JEE Main April 2023

Question Paper With Text Solution

13 April | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911
Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in

JEE MAIN APRIL 2023 | 13TH APRIL SHIFT-2

SECTION – A

Question ID : 7155053790

1. The random variable X follows binomial distribution B(n, p), for which the difference of the mean and the variance is 1 . If 2P(X = 2) = 3P(X = 1), then $n^2P(X > 1)$ is equal to :

एक यादृच्छिक चर X एक द्विपद B(n, p) जिसके माध्य तथा प्रसरण का अंतर 1 है, का अनुसरण करता है। यदि 2P(X = 2) = 3P(X = 1), है, तो $n^2P(X > 1)$ बराबर है :

(1) 11 (2) 16 (3) 15 (4) 12

Ans. Official Answer NTA (1)

Sol. $np - npq = 1$

$\Rightarrow np(1 - q) = 1$

$\Rightarrow np^2 = 1$

$2P(X = 2) = 3P(X = 1)$

$2 \cdot {}^nC_2 p^2 q^{n-2} = 3 \cdot {}^nC_1 p \cdot q^{n-1}$

$$\Rightarrow 2 \cdot \frac{n \cdot (n-1)}{2} \cdot p = 3 \cdot n \cdot q$$

$$\Rightarrow (n-1)p = 3(1-p)$$

$$\Rightarrow \left(\frac{1}{p^2} - 1\right)p = 3(1-p)$$

$$\Rightarrow \frac{(1-p)(1+p)}{p} = 3(1-p)$$

$\Rightarrow 1 + p = 3p$

$$\Rightarrow p = \frac{1}{2}$$

$\therefore n = 4$

$n^2P(x > 1) = n^2(1 - P(x = 1) - P(x = 0))$

$$= 16\left(1 - {}^4C_1 \cdot \left(\frac{1}{2}\right)^4 - \left(\frac{1}{2}\right)^4\right) = 11$$

Question ID : 7155053780

2. If $\lim_{x\to 0}\frac{e^{ax}-\cos(bx)-\frac{cxe^{-cx}}{2}}{1-\cos(2x)}=17$, then $5a^2+b^2$ is equal to :

यदि $\lim_{x\to 0}\frac{e^{ax}-\cos(bx)-\frac{cxe^{-cx}}{2}}{1-\cos(2x)}=17$, है, तो $5a^2+b^2$ बराबर है :

(1) 72 (2) 76 (3) 64 (4) 68

Ans. Official Answer NTA (4)

Sol. $$\lim_{x\to 0}\frac{e^{ax}-\cos(bx)-\frac{cxe^{-cx}}{2}}{\frac{(1-\cos 2x)}{4x^2}\times 4x^2}=17$$

On expansion,

$$\lim_{x\to 0}\frac{\left(1+ax+\frac{a^2x^2}{2}\right)-\left(1-\frac{b^2x^2}{2}\right)-\frac{cx}{2}(1-cx)}{2x^2}=17$$

$$\lim_{x\to 0}\frac{\left(a-\frac{c}{2}\right)x+x^2\left(\frac{a^2}{2}+\frac{b^2}{2}+\frac{c^2}{2}\right)}{2x^2}=17$$

For limit to be exist $a-\frac{c}{2}=0$

$a=\frac{c}{2}$

and $\frac{a^2+b^2+c^2}{4}=17$

$a^2+b^2+4a^2=17\times 4$

$5a^2+b^2=68$

Question ID : 7155053778

3. The coefficient of x^5 in the expansion of $\left(2x^3 - \frac{1}{3x^2}\right)^5$ is :

$\left(2x^3 - \frac{1}{3x^2}\right)^5$ के प्रसार में x^5 का गुणांक है :

(1) 9 (2) 8 (3) $\frac{26}{3}$ (4) $\frac{80}{9}$

Ans. Official Answer NTA (4)

Sol. General term for $\left(2x^3 - \frac{1}{3x^2}\right)^5$

$$T_{r+1} = {}^5C_r\left(-\frac{1}{3x^2}\right)^r\left(2x^3\right)^{5-r}$$

$= {}^5C_r(-1)^r 3^{-r} 2^{5-r} \cdot x^{15-5r}$

$15 - 5r = 5 \Rightarrow r = 2$

Coeff. of $x^5 = {}^5C_2(-1)^2 3^{-2} 2^3$

$$= 10 \times \frac{1}{9} \times 8$$

$$= \frac{80}{9}$$

Question ID : 7155053776

4. Let for $A = \begin{bmatrix} 1 & 2 & 3 \\ \alpha & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$, $|A| = 2$. If $|(2\text{ adj }(2A))| = 32^n$, then $3n + \alpha$ is equal to :

माना $A = \begin{bmatrix} 1 & 2 & 3 \\ \alpha & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ के लिए $|A| = 2$ है। यदि $|(2\text{ adj }(2A))| = 32^n$ है, तो $3n + \alpha$ बराबर है :

(1) 10 (2) 11 (3) 12 (4) 9

Ans. Official Answer NTA (2)

Sol. $A = \begin{bmatrix} 1 & 2 & 3 \\ \alpha & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$, $|A| = 2$

$\Rightarrow |A| = 1(6-1) - \alpha(4-3) + 1(2-9) = 2$

$\Rightarrow 5 - \alpha - 7 = 2 \Rightarrow \alpha = -4$

Now $|2\text{ adj }(2\text{adj }2A)| = 32^n$

$\Rightarrow 2^3|2\text{adj }2A|^2 = 32^n$

$\Rightarrow 2^3 \cdot 2^6|2A|^4 = 2^{5n}$

$\Rightarrow 2^9 \cdot 2^{12}|A|^4 = 2^{5n}$

$\Rightarrow 2^{25} = 2^{5n} \Rightarrow n = 5$

$\therefore 3n + \alpha = 15 - 4 = 11$

Question ID : 7155053779

5. Let $a_1, a_2, a_3, \ldots\ldots$ be a G. P. of increasing positive numbers. Let the sum of its 6^{th} and 8^{th} terms be 2 and the product of its 3^{rd} and 5^{th} terms be $\frac{1}{9}$. Then $6(a_2 + a_4)(a_4 + a_6)$ is equal to :

माना $a_1, a_2, a_3, \ldots\ldots$ वर्धमान धनात्मक संख्याओं की एक G. P. है। माना इसके छठे और आठवें पदों का योग 2 है तथा इसके तीसरे और पाँचवें पदों का गुणनफल $\frac{1}{9}$ है, तो $6(a_2 + a_4)(a_4 + a_6)$ बराबर है :

(1) 3 (2) 2 (3) $3\sqrt{3}$ (4) $2\sqrt{2}$

Ans. Official Answer NTA (1)

Sol. $a_6 + a_8 = 2$

$\Rightarrow ar^5 + ar^7 = 2$ ...(i)

$a_3 \cdot a_5 = \frac{1}{9} \Rightarrow a^2 \cdot r^2 \cdot r^4 = \frac{1}{9}$

$\Rightarrow ar^3 = \frac{1}{3}$

$\frac{r^2}{3} + \frac{r^4}{3} = 2$

$\Rightarrow r^4 + r^2 = 6$

$\Rightarrow (r^2 + 3)(r^2 - 2) = 0$

$\Rightarrow r^2 = 2$

$\therefore ar \cdot 2 = \frac{1}{3} \Rightarrow ar = \frac{1}{6}$

Now, $6(a_2 + a_4)(a_4 + a_6)$
$= 6(ar + ar^3)(ar^3 + ar^5)$

$$= 6\left(\frac{1}{6}+\frac{1}{3}\right)\left(\frac{1}{3}+\frac{2}{3}\right)$$

$$= 6\cdot\frac{1}{2}\cdot 1 = 3$$

Question ID : 7155053774

6. Let α, β be the roots of the equation $x^2 - \sqrt{2}x + 2 = 0$. Then $\alpha^{14} + \beta^{14}$ is equal to :

माना समीकरण $x^2 - \sqrt{2}x + 2 = 0$ के मूल α, β हैं। तो $\alpha^{14} + \beta^{14}$ बराबर है :

(1) –64 (2) –128 (3) $-64\sqrt{2}$ (4) $-128\sqrt{2}$

Ans. Official Answer NTA (2)

Sol. $x^2 - \sqrt{2}x + 2 = 0$

$$x = \frac{\sqrt{2} \pm \sqrt{2-8}}{2} = \frac{\sqrt{2} \pm \sqrt{6}i}{2}$$

$$\alpha = \frac{\sqrt{2}+\sqrt{6}i}{2} = \sqrt{2}e^{\frac{i\pi}{3}} \ \& \ \beta = \sqrt{2}e^{\frac{-i\pi}{3}}$$

$$\alpha^{14} = 2^7 e^{\frac{i14\pi}{3}} = 128\left[e^{\frac{i2\pi}{3}}\right]$$

$$\beta^{14} = 128\left[e^{\frac{-i2\pi}{3}}\right]$$

$$\alpha^{14} + \beta^{14} = 128(2)\cos\left(\frac{2\pi}{3}\right) = -128$$

Question ID : 7155053785

7. The plane, passing through the points (0, –1, 2) and (–1, 2, 1) and parallel to the line passing through (5, 1, –7) and (1, –1, –1), also passes through the point :

बिन्दुओं (5, 1, –7) तथा (1, –1, –1) से होकर जाने वाली रेखा के समांतर तथा बिन्दुओं (0, –1, 2) तथा (–1, 2, 1) से होकर जानेवाला समतल, निम्न में से किस बिन्दु से होकर जाता हैं?

(1) (0, 5, –1) (2) (–2, 5, 0) (3) (2, 0, 1) (4) (1, –2, 1)

Ans. Official Answer NTA (2)

Sol. Plane passing through (0, –1, 0) and (–1, 2, 1)

Then vector in plane $\langle -1,3,-1\rangle$ vector parallel to plane is $\langle 4,2,-6\rangle$

Normal vector to plane $(\vec{n}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & -1 \\ 4 & 2 & -6 \end{vmatrix}$

$= \hat{i}(16) - \hat{j}(10) + \hat{k}(-14)$

$\vec{n} = \langle 8,5,7\rangle$

Equation of plane

$8(x-0) + 5(y+1) + 7(z-2) = 0$

$\Rightarrow 8x + 5y + 7z = 9$

From given options point (–2, 5, 0) lies on plane.

Question ID : 7155053782

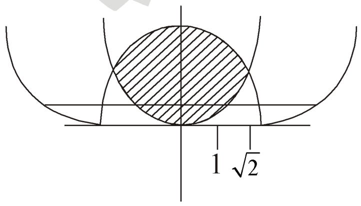
8. The area of the region $\{(x,y) : x^2 \le y \le |x^2-4|, y \ge 1\}$ is :

क्षेत्र $\{(x,y) : x^2 \le y \le |x^2-4|, y \ge 1\}$ का क्षेत्रफल है :

(1) $\frac{3}{4}(4\sqrt{2}-1)$ (2) $\frac{4}{3}(4\sqrt{2}+1)$ (3) $\frac{3}{4}(4\sqrt{2}+1)$ (4) $\frac{4}{3}(4\sqrt{2}-1)$

Ans. Official Answer NTA (4)

Sol.



$$A = 2\left(\int_0^1 (3-x^2)dx + \int_1^{\sqrt{2}} (4-2x^2)dx\right) = 2\left(\left(3x - \frac{x^3}{3}\right)_0^1 + \left(4x - \frac{2x^3}{3}\right)_1^{\sqrt{2}}\right)$$

$$= 2\left(3 - \frac{1}{3} + \left(4\sqrt{2} - \frac{4\sqrt{2}}{3} - 4 + \frac{2}{3}\right)\right) = 2\left(\frac{-2}{3} + \frac{2}{3}(4\sqrt{2})\right) = \frac{4(4\sqrt{2}-1)}{3}$$

Question ID : 7155053773

9. Let $S=\left\{z\in\mathbb{C}:\overline{z}=i\left(z^2+\text{Re}(\overline{z})\right)\right\}$. The $\sum_{z\in S}|z|^2$ is equal to :

माना $S=\left\{z\in\mathbb{C}:\overline{z}=i\left(z^2+\text{Re}(\overline{z})\right)\right\}$ है। तो $\sum_{z\in S}|z|^2$ बराबर है :

(1) $\frac{5}{2}$ (2) 4 (3) 3 (4) $\frac{7}{2}$

Ans. Official Answer NTA (2)

Sol. Let $z = x + iy$

$\overline{z}=i\left(z^2+\text{Re}\left(\overline{z}\right)\right)$

$\Rightarrow x - iy = i\,(x^2 - y^2 + 2i\,xy + x)$

$x - iy = i\,(x^2 - y^2 + x) - 2xy$

$x = -2xy \Rightarrow x(2y + 1) = 0$

$\Rightarrow x = 0, y = \frac{-1}{2}$ ....(i)

$-y = x^2 - y^2 + x$ ....(ii)

Case (1) $x = 0$

(ii) $\Rightarrow -y = -y^2 \Rightarrow y^2 - y = 0 \Rightarrow y = 0, 1$

$z = 0, i$

Case (2) $y = \frac{-1}{2}$

(ii) $\Rightarrow \frac{1}{2} = x^2 - \frac{1}{4} + x \Rightarrow x^2 + x - \frac{3}{4} = 0$

$4x + 4x - 3 = 0 \Rightarrow (2x - 1)\,(2x + 3) = 0$

$x = \frac{1}{2}, \frac{-3}{2}$

$z = \frac{1}{2} - \frac{1}{2}i, \frac{-3}{2} - \frac{1}{2}i$

$\sum|z|^2 = 0 + 1 + \frac{1}{2} + \frac{5}{2} = 4$

Question ID : 7155053789

10. Let for a triangle ABC,

$\overrightarrow{AB} = -2\hat{i} + \hat{j} + 3\hat{k}$

$\overrightarrow{CB} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

$\overrightarrow{CA} = 4\hat{i} + 3\hat{j} + \delta\hat{k}$

If $\delta > 0$ and the ara of the triangle ABC is $5\sqrt{6}$, the $\overrightarrow{CB} \cdot \overrightarrow{CA}$ is equal to :

माना एक त्रिभुज ABC के लिए,

$\overrightarrow{AB} = -2\hat{i} + \hat{j} + 3\hat{k}$

$\overrightarrow{CB} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

$\overrightarrow{CA} = 4\hat{i} + 3\hat{j} + \delta\hat{k}$

है। यदि $\delta > 0$ है तथा त्रिभुज ABC का क्षेत्रफल $5\sqrt{6}$ है, तो $\overrightarrow{CB} \cdot \overrightarrow{CA}$ बराबर है :

(1) 60 (2) 54 (3) 120 (4) 108

Ans. Official Answer NTA (1)

Sol. $\overline{AB} + \overline{BC} + \overline{CA} = \vec{0}$

$\alpha = 2,\ \beta = 4,\ \gamma - \delta = 3$

$\frac{1}{2}\left|\overline{AB} \times \overline{AC}\right| = 5\sqrt{6}$

$(\delta - 9)^2 + (2\delta + 12)^2 + 100 = 600$

$\Rightarrow \delta = 5, \gamma = 8$

Hence $\overline{CB} \cdot \overline{CA} = 60$

Question ID : 7155053772

11. The range of $f(x) = 4\sin^{-1}\left(\frac{x^2}{x^2+1}\right)$ is :

$f(x) = 4\sin^{-1}\left(\frac{x^2}{x^2+1}\right)$ का परिसर है :

(1) $[0, \pi]$ (2) $[0, 2\pi)$ (3) $[0, 2\pi]$ (4) $[0, \pi)$

Ans. Official Answer NTA (2)

Sol. $f(x) = 4\sin^{-1}\left(\frac{x^2}{1+x^2}\right)$

$0 \le \frac{x^2}{1+x^2} < 1$

$\Rightarrow 0 \le \sin^{-1}\left(\frac{x^2}{1+x^2}\right) < \frac{\pi}{2}$

$\Rightarrow 0 \le 4\sin^{-1}\left(\frac{x^2}{1+x^2}\right) < 2\pi$

Range : $[0, 2\pi)$

Question ID : 7155053775

12. If the system of equations

$2x + y - z = 5$
$2x - 5y + \lambda z = \mu$
$x + 2y - 5z = 7$

has infinitely many solutions, then $(\lambda+\mu)^2 + (\lambda-\mu)^2$ is equal to :

यदि समीकरण निकाय
$2x + y - z = 5$
$2x - 5y + \lambda z = \mu$
$x + 2y - 5z = 7$

के अनंत हल हैं, तो $(\lambda+\mu)^2 + (\lambda-\mu)^2$ बराबर है :

(1) 916 (2) 904 (3) 912 (4) 920

Ans. Official Answer NTA (1)

Sol. $2x + y - z = 5$

$2x - 5y + \lambda z = \mu$

$x + 2y - 5z = 7$

for infinite solution $D = D_1 = D_2 = D_3 = 0$

$$D = \begin{vmatrix} 2 & 1 & -1 \\ 2 & -5 & \lambda \\ 1 & 2 & -5 \end{vmatrix} = 0$$

$2(25 - 2\lambda) - (-10 - \lambda) - 1(4 + 5) = 0$

$50 - 4\lambda + 10 + \lambda - 9 = 0 \qquad \Rightarrow \lambda = 17$

$$D_1 = \begin{vmatrix} 5 & 1 & -1 \\ \mu & -5 & 17 \\ 7 & 2 & -5 \end{vmatrix} = 0$$

$5(25 - 34) - \mu(-5 + 2) + 7(17 - 5) = 0$

$-45 + 3\mu + 84 = 0 \Rightarrow \mu = -13$

Now $(\lambda+\mu)^2 + (\lambda-\mu)^2 = (17-13)^2 + (17+13)^2$

$= 16 + 900 = 916$

Question ID : 7155053783

13. Let (α, β) be the centroid of the triangle formed by the lines $15x - y = 82$, $6x - 5y = -4$ and $9x + 4y = 17$. Then $\alpha + 2\beta$ and $2\alpha - \beta$ are the roots of the equation :

माना रेखाओं $15x - y = 82$, $6x - 5y = -4$ तथा $9x + 4y = 17$ द्वारा बने त्रिभुज का केन्द्रक (α, β) है। तो $\alpha + 2\beta$ तथा $2\alpha - \beta$ किस समीकरण के मूल हैं :

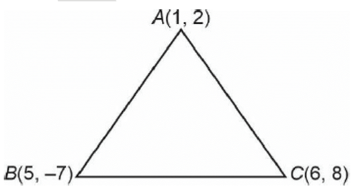
(1) $x^2 - 7x + 12 = 0$ (2) $x^2 - 13x + 42 = 0$ (3) $x^2 - 10x + 25 = 0$ (4) $x^2 - 14x + 48 = 0$

Ans. Official Answer NTA (2)

Sol. $15x - y = 82$

$6x - 5y = -4$

$9x + 4y = 17$



$$(\alpha, \beta) \equiv \left(\frac{1+5+6}{3}, \frac{2-7+8}{3}\right) \equiv (4, 1)$$

$\alpha + 2\beta = 6$ and $2\alpha - \beta = 7$

Equation $x^2 - 13x + 42 = 0$

Question ID : 7155053784

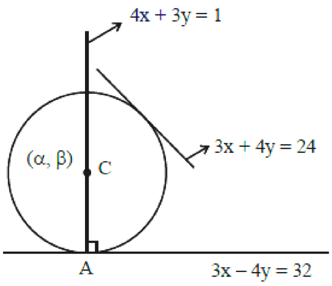
14. Let the centre of a circle be (α, β) and its radius $r < 8$. Let $3x + 4y = 24$ and $3x - 4y = 32$ be two tangents and $4x + 3y = 1$ be a normal to C. Then $(\alpha - \beta + r)$ is equal to :

माना एक वृत्त C का केन्द्र (α, β) है तथा इसकी त्रिज्या $r < 8$ है। माना C की दो स्पर्श रेखाएँ $3x + 4y = 24$ तथा $3x - 4y = 32$ हैं तथा एक अभिलंब $4x + 3y = 1$ है। तो $(\alpha - \beta + r)$ बराबर है :

(1) 7 (2) 5 (3) 9 (4) 6

Ans. Official Answer NTA (1)

Sol.



First find point A by solving $4x + 3y = 1$ and $3x - 4y = 32$
After solving, point A is $(4, -5)$
centre (α, β) lie on $4x + 3y = 1$

$$4\alpha + 3\beta = 1 \Rightarrow \beta = \frac{1-4\alpha}{3}$$

Now distance from centre to line $3x - 4y - 32 = 0$ and $3x + 4y - 24 = 0$ are equal.

$$\left|\frac{3\alpha - 4\left(\frac{1-4\alpha}{3}\right) - 32}{5}\right| = \left|\frac{3\alpha + 4\left(\frac{1-4\alpha}{3}\right) - 24}{5}\right|$$

after solving $\alpha = 1$ and $\alpha = \frac{28}{3}$

For $\alpha = 1$, centre $(1, -1) \Rightarrow$ radius = 5

For $\alpha = \frac{28}{3}$, centre $\left(\frac{28}{3}, \frac{-109}{2}\right)$

$\Rightarrow$ radius ≈ 49.78 (rejected)

Hence, $\alpha = 1, \beta = -1, r = 5$

$\alpha - \beta + r = 7$

Question ID : 7155053787

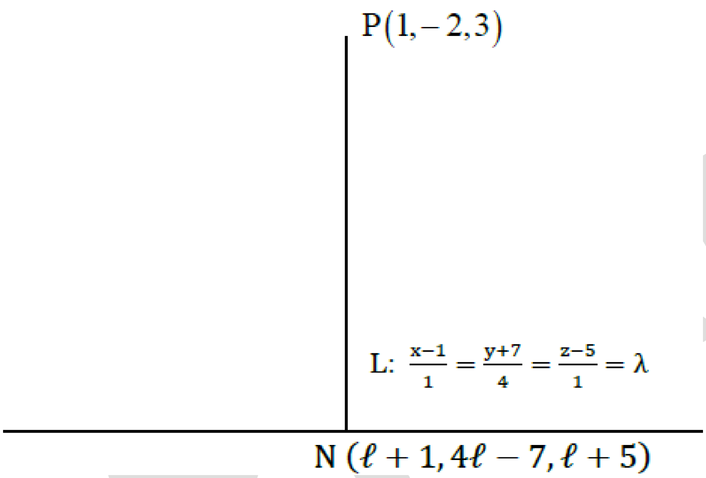
15. Let N be the foot of perpendicular from the point P(1, –2, 3) on the line passing through the points (4, 5, 8) and (1, –7, 5). Then the distance of N from the plane $2x - 2y + z + 5 = 0$ is :

माना बिन्दुओं (4, 5, 8) तथा (1, –7, 5) से होकर जाने वाली रेखा पर बिन्दु P(1, –2, 3) से डाले गए लंब का पाद N है। तो समतल $2x - 2y + z + 5 = 0$ से N की दूरी है :

(1) 7 (2) 8 (3) 9 (4) 6

Ans. Official Answer NTA (1)

Sol.



$\overline{PN} = \langle \lambda, 4\lambda - 5, \lambda + 2 \rangle$

$\overline{PN} \cdot \langle 1, 4, 1 \rangle = 0$

$\Rightarrow \lambda + 16\lambda - 20 + \lambda + 2 = 0$

$\Rightarrow \lambda = 1$

N (2, –3, 6)

Distance of N from $2x - 2y + z + 5 = 0$

$$d = \left| \frac{2(2) - 2(-3) + 6 + 5}{\sqrt{2^2 + (-2)^2 + (1)^2}} \right|$$

$$= \left| \frac{21}{3} \right| = 7$$

Question ID : 7155053788

16. Let $|\vec{a}| = 2, |\vec{b}| = 3$ and the angle between the vectors $\vec{a}$ and $\vec{b}$ be $\frac{\pi}{4}$. Then $|(\vec{a}+2\vec{b})\times(2\vec{a}-3\vec{b})|^2$ is equal to:

माना $|\vec{a}| = 2, |\vec{b}| = 3$ है तथा सदिशों $\vec{a}$ और $\vec{b}$ के बीच का कोण $\frac{\pi}{4}$ है। तो $|(\vec{a}+2\vec{b})\times(2\vec{a}-3\vec{b})|^2$ बराबर है :

(1) 882 (2) 841 (3) 441 (4) 482

Ans. Official Answer NTA (1)

Sol. $|\vec{a}| = 2, |\vec{b}| = 3$, $\vec{a} \wedge \vec{b} = \frac{\pi}{4}$

Now $\left|\left(\vec{a}+2\vec{b}\right)\times\left(2\vec{a}-3\vec{b}\right)\right|^2 = \left|2\vec{a}\times\vec{a} - 3\vec{a}\times\vec{b} + 4\vec{b}\times\vec{a} - 6\vec{b}\times\vec{b}\right|^2$

$\left|7\left(\vec{a}\times\vec{b}\right)\right|^2 = 49|\vec{a}|^2|\vec{b}|^2\sin^2\frac{\pi}{4} = 49\cdot4\cdot9\frac{1}{2} = 882$

Question ID : 7155053791

17. The statement $(p \wedge (\sim q)) \vee ((\sim p) \wedge q) \vee ((\sim p) \wedge (\sim q))$ is equivalent to :

कथन $(p \wedge (\sim q)) \vee ((\sim p) \wedge q) \vee ((\sim p) \wedge (\sim q))$ किस के तुल्य है :

(1) $(\sim p) \vee q$ (2) $p \vee (\sim q)$ (3) $(\sim p) \vee (\sim q)$ (4) $p \vee q$

Ans. Official Answer NTA (3)

Sol. $(p \wedge (\sim q)) \vee ((\sim p) \wedge q) \vee ((\sim p) \wedge (\sim q))$

$= (\sim p \wedge (q \vee \sim q)) \vee (p \wedge \sim q)$

$= \sim p \vee (p \wedge \sim q)$

$= (\sim p \vee p) \wedge (\sim p \vee \sim q)$

$= \sim p \vee \sim q$

Question ID : 7155053777

18. All words, with or without meaning, are made using all the letters of the word MONDAY. These words are written as in a dictionary with serial numbers. The serial number of the word MONDAY is :

शब्द MONDAY के सभी अक्षरों के प्रयोग से सारे शब्द अर्थपूर्ण या अर्थहीन, बनाए गए हैं। इन शब्दों को शब्दकोश के अनुसार क्रमांक संख्या के साथ लिखा गया है। शब्द MONDAY की क्रमांक संख्या है :

(1) 326 (2) 327 (3) 324 (4) 328

Ans. Official Answer NTA (2)

Sol. First arrange in alphabetical order

i.e. ADMNOY

A _ _ _ _ _ = 5!

D _ _ _ _ _ = 5!

M A _ _ _ _ = 4!

M D _ _ _ _ = 4!

M N _ _ _ _ = 4!

M O A _ _ _ = 3!

M O D _ _ _ = 3!

M O N A _ _ = 2!

M O N D A Y =1

= 327

Question ID : 7155053781

19. The value of $\dfrac{e^{-\frac{\pi}{4}+\int_0^{\frac{\pi}{4}} e^{-x}\tan^{50}x\,dx}}{\int_0^{\frac{\pi}{4}} e^{-x}\left(\tan^{49}x+\tan^{51}x\right)dx}$ is :

$\dfrac{e^{-\frac{\pi}{4}+\int_0^{\frac{\pi}{4}} e^{-x}\tan^{50}x\,dx}}{\int_0^{\frac{\pi}{4}} e^{-x}\left(\tan^{49}x+\tan^{51}x\right)dx}$ का मान है :

(1) 51 (2) 49 (3) 25 (4) 50

Ans. Official Answer NTA (4)

Sol. Let $I_1 = e^{\frac{-\pi}{4}} + \int_0^{\frac{\pi}{4}} e^{-x} \tan^{50} x dx$

$$I_2 = \int_0^{\frac{\pi}{4}} e^{-x} \left(\tan^{49} x + \tan^{51} x\right) dx$$

$$= \int_0^{\frac{\pi}{4}} e^{-x} \tan^{49} x \left(\sec^2 x\right) dx$$

$$= \left| e^{-x} \frac{\tan^{50} x}{50} \right|_0^{\frac{\pi}{4}} + \frac{1}{50} \int_0^{\frac{\pi}{4}} e^{-x} \tan^{50} x dx$$

$$= \frac{e^{-\frac{\pi}{4}}}{50} + \frac{1}{50} \int_0^{\frac{\pi}{4}} e^{-x} \tan^{50} x dx = \frac{I_1}{50}$$

then $\frac{I_1}{I_2} = 50$

Question ID : 7155053786

20. The line, that is coplanar to the line $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$, is :

रेखा $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$, के सहतलीय रेखा का समीकरण है :

(1) $\frac{x+1}{1} = \frac{y-2}{2} = \frac{z-5}{5}$

(2) $\frac{x-1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$

(3) $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$

(4) $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{4}$

Ans. Official Answer NTA (3)

Sol. Check by option

option 3 $\Rightarrow \begin{vmatrix} -3+1 & 1-2 & 5-5 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = \begin{vmatrix} -2 & -1 & 0 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix}$

$= \begin{vmatrix} -2 & -1 & 0 \\ -2 & -1 & 0 \\ -1 & 2 & 5 \end{vmatrix} = 0$

$\therefore$ option (3) is correct.

SECTION – B

Question ID : 7155053794

21. The remainder, when 7^{103} is divided by 17, is ________.

7^{103} को 17 से विभाजित करने पर शेषफल है ________

Ans. Official Answer NTA (12)

Sol. 7^{103} divided by 17

$7 \equiv 7 \pmod{17}$

$7^2 \equiv -2 \pmod{17}$

$7^6 \equiv -8 \pmod{17}$

$7^8 \equiv -1 \pmod{17}$

$7^{16} \equiv 1 \pmod{17}$

$7^{103} \equiv 12 \pmod{17}$

$\therefore$ Remainder = 12

Question ID : 7155053792

22. Let A = {–4, –3, –2, 0, 1, 3, 4} and $R = \{(a,b) \in A \times A : b = |a| \text{ or } b^2 = a+1\}$ be a relation on A. Then the minimum number of elements, that must be added to the relation R so that it becomes reflexive and symmetric, is ________.

माना A = {–4, –3, –2, 0, 1, 3, 4} है तथा A पर एक संबंध $R = \{(a,b) \in A \times A : b = |a| \text{ या } b^2 = a+1\}$ है। तो संबंध R में कम से कम कितने अवयव जोड़े जांए, जिससे कि यह स्वतुल्य तथा सममित हो जाए ________

Ans. Official Answer NTA (7)

Sol. R = [(–4,4), (–3,3), (3, –2), (0,1), (0,0), (1,1), (4,4), (3,3)}

For reflexive, add $\Rightarrow$ (–2,–2), (–4,–4), (–3, –3)

For symmetric, add $\Rightarrow$ (4, –4), (3, –3), (–2, 3), (1, 0)

Question ID : 7155053799

23. The foci of a hyperbola are (±2, 0) and its eccentricity is $\frac{3}{2}$. A tangent, perpendicular to the line $2x + 3y = 6$, is drawn at a point in the first quadrant on the hyperbola. If the intercepts made by the tangent on the x- and y-axes are a and b respectively, then |6a| + |5b| is equal to ________.

एक अतिपरवलय की नाभियाँ (±2, 0) हैं तथा इसकी उत्केन्द्रता $\frac{3}{2}$ है। प्रथम चतुर्थांश में अतिपरवलय के एक बिन्दु पर एक स्पर्श रेखा, जो $2x + 3y = 6$ के लंबवत है, खींची जाती है। यदि यह स्पर्श रेखा, x- तथा y-अक्षों पर क्रमशः अंतःखण्ड a तथा b बनाती है, तो |6a| + |5b| बराबर है ________

Ans. Official Answer NTA (12)

Sol. $2ae = 4$

$$2a\left(\frac{3}{2}\right) = 4$$

$$\Rightarrow a = \frac{4}{3}$$

$$e^2 = 1 + \frac{b^2}{a^2}$$

$$\Rightarrow \frac{9}{4} = 1 + b^2\left(\frac{9}{16}\right)$$

$$\Rightarrow b^2 = \left(\frac{5}{4}\right)\left(\frac{16}{9}\right) = \frac{20}{9}$$

slope of tangent $m = \frac{3}{2}$

equation of tangent is

$$y = mx \pm \sqrt{a^2m^2 - b^2}$$

$$y = \frac{3}{2}x \pm \sqrt{\frac{16}{9}\left(\frac{9}{4}\right) - \frac{20}{9}}$$

$$\Rightarrow y = \frac{3x}{2} \pm \frac{4}{3}$$

$$y = 0 \Rightarrow a = \pm\frac{8}{9}$$

$x = 0 \Rightarrow b = \pm\frac{4}{3}$

$|6a| + |5b| = \frac{16}{3} + \frac{20}{3} = 12$

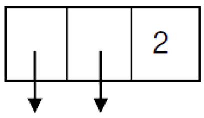
Question ID : 7155053793

24. Total numbers of 3-digit numbers that are divisible by 6 and can be formed by using the digits 1, 2, 3 4, 5 with repetition, is ________.

अंको 1, 2, 3, 4, 5 के प्रयोग से, पुनरावृत्ति के साथ, बनाई जा सकने वाली 6 से विभाज्य 3 अंकों की संख्याओं की संख्या है ________

Ans. Official Answer NTA (16)

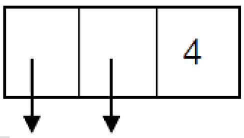
Sol. Case–1 Unit digit is 2



(i) 2 2 = 4

(ii) 2 2 = 4

Case–2 Unit digit is 4



(i) 2 2 = 4

(ii) 2 2 = 4

Total number = 16

Question ID : 7155053801

25. For $x \in (-1, 1]$, the number of solutions of the equation $\sin^{-1} x = 2\tan^{-1} x$ is equal to ________.

$x \in (-1, 1]$ के लिए, समीकरण $\sin^{-1} x = 2\tan^{-1} x$ के हलों की संख्या है ________

Ans. Official Answer NTA (2)

Sol. $\sin(\sin^{-1} x) = \sin(2\tan^{-1} x)$

$x = \frac{2x}{1+x^2}$

$x = 0, \pm 1$

$\Rightarrow x = 0$ and 1 are possible

Question ID : 7155053797

26. Let $f_n = \int_0^{\frac{\pi}{2}} \left(\sum_{k=1}^{n} \sin^{k-1} x\right)\left(\sum_{k=1}^{n} (2k-1)\sin^{k-1} x\right)\cos x dx, n \in \mathbb{N}$. Then $f_{21} - f_{20}$ is equal to ________.

माना $f_n = \int_0^{\frac{\pi}{2}} \left(\sum_{k=1}^{n} \sin^{k-1} x\right)\left(\sum_{k=1}^{n} (2k-1)\sin^{k-1} x\right)\cos x dx, n \in \mathbb{N}$ है। तो $f_{21} - f_{20}$ बराबर है ________

Ans. Official Answer NTA (41)

Sol. $f_n(x) = \int_0^{\frac{\pi}{2}} \left(1 + \sin x + \sin^2 x + \sin^3 x + ... + \sin^{n-1}(x)\right)\left(1 + 3\sin x + 5\sin^2 x + .. + (2n-1)\right)\sin^{n-1} x . \cos x dx$

Multiply & divide by $\sqrt{\sin x}$

$$\int_0^{\frac{\pi}{2}} \left((\sin x)^{\frac{1}{2}} + (\sin x)^{\frac{3}{2}} + (\sin x)^{\frac{5}{2}} + (\sin x)^{\frac{7}{2}} + ...(\sin x)^{\frac{2n-1}{2}}\right)$$

$$\left(1 + 3\sin x + 5\sin^2 x + ... + (2n-1)\sin^{n-1}(x)\right)\frac{\cos x}{\sqrt{\sin x}} dx$$

Put $(\sin x)^{1/2} + (\sin x)^{3/2} + (\sin x)^{5/2} + ... (\sin x)^{n-1/2} = t$

$$\frac{1}{2}\frac{\left(1 + 3\sin x + 5\sin^2 x + ...(2n-1)\sin^{n-1} x\right)}{\sqrt{\sin x}}\cos x dx = dt$$

$$f_n = 2\int_0^n t dt$$

$f_n = n^2$

$f_{21} - f_{20} = (21)^2 - (20)^2$

$= 441 - 400$

$= 41$

Question ID : 7155053795

27. Let $[\alpha]$ denote the greatest integer $\le \alpha$. Then $\left[\sqrt{1}\right] + \left[\sqrt{2}\right] + \left[\sqrt{3}\right] + ... + \left[\sqrt{120}\right]$ is equal to ________.

माना $[\alpha]$ महत्तम पूर्णांक $\le \alpha$ है। तो $\left[\sqrt{1}\right] + \left[\sqrt{2}\right] + \left[\sqrt{3}\right] + ... + \left[\sqrt{120}\right]$ बराबर है ________

Ans. Official Answer NTA (825)

Sol. $S=\left[\sqrt{1}\right]+\left[\sqrt{2}\right]+\left[\sqrt{3}\right]+....+\left[\sqrt{120}\right]$

$\left[\sqrt{1}\right]\to\left[\sqrt{3}\right]=1\times3$

$\left[\sqrt{4}\right]\to\left[\sqrt{8}\right]=2\times5$

$\left[\sqrt{9}\right]\to\left[\sqrt{15}\right]=3\times7$

$\vdots$

$\left[\sqrt{100}\right]\to\left[\sqrt{120}\right]=10\times21$

$S=1\times3+2\times5+3\times7+....+10\times21$

$=\sum_{r=1}^{10} r(2r+1)$

$=2\sum_{r=1}^{10} r^2+\sum_{r=1}^{10} r$

$=\frac{2\times10\times11\times21}{6}+\frac{10\times11}{2}$

$= 770 + 55$

$= 825$

Question ID : 7155053798

28. If y = y(x) is the solution of the differential equation $\frac{dy}{dx}+\frac{4x}{\left(x^2-1\right)}y=\frac{x+2}{\left(x^2-1\right)^{\frac{5}{2}}}, x>1$ such that $y(2)=\frac{2}{9}\log_e(2+\sqrt{3})$ and $y(\sqrt{2})=\alpha\log_e(\sqrt{\alpha}+\beta)+\beta-\sqrt{\gamma}, \alpha,\beta,\gamma\in\mathbb{N}$, then $\alpha\beta\gamma$ is equal to ________.

यदि $\frac{dy}{dx}+\frac{4x}{\left(x^2-1\right)}y=\frac{x+2}{\left(x^2-1\right)^{\frac{5}{2}}}, x>1$ का हल $y=y(x)$ है तथा $y(2)=\frac{2}{9}\log_e(2+\sqrt{3})$ और $y(\sqrt{2})=\alpha\log_e(\sqrt{\alpha}+\beta)+\beta-\sqrt{\gamma}, \alpha,\beta,\gamma\in\mathbb{N}$ हैं, तो $\alpha\beta\gamma$ बराबर है ________

Ans. Official Answer NTA (6)

Sol. $\frac{dy}{dx}+\frac{4x}{\left(x^2-1\right)}y=\frac{x+2}{\left(x^2-1\right)^{\frac{5}{2}}}, x>1$ which is linear in y

I.f. $e^{\int\frac{4x}{x^2-1}dx}=e^{2\ln\left(x^2-1\right)}=\left(x^2-1\right)^2$

Solution is $y\cdot\left(x^2-1\right)^2=\int\frac{x+2}{\left(x^2-1\right)^{\frac{5}{2}}}\left(x^2-1\right)^2dx+c$

$\Rightarrow y\left(x^2-1\right)^2=\int\frac{x+2}{\sqrt{x^2-1}}dx+c$

$\Rightarrow y\left(x^2-1\right)^2=\sqrt{x^2-1}+2\ln\left(x+\sqrt{x^2-1}\right)+c$

$\therefore y(2)=\frac{2}{9}\ln\left(2+\sqrt{3}\right)$

$\therefore \frac{2}{9}\ln\left(2+\sqrt{3}\right)\cdot 9=\sqrt{3}+2\ln\left(2+\sqrt{3}\right)+c$

$c=-\sqrt{3}$

$\therefore y\left(x^2-1\right)^2=\sqrt{x^2-1}+2\ln\left(x+\sqrt{x^2-1}\right)-\sqrt{3}$

Put $x=\sqrt{2}$

$\therefore y\left(\sqrt{2}\right)\cdot 1=1+2\ln\left(\sqrt{2}+1\right)-\sqrt{3}$

$y\left(\sqrt{2}\right)=2\ln\left(\sqrt{2}+1\right)+1-\sqrt{3}$

$=\alpha\ln\left(\sqrt{\alpha}+\beta\right)+\beta-\sqrt{\gamma}$

$\alpha = 2, \beta = 1, \gamma = 3$ $\quad\quad\therefore \alpha\cdot\beta\cdot\gamma = 6$

Question ID : 7155053800

29. The mean and standard deviation of the marks of 10 students were found to be 50 and 12 respectively. Later, it was observed that two marks 20 and 25 were wrongly read as 45 and 50 respectively. Then the correct variance is ________.

10 छात्रों के अंकों के माध्य तथा मानक विचलन क्रमशः 50 तथा 12 ज्ञात किए गए। बाद में यह देखा गया कि दो छात्रों के अंक 20 तथा 25 गलती से क्रमशः 45 तथा 50 पढ़े गए थे। तो सही प्रसरण है ________

Ans. Official Answer NTA (269)

Sol. $\sum x_{i(\text{wrong})} = 500$

$$144 = \frac{\sum x^2_{i_{(\text{wrong})}}}{10} - (50)^2$$

$$26440 = \sum x^2_{i_{(\text{wrong})}}$$

$$\sum x_{i_{(\text{correct})}} = 450$$

$$\Rightarrow \bar{x}_{i_{(\text{correct})}} = 45$$

$$\sum x^2_{i\,(\text{correct})} = 26440 - 3500$$

$= 22940$

$$\text{Variance} = \frac{22940}{10} - (45)^2$$

$= 2294 - 2025$

$= 269$

Question ID : 7155053796

30. Let $f(x) = \sum_{k=1}^{10} k\,x^k, x \in \mathbb{R}$. If $2f(2) + f'(2) = 119(2)^n + 1$ then n is equal to ________.

माना $f(x) = \sum_{k=1}^{10} k\,x^k, x \in \mathbb{R}$ है। यदि $2f(2) + f'(2) = 119(2)^n + 1$ है, तो n बराबर है ________

Ans. Official Answer NTA (10)

Sol. $f(x) = \sum_{k=1}^{10} kx^k$

$f(x) = x + 2x^2 + ... + 10\,x^{10}$

$f(x) \cdot x = x^2 + 2x^3 + ... + 9x^{10} + 10x^{11}$

$f(x)\,(1 - x) = x + x^2 + x^3 + ... + x^{10} - 10\,x^{11}$

$$f(x) = \frac{x(1-x^{10})}{(1-x)^2} - \frac{10x^{11}}{(1-x)}$$

$$f(x) = \frac{x - x^{11} - 10x^{11} + 10x^{12}}{(1-x)^2} \Rightarrow \frac{10x^{12} - 11x^{11} + x}{(1-x)^2}$$

Hence $2f(2) + f'(2) = 119.2^{10} + 1$

$\Rightarrow$ So, $n = 10$