and 5 cm, respectively. If the area of Δ ABC is 30 $cm^2$ and R and r are respectively the radii of circumcircle and incircle of Δ ABC, then the value of 2R + r (in cm) is equal to _______.

Ans. Offical Answer NTA (15)

Sol. 


$\Delta = \frac{1}{2} \times 5 \times 12 \times \sin\theta = 30$

$\sin\theta = 1 \Rightarrow \theta = \pi/2$

BC = 13

$s = \frac{5+12+13}{2}$

s = 15

$r = \frac{\Delta}{s} = 2$

$R = \frac{abc}{4\Delta} = \frac{5 \times 12 \times 13}{4 \times 30} = \frac{13}{2}$

2R + r = 13 + 2 = 15

3. Let $\frac{1}{16}$, a and b be in G.P. and $\frac{1}{a}, \frac{1}{b}, 6$ be in A.P in A.P where a, b > 0. then 72(a + b) is equal to _____.

Ans. Offical Answer NTA (14)

Sol. $\frac{1}{16}, a, b$ are in G.P.

$a^2 = \frac{b}{16} \Rightarrow b = 16a^2$ ... (i)

$\frac{1}{a}, \frac{1}{b}, 6$ are in A.P.

$\frac{2}{b} = \frac{1}{a} + 6$

$\frac{2}{16a^2} = \frac{1}{a} + 6$

$48a^2 + 8a - 1 = 0$

$a = \frac{1}{12}$ or $\frac{-1}{4} (a > 0)$

$a = \frac{1}{12}, \ b = \frac{1}{9}$

$72 (a + b) = 14$

4. Consider the statistics of two sets of observations as follows:

| | Size | Mean | Variance |
|---|---|---|---|
| Observation I | 10 | 2 | 2 |
| Observation II | n | 3 | 1 |

If the variance of the combined set of these two observations is $\frac{17}{9}$, then the value of n is equal to _______ .

Ans. Offical Answer NTA (5)

Sol. For observation I :

$\sum x_i = 10 \times 2 = 20$

$\sigma^2 = \frac{\Sigma x_i^2}{10} - (2)^2 = 2$

$\Sigma x_i^2 = 60$

For observation II :

$\Sigma x_i = n \times 3 = 3n$

$\sigma^2 = \frac{\Sigma x_i^2}{n} - (3)^2 = 1$

$\Sigma x_i^2 = 10n$

For combined (n + 10) observations

$\Sigma x_i = 20 + 3n$

$\Sigma x_i^2 = 60 + 10n$

$$\sigma^2 = \left(\frac{60+10n}{n+10}\right) - \left(\frac{20+3n}{n+10}\right)^2 = \frac{17}{9}$$

$n = 5$

5. Let $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ be two $2\times 1$ matrices with real entries such that $A = XB$, where $X = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$ and $k \in R$. If $a_1^2 + a_2^2 = \frac{2}{3}\left(b_1^2 + b_2^2\right)$ and $\left(k^2+1\right)b_2^2 \neq -2b_1b_2$ then the value of k is ______.

Ans. Offical Answer NTA (1)

Sol. $A = XB$

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$a_1 = \frac{1}{\sqrt{3}}\left(b_1 - b_2\right)$$

$$a_2 = \frac{1}{\sqrt{3}}\left(b_1 + kb_2\right)$$

$$a_1^2 + a_2^2 = \frac{1}{3}\left\{\left(b_1 - b_2\right)^2 + \left(b_1 + kb_2\right)^2\right\}$$

$$\frac{2}{3}\left(b_1^2 + b_2^2\right) = \frac{1}{3}\left\{\left(b_1 - b_2\right)^2 + \left(b_1 + kb_2\right)^2\right\}$$

$$2\left(b_1^2 + b_2^2\right) = \left\{\left(b_1 - b_2\right)^2 + \left(b_1 + kb_2\right)^2\right\}$$

$k = 1$

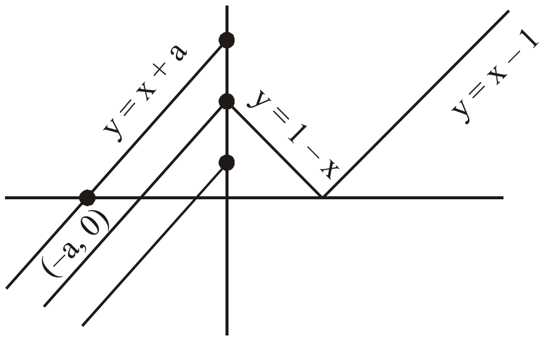
6. Let $f : R \to R$ and $g : R \to R$ be defined as $f(x) = \begin{cases} x + a, & x < 0 \\ |x - a|, & x \geq 0 \end{cases}$ and $g(x) = \begin{cases} x+1, & x < 0 \\ (x-1)^2 + b, & x \geq 0, \end{cases}$ where a, b are non- negative real numbers. If $(gof)(x)$ is continuous for all $x \in R$, then a + b is equal to ____.

Ans. Offical Answer NTA (1)

Sol. $f(x) = \begin{cases} x + a & x < 0 \\ |x-1| & x \geq 0 \end{cases}$

$$g(x) = \begin{cases} x+1 & x < 0 \\ \left(x-1\right)^2 + b & x \geq 0 \end{cases}$$

$$g(f(x)) = \begin{cases} f(x)+1 & f(x) < 0 \\ (f(x)-1)^2 + b & f(x) \geq 0 \end{cases}$$




$$g(f(x)) = \begin{cases} (x+a)+1 & x < -a \\ ((x+a)-1)^2 + b & -a \leq x < 0 \\ ((|x-1|)-1)^2 + b & 0 \leq x \end{cases}$$

For g(f(x)) to be continuous
$g(f(-a^-)) = g(f(-a^+)) = g(f(-a))$
and $g(f(o^-)) = g(f(o^+)) = g(f(o))$

$1 = 1 + b \Rightarrow b = 0$

$(a-1)^2 + b = b \Rightarrow a = 1$

$a + b = 1$

7. Let n be a positive integer . Let $A = \sum_{k=0}^{n} (-1)^k n_{C_k} \left[\left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \left(\frac{15}{16}\right)^k + \left(\frac{31}{32}\right)^k\right]$.

If $63A = 1 - \frac{1}{2^{30}}$, then n is equal to :

Ans. Offical Answer NTA (6)

Sol. $A = \sum_{k=0}^{n} (-1)^k n_{C_k} \left[\left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \left(\frac{15}{16}\right)^k + \left(\frac{31}{32}\right)^k\right]$

$A = \left(1-\frac{1}{2}\right)^n + \left(1-\frac{3}{4}\right)^n + \left(1-\frac{7}{8}\right)^n + \left(1-\frac{15}{16}\right)^n + \left(1-\frac{31}{32}\right)^n$

$A = \frac{1}{2^n} + \frac{1}{4^n} + \frac{1}{8^n} + \frac{1}{16^n} + \frac{1}{32^n}$

$$A = \frac{1}{2^n} + \frac{1}{2^{2n}} + \frac{1}{2^{3n}} + \frac{1}{2^{4n}} + \frac{1}{2^{5n}}$$

$$A = \frac{\frac{1}{2^n}\left(\left(\frac{1}{2n}\right)^5 - 1\right)}{\left(\frac{1}{2n} - 1\right)}$$

$$A = \frac{1-2^{5n}}{2^{5n}\left(1-2^n\right)} = \frac{2^{5n}-1}{2^{5n}\left(2^n-1\right)}$$

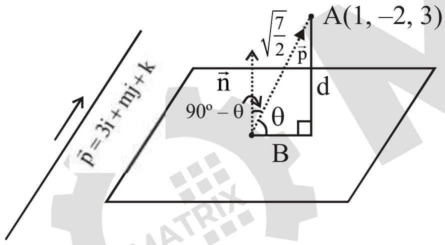
$$63A = \left(\frac{63}{2^n-1}\right)\left(1-\frac{1}{2^{5n}}\right) = 1 - \frac{1}{2^{30}}$$

$n = 6$

8. If the distance of the point (1, –2, 3) from the plane x + 2y –3z + 10 = 0 measured parallel to the line, $\frac{x-1}{3} = \frac{2-y}{m} = \frac{z+3}{1}$ is $\sqrt{\frac{7}{2}}$, then the value of |m| is equal to ______ .

Ans. Offical Answer NTA (2)

Sol.




$$L: \frac{x-1}{3} = \frac{2-y}{m} = \frac{z+3}{1}$$

$\vec{n} = i + 2j - 3k$

x + 2y – 3z + 10 = 0

$$d = \left|\frac{1-4-9+10}{\sqrt{14}}\right| = \frac{2}{\sqrt{14}}$$

$$\sin\theta = \frac{d}{\sqrt{\frac{7}{2}}} = \frac{\frac{2}{\sqrt{14}}}{\sqrt{\frac{7}{2}}} = \frac{2}{7}$$

$$\cos\left(90^\circ - \theta\right) = \frac{\vec{n}\cdot\vec{p}}{|\vec{n}||\vec{p}|}$$

$$\sin\theta = \frac{3+2m-3}{\sqrt{14}\cdot\sqrt{10+m^2}} = \frac{2}{7}$$

$|m| = 2$

9. Let $S_n(x) = \log_{a^{1/2}} x + \log_{a^{1/3}} x + \log_{a^{1/6}} x + \log_{a^{1/11}} x + \log_{a^{1/18}} x + \log_{a^{1/27}}$ + ...... up to n-terms, where $a > 1$. If $S_{24}$ $(x) = 1093$ and $S_{12}(2x) = 265$, then value of a is equal to _______ .

Ans. Offical Answer NTA (16)

Sol. $S_n(x) = 2\log_a x + 3\log_a x + 6\log_a x + 11\log_a x + .....\text{ n terms}$

$S_n(x) = (2+3+6+11+...+\text{n terms})\log_a^x$

Let, $S = 2+3+6+11+...+T_n$

$S = \quad 2+3+6+...+T_{n-1}+T_n$

$0 = 2+(1+3+5+....+(n-1)\text{ terms})+T_n$

$T_n = 2+(n-1)^2 = n^2-2n+3$

$$S = \Sigma(n^2-2n+3) = \frac{n}{6}\left[2n^2-3n+13\right]$$

$$S_n(x) = \frac{n}{6}(2n^2-3n+13)\log_a x$$

$S_{24}(x) = 4\times 1093\log_a x = 1093$

$$\log_a x = \frac{1}{4}$$

$S_{12}(2x) = 2\times 265(\log_a 2 + \log_a x) = 265$

$$2\left(\log_a 2 + \frac{1}{4}\right) = 1$$

$$\log_a 2 = \frac{1}{4}$$

$a = 2^4 = 16$

10. Let $\vec{c}$. be a vector perpendicular to the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ if $\vec{c}.\left(\hat{i} + \hat{j} + 3\hat{k}\right) = 8$ then the value of $\vec{c}.\left(\vec{a} \times \vec{b}\right)$ is equal to ______.

Ans. Offical Answer NTA (28)

Sol. $\vec{c} = \lambda\left(\vec{a} \times \vec{b}\right)$

$\vec{a} = i + j - k$

$\vec{b} = i + 2j + k$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = 3i - 2j + k$$

$\vec{c} = \lambda\left(3i - 2j + k\right)$

$\vec{c} \cdot \left(i + j + 3k\right) = 8$

$\lambda\left(3 - 2 + 3\right) = 8 \Rightarrow \lambda = 2$

$\vec{c} = 2\left(\vec{a} \times \vec{b}\right)$

$\vec{c} \cdot \left(\vec{a} \times \vec{b}\right) = 2\,|\vec{a} \times \vec{b}|^2$

$= 28$