JEE Main April 2023

Question Paper With Text Solution

12 April | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN APRIL 2023 | 12TH APRIL SHIFT-1

SECTION – A

Question ID : 7155054313

1. Let α, β be the roots of the quadratic equation $x^2+\sqrt{6}x+3=0$. Then $\frac{\alpha^{23}+\beta^{23}+\alpha^{14}+\beta^{14}}{\alpha^{15}+\beta^{15}+\alpha^{10}+\beta^{10}}$ is equal to :

माना द्विघातीय समीकरण $x^2+\sqrt{6}x+3=0$ के मूल α, β हैं। तो $\frac{\alpha^{23}+\beta^{23}+\alpha^{14}+\beta^{14}}{\alpha^{15}+\beta^{15}+\alpha^{10}+\beta^{10}}$ बराबर है :

(1) 72 (2) 81 (3) 729 (4) 9

Ans. Official Answer NTA (4)

Sol. $\frac{\alpha^{23}+\beta^{23}+\alpha^{14}+\beta^{14}}{\alpha^{15}+\beta^{15}+\alpha^{10}+\beta^{10}}$

Let $a_n=\alpha^n+\beta^n$

$=\frac{a_{23}+a_{14}}{a_{15}+a_{10}}$

$x^2+\sqrt{6}x+3=0 \overset{\alpha}{\underset{\beta}{<}}$

$x^2+\sqrt{6}x+3=0\sum_{\beta}^{\alpha}$

$x=\frac{-\sqrt{6}\pm\sqrt{-6}}{2}$

$=\sqrt{6}\left(\frac{-1\pm i}{2}\right)$

$=\sqrt{3}\left(\frac{-1\pm i}{\sqrt{2}}\right)$

$\alpha=\sqrt{3}e^{\frac{i3\pi}{4}}$

Question ID : 7155054329

2. Two dice A and B are rolled. Let the numbers obtained on A and B be α and β respectively. If the variance of $\alpha-\beta$ is $\frac{p}{q}$, where p and q are co-prime, then the sum of the positive divisors of p is equal to :

दो पासे A तथा B फेंको जाते हैं। माना A तथा B पर प्राप्त संख्याएँ α तथा β हैं। यदि $\alpha-\beta$ का प्रसरण $\frac{p}{q}$ है, जहाँ p तथा q असहभाज्य हैं, तो p के धनात्मक भाजकों का योग बराबर है :

(1) 36 (2) 72 (3) 48 (4) 31

Ans. Official Answer NTA (3)

Sol.

$\alpha-\beta$	Case	P
5	$(6,1)$	$1/36$
4	$(6,2)(5,1)$	$2/36$
3	$(6,3)(5,2)(4,1)$	$3/36$
2	$(6,4)(5,3)(4,3)(3,1)$	$4/36$
1	$(6,5)(5,4)(4,3)(3,2)(2,1)$	$5/36$
0	$(6,6)(5,5)........(1,1)$	$6/36$
–1	______	$5/36$
–2	______	$4/36$
–3	______	$3/36$
–4	$(2,6)(1,5)$	$2/36$
–5	$(1,6)$	$1/36$

$$\sum\left(x^2\right)=\sum x^2P(x)=2\left[\frac{25}{36}+\frac{32}{36}+\frac{27}{36}+\frac{16}{36}+\frac{5}{36}\right]$$

$$=\frac{105}{18}=\frac{35}{6}$$

$\mu=\sum(x)=0$ as data is symmetric

$$\Rightarrow \sigma^2=\sum\left(x^2\right)=\sum x^2p(x)=\frac{35}{6}$$

$\Rightarrow\ P=35=5\times7$

Sum of divisor = $(5^0+5^1)(7^0+7^1)=48$

Question ID : 7155054328

3. Let $\lambda \in \mathbb{Z}, \vec{a} = \lambda\hat{i}+\hat{j}-\hat{k}$ and $\vec{b} = 3\hat{i}-\hat{j}+2\hat{k}$. Let $\vec{c}$ be a vector such that $(\vec{a}+\vec{b}+\vec{c})\times\vec{c}=\vec{0}$, $\vec{a}\cdot\vec{c}=-17$ and $\vec{b}\cdot\vec{c}=-20$. Then $|\vec{c}\times(\lambda\hat{i}+\hat{j}+\hat{k})|^2$ is equal to :

माना $\lambda \in \mathbb{Z}, \vec{a} = \lambda\hat{i}+\hat{j}-\hat{k}$ तथा $\vec{b} = 3\hat{i}-\hat{j}+2\hat{k}$ है। माना $\vec{c}$ के लिए $(\vec{a}+\vec{b}+\vec{c})\times\vec{c}=\vec{0}$, $\vec{a}\cdot\vec{c}=-17$ तथा $\vec{b}\cdot\vec{c}=-20$ हैं। तो $|\vec{c}\times(\lambda\hat{i}+\hat{j}+\hat{k})|^2$ बराबर है :

(1) 62 (2) 46 (3) 49 (4) 53

Ans. Official Answer NTA (2)

Sol. $(\vec{a}+\vec{b}+\vec{c})\times\vec{c}=0$

$(\vec{a}+\vec{b})\times\vec{c}=0$

$\vec{c}=\alpha(\vec{a}+\vec{b})=\alpha(\lambda+3)\hat{i}+\alpha\hat{k}$

$\vec{b}\cdot\vec{c}=-20 \Rightarrow 3\alpha(\lambda+3)+2\alpha=-20$

$\vec{a}\cdot\vec{c}=-17 \Rightarrow \alpha\lambda(\lambda+3)-\alpha=-17$

$\Rightarrow \alpha(3\lambda+9+2)=-20$

$\alpha\left(\lambda^2+3\lambda-1\right)=-17$

$17(3\lambda+11)=20\left(\lambda^2+3\lambda-1\right)$

$20\lambda^2+9\lambda-207=0$

$\lambda=3 \quad (\lambda \in Z)$

$\Rightarrow \alpha=-1 \Rightarrow \vec{c}=-(6\hat{i}+\hat{k})$

$\vec{v}=\vec{c}\times(3\hat{i}+\hat{j}+\hat{k})$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -6 & 0 & -1\\ 3 & 1 & 1\end{vmatrix}=\hat{i}+3\hat{j}-6\hat{k}$$

$|\vec{v}|^2=(-1)^2+3^2+6^2=46$

Question ID : 7155054315

4. The number of five digit numbers, greater than 40000 and divisible by 5, which can be formed using the digits 0, 1, 3, 5, 7 and 9 without repetition, is equal to :

बिना पुनरावृत्ति के, अंकों 0, 1, 3, 5, 7 तथा 9 के प्रयोग से पाँच अंकों की संख्यओं, जो 40000 से बड़ी हों तथा 5 से विभाज्य हों,

की संख्या है :

(1) 72 (2) 120 (3) 132 (4) 96

Ans. Official Answer NTA (2)

Sol.

10,000th	1000th	100th	10th	unit		
3	4	3	2	0	=	72
2	4	3	2	5	=	48

Total = 120

Question ID : 7155054317

5. If $\frac{1}{n+1}{}^{n}C_{n}+\frac{1}{n}{}^{n}C_{n-1}+\ldots+\frac{1}{2}{}^{n}C_{1}+{}^{n}C_{0}=\frac{1023}{10}$ then n is equal to :

यदि $\frac{1}{n+1}{}^{n}C_{n}+\frac{1}{n}{}^{n}C_{n-1}+\ldots+\frac{1}{2}{}^{n}C_{1}+{}^{n}C_{0}=\frac{1023}{10}$ तो n बराबर है :

(1) 7 (2) 6 (3) 8 (4) 9

Ans. Official Answer NTA (4)

Sol. $\sum_{r=0}^{n}\frac{1}{r+1}{}^{n}C_{r}=\frac{1023}{10}$ $\left(\therefore {}^{n+1}C_{r+1}=\frac{n+1}{r+1}{}^{n}C_{r}\right)$

$\Rightarrow \sum_{r=0}^{n}\frac{1}{n+1}{}^{n+1}C_{r+1}=\frac{1023}{10}$

$\Rightarrow \frac{1}{n+1}\left[{}^{n+1}C_{1}+{}^{n+1}C_{2}+\ldots+{}^{n+1}C_{n+1}\right]=\frac{1023}{10}$

$\Rightarrow \frac{2^{n+1}-1}{n+1}=\frac{1023}{10}=\frac{2^{10}-1}{10}$

$\Rightarrow n+1=10$

$\Rightarrow n=9$

Question ID : 7155054316

6. The sum, of the coefficients of the first 50 terms in the binomial expansion of $(1-x)^{100}$, is equal to :

$(1-x)^{100}$ के द्विपद प्रसार में प्रथम 50 पदों के गुणांकों का योग बराबर है :

(1) ${}^{99}C_{49}$ (2) $-{}^{99}C_{49}$ (3) $-{}^{101}C_{50}$ (4) ${}^{101}C_{50}$

Ans. Official Answer NTA (2)

Sol. $(1-x)^{100} = Co - C_1x + C_2x^2 - C_3x^3 + \dots C_{99}x^{99} + C_{100}x^{100}$

$\Rightarrow Co - C_1 + C_2 - C_3 + \dots\dots - C_{99} + C_{100} = 0$

$2\left(Co - C_1 + C_2 + \dots\dots - C_9\right) + C_{50} = 0$

$C_0 - C_1 + C_2 + \dots . C_{99} = -\frac{1}{2}\,{}^{100}C_{50}$

$-\frac{1}{2}\frac{100!}{50!50!} = -\frac{1}{2}\times\frac{100\times 99!}{50!50!} = -{}^{99}C_{49}$

Question ID : 7155054331

7. Among the two statements

(S1) : $(p \Rightarrow q) \wedge (p \wedge (\sim q))$ is a contradiction and

(S2) : $(p \wedge q) \vee ((\sim p) \wedge q) \vee (p \wedge (\sim q)) \vee ((\sim p) \wedge (\sim q))$ is a tautology :

(1) only (S2) is true (2) both are true (3) only (S1) is true (4) both are false

दो कथनों

(S1) : $(p \Rightarrow q) \wedge (p \wedge (\sim q))$ एक विरोधोक्ति है तथा

(S2) : $(p \wedge q) \vee ((\sim p) \wedge q) \vee (p \wedge (\sim q)) \vee ((\sim p) \wedge (\sim q))$ एक पुनरूक्ति है, इनमें से

(1) केवल (S2) सत्य है (2) दोनों सत्य है (3) केवल (S1) सत्य है (4) दोनों असत्य है

Ans. Official Answer NTA (2)

Sol. $S_1 : (p \rightarrow q) \wedge (p \wedge (\sim q))$

P	Q	$p \rightarrow q$	$p \wedge (\sim q)$	S1
T	T	T	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	F	F

$\Rightarrow S_1$ is Contradiction

S_2

p	q	$p \wedge q$	$(p \wedge \sim q)$	$(p \wedge \sim q)$	$(\sim p) \wedge (\sim q)$	S_2
T	T	T	F	F	F	T
T	F	F	F	T	F	T
F	T	F	T	F	F	T
F	F	F	F	F	T	T

S_2 is tautology

Question ID : 7155054324

8. If the point $\left(\alpha, \frac{7\sqrt{3}}{3}\right)$ lies on the curve traced by the mid-points of the line segments of the lines $x\cos\theta + y\sin\theta = 7, \theta \in \left(0, \frac{\pi}{2}\right)$ between the co-ordinates axes, then α is equal to :

यदि रेखाओं $x\cos\theta + y\sin\theta = 7, \theta \in \left(0, \frac{\pi}{2}\right)$ के निर्देशांक अक्षों के बीच रेखाखंडों के मध्य बिंदुओं द्वारा बने वक्र पर एक बिंदु $\left(\alpha, \frac{7\sqrt{3}}{3}\right)$ है, तो α बराबर है :

(1) $7\sqrt{3}$ (2) $-7\sqrt{3}$ (3) -7 (4) 7

Ans. Official Answer NTA (4)

Sol. $x\cos\theta + y\sin\theta = 7; \quad \theta \in \left(0, \frac{\pi}{2}\right)$

Mid point of line segment between coordinate axes

$$(h, k) \equiv \left(\frac{7}{2\cos\theta}, \frac{7}{2\sin\theta}\right)$$

$$\Rightarrow \text{Locus of } (h, k) \text{ is } \frac{49}{y^2} + \frac{49}{x^2} = 4 \Rightarrow \frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{49}$$

Point $\left(\alpha, \frac{7\sqrt{3}}{3}\right)$ lies on it

$$\Rightarrow \frac{1}{\alpha^2}+\frac{3}{49}=\frac{4}{49}$$

$$\Rightarrow \frac{1}{\alpha^2}=\frac{1}{49}$$

$$\Rightarrow \alpha = 7$$

Question ID : 7155054312

9. Let D be the domain of the function $f(x)=\sin^{-1}\left(\log_{3x}\left(\frac{6+2\log_3 x}{-5x}\right)\right)$. If the range of the function $g : D \to \mathbb{R}$ defined by $g(x)=x-[x]$ ([x] is the greatest integer function), is (α, β), then $\alpha^2+\frac{5}{\beta}$ is equal to :

माना फलन $f(x)=\sin^{-1}\left(\log_{3x}\left(\frac{6+2\log_3 x}{-5x}\right)\right)$ का प्रॉंत D है। यदि $g(x)=x-[x]$, ([x] महत्तम पूर्णांक फलन है), द्वारा परिभाषित फलन $g : D \to \mathbb{R}$ का परिसर (α, β) है, तो $\alpha^2+\frac{5}{\beta}$ बराबर है :

(1) 46 (2) 136 (3) 45 (4) 135

Ans. Official Answer NTA ()

Sol. $\Rightarrow$ From (2), $x > 0$, from (3), $x \neq \frac{1}{3}$

From (4), $6+2\log_3 x < 0 \quad (\because x > 0)$

$\log_3 x < -3n$

$\Rightarrow x < 3^{-3}n$

$\Rightarrow \quad x < \frac{1}{27}$

$\Rightarrow$ From (2), (3), (4), $0 < x < \frac{1}{27}$ .....(5)

From (1), $-1 \le \log_{3x}\left(\frac{6+2\log_3 x}{-5x}\right) \le 1$

$\because \quad 3x \in \left(0, \frac{1}{9}\right)$

$$\Rightarrow \frac{1}{3x} \ge \frac{6+2\log_3 x}{-5x} \ge 3x$$

$$\Rightarrow -\frac{5}{3} \le 6 + 2\log_3 x \le -15x^2$$

$$\Rightarrow -\frac{23}{6} \le \log_3 x \le \frac{-15x^2 - 6}{2}$$

$$\Rightarrow 3^{-\frac{23}{6}} \le x \le 3^{\frac{-15x^2-6}{2}}$$

$$\Rightarrow 3^{\frac{1}{23}} \le x \le \frac{1}{3^{\frac{15x^2+6}{2}}}$$

From (5) & (6), $0 < x < \frac{1}{27}, [x] = 0$

$\Rightarrow g(x) = x$

$\Rightarrow$ Range g(x) is domain of f(x)

Question ID : 7155054318

10. Let C be the circle in the complex plane with centre $z_0 = \frac{1}{2}(1+3i)$ and radius r = 1. Let $z_1 = 1 + i$ and the complex number z_2 be outside the circle C such that $|z_1 - z_0||z_2 - z_0| = 1$. If z_0, z_1 and z_2 are collinear, then the smaller value of $|z_2|^2$ is equal to :

माना सम्मिश्र तल में वृत C का केन्द्र $z_0 = \frac{1}{2}(1+3i)$ तथा त्रिज्या r = 1 हैं। माना $z_1 = 1 + i$ है तथा वृत्त के बाहर सम्मिश्र संख्या z_2 इस प्रकार है कि $|z_1 - z_0||z_2 - z_0| = 1$ है। यदि z_0, z_1 तथा z_2 संरेख हैं, तो $|z_2|^2$ का छोटा मान बराबर है :

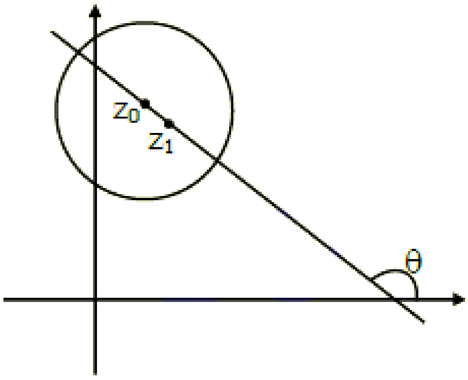
(1) $\frac{7}{2}$ (2) $\frac{13}{2}$ (3) $\frac{3}{2}$ (4) $\frac{5}{2}$

Ans. Official Answer NTA (4)

Sol. $|z_1 - z_0| = \left|\frac{1-i}{2}\right| = \frac{1}{\sqrt{2}}$

$\Rightarrow |z_2 - z_\circ| = \sqrt{2}$; centre $\left(\frac{1}{2}, \frac{3}{2}\right)$

$z_\circ\left(\frac{1}{2}, \frac{3}{2}\right)$ and $z_1(1,1)$



$\tan\theta = -1 \Rightarrow \theta = 135^{\circ}$

$z_2\left(\frac{1}{2}+\sqrt{2}\cos135^{\circ}, \frac{3}{2}+\sqrt{2}\sin135^{\circ}\right)$

or

$\left(\frac{1}{2}-\sqrt{2}\cos135^{\circ}, \frac{3}{2}-\sqrt{2}\sin135^{\circ}\right)$

$\Rightarrow z_2\left(-\frac{1}{2}, \frac{5}{2}\right)$ or $z_2\left(\frac{3}{2}, \frac{1}{2}\right)$

$\Rightarrow |z_2|^2 = \frac{26}{4}, \frac{5}{2}$

$\Rightarrow |z_2|_{\min^2} = \frac{5}{2}$

Question ID : 7155054326

11. Let the lines $l_1 : \frac{x+5}{3} = \frac{y+4}{1} = \frac{z-\alpha}{-2}$ and $l_2 : 3x+2y+z-2=0=x-3y+2z-13$ be coplanar. If the point P(a, b, c) on l_1 is nearest to the point Q(–4, –3, 2), then $|a|+|b|+|c|$ is equal to :

माना रेखाएँ $l_1 : \frac{x+5}{3} = \frac{y+4}{1} = \frac{z-\alpha}{-2}$ तथा $l_2 : 3x+2y+z-2=0=x-3y+2z-13$ सहतलीय हैं। यदि l_1 पर बिंदु P(a, b, c), बिंदु Q(–4, –3, 2) के निकटतम है, तो $|a|+|b|+|c|$ बराबर है :

(1) 8 (2) 10 (3) 14 (4) 12

Ans. Official Answer NTA (2)

Sol. $(3x + 2y + z - 2) + \mu(x - 3y + 2z - 13) = 0$

$3(3 + \mu) + 1.(2 - 3\mu) - 2(1+2\mu) = 0$

$9 - 4\mu = 0$

$\mu = \frac{9}{4}$

$4(-15 - 8 + \alpha - 2) + 9(-5 + 12 + 2\alpha - 13) = 0$

$-100 + 4\alpha - 54 + 18\alpha = 0$

$\Rightarrow \alpha = 7$

Let $P \equiv (3\lambda - 5, \lambda - 4, -2\lambda + 7)$

Directrion ratio of PQ $(3\lambda - 1, \lambda - 1, -2\lambda + 5)$

But $PQ \perp l_1$

$\Rightarrow 3(3\lambda - 1) + 1.(\lambda - 1) - 2(-2\lambda + 5) = 0$

$\Rightarrow \lambda = 1$

$P(-2, -3, 5) \Rightarrow |a| + |b| + |c| = 10$

Question ID : 7155054322

12. Let $y = y(x), y > 0$, be a solution curve of the differential equation $(1 + x^2)dy = y(x - y)dx$. If $y(0) = 1$ and $y(2\sqrt{2}) = \beta$, then :

माना अवकल समीकरण $(1 + x^2)dy = y(x - y)dx$ का एक हल वक्र $y = y(x), y > 0$ है। यदि $y(0) = 1$ तथा $y(2\sqrt{2}) = \beta$ हैं, तो :

(1) $e^{\beta^{-1}} = e^{-2}(3 + 2\sqrt{2})$
(2) $e^{3\beta^{-1}} = e(3 + 2\sqrt{2})$
(3) $e^{3\beta^{-1}} = e(5 + \sqrt{2})$
(4) $e^{\beta^{-1}} = e^{-2}(5 + \sqrt{2})$

Ans. Official Answer NTA (2)

Sol. $y(x - y)dx = (1 + x^2)dy$

$$\frac{y(x - y)}{1 + x^2} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{x}{1 + x^2}y - \frac{y^2}{1 + x^2}$$

$$\frac{dy}{dx} - \frac{x}{1 + x^2}y = \frac{-y^2}{1 + x^2}$$

$$\frac{-1}{y^2}\frac{dy}{dx} + \frac{x}{1 + x^2}\frac{1}{y} = \frac{1}{1 + x^2}$$

Let $\frac{1}{y}=t$

$\therefore \frac{-1}{y^2}\frac{dy}{dx}=\frac{dt}{dx}$

$\therefore \frac{dt}{dx}+\frac{x}{1+x^2}t=\frac{1}{1+x^2}$ linear in t

I.F. $=e^{\int\frac{x}{1+x^2}dx}=e^{\frac{1}{2}\ln\left(1+x^2\right)}=\sqrt{1+x^2}$

$\therefore$ solution is

$$t\sqrt{1+x^2}=\int\frac{\sqrt{1+x^2}}{1+x^2}dx+c$$

$$\frac{1}{y}\sqrt{1+x^2}=\int\frac{1}{\sqrt{1+x^2}}dx+c$$

$$\frac{1}{y}\sqrt{1+x^2}=\ln\left(x+\sqrt{x^2+1}\right)+c$$

$y(0)=1 \quad \Rightarrow \quad 1=0+c \quad \Rightarrow \quad c=1$

So at $x=2\sqrt{2}$ we have $\frac{3}{\beta}=\ln(2\sqrt{2}+3)+1 \Rightarrow 3\beta^{-1}=\ln e(2\sqrt{2}+3)$

$\Rightarrow e^{3\beta^{-1}}=e(3+2\sqrt{2})$

Question ID : 7155054314

13. Let $A=\begin{bmatrix}1 & \frac{1}{51}\\ 0 & 1\end{bmatrix}$. If $B=\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}A\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}$, then the sum of all the elements of the matrix $\sum_{n=1}^{50}B^n$ is equal to :

माना $A=\begin{bmatrix}1 & \frac{1}{51}\\ 0 & 1\end{bmatrix}$ है। यदि $B=\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}A\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}$ है, तो $\sum_{n=1}^{50}B^n$ के सभी अवयवों का योग है :

(1) 75 (2) 100 (3) 50 (4) 125

Ans. Official Answer NTA (2)

Sol. $$\because \overset{(M)}{\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}} \overset{(N)}{\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \quad MN = I = NM$$

$$B = MAN$$

$$B^n = (MAN)^n = (MAN)(MAN)\ldots$$

$$= \left(MA^2N\right)MAN$$

$$= MA^nN$$

$$\text{Now } A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{51} \\ 0 & 0 \end{bmatrix} = I + E$$

$$E^2 = 0$$

$$\therefore A^n = (I+E)^n = I + nE + \underbrace{{}^nC_2E^2}_{0}$$

$$= I + nE$$

$$= \begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}$$

$$\text{So, } B^n = MA^nN = \begin{bmatrix} 1+\frac{n}{51} & \frac{n}{51} \\ \frac{-n}{51} & 1-\frac{n}{51} \end{bmatrix}$$

$$\sum_{n=1}^{50} B^n = \begin{bmatrix} 50+\frac{50\cdot 51}{2\cdot 51} & \frac{50\cdot 51}{2\cdot 51} \\ \frac{-50\cdot 51}{2\cdot 51} & 50-\frac{50\cdot 51}{2\cdot 51} \end{bmatrix} = \begin{bmatrix} 75 & 25 \\ -25 & 25 \end{bmatrix}$$

$$\therefore \text{ sum} = 100$$

Question ID : 7155054323

14. Let $P\left(\frac{2\sqrt{3}}{\sqrt{7}}, \frac{6}{\sqrt{7}}\right)$, Q, R and S be four points on the ellipse $9x^2 + 4y^2 = 36$. Let PQ and RS be mutually

perpendicular and pass through the origin. If $\frac{1}{(PQ)^2}+\frac{1}{(RS)^2}=\frac{p}{q}$, where p and q are coprime, then $p+q$ is equal to :

माना दीर्घवृत $9x^2 + 4y^2 = 36$ पर चार बिंदु $P\left(\frac{2\sqrt{3}}{\sqrt{7}},\frac{6}{\sqrt{7}}\right)$, Q, R तथा S हैं। माना रेखाखंड PQ तथा RS परस्पर लंबवत है तथा मूलबिंदु से होकर जाते हैं। यदि $\frac{1}{(PQ)^2}+\frac{1}{(RS)^2}=\frac{p}{q}$, जहाँ p तथा q सहअभाज्य हैं, तो p + q बराबर है :

(1) 143 (2) 147 (3) 157 (4) 137

Ans. Official Answer NTA (3)

Sol. Let $R(2\cos\theta, 3\sin\theta)$

as $OP \perp OR$

so $\frac{3\sin\theta}{2\cos\theta}\times\frac{\frac{6}{\sqrt{7}}}{\frac{2\sqrt{3}}{\sqrt{7}}}=-1$

$\Rightarrow \tan\theta=\frac{-2}{3\sqrt{3}}$

$\Rightarrow R\left(\frac{-6\sqrt{3}}{\sqrt{31}},\frac{6}{\sqrt{31}}\right)$ or $R\left(\frac{6\sqrt{3}}{\sqrt{31}},\frac{-6}{\sqrt{31}}\right)$

Now $=\frac{1}{(PQ)^2}+\frac{1}{(RS)^2}=\frac{1}{4}\left(\frac{1}{(OP)^2}+\frac{1}{(OR)^2}\right)$

$=\frac{1}{4}\left(\frac{1}{48}+\frac{1}{\frac{144}{31}}\right)=\frac{1}{4}\left(\frac{7}{48}+\frac{31}{144}\right)$

$=\frac{13}{144}$

$\Rightarrow p+q=157$

Question ID : 7155054325

15. Let the plane P: $4x - y + z = 10$ be rotated by an angle $\frac{\pi}{2}$ about its line of intersection with the plane $x + y - z = 4$. If α is the distance of the point $(2, 3, -4)$ from the new position of the plane P, then 35α is equal to :

माना समतल P : $4x - y + z = 10$ को इसकी, समतल $x + y - z = 4$ से प्रतिच्छेदन रेखा के सापेक्ष $\frac{\pi}{2}$ के कोण तक घुमाया जाता है। यदि बिंदु $(2, 3, -4)$ की समतल P की नई स्थिति दूरी α है, तो 35α बराबर है :

(1) 105 (2) 85 (3) 90 (4) 126

Ans. Official Answer NTA (4)

Sol. Let equation in new position is

$(4x - y + z - 10) + \lambda(x + y - z - 4) = 0$

$4(4 + \lambda) - 1.(-1 + \lambda) + 1.(1 - \lambda) = 0$

$\Rightarrow \lambda = -9$

So equation in new position is

$-5x - 10y + 10z + 26 = 0$

$$\Rightarrow \alpha = \frac{54}{15}$$

Question ID : 7155054319

16. Let $<a_n>$ be a sequence such that $a_1 + a_2 + \ldots + a_n = \frac{n^2 + 3n}{(n+1)(n+2)}$. If $28\sum_{k=1}^{10}\frac{1}{a_k} = p_1p_2p_3\ldots p_m$, where $p_1, p_2, \ldots\ldots.p_m$ are the first m prime numbers, then m equal to :

माना $<a_n>$ एक अनुक्रम है जिसके लिए $a_1 + a_2 + \ldots + a_n = \frac{n^2 + 3n}{(n+1)(n+2)}$ है। यदि $28\sum_{k=1}^{10}\frac{1}{a_k} = p_1p_2p_3\ldots p_m$ है, जहाँ $p_1, p_2, \ldots\ldots.p_m$ प्रथम m अभाज्य संख्या है तो m बराबर है :

(1) 5 (2) 7 (3) 6 (4) 8

Ans. Official Answer NTA (3)

Sol. $$a_n = \frac{n^2 + 3n}{(n+1)(n+2)} - \frac{(n-1)^2 + 3(n-1)}{n(n+1)}$$

$$a_n = \frac{n^3 + 3n^2 - (n+2)\left(n^2 + n - 2\right)}{n(n+1)(n+2)}$$

$$a_n = \frac{4}{n(n+1)(n+2)}$$

$$\sum_{k=1}^{10}\frac{1}{a_k} = \frac{1}{4}\sum_{k=1}^{10}(k(k+1)(k+2))$$

$$\sum_{k=1}^{10}\frac{1}{a_k} = \frac{1}{16}\sum_{k=1}^{10}(k(k+1)(k+2)(k+3)-(k-1)k(k+1)(k+2))$$

$$\sum_{k=1}^{10}\frac{1}{a_k} = \frac{1}{16}(10\cdot 11\cdot 12\cdot 13)$$

$$28\sum_{k=1}^{10}\frac{1}{a_k} = \frac{28}{16}\cdot 2\cdot 5\cdot 11\cdot 3\cdot 4\cdot 13 \quad \Rightarrow m = 6$$

Question ID : 7155054327

17. Let a, b, c be three distinct real numbers, none equal to one. If the vectors $a\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+c\hat{k}$ are coplanar, then $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$ is equal to :

माना a, b, c तीन भिन्न वास्तविक संख्याएँ है तथा इनमें से कोई भी एक के बराबर नहीं है। यदि सदिश $a\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+b\hat{j}+\hat{k}$ तथा $\hat{i}+\hat{j}+c\hat{k}$ समतलीय हैं, तो $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$ बराबर है :

(1) –2 (2) 1 (3) 2 (4) –1

Ans. Official Answer NTA (2)

Sol.
$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

$$\begin{vmatrix} a-1 & 1-b & 0 \\ 0 & b-1 & 1-c \\ 1 & 1 & c \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$
$R_2 \to R_2 - R_3$
$\Rightarrow (a-1)\,[c(b-1)-(1-c)] + 1\,[(1-b)\,(1-c)] = 0$
$\Rightarrow c(a-1)\,(b-1) - (a-1)\,(1-c) + (1-b)\,(1-c) = 0$

$$\Rightarrow \frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$$

Question ID : 7155054330

18. In a triangle ABC, if $\cos A+2\cos B+\cos C=2$ and the lengths of the sides opposite to the angles A and C are 3 and 7 respectively, then $\cos A-\cos C$ is equal to :

यदि एक त्रिभुज ABC में $\cos A+2\cos B+\cos C=2$ है तथा कोणों A तथा C के सम्मुख भुजाओं की लंबाई क्रमश: 3 तथा 7 है, तो $\cos A-\cos C$ बराबर है :

(1) $\frac{9}{7}$ (2) $\frac{5}{7}$ (3) $\frac{3}{7}$ (4) $\frac{10}{7}$

Ans. Official Answer NTA (4)

Sol. $\cos A+\cos C=2(1-\cos B)$

$$2\cos\frac{A+C}{2}\cos\frac{A-C}{2}=4\sin^2\frac{B}{2}$$

as $\cos\left(\frac{A+C}{2}\right)=\sin\frac{B}{2}$

so $\cos\frac{A-C}{2}=2\sin\frac{B}{2}$

$$2\cos\frac{B}{2}\cos\frac{A-C}{2}=4\sin\frac{B}{2}\cos\frac{B}{2}$$

$$2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right)=4\sin\frac{B}{2}\cos\frac{B}{2}$$

$\sin A+\sin C=2\sin B$

$a+c=2b\Rightarrow a=3, c=7, b=5$

$$\cos A-\cos C=\frac{b^2+c^2-a^2}{2bc}-\frac{a^2+b^2-c^2}{2ab}$$

$$=\frac{25+49-9}{70}-\frac{9+25-49}{30}$$

$$=\frac{65}{70}+\frac{1}{2}=\frac{20}{14}=\frac{10}{7}$$

Question ID : 7155054320

19. If the local maximum value of the function $f(x)=\left(\frac{\sqrt{3}\,e}{2\sin x}\right)^{\sin^2 x}$, $x\in\left(0,\frac{\pi}{2}\right)$, is $\frac{k}{e}$, then $\left(\frac{k}{e}\right)^8+\frac{k^8}{e^5}+k^8$ is equal to :

यदि फलन $f(x)=\left(\frac{\sqrt{3}\,e}{2\sin x}\right)^{\sin^2 x}$, $x\in\left(0,\frac{\pi}{2}\right)$, का स्थानीय उच्चतम मान $\frac{k}{e}$ है, तो $\left(\frac{k}{e}\right)^8+\frac{k^8}{e^5}+k^8$ बराबर है :

(1) $e^5+e^6+e^{11}$ (2) $e^3+e^6+e^{10}$ (3) $e^3+e^5+e^{11}$ (4) $e^3+e^6+e^{11}$

Ans. Official Answer NTA (1)

Sol. Let $y=\left(\frac{\sqrt{3}e}{2\sin x}\right)^{\sin^2 x}$

$$\ln y=\sin^2 x\cdot\ln\left(\frac{\sqrt{3}e}{2\sin x}\right)$$

$$\frac{1}{y}y'=\ln\left(\frac{\sqrt{3}e}{2\sin x}\right)2\sin x\cos x+\sin^2 x\frac{2\sin x}{\sqrt{3}e}\frac{\sqrt{3}e}{2}(-\operatorname{cosec}x\cot x)$$

$$\Rightarrow \sin x\cos\left[2\ln\left(\frac{\sqrt{3}e}{2\sin x}\right)-1\right]=0$$

$$\Rightarrow \ln\left(\frac{3e}{4\sin^2 x}\right)=1\Rightarrow\frac{3e}{4\sin^2 x}=e\Rightarrow\sin^2 x=\frac{3}{4}$$

$$\Rightarrow \sin x=\frac{\sqrt{3}}{2}\left(\text{as } x\in\left(0,\frac{\pi}{2}\right)\right)$$

$$\Rightarrow \text{local max value}=\left(\frac{\sqrt{3}e}{\sqrt{3}}\right)^{3/4}=e^{3/8}=\frac{k}{e}$$

$$\Rightarrow k^8=e^{11}$$

$$\Rightarrow \left(\frac{k}{e}\right)^8+\frac{k^8}{e^5}+k^8=e^3+e^6+e^{11}$$

Question ID : 7155054321

20. The area of the region enclosed by the curve $y = x^3$ and its tangent at the point $(-1, -1)$ is :

वक्र $y = x^3$ तथा इसके बिंदु $(-1, -1)$ पर स्पर्श रेखा से घिरे क्षेत्र का क्षेत्रफल है :

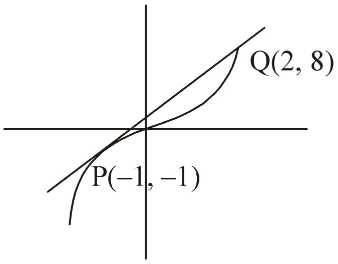
(1) $\frac{23}{4}$ (2) $\frac{19}{4}$ (3) $\frac{27}{4}$ (4) $\frac{31}{4}$

Ans. Official Answer NTA (3)

Sol. Given $y = x^3$ __________(1)

$$\Rightarrow \frac{dy}{dx} = 3x^2$$

$$\left(\frac{dy}{dx}\right)_{(-1,-1)} = 3$$



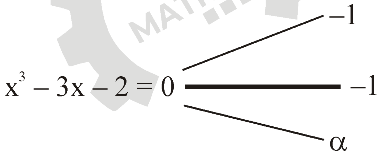
Equation of tangent at $(-1, -1)$

$(y + 1) = 3(x + 1)$

$y = 3x + 2$ __________(2)

Solving (1) and (2)

$x^3 = 3x + 2$



$-2 + \alpha = 0$

$\alpha = 2$

$Q \equiv (2, 8)$

$$\text{Required area} = \int_{-1}^{2}\left(3x + 2 - x^3\right)dx$$

$$= \frac{3}{2}(4-1) + 2(2+1) - \frac{1}{4}(16-1)$$

$$= \frac{9}{2} + 6 - \frac{15}{4} = \frac{18+24-15}{4} = \frac{27}{4}$$

SECTION – B

Question ID : 7155054337

21. If $\int_{-0.15}^{0.15} \left|100x^2 - 1\right| dx = \frac{k}{3000}$, then k is equal to ________.

यदि $\int_{-0.15}^{0.15} \left|100x^2 - 1\right| dx = \frac{k}{3000}$ है, तो k बराबर है ________

Ans. Official Answer NTA (575)

Sol. $\int_{-0.15}^{0.15} \left|100x^2 - 1\right| dx = 2\int_{0}^{0.15} \left|100x^2 - 1\right| dx$

$$= 2\int_{0}^{0.1} \left(-100x^2 + 1\right) dx + 2\int_{0.1}^{0.15} \left(100x^2 - 1\right) dx$$

$$= 2 \cdot \left(\frac{-100x^3}{3} + x\right)_0^{0.1} + 2\left(\frac{100x^3}{3} - x\right)_{0.1}^{0.15}$$

$$= 4\left(-\frac{1}{30} + \frac{1}{10}\right) + 2\left(\frac{9}{80} - \frac{3}{20}\right)$$

$$= \frac{8}{30} - \frac{3}{40} = \frac{575}{3000} \Rightarrow k = 575$$

Question ID : 7155054332

22. The number of relations, on the set {1, 2, 3} containing (1, 2) and (2, 3), which are reflexive and transitive but not symmetric, is ________.

समुच्चय {1, 2, 3} पर संबंधों, जिनमें (1, 2) तथा (2, 3) हैं, तथा जो स्वतुल्य और संक्रामक हैं परन्तु सममित नहीं हैं, की संख्या है ________

Ans. Official Answer NTA (4)

Sol. A = {1,2, 3}

For Reflexive (1, 1) (2, 2), (3, 3) $\in$ R

For transitive : (1, 2) and (2, 3) $\in$ R $\Rightarrow$ (1, 3) $\in$ R

Not symmetric : (2, 1) and (3, 2) $\notin$ R

R_1 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}

R_2 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3) (2, 1)}

R_3 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3) (3, 2)}

Question ID : 7155054340

23. Let the plane $x + 3y - 2z + 6 = 0$ meet the co-ordinate axes at the points A, B, C. If the orthocenter of the triangle ABC is $\left(\alpha, \beta, \frac{6}{7}\right)$, then $98(\alpha + \beta)^2$ is equal to ________.

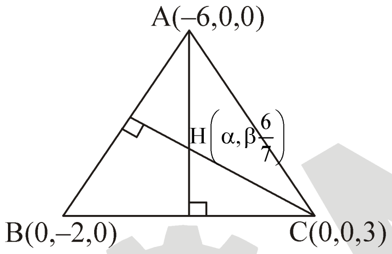
माना समतल $x + 3y - 2z + 6 = 0$ निर्देशांक अक्षों को बिंदुओं A, B, C पर मिलता है। यदि त्रिभुज ABC का लंबकेन्द्र $\left(\alpha, \beta, \frac{6}{7}\right)$ है, तो $98(\alpha + \beta)^2$ बराबर है ________

Ans. Official Answer NTA (288)

Sol. $A(-6, 0, 0)\ B(0, -2, 0)\ C = (0, 0, 3)$

$\overrightarrow{AB} = 6\hat{i} - 2\hat{j}, \overrightarrow{BC} = 2\hat{j} + 3\hat{k}$,

$\overrightarrow{AC} = 6\hat{i} + 3\hat{k}$



$\overrightarrow{AH} \cdot \overrightarrow{BC} = 0$

$\left(\alpha + 6, \beta, \frac{6}{7}\right).(0, 2, 3) = 0$

$\beta = \frac{-9}{7}$

$\overrightarrow{CH} \cdot \overrightarrow{AB} = 0$

$\left(\alpha, \beta, \frac{-15}{7}\right).(6, -2, 0) = 0$

$6\alpha - 2\beta = 0$

$\alpha = \frac{-3}{7}$

$98(\alpha + \beta)^2 = (98)\frac{(144)}{49} = 288$

Question ID : 7155054338

24. Let $I(x)=\int\sqrt{\frac{x+7}{x}}dx$ and $I(9)=12+7\log_e 7$. If $I(1)=\alpha+7\log_e(1+2\sqrt{2})$, α^4 is equal to ________.

माना $I(x)=\int\sqrt{\frac{x+7}{x}}dx$ तथा $I(9)=12+7\log_e 7$ हैं। यदि $I(1)=\alpha+7\log_e(1+2\sqrt{2})$ है, तो α^4 बराबर है ________

Ans. Official Answer NTA (64)

Sol. $I(x)=\int\sqrt{\frac{x+7}{x}}dx$

$$=\int\frac{\sqrt{\left(\sqrt{x}\right)^2+\left(\sqrt{7}\right)^2}}{\sqrt{x}}dx$$

Let $\sqrt{x}=t\ \therefore \frac{1}{2\sqrt{x}}dx=dt$

$$=2\int\sqrt{t^2+(\sqrt{7})^2}dt=2\left(\frac{t}{2}\sqrt{t^2+7}+\frac{7}{2}\ln\left(t+\sqrt{t^2+7}\right)\right)+c$$

$I(x)=\sqrt{x}\sqrt{x+7}+7\ln(\sqrt{x}+\sqrt{x+7})+c$

$I(9)=12+7\ln 7=12+7\ln(3+4)+c$

$\Rightarrow c=0$

$\therefore I(x)=\sqrt{x}\sqrt{x+7}+7\ln(\sqrt{x}+\sqrt{x+7})$

at $x=1$

$I(1)=\sqrt{8}+7\ell n(1+\sqrt{8})=7\ell n(1+2\sqrt{2})+\alpha$

$\alpha=\sqrt{8}$

$\alpha^4=64$

Question ID : 7155054336

25. Let [x] be the greatest integer $\le x$. Then the number of points in the interval (–2, 1), where the function $f(x)=|[x]|+\sqrt{x-[x]}$ is discontinuous, is ________.

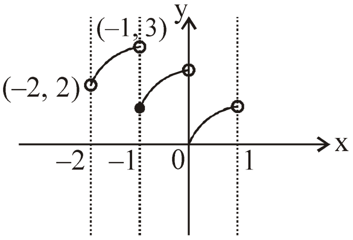
माना [x] महत्तम पूर्णांक $\le x$ है। तो अंतराल (–2, 1) में उन बिंदुओं, जहाँ फलन $f(x)=|[x]|+\sqrt{x-[x]}$ असंतत है, की संख्या है ________

Ans. Official Answer NTA (2)

Sol. $f(x) = \left|[x]\right| + \sqrt{x-[x]}$

$$x-[x] \geq 0 \Rightarrow x \in R$$

$$f(x) = \begin{cases} 2+\sqrt{x+2}, & -2 < x < -1 \\ 1+\sqrt{x+1}, & -1 \leq x < 0 \\ \sqrt{x}, & 0 \leq x < 1 \end{cases}$$



f(x) is discontinuous at two points x = {–1, 0}

Question ID : 7155054334

26. Let the digits a, b, c be in A. P. Nine-digit numbers are to be formed using each of these three digits thrice such that three consecutive digits are in A.P. at least once. How many such numbers can be formed? ________.

माना तीन अंक a, b, c, A. P. में हैं। इनमें से प्रत्येक अंक को तीन बार प्रयोग कर 9 अंकों की संख्याएँ इस प्रकार बनाई जाती हैं कितीन क्रमागत संख्याएँ कम से कम एक बार A.P. में हों। इस प्रकार की कितनी संख्याएँ बनाई जा सकती हैं ?

Ans. Official Answer NTA (1260)

Sol. abc or cba

$$\_\,\_\,\frac{a}{c}\,\frac{b}{b}\,\frac{c}{a}\,\_\,\_$$

$$\frac{{}^{7}C_{1} \times 2 \times 6!}{2!2!2!} = 1260$$

Question ID : 7155054333

27. Let $D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$. If $\sum_{k=1}^{n} D_k = 96$, then n is equal to ________.

माना $D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$ है। यदि $\sum_{k=1}^{n} D_k = 96$ तो n बराबर है ________

Ans. Official Answer NTA (6)

Sol. $D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$

$$\sum_{k=1}^{n} D_k = 96 \Rightarrow$$

$$\begin{vmatrix} \sum_{k=1}^{n} 1 & \sum 2k & \sum (2k-1) \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix} = 96$$

$$\Rightarrow \begin{vmatrix} n & n^2+n & n^2 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix} = 96$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\begin{vmatrix} n & n^2+n & n^2 \\ 0 & 2 & 0 \\ 0 & 0 & n+2 \end{vmatrix} = 96$$

$$\Rightarrow n(2n+4) = 96 \Rightarrow n(n+2) = 48 \Rightarrow n = 6$$

Question ID : 7155054339

28. Two circles in the first quadrant of radii r_1 and r_2 touch the coordinate axes. Each of them cuts off an intercept of 2 units with the line $x + y = 2$. Then $r_1^2 + r_2^2 - r_1 r_2$ is equal to ________.

प्रथम चतुर्थांश में r_1 तथा r_2 त्रिज्या के दो वृत्त निर्देशांक अक्षों को स्पर्श करते हैं। इनमें से प्रत्येक रेखा $x + y = 2$ से 2 इकाई का अंतःखंड काटता है। तो $r_1^2 + r_2^2 - r_1 r_2$ बराबर है ________

Ans. Official Answer NTA (7)

Sol. Let equation of circle is

$(x-r)^2 + (y-r)^2 = r^2$

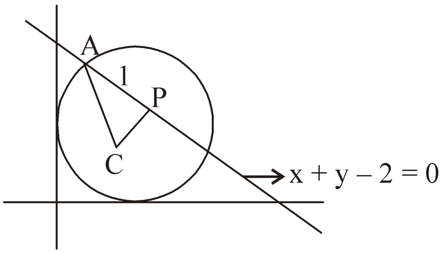
$$CP = \frac{|r+r-2|}{\sqrt{2}} = \sqrt{r^2-1}$$

$$\left(\sqrt{2}(r-1)\right)^2 = r^2-1$$

$$r^2-4r+3=0$$

$$r_1=1, r_2=3$$

$$r_1^2+r_2^2-r_1r_2=1+9-3=7$$



Question ID : 7155054335

29. Let the positive numbers a_1, a_2, a_3, a_4 and a_5 be in a G.P. Let their mean and variance be $\frac{31}{10}$ and $\frac{m}{n}$ respectively, where m and n are co-prime. If the mean of their reciprocals is $\frac{31}{40}$ and $a_3+a_4+a_5=14$, then m + n is equal to ________.

माना धनात्मक संख्याएँ a_1, a_2, a_3, a_4 तथा a_5 एक G.P. में हैं। माना इनके माध्य तथा प्रसरण क्रमश: $\frac{31}{10}$ तथा $\frac{m}{n}$ हैं, जहाँ m तथा n सहअभाज्य हैं। यदि इन संख्याओं के व्युत्क्रमों का माध्य $\frac{31}{40}$ है तथा $a_3 + a_4 + a_5 = 14$ है, तो m + n बराबर है ________

Ans. Official Answer NTA (211)

Sol. Let $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

Given $\frac{a}{r^2}+\frac{a}{r}+a+ar+ar^2=5\times\frac{31}{10}$ __________(1)

and $\frac{r^2}{a}+\frac{r}{a}+\frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2}=5\times\frac{31}{40}$ __________(2)

$(1)\div(2)\, a^2=4 \Rightarrow a=2$

$\therefore r+\frac{1}{r}=\frac{5}{2}(a\neq-2)$

$\Rightarrow r=2$

$\therefore$ Now $\frac{1}{2},1,2.4,8$

$$\therefore \sigma^2 = \frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2$$

$$= \frac{186}{25} = \frac{M}{N} \Rightarrow 211 = m + n$$

Question ID : 7155054341

30. A fair n(n > 1) faces die is rolled repeatedly until a number less than n appears. If the mean of the number of tosses required is $\frac{n}{9}$, then n is equal to ________.

एक न्याय n(n > 1) फलकों के पासे को बार बार फेंका जाता है जब तक कि n से कम संख्या प्राप्त न हो जाए। यदि पासे को फेंकने की आवश्यक संख्याओं का माध्य $\frac{n}{9}$ है, तो n बराबर है ________

Ans. Official Answer NTA (10)

Sol. Let in r^{th} thrown, L^{st} number appears which is smaller than n.(where $r \geq 1$)

$$\text{hence mean} = \sum_{r=1}^{\infty} r.\left(\frac{n-1}{n}\right)\left(\frac{1}{n}\right)^{r-1} = \frac{n-1}{n}\sum_{r=1}^{\infty} r.\left(\frac{1}{n}\right)^{r-1}$$

$$= \frac{n-1}{n}\left[1 + 2x + 3x^2 + 4x^3 + ...\right] \text{ where } x = \frac{1}{n}$$

$$= \frac{n-1}{n}\frac{1}{(1-x)^2} = \frac{n-1}{n}\left(\frac{n^2}{(n-1)^2}\right) = \frac{n}{n-1}$$

$$\text{But given mean} = \frac{n}{9} \quad \Rightarrow n - 1 = 9$$

$$\Rightarrow n = 10$$