JEE Main April 2023

Question Paper With Text Solution

13 April | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN APRIL 2023 | 13TH APRIL SHIFT-1

SECTION – A

Question ID : 3666947183

1. For the differentiable function $f:\mathbb{R}-\{0\}\to\mathbb{R}$, let $3f(x)+2f\left(\frac{1}{x}\right)=\frac{1}{x}-10$, then $\left|f(3)+f'\left(\frac{1}{4}\right)\right|$ is equal to :

अवकलनीय फलन $f:\mathbb{R}-\{0\}\to\mathbb{R}$ के लिए माना $3f(x)+2f\left(\frac{1}{x}\right)=\frac{1}{x}-10$, है, तो $\left|f(3)+f'\left(\frac{1}{4}\right)\right|$ बराबर है :

(1) $\frac{33}{5}$ (2) 7 (3) $\frac{29}{5}$ (4) 13

Ans. Official Answer NTA (4)

Sol. $3f(x)+2f\left(\frac{1}{x}\right)=\frac{1}{x}-10$ ...(i)

$x\to\frac{1}{x}$

$3f\left(\frac{1}{x}\right)+2f(x)=x-10$ ...(ii)

From (i) & (ii)

$5f(x)=\frac{3}{x}-2x-10$

Now, $f(3)=-3$

$5f'(x)=\frac{-3}{x^2}-2$

$5f'\left(\frac{1}{4}\right)=-48-2$

$f'\left(\frac{1}{4}\right)=-10$

$\left|f(3)+f'\left(\frac{1}{4}\right)\right|=13$

Question ID : 3666947189

2. Among

$(S1): \lim\limits_{n\to\infty}\frac{1}{n^2}(2+4+6+.....+2n)=1$

$(S2): \lim\limits_{n\to\infty}\frac{1}{n^{16}}\left(1^{15}+2^{15}+3^{15}+.....+n^{15}\right)=\frac{1}{16}$

(1) Only (S2) is true (2) Both (S1) and (S2) are true

(3) Only (S1) is true (4) Both (S1) and (S2) are false

$(S1): \lim\limits_{n\to\infty}\frac{1}{n^2}(2+4+6+.....+2n)=1$

$(S2): \lim\limits_{n\to\infty}\frac{1}{n^{16}}\left(1^{15}+2^{15}+3^{15}+.....+n^{15}\right)=\frac{1}{16}$ में से :

(1) केवल (S2) सत्य है (2) दोनों (S1) तथा (S2) सत्य हैं

(3) केवल (S1) सत्य है (4) दोनों (S1) तथा (S2) असत्य हैं

Ans. Official Answer NTA (2)

Sol. $S_1: \lim\limits_{n\to\infty}\frac{n(n+1)}{n^2}=1\Rightarrow$ True

$S_2: \lim\limits_{n\to\infty}n^{\frac{1}{16}}\left(\sum r^{15}\right)=\lim\limits_{n\to\infty}\frac{1}{n}\sum\left(\frac{r}{n}\right)^{15}$

$=\int_0^1 x^{15}dx=\frac{1}{16}\Rightarrow$ True

Question ID : 3666947191

3. Let $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}$, $\vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$. If a vector $\vec{d}$ satisfies $\vec{d}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{d}\cdot\vec{a}=24$, then $|\vec{d}|^2$ is equal to :

माना $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}$, $\vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$ तथा $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$ हैं। यदि सदिश $\vec{d}$, $\vec{d}\times\vec{b}=\vec{c}\times\vec{b}$ तथा $\vec{d}\cdot\vec{a}=24$ को संतुष्ट करता है, तो $|\vec{d}|^2$ बराबर है :

(1) 313 (2) 323 (3) 423 (4) 413

Ans. Official Answer NTA (4)

Sol. $\vec{d}\times\vec{b}=\vec{c}\times\vec{b}$

$\Rightarrow\left(\vec{d}-\vec{c}\right)\times\vec{b}=0$

$\Rightarrow \vec{d} = \vec{c} + \lambda\vec{b}$

Also $\vec{d}\vec{a} = 24$

$\Rightarrow \left(\vec{c} + \lambda\vec{b}\right)\cdot\vec{a} = 24$

$\lambda = \dfrac{24 - \vec{a}\cdot\vec{c}}{\vec{b}\cdot\vec{a}} = \dfrac{24-6}{9} = 2$

$\Rightarrow \vec{d} = \vec{c} + 2\left(\vec{b}\right)$

$= 8\hat{i} - 5\hat{j} + 18\hat{k}$

$\Rightarrow \left|\vec{d}\right|^2 = 64 + 25 + 324 = 413$

Question ID : 3666947188

4. Let $s_1, s_2, s_3, \ldots\ldots\ldots s_{10}$ respectively be the sum to 12 terms of 10 A.P. s whose first terms are 1, 2, 3, ..., 10 and the common differences are 1, 3, 5, ...19 respectively. Then $\sum_{i=1}^{10} s_i$ is equal to :

माना 10 A.P., जिनके प्रथम पद 1, 2, 3, ..., 10 तथा सार्व अंतर क्रमशः 1, 3, 5, ..., 19 हैं, के 12 पदों का योग क्रमशः $s_1, s_2, s_3, \ldots\ldots\ldots s_{10}$ है। तो $\sum_{i=1}^{10} s_i$ बराबर है :

(1) 7380 (2) 7360 (3) 7260 (4) 7220

Ans. Official Answer NTA (3)

Sol. $1 \le r \le 10$

$S_r = \dfrac{12}{2}\left[2r + (12-1)\cdot(2r-1)\right] = 6\left(2r + 22r - 11\right)$

$= 6\,(24r - 11)$

$\sum_{r=1}^{10} S_r = 6\sum_{r=1}^{10}(24r-11) = 6 \times \dfrac{10}{2}[24 - 11 + 240 - 11]$

$= 30\,[242] = 7260$

Question ID : 3666947184

5. The set of all $a \in \mathbb{R}$ for which the equation $x|x-1| + |x+2| + a = 0$ has exactly one real root, is :

सभी $a \in \mathbb{R}$, जिनके लिए समीकरण $x|x-1| + |x+2| + a = 0$ का मात्र एक वास्तविक मूल है, का समुच्चय है :

(1) $(-6, \infty)$ (2) $(-6, -3)$ (3) $(-\infty, -3)$ (4) $(-\infty, -\infty)$

Ans. Official Answer NTA (4)

Sol. $x|x-1|+|x+2|+a=0$

Case–I : $x < -2$

$-x^2 + x - x - 2 + a = 0$

$a = x^2 + 2$

$y = x^2 + 2$ is decreasing $\forall x \in (-\infty, -2)$

Case–II : $-2 \le x < 1$

$-x^2 + x + x + 2 + a = 0$

$a = x^2 - 2x - 2$

$y = x^2 - 2x - 2$ is decreasing $\forall x \in [-2, 1)$

Case III : $x \ge 1$

$x^2 - x + x + 2 + a = 0$

$a = -(x^2 + 2)$

$y = -(x^2 + 2)$ is decreasing $\forall x \in [1, \infty)$

$\therefore$ Exactly one real root $\forall x \in R$

Question ID : 3666947192

6. $\max_{0 \le x \le \pi} \left\{ x - 2\sin x \cos x + \frac{1}{3}\sin 3x \right\} =$

(1) π (2) $\frac{5\pi + 2 + 3\sqrt{3}}{6}$ (3) 0 (4) $\frac{\pi + 2 - 3\sqrt{3}}{6}$

Ans. Official Answer NTA (2)

Sol. $f(x) = x - \sin 2x + \frac{1}{3}\sin 3x$

$f'(x) = 1 - 2\cos 2x + \cos 3x = 0$

$x = \frac{5\pi}{6}, \frac{\pi}{6}$

$\therefore f''(x) = 4\sin 2x - 3\sin 3x$

$f''\left(\frac{5\pi}{6}\right) < 0$

$\Rightarrow \left(\frac{5\pi}{6}\right)$ is point of maxima

$$\Rightarrow f\left(\frac{5\pi}{6}\right)=\frac{5\pi}{6}+\frac{\sqrt{3}}{2}+\frac{1}{3}$$

Question ID : 3666947196

7. Let $y=y_1(x)$ and $y=y_2(x)$ be the solution curves of the differential equation $\frac{dy}{dx}=y+7$ with initial conditions $y_1(0)=0$ and $y_2(0)=1$ respectively. Then the curves $y=y_1(x)$ and $y=y_2(x)$ intersect at :

(1) two points (2) infinite number of points

(3) no point (4) one point

माना अवकल समीकरण $\frac{dy}{dx}=y+7$ के प्रारंभिक प्रतिबंधो $y_1(0)=0$ तथा $y_2(0)=1$ के साथ हल वक्र क्रमशः $y=y_1(x)$ तथा $y=y_2(x)$ हैं। तो वक्र $y=y_1(x)$ तथा $y=y_2(x)$:

(1) दो बिन्दुओं पर मिलते हैं (2) अनंत बिन्दुओं पर मिलते हैं

(3) किसी भी बिन्दु पर नहीं मिलते (4) एक बिन्दु पर मिलते हैं

Ans. Official Answer NTA (3)

Sol. $\frac{dy}{dx}=y+7\Rightarrow\frac{dy}{dx}-y=7$

I.F. $=e^{-x}$

$ye^{-x}=\int 7e^{-x}dx$

$\Rightarrow ye^{-x}=-7e^{-x}+c$

$\Rightarrow y=-7+ce^{x}$

$-7+7e^{x}=-7+8e^{x}\Rightarrow e^{x}=0$

No solution.

Question ID : 3666947199

8. The distance of the point $(-1, 2, 3)$ from the plane $\vec{r}\cdot(\hat{i}-2\hat{j}+3\hat{k})=10$ parallel to the line of the shortest distance between the lines $\vec{r}=(\hat{i}-\hat{j})+\lambda(2\hat{i}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j})+\mu(\hat{i}-\hat{j}+\hat{k})$ is :

बिन्दु $(-1, 2, 3)$ की रेखाओं $\vec{r}=(\hat{i}-\hat{j})+\lambda(2\hat{i}+\hat{k})$ तथा $\vec{r}=(2\hat{i}-\hat{j})+\mu(\hat{i}-\hat{j}+\hat{k})$ के बीच न्यूनतम दूरी की रेखा के समान्तर समतल $\vec{r}\cdot(\hat{i}-2\hat{j}+3\hat{k})=10$ से दूरी है :

(1) $2\sqrt{5}$ (2) $3\sqrt{6}$ (3) $3\sqrt{5}$ (4) $2\sqrt{6}$

Ans. Official Answer NTA (4)

Sol. Let DR's of line of shortest distance between given lines L_1 and L_2 are a, b, c then

$$a\hat{i}+b\hat{j}+c\hat{k}=t\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 0 & 1\\ 1 & -1 & 1\end{vmatrix}=t\left(\hat{i}-\hat{j}-2\hat{k}\right)$$

Now equation of line passes through A (–1,2,3) and DR's are (1, –1, –2)

$$\vec{r}=\left(-\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}-2\hat{k}\right) \quad ...(1)$$

point of intersection of line (1) and plane $\vec{r}\cdot\left(\hat{i}-2\hat{j}+3\hat{k}\right)=10$ ...(2)

$(\lambda-1)-2(2-\lambda)+3(3-2\lambda)=10$

$\Rightarrow 4-3\lambda=10$

$\Rightarrow \lambda=-2$

So, B (–3, 4, 7)

Distance $AB=\sqrt{4+4+16}=\sqrt{24}=2\sqrt{6}$

Question ID : 3666947202

9. The negation of the statement $((A\wedge(B\vee C))\Rightarrow(A\vee B))\Rightarrow A$ is :

(1) equivalent to $\sim C$ (2) a fallacy

(3) equivalent to $\sim A$ (4) equivalent to $B\vee\sim C$

कथन $((A\wedge(B\vee C))\Rightarrow(A\vee B))\Rightarrow A$ का निषेधन :

(1) $\sim C$ के तुल्य है (2) हेत्वा भास (fallacy) है

(3) $\sim A$ के तुल्य है (4) $B\vee\sim C$ के तुल्य है

Ans. Official Answer NTA (3)

Sol. $(A\wedge(B\vee C))\Rightarrow(A\vee B)\Rightarrow A$

$\sim(\sim(A\wedge(B\vee C))\vee(A\vee B))\vee A$

$(A\wedge(B\vee C))\wedge\sim(A\vee B)\vee A$

$=A$

$\therefore$ Negation of statement $=\sim A$

Question ID : 3666947190

10. Let the equation of plane passing through the line of intersection of the planes $x + 2y + az = 2$ and $x - y + z = 3$ be $5x - 11y + bz = 6a - 1$. For $c \in \mathbb{Z}$, if the distance of this plane from the point $(a, -c, c)$ is $\frac{2}{\sqrt{a}}$, then $\frac{a+b}{c}$ is equal to :

माना समतलों $x + 2y + az = 2$ तथा $x - y + z = 3$ की प्रतिच्छेदन रेखा से होकर जाने वाले समतल का समीकरण $5x - 11y + bz = 6a - 1$ है। $c \in \mathbb{Z}$ के लिए, यदि इस समतल की बिन्दु $(a, -c, c)$ से दूरी $\frac{2}{\sqrt{a}}$ है, तो $\frac{a+b}{c}$ बराबर है :

(1) -4 (2) -2 (3) 4 (4) 2

Ans. Official Answer NTA (1)

Sol. $(x + 2y + az - 2) + \lambda (x - y + z - 3) = 0$

$$\frac{1+\lambda}{5} = \frac{2-\lambda}{-11} = \frac{a+\lambda}{b} = \frac{2+3\lambda}{6a-1}$$

$$\lambda = -\frac{7}{2}, a = 3, b = 1$$

$$\frac{2}{\sqrt{a}} = \left|\frac{5a + 11c + bc - 6a + 1}{\sqrt{25 + 121 + 1}}\right|$$

$c = -1$

$$\therefore \frac{a+b}{c} = \frac{3+1}{-1} = -4$$

Question ID : 3666947194

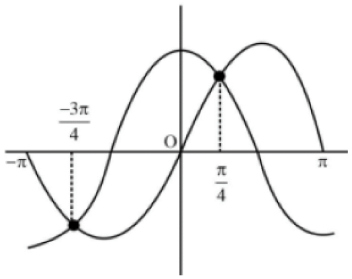
11. The area of the region enclosed by the curve $f(x) = \max\{\sin x, \cos x\}, -\pi \le x \le \pi$ and the x-axis is :

वक्र $f(x) = \max\{\sin x, \cos x\}, -\pi \le x \le \pi$ तथा x-अक्ष से घिरे क्षेत्र का क्षेत्रफल है :

(1) 4 (2) $2(\sqrt{2} + 1)$ (3) $2\sqrt{2}(\sqrt{2} + 1)$ (4) $4(\sqrt{2})$

Ans. Official Answer NTA (1)

Sol.



Area =

$$\left|\int_{-\pi}^{\frac{-3\pi}{4}} \sin x dx\right| + \left|\int_{\frac{-3\pi}{4}}^{\frac{-\pi}{2}} \cos x dx\right| + \int_{\frac{-\pi}{2}}^{\frac{\pi}{4}} \cos x dx + \int_{\frac{\pi}{4}}^{\pi} \sin x dx = 4$$

Question ID : 3666947186

12. For the system of linear equations

$2x + 4y + 2az = b$
$x + 2y + 3z = 4$
$2x - 5y + 2z = 8$

which of the following is NOT correct?

(1) has unique solution if $a = b = 8$

(2) It has infinitely many solutions if $a = 3, b = 8$

(3) It has infinitely many solutions if $a = 3, b = 6$

(4) It has unique solution if $a = b = 6$

रैखिक समीकरण निकाय

$2x + 4y + 2az = b$
$x + 2y + 3z = 4$
$2x - 5y + 2z = 8$

के लिए निम्न में से कौनसा सही नहीं है?

(1) इसका अद्वितीय हल है यदि $a = b = 8$ हैं

(2) इसके अनंत हल हैं यदि $a = 3, b = 8$ हैं

(3) इसके अनंत हल हैं यदि $a = 3, b = 6$ हैं

(4) इसका अद्वितीय हल है यदि a = b = 6 हैं

Ans. Official Answer NTA (3)

Sol. $D = \begin{vmatrix} 2 & 4 & 2a \\ 1 & 2 & 3 \\ 2 & -5 & 2 \end{vmatrix}$

$= 2(4 + 15) - 4(2 - 6) + 2a(-5 - 4)$

$= 38 + 16 - 18a = 54 - 18a = 18(3 - a)$

$D_1 = \begin{vmatrix} b & 4 & 2a \\ 4 & 2 & 3 \\ 8 & -5 & 2 \end{vmatrix}$

$= b(4 + 15) - 4(8 - 24) + 2a(-20 - 16)$

$= 19b + 64 - 72a$

$= 19b - 72a + 64$

$D_2 = \begin{vmatrix} 2 & b & 2a \\ 1 & 4 & 3 \\ 2 & 8 & 2 \end{vmatrix}$

$= 2(8 - 24) - b(2 - 6) + 2a(8 - 8)$

$= -32 + 4b = 4(b - 8)$

$D_3 = \begin{vmatrix} 2 & 4 & b \\ 1 & 2 & 4 \\ 2 & -5 & 8 \end{vmatrix}$

$= 2(16 + 20) - 4(8 - 8) + b(-5 - 4)$

$= 72 - 9b = 9(8 - b)$

Now when $a \neq 3, b \in R$ system have unique solution.

when $a = 3, b \neq 8$, system have no solution

when $a = 3, b = 8$, system have infinitely many solutions.

Question ID : 3666947195

13. Fractional part of the number $\frac{4^{2022}}{15}$ is equal to :

संख्या $\frac{4^{2022}}{15}$ का भिन्न भाग है :

(1) $\frac{4}{15}$ (2) $\frac{14}{15}$ (3) $\frac{1}{15}$ (4) $\frac{8}{15}$

Ans. Official Answer NTA (3)

Sol. $\frac{4^{2022}}{15}$

$\because 4 \equiv 4(\text{mod}15)$

$4^2 \equiv 1(\text{mod}15)$

$4^{2022} \equiv 1(\text{mod}15)$

$\therefore$ Fractional part of $\frac{4^{2022}}{15} = \frac{1}{15}$

Question ID : 3666947187

14. The number of symmetric matrices of order 3, with all the entries from the set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, is:

कोटि 3 के समामित आव्यूहों, जिनके सभी अवयव समुच्चय {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} से हैं, की संख्या है :

(1) 6^{10} (2) 10^6 (3) 9^{10} (4) 10^9

Ans. Official Answer NTA (2)

Sol. $A = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}, a,b,c,d,e,f \in \{0,1,2,.....9\}$

Number of matrices = 10^6

Question ID : 3666947201

15. For $x \in \mathbb{R}$, two real valued functions f(x) and g(x) are such that, $g(x) = \sqrt{x} + 1$ and $fog(x) = x + 3 - \sqrt{x}$. Then f(0) is equal to :

$x \in \mathbb{R}$ के लिए दो वास्तविक फलन f(x) तथा g(x) इस प्रकार है कि $g(x) = \sqrt{x} + 1$ तथा $fog(x) = x + 3 - \sqrt{x}$ हैं। तो f(0) बराबर है :

(1) 5 (2) 0 (3) –3 (4) 1

Ans. Official Answer NTA (Bonus)

Sol. $g(x)=\sqrt{x}+1$

$fog(x)=x+3-\sqrt{x}$

$=\left(\sqrt{x}+1\right)^2-3\left(\sqrt{x}+1\right)+5$

$=g^2(x)-3g(x)+5$

$\Rightarrow f(x)=x^2-3x+5$

$\therefore f(0)=5$

But, if we consider the domain of the composite function fog(x) then in that case f(0) will not be defined as g(x) cannot be equal to zero.

Question ID : 3666947193

16. $\int_0^{\infty}\frac{6}{e^{3x}+6e^{2x}+11e^{x}+6}dx=$

(1) $\log_e\left(\frac{256}{81}\right)$ (2) $\log_e\left(\frac{512}{81}\right)$ (3) $\log_e\left(\frac{32}{27}\right)$ (4) $\log_e\left(\frac{64}{27}\right)$

Ans. Official Answer NTA (3)

Sol. Let $e^x=t\Rightarrow e^x\,dx=dt$

$$I=\int_1^{\infty}\frac{6}{t(t+1)(t+2)(t+3)}dt$$

$$\Rightarrow I=6\int_0^{\infty}\left\{\frac{A}{t}+\frac{B}{t+1}+\frac{C}{t+2}+\frac{D}{t+3}\right\}dt$$

$$\Rightarrow I=6\int_1^{\infty}\left(\frac{1}{6t}+\frac{1}{(-2)(t+1)}+\frac{1}{2(t+2)}-\frac{1}{6(t+3)}\right)dt$$

$$\Rightarrow I=\left[\ell nt-3\ell n(t+1)+3\ell n(t+2)-\ell n(t+3)\right]_1^{\infty}$$

$$\Rightarrow I=\left[\ell n\frac{t(t+2)^3}{(t+1)^3(t+3)}\right]_1^{\infty}$$

$$\Rightarrow I=0-\ell n\frac{3^3}{2^3.4}=\ell n\frac{32}{27}$$

Question ID : 3666947197

17. Let the tangent and normal at the point $(3\sqrt{3},1)$ on the ellipse $\frac{x^2}{36}+\frac{y^2}{4}=1$ meet the y-axis at the points A and B respectively. Let the circle C be drawn taking AB as a diameter and the line $x=2\sqrt{5}$ intersect C at the points P and Q. If the tangents at the points P and Q on the circle intersect at the point (α, β), then $\alpha^2-\beta^2$ is equal to :

माना दीर्घवृत्त $\frac{x^2}{36}+\frac{y^2}{4}=1$ के बिन्दु $(3\sqrt{3},1)$ पर स्पर्श तथा अभिलंब y-अक्ष को क्रमशः बिन्दुओं A तथा B पर मिलते हैं। माना AB को एक व्यास लेकर खींचा गया वृत्त C है तथा रेखा $x=2\sqrt{5}$, वृत्त C को बिन्दुओं P तथा Q पर काटती है। यदि वृत्त के बिन्दुओं P तथा Q पर स्पर्श रेखाओं का प्रतिच्छेदन बिन्दु (α, β) है, तो $\alpha^2-\beta^2$ बराबर है :

(1) $\frac{314}{5}$ (2) 61 (3) $\frac{304}{5}$ (4) 13

Ans. Official Answer NTA (3)

Sol. $\frac{x^2}{36}+\frac{y^2}{4}=1$

$T: \frac{3\sqrt{3}x}{36}+\frac{y}{4}=1$

$T: \frac{\sqrt{3}x}{12}+\frac{y}{4}=1$

$N: \frac{x-3\sqrt{3}}{\frac{3\sqrt{3}}{36}}=\frac{y-1}{\frac{1}{4}}$

$\frac{12x-36\sqrt{3}}{\sqrt{3}}=4y-4$

$3x-9\sqrt{3}=\sqrt{3}y-\sqrt{3}$

$N: 3x-\sqrt{3}y=8\sqrt{3}$

A(0, 4) B(0, –8)

$C: x^2+(y-4)(y+8)=0$

Line $x=2\sqrt{5}$

$20+y^2+4y-32=0$

$y^2+4y-12=0$

$(y+6)(y-2)=0$

$P\left(2\sqrt{5},-6\right)$ $Q\left(2\sqrt{5},2\right)$

$C : x^2 + y^2 + 4y - 32 = 0$

$P\left(2\sqrt{5},-6\right)$ $Q\left(2\sqrt{5},2\right)$

$T : xx_1 + yy_1 + 2y + 2y_1 - 32 = 0$

$T_1 : 2\sqrt{5}x - 6y + 2y - 12 - 32 = 0$

$2\sqrt{5}x - 4y = 44$

$T_1 : \sqrt{5}x - 2y = 22$ ...(i)

$T_2 : 2\sqrt{5}x + 2y + 2y + 4 - 32 = 0$

$2\sqrt{5}x + 4y = 28$

$T_2 : \sqrt{5}x + 2y = 14$ ...(ii)

From (i) & (ii)

$\alpha = \frac{18}{\sqrt{5}}$ $\beta = -2$

$\alpha^2 - \beta^2 = \frac{304}{5}$

Question ID : 3666947198

18. Let PQ be a focal chord of the parabola $y^2 = 36x$ of length 100 , making an acute angle with the positive x-axis. Let the ordinate of P be positive and M be the point on the line segment PQ such that PM:MQ = 3:1. Then which of the following points does NOT lie on the line passing through M and perpendicular to the line PQ ?

माना परवलय $y^2 = 36x$ की एक नाभीय जीवा PQ की लंबाई 100 है, जो धनात्मक x अक्ष से एक न्यून कोण बनाती है। माना P की कोटी धनात्मक है तथा रेखाखण्ड PQ पर बिन्दु M इस प्रकार है कि PM:MQ = 3:1 है। तो बिन्दु M से होकर जाने वाली तथा रेखा PQ के लंबवत रेखा पर निम्न में से कौनसा बिन्दु स्थित नहीं है?

(1) (3, 33) (2) (–3, 43) (3) (6, 29) (4) (–6, 45)

Ans. Official Answer NTA (2)

Sol. $9\left(t + \frac{1}{t}\right)^2 = 100$

$t = 3$

$\Rightarrow$ P(81, 54) & Q(1, –6)

M (21, 9)

$\Rightarrow L \text{ is } (y-9)=\frac{-4}{3}(x-21)$

$3y-27=-4x+84$

$4x+3y=111$

Question ID : 3666947200

19. A coin is biased so that the head is 3 times as likely to occur as tail. This coin is tossed until a head or three tails occur. If X denotes the number of tosses of the coin, then the mean of X is :

एक सिक्का इस प्रकार अभिनत है कि चित्त के आने की संभावना पट् के आने की संभावना की तीन गुना है। इस सिक्के को तब तक उछाला जाता है जब तक कि एक चित्त या तीन पट् दिख जाएँ। यदि X सिक्के को उछालने की संख्या को दर्शाता है, तो X का माध्य है :

(1) $\frac{37}{16}$ (2) $\frac{15}{16}$ (3) $\frac{81}{64}$ (4) $\frac{21}{16}$

Ans. Official Answer NTA (4)

Sol. $P(H)=\frac{3}{4}$

$P(T)=\frac{1}{4}$

X	1	2	3
P(X)	$\frac{3}{4}$	$\frac{1}{4}\times\frac{3}{4}$	$\left(\frac{1}{4}\right)^3+\left(\frac{1}{4}\right)^2\times\frac{3}{4}$

Mean $\overline{X}=\frac{3}{4}+\frac{3}{8}+3\left(\frac{1}{64}+\frac{3}{64}\right)$

$=\frac{3}{4}+\frac{3}{8}+\frac{3}{16}$

$=3\left(\frac{7}{16}\right)=\frac{21}{16}$

Question ID : 3666947185

20. Let $B=\begin{bmatrix}1 & 3 & \alpha\\ 1 & 2 & 3\\ \alpha & \alpha & 4\end{bmatrix}$, $\alpha>2$ be the adjoint of a matrix A and $|A|=2$. Then $\begin{bmatrix}\alpha-2\alpha & \alpha\end{bmatrix}B\begin{bmatrix}\alpha\\ -2\alpha\\ \alpha\end{bmatrix}$ is equal to :

माना $B=\begin{bmatrix}1 & 3 & \alpha\\ 1 & 2 & 3\\ \alpha & \alpha & 4\end{bmatrix}$, $\alpha>2$, एक आव्यूह A का सहखण्डज है तथा $|A|=2$ है। तो $\begin{bmatrix}\alpha-2\alpha & \alpha\end{bmatrix}B\begin{bmatrix}\alpha\\ -2\alpha\\ \alpha\end{bmatrix}$ बराबर है :

(1) –16 (2) 16 (3) 0 (4) 32

Ans. Official Answer NTA (1)

Sol. $|B| = |\text{adj } A| = |A|^{3-1} = |A|^2 = 4$

$1(8-3\alpha) - 3(4-3\alpha) + \alpha(\alpha-2\alpha) = 4$

$\Rightarrow 8 - 3\alpha - 12 + 9\alpha - \alpha^2 = 4$

$\Rightarrow \alpha^2 - 6\alpha + 8 = 0$

$\Rightarrow \alpha = 2, 4$

$\Rightarrow \alpha = 4 \qquad (\because \alpha > 2)$

Now

$$\begin{bmatrix}4 & -8 & 4\end{bmatrix}\begin{bmatrix}1 & 3 & 4\\ 1 & 2 & 3\\ 4 & 4 & 4\end{bmatrix}\begin{bmatrix}4\\ -8\\ 4\end{bmatrix}$$

$$=16\begin{bmatrix}1 & -2 & 1\end{bmatrix}\begin{bmatrix}1 & 3 & 4\\ 1 & 2 & 3\\ 4 & 4 & 4\end{bmatrix}\begin{bmatrix}1\\ -2\\ 1\end{bmatrix}$$

$$=16\begin{bmatrix}1-2+4 & 3-4+4 & 4-6+4\end{bmatrix}\begin{bmatrix}1\\ -2\\ 1\end{bmatrix}$$

$= 16[3 - 6 + 2]$

$= 16[-1] = [-16]$

SECTION – B

Question ID : 3666947204

21. The number of seven digit positive integers formed using the digits 1, 2, 3 and 4 only and sum of the digits equal to 12 is _____.

केवल अंको 1, 2, 3 तथा 4 के प्रयोग से बनने वाले सात अंकों के धनात्मक पूर्णांकों, जिनके अंकों का योग 12 है, की संख्या है _____

Ans. Official Answer NTA (413)

Sol. $x_1 + x_2 + x_3 + ... x_7 = 12$

Number of solutions

$= \text{Coefficient of } x^{12} \text{ in } (x^1 + x^2 + x^3 + x^4)^7$

$= \text{Coefficient of } x^5 \text{ in } (1 + x + x^2 + x^3)^7$

$= \text{Coefficient of } x^5 \text{ in } (1 - x^4)^7 (1 - x)^{-7}$

$= \text{Coefficient of } x^5 \text{ in } (1 - 7x^4)(1 - x)^{-7}$

$= \text{Coefficient of } x^5 \text{ in } \left(1 - 7x^4\right)\sum_{r=0}^{\infty} {}^{7+r-1}C_r \cdot x^r$

$= {}^{11}C_5 - 7 \times {}^{7}C_1$

$= 462 - 49 = 413$

Question ID : 3666947209

22. Let the image of the point $\left(\frac{5}{3}, \frac{5}{3}, \frac{8}{3}\right)$ in the plane $x - 2y + z - 2 = 0$ be P. If the distance of the point $Q(6, -2, \alpha), \alpha > 0$, from P is 13, then α is equal to _____.

माना समतल $x - 2y + z - 2 = 0$ में बिन्दु $\left(\frac{5}{3}, \frac{5}{3}, \frac{8}{3}\right)$ का प्रतिबिंब बिन्दु P है। यदि बिन्दु $Q(6, -2, \alpha), \alpha > 0$ की बिन्दु P से 13 है, तो α बराबर है _____.

Ans. Official Answer NTA (15)

Sol. $\frac{x - \frac{5}{3}}{1} = \frac{y - \frac{5}{3}}{-2} = \frac{2 - \frac{8}{3}}{1} = \frac{-2(-1)}{6}$

$(x, y, z) = (2, 1, 3)$

$PQ = 13$

$4^2 + 3^2 + (\alpha - 3)^2 = 169$

$|\alpha - 3| = 12$

$\alpha = 15$

Question ID : 3666947208

23. Let m_1 and m_2 be the slopes of the tangents drawn from the point P(4, 1) to the hyperbola $H : \frac{y^2}{25} - \frac{x^2}{16} = 1$. If Q is the point from which the tangents drawn to H have slopes $|m_1|$ and $|m_2|$ and they make positive intercepts α and β on the x-axis, then $\frac{(PQ)^2}{\alpha\beta}$ is equal to _____.

माना बिन्दु P(4, 1) से अतिपरवलय $H : \frac{y^2}{25} - \frac{x^2}{16} = 1$ पर खींची गई स्पर्श रेखाओं की प्रवणताएं m_1 तथा m_2 हैं। यदि Q वह बिन्दु है, जिससे H पर खींची गई स्पर्श रेखाओं की प्रवणताएं $|m_1|$ तथा $|m_2|$ हैं तथा यह स्पर्श रेखाएं x-अक्ष पर धनात्मक अंतःखण्ड α तथा β बनाती है, तो $\frac{(PQ)^2}{\alpha\beta}$ बराबर है _____

Ans. Official Answer NTA (8)

Sol. Equation of tangent to the hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$y = mx \pm \sqrt{a^2 - b^2m^2}$

passing through (4, 1)

$1 = 4m \pm \sqrt{25 - 16m^2} \Rightarrow 4m^2 - m - 3 = 0$

$\Rightarrow m = 1, \frac{-3}{4}$

Equation of tangent with positive slopes 1 & $\frac{3}{4}$.

$\left.\begin{matrix} 4y = 3x - 16 \\ y = x - 3 \end{matrix}\right\}$ with positive intercept on x-axis

$\alpha = \frac{16}{3}, \beta = 3$

Intersection points :

Q: (–4, – 7)

P: (4, 1)

$PQ^2 = 128$

$$\frac{PQ^2}{\alpha\beta}=\frac{128}{16}=8$$

Question ID : 3666947203

24. Let $w=z\bar{z}+k_1z+k_2iz+\lambda(1+i), k_1,k_2\in\mathbb{R}$. Let Re(w) = 0 be the circle C of radius 1 in the first quadrant touching the line y = 1 and the y-axis. If the curve Im(w) = 0 intersects C at A and B, then $30(AB)^2$ is equal to _____.

माना $w=z\bar{z}+k_1z+k_2iz+\lambda(1+i), k_1,k_2\in\mathbb{R}$ है। माना Re(w) = 0 प्रथम चतुर्थांश में इकाई त्रिज्या का एक वृत्त C है, जो रेखा y = 1 तथा y-अक्ष को स्पर्श करता है। यदि वक्र Im(w) = 0, वृत्त C को A तथा B पर काटता है, तो $30(AB)^2$ बराबर है _____

Ans. Official Answer NTA (24)

Sol. $\omega=z\bar{z}+k_1z+k_2iz+\lambda(1+i)$

$\text{Re}(\omega)=x^2+y^2+k_1x-x_2y+\lambda=0$

$\text{Centre}\equiv\left(\frac{-k_1}{2},\frac{k_2}{2}\right)\equiv(1,2)$

$\Rightarrow k_1=-2, k_2=4$

radius $=1\Rightarrow\lambda-4$

Im $=k_1y+k_2x+\lambda=$

$\therefore 2x-y+2=0$

Question ID : 3666947212

25. If $S=\left\{x\in\mathbb{R}:\sin^{-1}\left(\frac{x+1}{\sqrt{x^2+2x+2}}\right)-\sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right)=\frac{\pi}{4}\right\}$, then

$\sum_{x\in s}\left(\sin\left(\left(x^2+x+5\right)\frac{\pi}{2}\right)-\cos\left(\left(x^2+x+5\right)\pi\right)\right)$ is equal to _____.

यदि $S=\left\{x\in\mathbb{R}:\sin^{-1}\left(\frac{x+1}{\sqrt{x^2+2x+2}}\right)-\sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right)=\frac{\pi}{4}\right\}$, है, तो

$\sum_{x\in s}\left(\sin\left(\left(x^2+x+5\right)\frac{\pi}{2}\right)-\cos\left(\left(x^2+x+5\right)\pi\right)\right)$ बराबर है _____.

Ans. Official Answer NTA (4)

Sol. $\sin^{-1}\left(\frac{x+1}{\sqrt{x^2+2x+2}}\right)-\sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right)=\frac{\pi}{4}$

$\tan^{-1}(x+1)-\tan^{-1}x=\frac{\pi}{4}\Rightarrow\tan^{-1}\left(\frac{(x+1)-x}{1+(x+1)x}\right)=\frac{\pi}{4}$

$\left(\frac{1}{1+x^2+x}\right)=\frac{\pi}{4}\Rightarrow\frac{1}{1+x^2+x}=1$

$x^2+x+1=1\Rightarrow x^2+x=0$

$\Rightarrow x=0,-1\Rightarrow S=\{0,-1\}$

Now, $\sum_{x\in S}\left(\sin\left(\left(x^2+x+5\right)\right)\frac{\pi}{2}\right)-\cos\left(\left(x^2+x+5\right)\pi\right)$

$=\left(\sin\frac{5\pi}{2}-\cos 5\pi\right)+\left(\sin\frac{5\pi}{2}-\cos 5\pi\right)=(1+1)+(1+1)=4$

Question ID : 3666947210

26. Let $\vec{a}=3\hat{i}+\hat{j}-\hat{k}$ and $\vec{c}=2\hat{i}-3\hat{j}+3\hat{k}$. If $\vec{b}$ is a vector such that $\vec{a}=\vec{b}\times\vec{c}$ and $|\vec{b}|^2=50$, then $\left|72-|\vec{b}+\vec{c}|^2\right|$ is equal to _____.

माना $\vec{a}=3\hat{i}+\hat{j}-\hat{k}$ तथा $\vec{c}=2\hat{i}-3\hat{j}+3\hat{k}$ हैं। यदि एक सदिश $\vec{b}$ इस प्रकार है कि $\vec{a}=\vec{b}\times\vec{c}$ तथा $|\vec{b}|^2=50$ हैं, तो $\left|72-|\vec{b}+\vec{c}|^2\right|$ बराबर है _____

Ans. Official Answer NTA (66)

Sol. $|\vec{a}|=|\vec{b}\times\vec{c}|$

$=\sqrt{11}=|\vec{b}|.\sqrt{22}.\sin\theta$

$\therefore\sin\theta=\frac{1}{10}$

$\text{or}\cos\theta=\frac{\sqrt{99}}{10}$

$\left|72-|\vec{b}+\vec{c}|^2\right|=\left|72-\left(50+22+2\times5\sqrt{2}.\sqrt{22}\frac{\sqrt{99}}{10}\right)\right|$

$=\left|72-\left(72+\frac{2\times5\times11\times3}{10}\right)\right|$

$=|66|=66$

Question ID : 3666947205

27. Let α be the constant term in the binomial expansion of $\left(\sqrt{x}-\frac{6}{x^{\frac{3}{2}}}\right)^n$, $n \le 15$. If the sum of the coefficients of the remaining terms in the expansion is 649 and the coefficient of x^{-n} is $\lambda\alpha$, then λ is equal to _____.

माना $\left(\sqrt{x}-\frac{6}{x^{\frac{3}{2}}}\right)^n$, $n \le 15$ के द्विपद प्रसार में अचर पद α है। यदि इस प्रसार में शेष पदों के गुणांकों का योग 649 है तथा x^{-n} का गुणांक $\lambda\alpha$ है, तो λ बराबर है _____

Ans. Official Answer NTA (36)

Sol. $T_{k+1} = {}^nC_k (x)^{\frac{n-k}{2}} (-6)^k (x)^{\frac{-3}{2}k}$

$\frac{n-k}{2} - \frac{3}{2}k = 0$

$n - 4k = 0$

$(-5)^n - \left({}^nC_{\frac{n}{4}} (-6)^{\frac{n}{4}}\right) = 649$

By observation $(625 + 24 = 649)$, we get $n = 4$

$\because n = 4 \& k = 1$

Required is coefficient of x^{-4} is $\left(\sqrt{4}-\frac{6}{x^{\frac{3}{2}}}\right)^4$

${}^4C_1 (-6)^3$

By calculating we will get $\lambda = 36$

Question ID : 3666947206

28. The sum to 20 terms of the series $2\cdot2^2-3^2+2\cdot4^2-5^2+2\cdot6^2-\ldots\ldots$ is equal to _____.

श्रेणी $2\cdot2^2-3^2+2\cdot4^2-5^2+2\cdot6^2-\ldots\ldots$ के 20 पदों का योग है _____

Ans. Official Answer NTA (1310)

Sol. $(2^2-3^2+4^2-5^2+20 \text{ terms})+(2^2+4^2+\ldots\ldots+\text{terms})$

$-2(2+3+4+5+\ldots\ldots+11)+4[1+2^2+\ldots\ldots10^2]$

$$-\left[\frac{21\times22}{2}-1\right]+4\times\frac{10\times11\times21}{6}$$

$=1-231+14\times11\times10$

$=1540+1-231$

$=1310$

Question ID : 3666947207

29. Let for $x\in\mathbb{R}, S_0(x)=x, S_k(x)=C_kx+k\int_0^x S_{k-1}(t)dt$, where $C_0=1, C_k=1-\int_0^1 S_{k-1}(x)dx, k=1,2,3,\ldots$

Then $S_2(3)+6C_3$ is equal to _____.

माना $x\in\mathbb{R}, S_0(x)=x, S_k(x)=C_kx+k\int_0^x S_{k-1}(t)dt$ है, जहाँ $C_0=1, C_k=1-\int_0^1 S_{k-1}(x)dx, k=1,2,3,\ldots$ हैं। तो

$S_2(3)+6C_3$ बराबर है _____

Ans. Official Answer NTA (18)

Sol. $C_0=1$

$$C_1=1-\int_0^1 S_0(x)dx=1-\int_0^1 xdx=1-\frac{1}{2}=\frac{1}{2}$$

$$S_1(x)=C_1x+\int_0^x tdt$$

$$=\frac{1}{2}x+\frac{x^2}{2}=\frac{x+x^2}{2}$$

$$C_2=1-\int_0^1\left(\frac{x+x^2}{2}\right)dx=1-\frac{1}{2}\left(\frac{1}{2}+\frac{1}{3}\right)=1-\frac{5}{12}=\frac{7}{12}$$

$$S_2(x)=C_2x+2\int_0^x\left(\frac{t+t^2}{2}\right)dt=\frac{7}{12}x+\left(\frac{x^2}{2}+\frac{x^3}{3}\right)=\frac{7x+6x^2+4x^3}{12}$$

$$S_2(3)=\frac{21+54+108}{12}=\frac{183}{12}=\frac{61}{4}$$

$$C_3 = 1 - \int_0^1 \left(\frac{7x + 6x^2 + 4x^3}{12} \right) dx$$

$$= 1 - \frac{1}{12}\left[\frac{7}{2} + \frac{6}{3} + \frac{4}{4}\right]$$

$$= 1 - \frac{1}{12}\left(\frac{7}{2} + 3\right) = \frac{24 - 7 - 6}{24} = \frac{11}{24}$$

Now, $S_2(3) + 6C_3 = \frac{61}{4} + \frac{11}{4} = \frac{72}{4} = 18$

Question ID : 3666947211

30. Let the mean of the data

x	1	3	5	7	9
Frequency (f)	4	24	28	α	8

be 5 . If m and σ^2 are respectively the mean deviation about the mean and the variance of the data, then $\frac{3\alpha}{m + \sigma^2}$ is equal to _____.

माना आकड़ों

x	1	3	5	7	9
बारंबारता (f)	4	24	28	α	8

का माध्य 5 है। यदि इन आंकड़ों के माध्य के सापेक्ष माध्य विचलन तथा प्रसरण क्रमशः m तथा σ^2 हैं, तो $\frac{3\alpha}{m + \sigma^2}$ बराबर है _____.

Ans. Official Answer NTA (8)

Sol. $\frac{4 + 72 + 28\times + 7\alpha + 72}{64 + \alpha} = 5$

$\Rightarrow \alpha = 16$

$\sum f_i = 80$

$$M.D = \frac{\sum f_i |x_i - 5|}{\sum f_i}$$

$$= \frac{4 + 4 + 24\times 2 + 0 + 16\times 2 + 8\times 4}{80}$$

$$= \frac{8}{5}$$

$$\sigma^2 = \frac{\sum f_i |x_i - 5|}{\sum f_i}$$

$$= \frac{4+16+24\times4+0+16\times4+8\times16}{80} = \frac{22}{5}$$

$$\frac{3\alpha}{m+\sigma^2} = \frac{3\times16}{\frac{8}{5}+\frac{22}{5}} = 8$$