JEE Main April 2023

Question Paper With Text Solution

15 April | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN APRIL 2023 | 15TH APRIL SHIFT-1

SECTION – A

Question ID : 3666943214

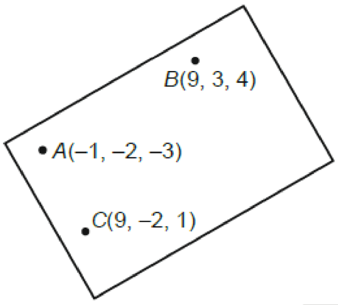
1. Let the foot of perpendicular of the point P(3, –2, –9) on the plane passing through the points (–1, –2, –3), (9, 3, 4), (9, –2, 1) be Q (α, β, γ). Then the distance of Q from the origin is :

यदि बिन्दु P(3, –2, –9) का बिन्दुओं (–1, –2, –3), (9, 3, 4), (9, –2, 1) से गुजरने वाले समतल पर लम्ब पाद Q(α, β, γ) है, तो Q की मूल बिन्दु से दूरी होगी :

(1) $\sqrt{42}$ (2) $\sqrt{38}$ (3) $\sqrt{29}$ (4) $\sqrt{35}$

Ans. Official Answer NTA (1)

Sol.



$\overrightarrow{AC} = 10i + 4k$

$\overrightarrow{AB} = 10i + 5j + 7k$

$$\overrightarrow{AC} \times \overrightarrow{AB} = \begin{vmatrix} i & j & k \\ 10 & 0 & 4 \\ 10 & 5 & 7 \end{vmatrix}$$

$= -20i - 30j + 50k$

Equation of plane

$2x + 3y - 5z = d$

Put (–1,–2,–3)

$-2 - 6 + 15 = d$

$d = 7$

$\therefore\ 2x + 3y - 5z = 7$

Foot of $\perp$r

$$\frac{x-3}{2} = \frac{y+2}{3} = \frac{z+9}{-5} = -\left(\frac{38}{38}\right)$$

$x = 1, y = -5, z = -4$

Q(1, –5, –4)

Distance from origin $= \sqrt{1+25+16}$

$= \sqrt{42}$

Question ID : 3666943210

2. If $\int_0^1 \frac{1}{\left(5+2x-2x^2\right)\left(1+e^{(2-4x)}\right)}dx = \frac{1}{\alpha}\log_e\left(\frac{\alpha+1}{\beta}\right)$, $\alpha, \beta, > 0$, then $\alpha^4 - \beta^4$ is equal to :

यदि $\int_0^1 \frac{1}{\left(5+2x-2x^2\right)\left(1+e^{(2-4x)}\right)}dx = \frac{1}{\alpha}\log_e\left(\frac{\alpha+1}{\beta}\right)$, $\alpha, \beta, > 0$ हो, तो $\alpha^4 - \beta^4$ का मान होगा :

(1) 0 (2) 21 (3) –21 (4) 19

Ans. Official Answer NTA (2)

Sol. $I = \int_0^1 \frac{dx}{\left(5+2x-2x^2\right)\left(1+e^{2-4x}\right)}$ ...(i)

$x \to 1-x$

$I = \int_0^1 \frac{e^{2-4x}dx}{\left(5+2x-2x^2\right)\left(1+e^{2-4x}\right)}$ ...(ii)

Add (i) and (ii)

$$2I = \int_0^1 \frac{dx}{5+2x-2x^2} = \int_0^1 \frac{dx}{2\left(\frac{11}{4}-\left(x-\frac{1}{2}\right)^2\right)}$$

$I = \frac{1}{\sqrt{11}}\ln\left(\frac{\sqrt{11}+1}{\sqrt{10}}\right)$ $\quad \alpha = \sqrt{11}$, $\beta = \sqrt{10}$

$\alpha^4 - \beta^4 = 121 - 100 = 21$

Question ID : 3666943211

3. Let $x = x(y)$ be the solution of the differential equation

$2(y+2)\log_e(y+2)dx + \left(x+4-2\log_e(y+2)\right)dy = 0, y > -1$ with $x\left(e^4-2\right)=1$. Then $x\left(e^9-2\right)$ is equal to :

यदि $x = x(y)$ अवकल समीकरण $2(y+2)\log_e(y+2)dx + \left(x+4-2\log_e(y+2)\right)dy = 0, y > -1$ का हल है जहाँ $x\left(e^4-2\right)=1$ हो, तो $x\left(e^9-2\right)$ का मान बराबर होगा :

(1) $\frac{10}{3}$ (2) $\frac{4}{9}$ (3) 3 (4) $\frac{32}{9}$

Ans. Official Answer NTA (4)

Sol. $2(y+2)\ln(y+2)\,dx + (x+4-2\ln(y+2))\,dy = 0$

$$2\ln(y+2) + \left(x+4-2\ln(y+2)\right)\frac{1}{y+2}\cdot\frac{dy}{dx} = 0$$

let $\ln(y+2) = t$

$$\frac{1}{y+2}\cdot\frac{dy}{dx} = \frac{dt}{dx}$$

$$2t + (x+4-2t)\cdot\frac{dt}{dx} = 0$$

$$(x+4-2t)\frac{dt}{dx} = -2t$$

$$\frac{dx}{dt} = \frac{2t-4-x}{2t}$$

$$\frac{dx}{dt} + \frac{x}{2t} = \frac{2t-4}{2t}$$

$$x\cdot t^{1/2} = \int \frac{2t-4}{2t}\cdot t^{1/2}\cdot dt$$

$$x\cdot t^{1/2} = \int\left(t^{1/2} - \frac{2}{t^{1/2}}\right)\cdot dt$$

$$xt^{\frac{1}{2}} = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - 2\cdot\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$x\cdot t^{\frac{1}{2}} = \frac{2t^{\frac{3}{2}}}{3} - 4t^{\frac{1}{2}} + C$$

$x = \frac{2}{3} \cdot t - 4 + C \cdot t^{-\frac{1}{2}}$

$x = \frac{2}{3} \ln(y+2) - 4 + C \cdot (\ln(y+2))^{-\frac{1}{2}}$

Put $y = e^4 - 2, x = 1$

$1 = \frac{2}{3} \times 4 - 4 + C \times \frac{1}{2}$

$\frac{C}{2} = 5 - \frac{8}{3} = \frac{7}{3}$

$\Rightarrow C = \frac{14}{3}$

$x = \frac{2}{3} \times 9 - 4 + \frac{14}{3} \times \frac{1}{3}$

$= 2 + \frac{14}{9}$

$= \frac{32}{9}$

Question ID : 3666943206

4. The total number of three-digit numbers, divisible by 3, which can be formed using the digits 1, 3, 5, 8, if repetition of digits is allowed, is :

अंकों 1, 3, 5, 8 को लेकर बनायी गई तीन अंकों की तीन से भाज्य कुल कितनी संख्याएं होगी, यदि अंकों की पुनरावृति संभव हो :

(1) 22 (2) 21 (3) 20 (4) 18

Ans. Official Answer NTA (1)

Sol. Sum of three digits is 3 then digits are 1,1,1

Sum of three digits is 9 then digits are 1,3,5 or 3,3,3

Sum of three digits is 12 then digits are 1,3,8

Sum of three digits is 15 then digits are 5,5,5

Sum of three digits is 18 then digits are 5,5,8

Sum of three digits is 21 then digits are 5,8,8

Sum of three digits is 24 then digits are 8,8,8

Now possible number are $= 1 + 3! + 1 + 3! + 1 + \frac{3!}{2!} + \frac{3!}{2!} + 1$

$= 1 + 6 + 1 + 6 + 1 + 3 + 3 + 1$

$= 22$

Question ID : 3666943218

5. A bag contains 6 white and 4 black balls. A die is rolled once and the number of balls equal to the number obtained on the die are drawn from the bag at random. The probability that all the balls drawn are white is :

एक थेले में 6 सफेद तथा 4 काली गेंद है। यदि पासे को एक बार उछाला जाता है तो पासे पर प्राप्त संख्या के बराबर गेंद थेले से यादृच्छया निकाली जाती है तो निकाली गई गेंदों के सफेद होने की प्रायिकता होगी :

(1) $\frac{1}{5}$ (2) $\frac{9}{50}$ (3) $\frac{1}{4}$ (4) $\frac{11}{50}$

Ans. Official Answer NTA (1)

Sol. Bag have 6 white and 4 black balls

Probability all drawn balls are white

$$= \frac{1}{6}\left[\frac{^6C_1}{^{10}C_1} + \frac{^6C_2}{^{10}C_2} + \frac{^6C_3}{^{10}C_3} + \frac{^6C_4}{^{10}C_4} + \frac{^6C_5}{^{10}C_5} + \frac{^6C_6}{^{10}C_6}\right]$$

$$= \frac{504}{2520} = \frac{1}{5}$$

Question ID : 3666943205

6. Let the system of linear equations

$-x + 2y - 9z = 7$

$-x + 3y + 7z = 9$

$-2x + y + 5z = 8$

$-3x + y + 13z = \lambda$

has a unique solution $x = \alpha$, $y = \beta$, $z = \gamma$. Then the distance of the point(α, β, γ) from the plane $2x - 2y + z = \lambda$ is :

यदि रेखिक समीकरण निकाय

$-x + 2y - 9z = 7$

$-x + 3y + 7z = 9$

$-2x + y + 5z = 8$
$-3x + y + 13z = \lambda$

के अद्वितीय हल $x = \alpha, y = \beta, z = \gamma$ हो, तो बिन्दु(α, β, γ) की समतल $2x - 2y + z = \lambda$ से दूरी होगी :

(1) 13 (2) 9 (3) 11 (4) 7

Ans. Official Answer NTA (4)

Sol. $-x + 2y - 9z = 7$ ...(1)

$-x + 3y + 7z = 9$ ...(2)

$-2x + y + 5z = 8$ ...(3)

(2) – (1)

$y + 16z = 2$ ...(4)

$(3) - 2 \times (1)$

$-3y + 23z = -6$ ...(5)

$3 \times (4) + (5)$

$71z = 0 \Rightarrow z = 0$

$y = 2$

$x = -3$

$(-3, 2, 0) \to (\alpha, \beta, \gamma)$

Put in $-3x + y + 13z = \lambda$

$\lambda = 9 + 2 = 11$

$$d = \left|\frac{-6-4-11}{3}\right| = 7$$

Question ID : 3666943202

7. The number of real roots of the equation $x|x| - 5|x + 2| + 6 = 0$, is :

समीकरण $x|x| - 5|x + 2| + 6 = 0$ के वास्तविक हलों की संख्या होगी :

(1) 5 (2) 4 (3) 3 (4) 6

Ans. Official Answer NTA (3)

Sol. $x|x| - 5|x + 2| + 6 = 0$

C–1 : $x \in [0,\infty]$

$x^2 - 5x - 4 = 0$

$$x = \frac{5 \pm \sqrt{25+16}}{2} = \frac{5+\sqrt{41}}{2}$$

$x = \frac{5 \pm \sqrt{41}}{2}$

C–2 :- $x \in [-2, 0)$

$-x^2 - 5x - 4 = 0$

$x^2 + 5x + 4 = 0$

$x = -1, -4$

$x = -1$

C–3 : $x \in (-\infty, -2)$

$-x^2 + 5x + 16 = 0$

$x^2 - 5x - 16 = 0$

$x = \frac{5 \pm \sqrt{25+64}}{2}$

$x = \frac{5 \pm \sqrt{89}}{2}$

$x = \frac{5 - \sqrt{89}}{2}$

The number of real roots = 3

Question ID : 3666943217

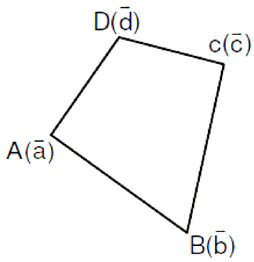
8. Let ABCD be a quadrilateral. If E and F are the mid points of the diagonals AC and BD respectively and $(\overrightarrow{AB} - \overrightarrow{BC}) + (\overrightarrow{AD} - \overrightarrow{DC}) = k\,\overrightarrow{FE}$, then k is equal to :

माना ABCD एक चतुर्भुज है जिसके विकर्ण AC तथा BD के मध्य बिन्दु क्रमश: E तथा F है, तथा $(\overrightarrow{AB} - \overrightarrow{BC}) + (\overrightarrow{AD} - \overrightarrow{DC}) = k\,\overrightarrow{FE}$ हो तो k का मान होगा :

(1) 4 (2) –2 (3) 2 (4) –4

Ans. Official Answer NTA (4)

Sol. Let P.V. & A, B, C, D are $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ resp. mid-point of AC is E.



$\Rightarrow$ P.V. of E are $\left(\frac{\vec{a}+\vec{c}}{2}\right)$

Mid point of BD is F

$\Rightarrow$ P.V. of F are $\left(\frac{\vec{b}+\vec{d}}{2}\right)$

$\left(\overrightarrow{AB} - \overrightarrow{BC}\right) + \left(\overrightarrow{AD} - \overrightarrow{DC}\right) = \left(\mathrm{b} - \vec{a}\right) - \left(\vec{c} - \vec{b}\right) + \left(\vec{d} - \vec{a}\right) - \left(\vec{c} - \vec{d}\right)$

$= 2\vec{b} - 2\vec{a} - 2\vec{c} + 2\vec{d}$

$= 2\left(\vec{b}+\vec{d}\right) - 2\left(\vec{a}+\vec{c}\right)$

$= 4\left[\left(\dfrac{\vec{b}+\vec{d}}{2}\right) - \left(\dfrac{\vec{a}+\vec{c}}{2}\right)\right] = 4\overrightarrow{EF} = -4\overrightarrow{FE}$

Question ID : 3666943209

9. Let [x] denote the greatest integer function and $f(x) = \max\{1+x+[x], 2+x, x+2[x]\}, 0 \le x \le 2$. Let m be the number of points in [0,2], where f is not continuous and n be the number of points in (0, 2), where f is not differentiable. Then $(m+n)^2+2$ is equal to :

यदि [x] महत्तम पूर्णांक फलन है तथा $f(x) = \max\{1+x+[x], 2+x, x+2[x]\}, 0 \le x \le 2$ है। यदि [0,2] में बिन्दुओं की संख्या m इस प्रकार है जहाँ f असतत् है तथा (0, 2) में बिन्दुओं की संख्या इस प्रकार है जहाँ f अवकलनीय नहीं है, तो $(m+n)^2+2$ का मान बराबर होगा :

(1) 6 (2) 11 (3) 2 (4) 3

Ans. Official Answer NTA (4)

Sol. $f(x) = \begin{cases} \max\{x+1, x+2, x\} & 0 \le x < 1 \\ \max\{x+2, x+2, x+2\} & 1 \le x < 2 \\ \max\{5, 4, 6\} & x = 2 \end{cases}$

$\Rightarrow f(x) = \begin{cases} x+2 & 0 \le x < 1 \\ x+2 & 1 \le x < 2 \\ 6 & x = 2 \end{cases}$

$\Rightarrow f(x) = \begin{vmatrix} x+2 & 0 \le x < 2 \\ 6 & x = 2 \end{vmatrix}$

$\Rightarrow f(x) = \begin{cases} x+2 & 0 \le x < 2 \\ 6 & x = 2 \end{cases}$

f is not continuous at $x = 2$
f is differentiable in (0, 2)
$\therefore$ m = 1, n = 0
$(m+n)^2 + 2 = 1 + 2 = 3$
option (4) is correct.

Question ID : 3666943203

10. If the set $\left\{\text{Re}\left(\frac{z-\bar{z}+z\bar{z}}{2-3z+5\bar{z}}\right): z\in\mathbb{C}, \text{Re}(z)=3\right\}$ is equal to the interval $(\alpha, \beta]$, then $24(\beta-\alpha)$ is equal to :

यदि समुच्चय $\left\{\text{Re}\left(\frac{z-\bar{z}+z\bar{z}}{2-3z+5\bar{z}}\right): z\in\mathbb{C}, \text{Re}(z)=3\right\}$ अन्तराल $(\alpha, \beta]$ के बराबर है, तो $24(\beta-\alpha)$ का मान होगा :

(1) 27 (2) 42 (3) 36 (4) 30

Ans. Official Answer NTA (4)

Sol. Let $z_1 = \left(\frac{z-\bar{z}+z\bar{z}}{2-3z+5\bar{z}}\right)$

Let $z = 3 + iy$

$\bar{z} = 3 - iy$

$$z_1 = \frac{2iy+\left(9+y^2\right)}{2-3\left(3+iy\right)+5\left(3-iy\right)}$$

$$= \frac{9+y^2+i\left(2y\right)}{8-8iy}$$

$$= \frac{\left(9+y^2\right)+i\left(2y\right)}{8\left(1-iy\right)}$$

$$\text{Re}\left(z_1\right) = \frac{\left(9+y^2\right)-2y^2}{8\left(1+y^2\right)}$$

$$= \frac{9-y^2}{8\left(1+y^2\right)}$$

$$= \frac{1}{8}\left[\frac{10-\left(1+y^2\right)}{\left(1+y^2\right)}\right]$$

$$= \frac{1}{8}\left[\frac{10}{1+y^2}-1\right]$$

$1 + y^2 \in [1, \infty]$

$\frac{1}{1+y^2}\in(0,1]$

$\frac{10}{1+y^2}\in(0,10]$

$\frac{10}{1+y^2}-1\in(-1,9]$

$\text{Re}(z_1)\in\left(\frac{-1}{8},\frac{9}{8}\right]$

$\alpha=\frac{-1}{8},\beta=\frac{9}{8}$

$24(\beta-\alpha)=24\left(\frac{9}{8}+\frac{1}{8}\right)=30$

Question ID : 3666943220

11. Negation of $p\wedge(q\wedge\sim(p\wedge q))$ is :

$p\wedge(q\wedge\sim(p\wedge q))$ का विरोधात्मक होगा :

(1) $(\sim(p\wedge q))\vee p$ (2) $\sim(p\vee q)$ (3) $p\vee q$ (4) $(\sim(p\wedge q))\wedge q$

Ans. Official Answer NTA (1)

Sol. $\sim[p\wedge(q\wedge\sim(p\wedge q))]$

$\sim p\vee(\sim q\vee(p\wedge q))$

$\sim p\vee((\sim q\vee p)\wedge(\sim q\vee q))$

$\sim p\vee(\sim q\vee p)$

$\sim(p\wedge q)\vee p$

Question ID : 3666943215

12. Let S be the set of all values of λ, for which the shortest distance between the lines $\frac{x-\lambda}{0}=\frac{y-3}{4}=\frac{z+6}{1}$ and $\frac{x+\lambda}{3}=\frac{y}{-4}=\frac{z-6}{0}$ is 13. Then $8\left|\sum_{\lambda\in S}\lambda\right|$ is equal to :

यदि λ के सभी मानों का समुच्चय S है, जिसके लिए रेखाओं $\frac{x-\lambda}{0}=\frac{y-3}{4}=\frac{z+6}{1}$ तथा $\frac{x+\lambda}{3}=\frac{y}{-4}=\frac{z-6}{0}$ के मध्य की न्यूनतम दूरी 13 है तो $8\left|\sum_{\lambda\in S}\lambda\right|$ का मान होगा :

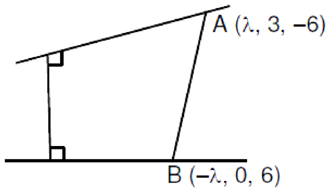
(1) 308 (2) 304 (3) 302 (4) 306

Ans. Official Answer NTA (4)

Sol. Given lines : $\frac{x-\lambda}{0}=\frac{y-3}{4}=\frac{z+6}{1}$ ...(i)

$\frac{x+\lambda}{3}=\frac{y}{-4}=\frac{z-6}{0}$ ...(ii)

DR of $\overline{AB}$: $2\lambda, 3, -12$



$\vec{p}\times\vec{q}$ is perpendicular both the lines

$$\vec{p}\times\vec{q}=\begin{vmatrix} i & j & k \\ 0 & 4 & 1 \\ 3 & -4 & 0 \end{vmatrix}$$

SD = 13

$$\left|\frac{\overline{AB}\cdot(\vec{p}\times\vec{q})}{\vec{p}\times\vec{q}}\right|=13$$

$$\Rightarrow\left|\frac{8\lambda+153}{13}\right|=13$$

Now, $8\left|\sum_{\lambda\in S}\lambda\right|=|16-322|=306$

Question ID : 3666943201

13. If the domain of the function $f(x)=\log_e\left(4x^2+11x+6\right)+\sin^{-1}(4x+3)+\cos^{-1}\left(\frac{10x+6}{3}\right)$ is $(\alpha,\beta]$, then $36|\alpha+\beta|$ is equal to :

यदि $f(x)=\log_e\left(4x^2+11x+6\right)+\sin^{-1}(4x+3)+\cos^{-1}\left(\frac{10x+6}{3}\right)$ का प्रांत $(\alpha,\beta]$ है, तो $36|\alpha+\beta|$ का मान बराबर होगा :

(1) 63 (2) 54 (3) 45 (4) 72

Ans. Official Answer NTA (3)

Sol. $4x^2+11x+6>0 \Rightarrow (4x+3)(x+2)>0$

$\Rightarrow x\in(-\infty,-2)\cup\left(\frac{-3}{4},\infty\right)$ ...(i)

$-1\le 4x+3\le 1 \Rightarrow -4\le 4x\le -2$

$-1\le x\le -\frac{1}{2}$ ...(ii)

$-1\le \frac{10x+6}{3}\le 1 \Rightarrow -3\le 10x+6\le 3$

$\Rightarrow -9\le 10x\le -3$

$\frac{-9}{10}\le x\le \frac{-3}{10}$ ...(iii)

$(i),(ii),(iii)\Rightarrow \frac{-3}{4}<x\le\frac{-1}{2}$

$\alpha=\frac{-3}{4},\beta=\frac{-1}{2}$

$36|\alpha+\beta|=36\times\frac{5}{4}=45$

Question ID : 3666943213

14. The number of common tangents, to the circles $x^2+y^2-18x-15y+131=0$ and $x^2+y^2-6x-6y-7=0$:

वृत्तों $x^2+y^2-18x-15y+131=0$ तथा $x^2+y^2-6x-6y-7=0$ की उभ्यनिष्ठ स्पर्श रेखाओं की संख्या होगी :

(1) 2 (2) 4 (3) 3 (4) 1

Ans. Official Answer NTA (3)

Sol. $C_1\left(9, \frac{15}{2}\right)$ $\quad r_1 = \sqrt{81 + \frac{225}{4} - 131} = \frac{5}{2}$

$C_2\ (3,3)$ $\quad r_2 = 5$

$C_1C_2 = \sqrt{6^2 + \frac{81}{4}} = \frac{15}{2}$

$r_1 + r_2 = \frac{15}{2}$

$C_1C_2 = r_1 + r_2$

Question ID : 3666943207

15. Let $\left(a + bx + cx^2\right)^{10} = \sum_{i=0}^{20} p_i x^i, a, b, c \in \mathbb{N}$. If $p_1 = 20$ and $p_2 = 210$, then 2(a+b+c) is equal to :

यदि $\left(a + bx + cx^2\right)^{10} = \sum_{i=0}^{20} p_i x^i, a, b, c \in \mathbb{N}$, $p_1 = 20$ तथा $p_2 = 210$ हो, तो 2(a+b+c) का मान बराबर होगा :

(1) 8 (2) 12 (3) 6 (4) 15

Ans. Official Answer NTA (2)

Sol. $\left(a + bx + cx^2\right)^{10} = \sum_{i=0}^{20} p_i x^i,$

Coefficient of $x^1 = 20$

$20 = \frac{10!}{9!1!} \times a^9 \times b^1$

$a^9 \cdot b = 2$

$a = 1, b = 2$

Coefficient of $x^2 = 210$

$210 = \frac{10!}{9!1!} \times a^9 \times c^1 + \frac{10!}{8!2!} \times a^8 b^2$

$210 = 10 \cdot c + 45 \times 4$

$10c = 30$

$c = 3$

$2\ (a + b + c) = 12$

Question ID : 3666943219

16. The mean and standard deviation of 10 observations are 20 and 8 respectively. Later on, it was observed that one observation was recorded as 50 instead of 40. Then the correct variance is :

यदि 10 प्रेक्षणों का माध्य तथा मानक विचलन क्रमशः 20 तथा 8 है, बाद में यह पाया गया कि एक प्रेक्षण 40 की जगह 50 लिया गया हो, तो सही विचलन होगा :

(1) 14 (2) 13 (3) 11 (4) 12

Ans. Official Answer NTA (2)

Sol. $\frac{\sum_{i=1}^{10} x_i}{10} = 20 \Rightarrow \sum x_i = 200$

Correct $\sum x_i = 200 - 50 + 40 = 190$

$\sigma^2 = \frac{\sum_{i=1}^{10} x_i^2}{10} - (20)^2 = 64$

$\sum x_i^2 = 4640 - 2500 + 1600 = 3740$

Correct $\sigma^2 = \frac{3740}{10} - \left(\frac{190}{10}\right)^2$

$= 374 - 361 = 13$

Question ID : 3666943208

17. Let A_1 and A_2 be two arithmetic means and G_1, G_2, G_3 be three geometric means of two distinct positive numbers. Then $G_1^4 + G_2^4 + G_3^4 + G_1^2G_3^2$ is equal to :

यदि दो धनात्मक विभिन्न संख्याओं के मध्य दो समान्तर माध्य क्रमशः A_1, A_2 तथा तीन गुणोत्तर माध्य G_1, G_2, G_3 है, तो $G_1^4 + G_2^4 + G_3^4 + G_1^2G_3^2$ का मान बराबर होगा :

(1) $(A_1 + A_2)G_1^2G_3^2$ (2) $2(A_1 + A_2)G_1G_3$ (3) $(A_1 + A_2)^2 G_1G_3$ (4) $2(A_1 + A_2)G_1^2G_3^2$

Ans. Official Answer NTA (3)

Sol. Let the two numbers are a, b.

$A_1 = a + \frac{b-a}{3} = \frac{2a+b}{3}$

$$A_2 = a + \frac{b-a}{3}\cdot 2 = \frac{a+2b}{3}$$

$$G_1 = a\left(\frac{b}{a}\right)^{\frac{1}{4}}$$

$$G_2 = a\left(\frac{b}{a}\right)^{\frac{2}{4}}$$

$$G_3 = a\left(\frac{b}{a}\right)^{\frac{3}{4}}$$

$(G_1)^4 + (G_2)^4 + (G_3)^4 + (G_1)^2 \cdot (G_3)^2$

$$\Rightarrow a^4\cdot\frac{b}{a} + a^4\cdot\frac{b^2}{a^2} + a^4\cdot\frac{b^3}{a^3} + a^4\cdot\frac{b^2}{a^2}$$

$= ba^3 + b^2a^2 + b^3a + a^2b^2$

$= ab\ (a^2 + b^2 + 2ab) = ab\ (a + b)^2$

$(A_1 + A_3)^2 \cdot G_1G_3 = (a + b)^2 \cdot ab$

Question ID : 3666943204

18. Let the determinant of a square matrix A of order m be m – n, where m and n satisfy $4m + n = 22$ and $17m + 4n = 93$. If $\det(n\,\text{adj}(\text{adj}(mA))) = 3^a5^b6^c$, then $a + b + c$ is equal to :

यदि m कोटि के वर्ग आव्यूह A के सारणिक का मान $m - n$ है, जहाँ m तथा n, $4m + n = 22$ तथा $17m + 4n = 93$ को संतुष्ट करते है। यदि $\det(n\,\text{adj}(\text{adj}(mA))) = 3^a5^b6^c$ हो, तो $a + b + c$ के मान बराबर होगा :

(1) 96 (2) 101 (3) 109 (4) 84

Ans. Official Answer NTA (1)

Sol. $|A| = m - n$

$4m + n = 22$

$17m + 4n = 93$

$m = 5, n = 2$

$|A| = 3$

$|2\ \text{adj}\ (\text{adj}\ 5A)| = 2^5\ |5A|^{16}$

$= 2^5 \cdot 5^{80}\ |A|^{16}$

$= 2^5 \cdot 5^{80} \cdot 3^{16}$

$= 3^{11} \cdot 5^{80} \cdot 6^5$

a + b + c = 96

Question ID : 3666943212

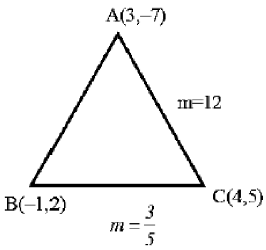
19. If (α, β) is the orthocenter of the triangle ABC with vertices A(3, –7), B(–1, 2) and C(4, 5), then 9α – 6β + 60 is equal to :

यदि त्रिभुज ABC का लम्ब केन्द्र (α, β) है जहाँ त्रिभुज ABC के शीर्ष क्रमशः A(3, –7), B(–1, 2) तथा C(4, 5) हो, तो 9α – 6β + 60 का मान बराबर होगा :

(1) 30 (2) 35 (3) 40 (4) 25

Ans. Official Answer NTA (4)

Sol.



Altitude of BC : $y+7=\frac{-5}{3}(x-3)$

3y + 21 = –5x + 15

5x + 3y + 6 = 0

Altitude of AC : $y-2=\frac{-1}{12}(x+1)$

12y – 24 = – x – 1

x + 12y = 23

$\alpha=\frac{-47}{19},\beta=\frac{121}{57}$

9α – 6β + 60 = 25

Question ID : 3666943216

20. Let S be the set of all (λ, μ) for which the vectors $\lambda\hat{i}-\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}+\mu\hat{k}$ and $3\hat{i}-4\hat{j}+5\hat{k}$, where $\lambda-\mu=5$, are coplanar, then $\sum_{(\lambda,\mu)\in S}80\left(\lambda^2+\mu^2\right)$ is equal to :

यदि (λ, μ) का समुच्चय S इस प्रकार है जिसके लिए सदिश $\lambda\hat{i}-\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}+\mu\hat{k}$ तथा $3\hat{i}-4\hat{j}+5\hat{k}$ समतलीय है, जहाँ $\lambda-\mu=5$ हो तो $\sum_{(\lambda,\mu)\in S} 80\left(\lambda^2+\mu^2\right)$ का मान होगा :

(1) 2210 (2) 2370 (3) 2290 (4) 2130

Ans. Official Answer NTA (3)

Sol. $\begin{vmatrix} \lambda & -1 & 1 \\ 1 & 2 & \mu \\ 3 & -4 & 5 \end{vmatrix} = 0$

$\lambda(10+4\mu)+1(5-3\mu)+1(-4-6)=0$

$10\lambda+4\lambda\mu+5-3\mu-10=0$

$10\lambda+4\lambda\mu-3\mu=5$ ...(i)

And $\lambda=5+\mu$ ...(ii)

$10(5+\mu)+4\mu(5+\mu)-3\mu=5$

$10\mu+20\mu-3\mu+4\mu^2+50=5$

$4\mu^2+27\mu+45=0$

$4\mu^2+12\mu+15\mu+45=0$

$4\mu(\mu+3)+15(\mu+3)=0$

$\mu=-3$ or $-\frac{15}{4}$

$\lambda=5-3=2$ or $\lambda=5-\frac{15}{4}=\frac{5}{4}$

$\sum_{\lambda,\mu} 80\left(\lambda^2+\mu^2\right)=\left(9+4+\frac{225}{16}+\frac{25}{16}\right)80=\left(13+\frac{250}{16}\right)80=1040+1250=2290$

SECTION – B

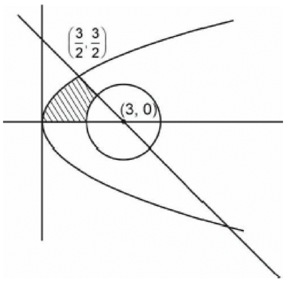
Question ID : 3666943227

21. If the area bounded by the curve $2y^2 = 3x$, lines $x + y = 3$, $y = 0$ and outside the circle $(x – 3)^2 + y^2 = 2$ is A, then $4(\pi + 4A)$ is equal to ________.

यदि वक्र $2y^2 = 3x$, रेखाओं $x + y = 3$, $y = 0$ तथा वृत्त $(x – 3)^2 + y^2 = 2$ के बाहर गिरा हुआ क्षेत्रफल A है, तो $4(\pi + 4A)$ मान होगा ________

Ans. Official Answer NTA (42)

Sol.



$$\int_0^{3/2}\left((3-y)-\frac{2y^2}{3}\right)dy-\pi\left(\sqrt{2}\right)^2\cdot\frac{1}{8}$$

$$=\frac{36-9-6}{8}-\frac{\pi}{4}=\frac{21}{8}-\frac{\pi}{4}$$

$4(\pi + 4A) = 42$

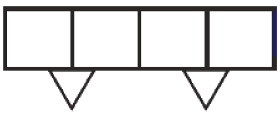
Question ID : 3666943222

22. A person forgets his 4-digit ATM pin code. But he remembers that in the code all the digits are different, the greatest digit is 7 and the sum of the first two digits is equal to the sum of the last two digits. Then the maximum number of trials necessary to obtain the correct code is ________.

एक व्यक्ति अपने चार अंक का ATM पिन कोड भुल जाता है, लेकिन उसे यह याद रहता है कि सभी अंक भिन्न–भिन्न है, सबसे बड़ा अंक 7 है तथा प्रथम दो अंको का योग अन्तिम दो अंकों के योग के बराबर है, तो सही कोड के लिए आवश्यक अधिकतम प्रयासों की संख्या होगी ________

Ans. Official Answer NTA (72)

Sol.



Sum of first two digits = Sum of last two digits = α

C-I : $\alpha = 7$

$2 \times 12 = 24$ ways

7	0		
0	7	1	6
		2	5
		3	4
		4	3
		5	2
		6	1

2×8 Ways

= 16 ways

Case-III : $\alpha = 9$

27	36
72	63
	45
	54

2×8 Ways

= 16 ways

Case-IV : $\alpha = 10$

37	46
73	64

2×4 ways

8 ways

Case-V : $\alpha = 11$

47	56
74	65

2×4 ways

= 8 ways

Ans. 24 + 16 + 16 + 8 + 8 = 72

Question ID : 3666943229

23. Let the plane P contain the line $2x + y - z - 3 = 0 = 5x - 3y + 4z + 9$ and be parallel to the line $\frac{x+2}{2} = \frac{3-y}{-4} = \frac{z-7}{5}$. Then the distance of the point A(8, –1, –19) from the plane P measured parallel to the line $\frac{x}{-3} = \frac{y-5}{4} = \frac{2-z}{-12}$ is equal to ________.

यदि समतल P रेखा $2x + y - z - 3 = 0 = 5x - 3y + 4z + 9$ को रखता है तथा रेखा $\frac{x+2}{2} = \frac{3-y}{-4} = \frac{z-7}{5}$ के समान्तर है, तो बिन्दु A(8, –1, –19) की रेखा $\frac{x}{-3} = \frac{y-5}{4} = \frac{2-z}{-12}$ के अनुदिश समतल P से दूरी होगी ________

Ans. Official Answer NTA (26)

Sol. Plane $\equiv P_1 + \lambda P_2 = 0$

$(2x + y - z - 3) + \lambda\,(5x - 3y + 4z + 9) = 0$

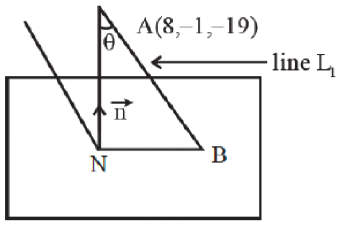
$(5\lambda + 2)\,x + (1 - 3\lambda)\,y + (4\lambda - 1)\,z + 9\lambda - 3 = 0$

$\vec{n}\cdot\vec{b}=0$ where $\vec{b}(2,4,5)$

$2(5\lambda+2)+4(1-3\lambda)+5(4\lambda-1)=0$

$\lambda=-\frac{1}{6}$

Plane $7x+9y-10z-27=0$



Equation of line AB is

$$\frac{x-8}{-3}=\frac{y+1}{4}=\frac{z+19}{12}=\lambda$$

Let $B=(8-3\lambda,-1+4\lambda,-19+12\lambda)$ lies on plane P

$\therefore\ 7(8-3\lambda)+9(4\lambda-1)-10(12\lambda-19)=27$

$\lambda=2$

$\therefore$ Point $B=(2,7,5)$

$AB=\sqrt{6^2+8^2+24^2}=26$

Question ID : 3666943226

24. Let $f(x)=\int\frac{dx}{(3+4x^2)\sqrt{4-3x^2}}$, $|x|<\frac{2}{\sqrt{3}}$. If $f(0)=0$ and $f(1)=\frac{1}{\alpha\beta}\tan^{-1}\left(\frac{\alpha}{\beta}\right)$, $\alpha,\beta>0$, then $\alpha^2+\beta^2$ is equal to ________.

यदि $f(x)=\int\frac{dx}{(3+4x^2)\sqrt{4-3x^2}}$, $|x|<\frac{2}{\sqrt{3}}$, $f(0)=0$ तथा $f(1)=\frac{1}{\alpha\beta}\tan^{-1}\left(\frac{\alpha}{\beta}\right)$, $\alpha,\beta>0$ हो, तो $\alpha^2+\beta^2$ का मान होगा ________

Ans. Official Answer NTA (28)

Sol. $f(x)=\int\frac{dx}{(3+4x^2)\sqrt{4-3x^2}}$

Let $x=\frac{1}{t}$

$$dx = -\frac{1}{t^2}dt$$

$$= \int \frac{-\frac{1}{t^2}dt}{\left(3+\frac{4}{t^2}\right)\sqrt{4-\frac{3}{t^2}}}$$

$$= \int \frac{-tdt}{\left(3t^2+4\right)\sqrt{4t^2-3}}$$

Now, let $4t^2 - 3 = p^2$

$4t\,dt = p\,dp$

$$= \int \frac{-\frac{p}{4}dp}{\left(\frac{3p^2+25}{5}\right)p} = -\int \frac{dp}{3p^2+25}$$

$$f(x) = -\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}p}{5}\right)+C$$

$$f(x) = -\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}\cdot\sqrt{4t^2-3}}{5}\right)+C$$

$$= -\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}\cdot\sqrt{\frac{4}{x^2}-3}}{5}\right)+C$$

$$f(x) = -\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}\cdot\sqrt{4-3x^2}}{5|x|}\right)+C$$

$x = 0, y = 0$

$$0 = -\frac{1}{5\sqrt{3}}\tan^{-1}(\infty)+C$$

$$C = \frac{1}{5\sqrt{3}}\times\frac{\pi}{2}$$

$$f(x) = -\frac{1}{5\sqrt{3}}\tan^{-1}\frac{\sqrt{3}\sqrt{4-3x^2}}{5x} + \frac{\pi}{2(5\sqrt{3})}$$

$$f(1) = -\frac{1}{5\sqrt{3}}\tan^{-1}\frac{\sqrt{3}}{5} + \frac{\pi}{2(5\sqrt{3})}$$

$$= \frac{1}{5\sqrt{3}}\left(\frac{\pi}{2} - \tan^{-1}\frac{\sqrt{3}}{5}\right)$$

$$= \frac{1}{5\sqrt{3}}\cot^{-1}\frac{\sqrt{3}}{5}$$

$$= \frac{1}{5\sqrt{3}}\tan^{-1}\frac{5}{\sqrt{3}}$$

Now, $\alpha = 5,\ \beta = \sqrt{3}$

$\alpha^2 + \beta^2 = 25 + 3 = 28$

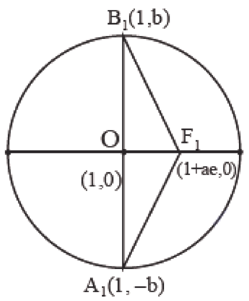
Question ID : 3666943228

25. Let an ellipse with centre (1, 0) and latus rectum of length $\frac{1}{2}$ have its major axis along x-axis. If its minor axis subtends an angle 60° at the foci, then the square of the sum of the lengths of its minor and major axes is equal to ________.

यदि दीर्घवृत्त का केन्द्र (1, 0) तथा नाभि लम्ब की लम्बाई $\frac{1}{2}$ है कि दीर्घ x-अक्ष है। यदि लघु अक्ष द्वारा नाभि पर बनाया गया कोण 60° तो इसकी लघु अक्ष तथा दीर्घ अक्ष की लम्बाइयों के योग का वर्ग होगा ________

Ans. Official Answer NTA (9)

Sol.



$$L.R. = \frac{2b^2}{a} = \frac{1}{2}$$

$4b^2 = a$ ...(i)

Ellipse $\frac{(x-1)^2}{a^2}+\frac{y^2}{b^2}=1$

$m_{B_1F_1}=\frac{1}{\sqrt{3}}$

$\frac{b}{ae}=\frac{1}{\sqrt{3}}$

$3b^2 = a^2e^2 = a^2 - b^2$

$4b^2 = a^2$ ...(ii)

from (i) and (ii)

$a = a^2$

$\therefore\ a = 1$

$b^2=\frac{1}{4}$

$((2a)+(2b))^2=9$

Question ID : 3666943224

26. If the sum of the series

$$\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{2^2}-\frac{1}{2\cdot 3}+\frac{1}{3^2}\right)+\left(\frac{1}{2^3}-\frac{1}{2^2\cdot 3}+\frac{1}{2\cdot 3^2}-\frac{1}{3^3}\right)+\left(\frac{1}{2^4}-\frac{1}{2^3\cdot 3}+\frac{1}{2^2\cdot 3^2}-\frac{1}{2\cdot 3^3}+\frac{1}{3^4}\right)+\ldots\ldots \text{ is } \frac{\alpha}{\beta},$$

where α and β are co-prime, then $\alpha + 3\beta$ is equal to ________.

यदि श्रेणी

$$\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{2^2}-\frac{1}{2\cdot 3}+\frac{1}{3^2}\right)+\left(\frac{1}{2^3}-\frac{1}{2^2\cdot 3}+\frac{1}{2\cdot 3^2}-\frac{1}{3^3}\right)+\left(\frac{1}{2^4}-\frac{1}{2^3\cdot 3}+\frac{1}{2^2\cdot 3^2}-\frac{1}{2\cdot 3^3}+\frac{1}{3^4}\right)+\ldots\ldots \text{ का योग}$$

$\frac{\alpha}{\beta}$ है, जहाँ α तथा β सहभाज्य है, तो $\alpha + 3\beta$ का मान होगा________

Ans. Official Answer NTA (7)

Sol. $P\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{2^2}-\frac{1}{2.3}+\frac{1}{3^2}\right)+\left(\frac{1}{2^3}+\frac{1}{2^2.3}+\frac{1}{2.3^2}-\frac{1}{3^3}\right)+\ldots P\left(\frac{1}{2}+\frac{1}{3}\right)=$

$$\left(\frac{1}{2^2}-\frac{1}{3^2}\right)+\left(\frac{1}{2^3}+\frac{1}{3^3}\right)+\left(\frac{1}{2^4}-\frac{1}{3^4}\right)+\ldots.$$

$$\frac{5P}{6}=\frac{\frac{1}{4}}{1-\frac{1}{2}}-\frac{\frac{1}{9}}{1+\frac{1}{3}}$$

$$\frac{5P}{6}=\frac{1}{2}-\frac{1}{12}=\frac{5}{12}$$

$$\therefore P=\frac{1}{2}=\frac{\alpha}{\beta} \qquad \therefore \alpha=1, \beta=2$$

$$\alpha+3\beta=7$$

Question ID : 3666943225

27. Consider the triangle with vertices A(2, 1), B(0, 0) and C(t, 4), $t \in [0,4]$. If the maximum and the minimum perimeters of such triangles are obtained at t= α and t = β respectively, then $6\alpha + 21\beta$ is equal to ________.

एक त्रिभुज इस प्रकार है जिसके शीर्ष A(2, 1), B(0, 0) तथा C(t, 4), $t \in [0,4]$ है। इस प्रकार के त्रिभुजों का अधिकतम तथा न्यूनतम परिमाप क्रमशः t= α तथा t = β पर प्राप्त होता है तो $6\alpha + 21\beta$ का मान होगा ________

Ans. Official Answer NTA (48)

Sol. A (2, 1), B(0, 0), C (t, 4), $t \in [0, 4]$

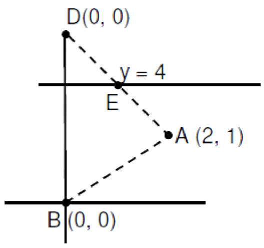


Image of B in y = 4

D(0, 8)

Equation of AD :

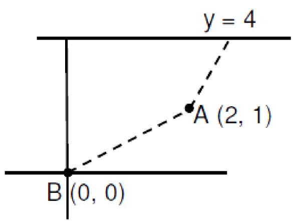
$$y-1=\frac{7}{-2}(x-2)$$

$$7x+2y=16$$

$$E\left(\frac{8}{7},4\right) \Rightarrow \beta=\frac{8}{7}$$

For maximum perimeter

Maximum perimeter at C (4, 4)



$$\therefore t \in [0,4]$$

Now, $6\alpha+21\beta=6\times4+21\left(\frac{8}{7}\right)=48$

Question ID : 3666943230

28. If the line x = y = z intersects the line

$x\sin A + y\sin B + z\sin C - 18 = 0 = x\sin 2A + y\sin 2B + z\sin 2C - 9$, where A, B, C are the angles of a triangle ABC, then $80\left(\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)$ is equal to ________.

यदि रेखा x = y = z रेखाओं

$x\sin A + y\sin B + z\sin C - 18 = 0 = x\sin 2A + y\sin 2B + z\sin 2C - 9$ को प्रतिच्छेद करती है जहाँ A, B, C त्रिभुज ABC के कोण है, तो $80\left(\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)$ का मान होगा ________

Ans. Official Answer NTA (5)

Sol. x = y = z = k (let

$\therefore k(\sin A + \sin B + \sin C) = 18$

$\Rightarrow k\left(4\cos\frac{A}{2}\cdot\cos\frac{B}{2}\cdot\cos\frac{C}{2}\right) = 18$ ...(i)

$k(\sin 2A + \sin 2B + \sin 2C) = 9$

$\Rightarrow k(4\sin A\cdot\sin B\cdot\sin C) = 9$ ...(ii)

(ii)/(i)

$8\sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2} = \frac{9}{18}$

$\Rightarrow 80\sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2} = 5$

Question ID : 3666943221

29. Let A = {1, 2, 3, 4} and R be a relation on the set A × A defined by $R = \{((a,b),(c,d)) : 2a + 3b = 4c + 5d\}$. Then the number of elements in R is ________.

यदि A = {1, 2, 3, 4} तथा समुच्चय A × A में एक संबंध R इस प्रकार परिभाषित है कि $R = \{((a,b),(c,d)) : 2a + 3b = 4c + 5d\}$ हो, तो R में अवयवों की संख्या होगी ________

Ans. Official Answer NTA (6)

Sol. A = {1,2,3,4}

R = {(a, b), (c, d)}

$2a + 3b = 4c + 5d = \alpha$ let

2a = {2,4,6,8}   4c = {4,8,12,16}

3b = {3,6,9,12}   5d = {5,10,15,20}

$$2a+3b=\begin{Bmatrix} 5,8,11,14 \\ 7,10,13,16 \\ 9,12,15,18 \\ 11,14,17,20 \end{Bmatrix} \quad 4c+5d\begin{Bmatrix} 9,14,19,24 \\ 13,18,....... \\ 17,22,....... \\ 21,26,....... \end{Bmatrix}$$

Possible value of $\alpha = 9,13,14,14,17,18$

Pairs of $\{(a, b), (c, d)\} = 6$

Question ID : 3666943223

30. The number of elements in the set $\{n \in \mathbb{N} : 10 \le n \le 100$ and $3^n - 3$ is a multiple of 7$\}$ is ________.

समुच्चय में अवयवों की संख्या होगी $\{n \in \mathbb{N} : 10 \le n \le 100$ तथा $3^n - 3$, 7 का गुणक $\}$________

Ans. Official Answer NTA (15)

Sol. $n \in [10, 100]$

$3^n - 3$ is multiple of 7

$3^n = 7\lambda + 3$

$n = 1,7,13,20,.....97$

Number of possible values of $n = 15$