MATHS
12 Jan. 2019 [Session : 9.30 AM to 12.00 PM]
JEE MAIN PAPER ONLINE

1. If the vertices of a hyperbola be at (–2, 0) and (2,0) and one of its foci be at (–3,0), then which one of the following points does not lie on this hyperbola :

यदि एक अतिपरवलय के शीर्ष (–2, 0) तथा (2,0) पर हैं तथा इसकी एक नाभि (–3,0) पर है, तो निम्न में से कौन सा बिन्दु इस अतिपरवलय पर स्थित नहीं है ?

(1) $\left(-6, 2\sqrt{10}\right)$ (2) $\left(2\sqrt{6}, 5\right)$ (3) $\left(6, 5\sqrt{2}\right)$ (4) $\left(4, \sqrt{15}\right)$

A. 3

Question ID : 4165299767

Option 1 ID : 41652938529
Option 2 ID : 41652938527
Option 3 ID : 41652938528
Option 4 ID : 41652938526

sol. Vertex of hyperbola (–2, 0) and (2,0)

So a = 2

–ae = –3

e = 3/2

$b^2 = a^2e^2 - a^2$

$b^2 = 5$

Equation of hyperbola

$$\frac{x^2}{4} - \frac{y^2}{5} = 1$$

2. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ and $Q = [q_{ij}]$ be two 3×3 matrices such that $Q - P^5 = I_3$. Then $\frac{q_{21} + q_{31}}{q_{32}}$ is equal to :

यदि $P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ तथा $Q = [q_{ij}]$ दो ऐसे 3×3 आव्यूह हैं, कि $Q - P^5 = I_3$ है, तो $\frac{q_{21} + q_{31}}{q_{32}}$ बराबर है –

(1) 10 (2) 9 (3) 15 (4) 135

A. 1

Question ID : 4165299749

Option 1 ID : 41652938455
Option 2 ID : 41652938454
Option 3 ID : 41652938456
Option 4 ID : 41652938457

sol. $P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$

MATRIX

$$P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 3+3 & 1 & 0 \\ 9+3\times3+1 & 3+3 & 1 \end{bmatrix}$$

$$P^5 = \begin{bmatrix} 1 & 0 & 0 \\ 15 & 1 & 0 \\ 135 & 15 & 1 \end{bmatrix}$$

$Q - P^5 = I_3$

$Q = I_3 + P^5$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{bmatrix}$$

$$\frac{q_{21} + q_{31}}{q_{32}} \Rightarrow 10$$

3. Let $S_k = \dfrac{1+2+3+.....+k}{k}$. If $S_1^2 + S_2^2 + ...+ S_{10}^2 = \dfrac{5}{12}A$, then A is equal to :

माना $S_k = \dfrac{1+2+3+.....+k}{k}$ है। यदि $S_1^2 + S_2^2 + ...+ S_{10}^2 = \dfrac{5}{12}A$ है, तो A बराबर है –

(1) 283 (2) 301 (3) 156 (4) 303

A. 4

Question ID : 4165299754

Option 1 ID : 41652938475

Option 2 ID : 41652938476

Option 3 ID : 41652938474

Option 4 ID : 41652938477

sol. $S_k = \dfrac{1+2+3+.....+k}{k}$

$$S_1^2 + S_2^2 + ...+ S_{10}^2 = \frac{5}{12}A$$

$$S_1 = 1^2 \qquad S^2{}_2 = \frac{(1+2)^2}{2^2} \qquad S_3 = \frac{(1+2+3)^2}{3^2}$$

$$\sum S_k^2 = \sum \frac{k^2(k+1)^2}{4k^2} = \sum_{k=1}^{10} \frac{(k+1)^2}{4}$$

$$= \frac{1}{4}\left[\frac{11(12)^{23}}{6} - 1\right] = \frac{5\times303}{12}$$

$A = 303$

4. Let P (4,–4) and Q(9,6) be two points on the parabola, $y^2 = 4x$ and let X be any point on the arc POQ of this parabola, where O is the vertex of this parabola, such that the area of Δ PXQ is maximum. Then this maximum area (in sq. units) is :

माना P (4,–4) तथा Q(9,6) परवलय $y^2 = 4x$ पर स्थित दो बिन्दु है। O इस परवलय का शीर्ष बिन्दु है तथा X इस परवलय की चाप POQ का कोई ऐसा बिन्दु है, जिसके लिए Δ PXQ का क्षेत्रफल अधिकतम है, तो यह अधिकतम क्षेत्रफल (वर्ग इकाइयों में) है–

(1) $\frac{125}{4}$ (2) $\frac{125}{2}$ (3) $\frac{625}{4}$ (4) $\frac{75}{2}$

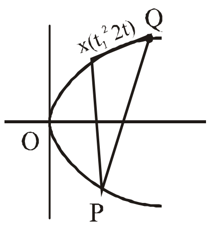
A. 1

Question ID : 4165299766

Option 1 ID : 41652938524 Option 2 ID : 41652938522

Option 3 ID : 41652938523 Option 4 ID : 41652938525

sol. P (4,–4) and Q(9,6)



Area of triangle PXQ

$$\frac{1}{2}\begin{vmatrix} 4 & -4 & 1 \\ 9 & 6 & 1 \\ t^2 & 2t & 1 \end{vmatrix}$$

$$\text{Area } A = \frac{1}{2}\left(60 + 10t - 10t^2\right)$$

$$\frac{dA}{dt} = 10 - 20t = 0$$

t = ½

$$\frac{d^2A}{dt^2} = -\text{ve (so maximum)}$$

$$A = \frac{1}{2}\left(60 + 5 - \frac{10}{4}\right)$$

$$= 2508 = \frac{125}{4}$$

5. If the sum of the deviations of 50 observations from 30 is 50, then the mean of these observations is :

यदि 50 प्रेक्षणों के 30 से विचलनों का योग 50 है, तो इन प्रेक्षणों का माध्य है –

(1) 31 (2) 30 (3) 50 (4) 51

A. 1

Question ID : 4165299771

Option 1 ID : 41652938545 Option 2 ID : 41652938544
Option 3 ID : 41652938542 Option 4 ID : 41652938543

sol. Given $\sum_{i=1}^{50}(x_i - 30) = 50$

$\sum x_i = 50 + 1500$

$\sum x_i = 1550$

Mean $\frac{\sum x_i}{50} = \frac{1550}{50} = 31$

6. The maximum value of $3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$ for any real value of θ is :

$3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$ का θ के किसी भी वास्तविक मान के लिए अधिकतम मान है –

(1) $\sqrt{34}$ (2) $\frac{\sqrt{79}}{2}$ (3) $\sqrt{31}$ (4) $\sqrt{19}$

A. 4

Question ID : 4165299773
Option 1 ID : 41652938553 Option 2 ID : 41652938550
Option 3 ID : 41652938552 Option 4 ID : 41652938551

sol. $3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$

$3\cos\theta + \frac{5\sqrt{3}}{2}\sin\theta - \frac{5}{2}\cos\theta$

$\frac{1}{2}\cos\theta + \frac{5\sqrt{3}}{2}\sin\theta$

Maximum value of this will be

$\sqrt{\frac{1}{4} + \frac{75}{4}} = \sqrt{\frac{76}{4}} = \sqrt{19}$

7. Let $S = \{1,2,3,......,100\}$. The number of non-empty subsets A of S such that the product of elements in A is even is :

माना $S = \{1,2,3,......,100\}$ तो S के उन सभी अरिक्त उपसमुच्चयों A जिनके अवयवों का गुणनफल सम है, की संख्या है –

(1) $2^{50}(2^{50} - 1)$ (2) $2^{50} - 1$ (3) $2^{100} - 1$ (4) $2^{50} + 1$

MATRIX

A. 1

Question ID : 4165299746

Option 1 ID : 41652938444 Option 2 ID : 41652938443

Option 3 ID : 41652938442 Option 4 ID : 41652938445

sol. Let S = {1,2,3,......,100}

Total subset = $2^{100} - 1$

non empty subset with product of element is even = Total Subsets - (Total Subsets when all elements are odd)

$= 2^{100} - 1 - (2^{50} - 1)$

$= 2^{100} - 2^{50}$

$= 2^{50}(2^{50} - 1)$

8. If the straight line, $2x - 3y + 17 = 0$ is perpendicular to the line passing through the points (7,17) and (15, β), then β equals :

यदि सरल रेखा $2x - 3y + 17 = 0$ बिन्दुओं (7,17) तथा (15, β) से होकर जाने वाली रेखा के लम्बवत् है, तो β बराबर है –

(1) 5 (2) –5 (3) $-\frac{35}{3}$ (4) $\frac{35}{3}$

A. 1

Question ID : 4165299763

Option 1 ID : 41652938512 Option 2 ID : 41652938510

Option 3 ID : 41652938513 Option 4 ID : 41652938511

sol. $2x - 3y + 17 = 0$

is perpendicular to line passing through (7, 17) and (15, β)

product of slope = –1

$$\frac{2}{3} \times \left(\frac{\beta - 17}{8}\right) = -1$$

$\beta - 17 = -12$

$\beta = 5$

9. Let S be the set of all points in $(-\pi, \pi)$ at which the function, f(x) = min {sin x, cos x} is not differentiable. Then S is a subset of which of the following :

माना S, अंतराल $(-\pi, \pi)$ के बीच में स्थित ऐसे सभी बिन्दुओं का समुच्चय है, जिन पर फलन, f(x) = min {sin x, cos x} अवकलनीय नहीं है, तो S निम्न में से किसका उपसमुच्चय है ?

(1) $\left\{-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}\right\}$ (2) $\left\{-\frac{\pi}{2}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}\right\}$

(3) $\left\{-\frac{3\pi}{4}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{4}\right\}$ (4) $\left\{-\frac{\pi}{4}, 0, \frac{\pi}{4}\right\}$

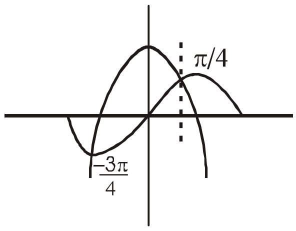
A. 1

Question ID : 4165299757

Option 1 ID : 41652938487 Option 2 ID : 41652938486

Option 3 ID : 41652938488 Option 4 ID : 41652938489

sol. f(x) = min {sin x, cos x} in $(-\pi, \pi)$



so function is not differentiable at

$\left(-\frac{3\pi}{4} \text{ and } \frac{\pi}{4}\right)$

10. If $\frac{z-\alpha}{z+\alpha}$ $(a \in R)$ is a purely imaginary number and $|z| = 2$, then a value of α is :

यदि $\frac{z-\alpha}{z+\alpha}$ $(a \in R)$ एक शुद्ध रूप से काल्पनिक संख्या है, तथा $|z| = 2$ है, तो α का एक मान है –

(1) $\frac{1}{2}$ (2) 1 (3) 2 (4) $\sqrt{2}$

A. 3

Question ID : 4165299747

Option 1 ID : 41652938446

Option 2 ID : 41652938449

Option 3 ID : 41652938447

Option 4 ID : 41652938448

sol. $\frac{z-\alpha}{z+\alpha}$ $|z| = 2$

$\frac{z-\alpha}{z+\alpha} + \frac{\bar{z}-\bar{\alpha}}{\bar{z}+\alpha} = 0$

$z\bar{z} + z\alpha - \alpha\bar{z} - \alpha^2 + z\bar{z} - z\alpha + \bar{z}\alpha - \alpha^2 = 0$

$z\bar{z} = \alpha^2$

$|z|^2 = \alpha^2 = 4$

$\alpha = \pm 2$

11. If a variable line, $3x + 4y - \lambda = 0$ is such that the two circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2 + y^2 - 18x - 2y + 78 = 0$ are on its opposite sides, then the set of all values of λ is the interval :

यदि एक चर रेखा $3x + 4y - \lambda = 0$ इस प्रकार है कि दो वृत $x^2 + y^2 - 2x - 2y + 1 = 0$ तथा $x^2 + y^2 - 18x - 2y + 78 = 0$ इसके दोनों ओर हैं, तो λ के सभी मानों का समुच्चय निम्न में से कौनसा अन्तराल है ?

(1) [12, 21] (2) (23,31) (3) [13,23] (4) (2,17)

A. 1

Question ID : 4165299764

Option 1 ID : 41652938517

Option 2 ID : 41652938514

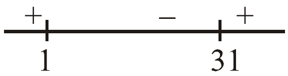
Option 3 ID : 41652938516 Option 4 ID : 41652938515

sol. $L : 3x + 4y - \lambda = 0$

$S_1\ (1, 1)$ $S_2\ (9, 1)$

$L(S_1)\ L(S_2) < 0$

$(7 - \lambda)\quad (31 - \lambda) < 0$



$\lambda \in (7, 31)$

$$\left|\frac{7-\lambda}{5}\right| > 1 \Rightarrow |7 - \lambda| > 5$$

$\lambda - 7 > 5 \quad \lambda > 12$

$$\left|\frac{31-\lambda}{5}\right| > 2 \Rightarrow |31 - \lambda| > 10$$

$31 - \lambda > 10$

$\lambda < 21$

$[12, 21]$

12. A ratio of the 5th term from the beginning to the 5th term from the end in the binomial expansion of $\left(2^{1/3} + \frac{1}{2(3)^{1/3}}\right)^{10}$ is :

$\left(2^{1/3} + \frac{1}{2(3)^{1/3}}\right)^{10}$ के द्विपद प्रसार में आरम्भ में 5 वें तथा अंत से (प्रथम की ओर) 5 वें पदों का एक अनुपात है –

(1) $1 : 2(6)^{\frac{1}{3}}$ (2) $1 : 4(16)^{\frac{1}{3}}$ (3) $2(36)^{\frac{1}{3}} : 1$ (4) $4(36)^{\frac{1}{3}} : 1$

A. 4

Question ID : 4165299752

Option 1 ID : 41652938466 Option 2 ID : 41652938469

Option 3 ID : 41652938467 Option 4 ID : 41652938468

sol. $$\left(2^{1/3} + \frac{1}{2(3)^{1/3}}\right)^{10}$$

$$T_{4+1} = {}^{10}C_4 \left(2^{1/3}\right)^6 \left(\frac{1}{2(3)^{1/3}}\right)^4$$

In reverse order

$$\frac{8}{r} + 8 + 8r$$

MATRIX

$$\frac{T_5}{T_{e^5}} = \frac{\left(2^{1/3}\right)^6}{\left(2.(3)^{1/3}\right)^4}\frac{\left(2.\left(3^{1/3}\right)\right)^6}{\left(2^{1/3}\right)^4}$$

$$= \left(2^{1/3}\right)^2.2^2.\left(3^{1/3}\right)^2$$

$$= \frac{4(6)^{1/3}}{1}$$

$$= \frac{4(6)^{2/3}}{1}$$

$$= \frac{4(36)^{1/3}}{1}$$

13. Let f and g be continuous functions on [0,a] such that f(x) = f(a – x) and g(x) + g(a – x) = 4, then $\int_0^a f(x)g(x)dx$ is equal to :

माना f तथा g, [0,a] पर ऐसे संतत फलन है कि f(x) = f(a – x) तथा g(x) + g(a – x) = 4 है, तो $\int_0^a f(x)g(x)dx$ बराबर है –

(1) $\int_0^a f(x)dx$ (2) $-3\int_0^a f(x)dx$ (3) $2\int_0^a f(x)dx$ (4) $4\int_0^a f(x)dx$

A. 3

Question ID : 4165299760

Option 1 ID : 41652938498

Option 2 ID : 41652938501

Option 3 ID : 41652938499

Option 4 ID : 41652938500

sol. f(x) = f(a – x)

g(x) + g(a – x) = 4

$$I = \int_0^a f(a-x)g(a-x)dx$$

$$= \int_0^a f(x)g(4-g(x))dx$$

$$I = 4\int_0^a f(x)dx - I$$

$$I = 2\int_0^a f(x)dx$$

14. An ordered pair (α,β) for which the system of linear equations :

$(1+\alpha)x + \beta y + z = 2$

$\alpha x + (1+\beta)y + z = 3$

$\alpha x + \beta y + 2z = 2$

has a unique solution, is :

एक ऐसा क्रमित युग्म (α,β) जिसके लिए रैखिक समीकरण निकाय

$(1+\alpha)x + \beta y + z = 2$

$\alpha x+(1+\beta)y+z=3$

$\alpha x+\beta y+2z=2$

का एकमात्र एक हल है, हैं –

(1) (–4, 2) (2) (2,4) (3) (–3,1) (4) (1,–3)

A. 2

Question ID : 4165299750

Option 1 ID : 41652938460

Option 2 ID : 41652938461

Option 3 ID : 41652938459

Option 4 ID : 41652938458

sol. For unique solution $\Delta \neq 0$

$$\begin{vmatrix} 1+\alpha & \beta & 1 \\ \alpha & 1+\beta & 1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0$$

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0$$

$1(2+\beta)+1(+\alpha)\neq 0$

$\alpha+\beta+2\neq 0$

15. The sum of the distinct real values of μ, for which the vectors, $\mu\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+\mu\hat{j}+\hat{k}$, $\hat{i}+\hat{j}+\mu\hat{k}$ are coplanar, is :

μ के उन भिन्न वास्तविक मानों का योग, जिनके लिए सदिश $\mu\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+\mu\hat{j}+\hat{k}$, $\hat{i}+\hat{j}+\mu\hat{k}$ समतलीय हैं, है –

(1) 0 (2) 1 (3) –1 (4) 2

A. 3

Question ID : 4165299770

Option 1 ID : 41652938538

Option 2 ID : 41652938540

Option 3 ID : 41652938539

Option 4 ID : 41652938541

sol. For coplanar

Determinant of coefficent should be zero

$$\begin{vmatrix} \mu & 1 & 1 \\ 1 & \mu & 1 \\ 1 & 1 & \mu \end{vmatrix} = 0$$

$$\begin{vmatrix} \mu-1 & 1-\mu & 0 \\ 0 & \mu-1 & 1-\mu \\ 1 & 1 & \mu \end{vmatrix} = 0$$

$(\mu-1)(\mu^2-1)+(\mu-1)(\mu-1)=0$

$(\mu-1)^2[\mu+1+1]=0$

$\mu=1, \mu=-2$

Sum of $\mu=-1$

16. Let $y = y(x)$ be the solution of the differential equation, $x\frac{dy}{dx} + y = x\log_e x, (x > 1)$. If $2y(2) = \log_e 4 - 1$, then y(e) is equal to :

माना $y = y(x)$ अवकल समीकरण $x\frac{dy}{dx} + y = x\log_e x, (x > 1)$ का हल है। यदि $2y(2) = \log_e 4 - 1$ है, तो y(e) बराबर है–

(1) $-\frac{e}{2}$ (2) $\frac{e^2}{4}$ (3) $-\frac{e^2}{2}$ (4) $\frac{e}{4}$

A. 4

Question ID : 4165299762

Option 1 ID : 41652938508

Option 2 ID : 41652938507

Option 3 ID : 41652938509

Option 4 ID : 41652938506

Sol. $x\frac{dy}{dx} + y = x\log x$

$$\frac{dy}{dx} + y.\frac{1}{x} = \log x$$

$$p = \frac{1}{x}$$

$$I.E = e^{\int \frac{1}{x}dx} = e^{\log x} = x$$

$$y(x)x = \int x\log x\,dx$$

$$y(x).x = \log x\frac{x^2}{2} - \int \frac{1}{x}.\frac{x^2}{2}$$

$$y(x) = \log x.\frac{x}{2} - \frac{x}{4} + \frac{c}{x}$$

$$2\log 2 - 1 + c = \log 4 - 1$$

$$c = 0$$

$$y(e) = \frac{e}{2} - \frac{e}{4} = \frac{e}{4}$$

17. The Boolean expression $((p \wedge q) \vee (p \vee \sim q)) \wedge (\sim p \wedge \sim q)$ is equivalent to :

बूलीय व्यंतक (Boolean expression) $((p \wedge q) \vee (p \vee \sim q)) \wedge (\sim p \wedge \sim q)$ निम्न में जिसके तुल्य है, वह है –

(1) $p \wedge (\sim q)$ (2) $(\sim p) \wedge (\sim q)$ (3) $p \vee (\sim q)$ (4) $p \wedge q$

A. 2

Question ID : 4165299775

Option 1 ID : 41652938558

Option 2 ID : 41652938560

Option 3 ID : 41652938559

Option 4 ID : 41652938561

sol. $((p \wedge q) \vee (p \vee \sim q)) \wedge (\sim p \wedge \sim q)$

p	q	$p \wedge q$	$p \vee (\sim q)$	$(p \wedge q) \vee (p \vee q) = A$	$(\sim p \wedge \sim q)$	Result
F	F	F	T	T	T	T

MATRIX

F	T	F	F	F	F	F
T	F	F	T	T	F	F
T	T	T	T	T	F	F

equivalent to $(\sim p)\wedge(\sim q)$

18. A tetrahedron has vertices P(1,2,1), Q(2,1,3), R(–1,1,2) and O(0,0,0). The angle between the faces OPQ and PQR is :

एक चतुष्फलक के शीर्ष P(1,2,1), Q(2,1,3), R(–1,1,2) तथा O(0,0,0) हैं। फलक OPQ तथा PQR के बीच का कोण है –

(1) $\cos^{-1}\left(\frac{17}{31}\right)$ (2) $\cos^{-1}\left(\frac{9}{35}\right)$ (3) $\cos^{-1}\left(\frac{19}{35}\right)$ (4) $\cos^{-1}\left(\frac{7}{31}\right)$

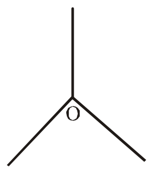
A. 3

Question ID : 4165299769

Option 1 ID : 41652938535 Option 2 ID : 41652938536

Option 3 ID : 41652938534 Option 4 ID : 41652938537

sol. P(1,2,1), Q(2,1,3), R(–1,1,2) and O(0,0,0)



OPQ PQR

$$\begin{array}{llll} OP & 1 & 2 & 1 \\ OQ & 2 & 1 & 3 \\ \hline & 5 & -1 & -3 \end{array} \qquad \begin{array}{llll} PQ & 1 & -1 & 2 \\ QR & -3 & 0 & -1 \\ \hline & 1 & -5 & -3 \end{array}$$

$$\text{Angle} = \cos^{-1}\left|\frac{5+5+9}{\sqrt{35}\sqrt{35}}\right|$$

$$\cos^{-1}\left(\frac{19}{35}\right)$$

19. The maximum area (in sq. units) of a rectangle having its base on the x-axis and its other two vertices on the parabola, $y = 12 - x^2$ such that the rectangle lies inside the parabola, is :

एक ऐसी आयत, जिसका आधार x-अक्ष पर है तथा अन्य दो शीर्ष परवलय $y = 12 - x^2$ पर इस प्रकार स्थित है कि आयत, परवलय के अन्त: भाग में है, का अधिकतम क्षेत्रफल (वर्ग इकाईयों में) है –

(1) $18\sqrt{3}$ (2) $20\sqrt{2}$ (3) 32 (4) 36

A. 3

Question ID : 4165299758

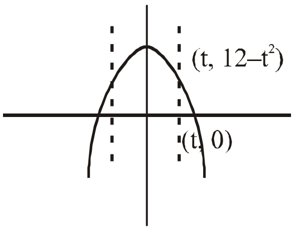
Option 1 ID : 41652938493 Option 2 ID : 41652938492

Option 3 ID : 41652938491 Option 4 ID : 41652938490

sol. $y = 12 - x^2$

$y - 12 = -x^2$

MATRIX



Area = 2t × (12 – t²)

A = 24t – 2t³

$\frac{dA}{dt} = 24 - 6t^2 = 0$

t² = 4

t = ±2

Area maximum at t = 2

= 4 (12–4)

= 32

20. The product of three consecutive terms of a G.P. is 512. If 4 is added to each of the first and the second of these terms, the three terms now form an A.P. Then the sum of the original three terms of the given G.P. is :

एक गुणोत्तर श्रेढ़ी के तीन क्रमागत पदों का गुणनफल 512 हैं यदि इसके पहले तथा दूसरे प्रत्येक पद में 4 जोड़ दें तो यह तीन संख्याएँ एक समांतर श्रेढ़ी बनाती है। तो दी हुई गुणोत्तर श्रेढ़ी के तीनों पदों का योग है –

(1) 24 (2) 28 (3) 36 (4) 32

A. 2

Question ID : 4165299753

Option 1 ID : 41652938473
Option 2 ID : 41652938472
Option 3 ID : 41652938470
Option 4 ID : 41652938471

sol. $\frac{a}{r}$ a ar

Product ⇒ a³ = 512

a = 8

$\frac{8}{r}+4,\quad 12,\quad 8r$

$\frac{8}{r}+4,+8r = 24$

$\text{Sum} = \frac{8}{r}+8+8r = 28$

21. If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2 + m(m-4)x + 2 = 0$, then the least value of m for which $\lambda + \frac{1}{\lambda} = 1$ is :

यदि x में द्विघात समीकरण $3m^2x^2 + m(m-4)x + 2 = 0$ के मूलों का अनुपात λ है, तो m का वह न्यूनतम मान जिसके लिए $\lambda + \frac{1}{\lambda} = 1$ है, है –

(1) $2-\sqrt{3}$ (2) $-2+\sqrt{2}$ (3) $4-3\sqrt{2}$ (4) $4-2\sqrt{3}$

A. 3

Question ID : 4165299748

Option 1 ID : 41652938452

Option 2 ID : 41652938453

Option 3 ID : 41652938450

Option 4 ID : 41652938451

sol. $3m^2x^2 + m(m-4)x + 2 = 0$

Ratio of roots $\Rightarrow \frac{\alpha}{\beta} = \lambda$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = 1$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = 1$

$= \left(\frac{m(m-4)}{3m^2}\right)^2 - \frac{4}{3m^2} = \frac{2}{3m^2}$

$= m^2(m-4)^2 - 6.3\,m^2 = 0$

$= (m-4)^2 - 18 = 0$

$m = 4 - 3\sqrt{2}$

22. Considering only the principal values of inverse functions, the set $A = \left\{x \geq 0 : \tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}\right\}$

(1) Contains two elements

(2) Is an empty set

(3) Is a singleton

(4) Contains more than two elements

प्रतिलोम फलनों के केवल मुख्य मान लेते हुए, समुच्चय $A = \left\{x \geq 0 : \tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}\right\}$

(1) में दो अवयव है

(2) एक रिक्त समुच्चय है

(3) एक एकल समुच्चय है

(4) दो से अधिक अवयव है

A. 3

MATRIX

Question ID : 4165299774

Option 1 ID : 41652938556 Option 2 ID : 41652938554

Option 3 ID : 41652938555 Option 4 ID : 41652938557

sol. $A = \left\{ x \geq 0 : \tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4} \right\}$

$\tan^{-1}\left(\frac{5x}{1-6x^2}\right) = \frac{\pi}{4}$

$\frac{5x}{1-6x^2} = 1$

$6x^2 + 5x - 1 = 0$

$6x^2 + 6x - x - 1 = 0$

$6x(x+1) - 1(x+1) = 0$

$x = 1/6$ or $x = -1$

but $x \geq 0$

$x = 1/6$ (singleton set)

23. In a random experiment, a fair die is rolled until two fours are obtained in succession. The probability that the experiment will end in the fifth throw of the die is equal to :

एक यादृच्छिक प्रयोग में, एक अनभिनत पासे को तब तक उछाला जाता है जब कि लगातार दो बार 4 न आए। तो इस प्रयोग के पाँचवी बार पासे के उछाल तक समाप्त होने की प्रायिकता है –

(1) $\frac{175}{6^5}$ (2) $\frac{200}{6^5}$ (3) $\frac{150}{6^5}$ (4) $\frac{225}{6^5}$

A. 1

Question ID : 4165299772

Option 1 ID : 41652938549 Option 2 ID : 41652938546

Option 3 ID : 41652938547 Option 4 ID : 41652938548

sol. $\underset{\not 4}{\sqcup}\ \underset{\not 4}{\sqcup}\ \overset{\not 4}{\underset{\not 4}{\sqcup}}\ \boxed{4}\ \boxed{4}$

$\boxed{}\ \boxed{4\ \ 4} \Rightarrow \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} \times \frac{1}{6}$

$\boxed{4}\boxed{}\boxed{}\ \boxed{4\ \ 4}\ 2 \times \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} \times \frac{1}{6}$

$\boxed{}\boxed{4}\boxed{}\ \boxed{4\ \ 4} = \frac{175}{6^5}$

24. The integral $\int \cos(\log_e x)dx$ is equal to :

(where C is a constant of integration)

समाकल $\int \cos(\log_e x)dx$ बराबर है –

(जहाँ C एक समाकलन अचर है)

(1) $x\left[\cos(\log_e x) + \sin(\log_e x)\right] + C$ (2) $\frac{x}{2}\left[\sin(\log_e x) - \cos(\log_e x)\right] + C$

(3) $\frac{x}{2}\left[\cos(\log_e x)+\sin(\log_e x)\right]+C$ (4) $x\left[\cos(\log_e x)-\sin(\log_e x)\right]+C$

A. 3

Question ID : 4165299759

Option 1 ID : 41652938494 Option 2 ID : 41652938496

Option 3 ID : 41652938497 Option 4 ID : 41652938495

sol. $\int \cos(\log_e x)dx$

$I=\cos(lnx).x+\int \sin(l n x)dx$

$=\cos(lnx)x+\sin(lnx).x-\int \cos(l n x)$

$I=\frac{x}{2}[\sin(lnx)+\cos(lnx).x]+C$

25. $\lim\limits_{x\to\frac{\pi}{4}}\frac{\cot^3 x-\tan x}{\cos\left(x+\frac{\pi}{4}\right)}$ is :

$\lim\limits_{x\to\frac{\pi}{4}}\frac{\cot^3 x-\tan x}{\cos\left(x+\frac{\pi}{4}\right)}$ बराबर है –

(1) $4\sqrt{2}$ (2) 8 (3) 4 (4) $8\sqrt{2}$

A. 2

Question ID : 4165299755

Option 1 ID : 41652938478 Option 2 ID : 41652938481

Option 3 ID : 41652938479 Option 4 ID : 41652938480

sol. $\lim\limits_{x\to\frac{\pi}{4}}\frac{\cot^3 x-\tan x}{\cos\left(x+\frac{\pi}{4}\right)}$

$x-\frac{\pi}{4}=t$

$\lim\limits_{x\to 0}\frac{\cot^3(t+\frac{\pi}{4})-\tan(t+\frac{\pi}{4})}{-\sin t}$

$\lim\limits_{x\to 0}\frac{1-\tan^4(t+\frac{\pi}{4})}{-\sin t}$

D'L Hospital rule

$\frac{-4\tan^3\left(t+\frac{\pi}{4}\right)\sec^2\left(t+\frac{\pi}{4}\right)}{-\cos t}$

$= \frac{-4.2}{-1}$

$= 8$

26. For $x > 1$, if $(2x)^{2y} = 4e^{2x-2y}$, then $\left(1+\log_e 2x\right)^2 \frac{dy}{dx}$ is equal to :

यदि $x > 1$ के लिए $(2x)^{2y} = 4e^{2x-2y}$ है, तो $\left(1+\log_e 2x\right)^2 \frac{dy}{dx}$ बराबर है –

(1) $\frac{x\log_e 2x + \log_e 2}{x}$ (2) $\log_e 2x$

(3) $x\log_e 2x$ (4) $\frac{x\log_e 2x - \log_e 2}{x}$

A. 4

Question ID : 4165299756

Option 1 ID : 41652938482 Option 2 ID : 41652938483

Option 3 ID : 41652938485 Option 4 ID : 41652938484

sol. $x > 1$

$(2x)^{2y} = 4e^{2x-2y}$

$2y\, ln2x = ln4 + 2x - 2y$

$y = \frac{x + ln2}{1 + ln2x}$

$\frac{dy}{dx} = \frac{(1+ln2x) - (x+ln2).\frac{1}{x}}{(1+ln2x)^2}$

$\frac{dy}{dx}(1+ln2x)^2 = \left[\frac{x\,ln2x - ln2}{x}\right]$

27. The area (in sq. units) of the region bounded by the parabola, $y = x^2 + 2$ and the lines, $y = x + 1$, $x = 0$ and $x = 3$, is :

परवलय $y = x^2 + 2$ तथा रेखाओं $y = x + 1$, $x = 0$ तथा $x = 3$ द्वारा घिरे हुए क्षेत्र का क्षेत्रफल (वर्ग इकाइयों में) है –

(1) $\frac{15}{2}$ (2) $\frac{17}{4}$ (3) $\frac{21}{2}$ (4) $\frac{15}{4}$

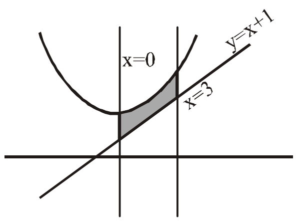
A. 1

Question ID : 4165299761

Option 1 ID : 41652938502 Option 2 ID : 41652938504

Option 3 ID : 41652938503 Option 4 ID : 41652938505

sol. $y = x^2 + 2$



MATRIX

Req. area $= \int_0^3 (x^2+2)\,dx - \frac{1}{2}5\times 3$

$= \frac{15}{2}$

28. Consider three boxes, each containing 10 balls labelled 1,2,.....10. Suppose one ball is randomly drawn from each of the boxes. Denote by n_i, the label of the ball drawn from the i^{th} box, (i = 1,2,3). Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is :

तीन ऐसे डिब्बों पर विचार कीजिए जिनमें प्रत्येक में 1,2,.....10 तक संख्याओं से अंकित 10 गेंदे है। माना कि प्रत्येक डिब्बे में से यादृच्छया एक गेंद निकाली गई। यदि i वें (i = 1, 2, 3) डिब्बे में से निकाली गई गेंद पर अंकित संख्या को n_i से प्रदर्शित किया जाए, तो जितने तरीकों से यह गेंदें निकाली जा सकती हैं, ताकि $n_1 < n_2 < n_3$ is :

(1) 240 (2) 120 (3) 164 (4) 82

A. 2

Question ID : 4165299751

Option 1 ID : 41652938465 Option 2 ID : 41652938462

Option 3 ID : 41652938463 Option 4 ID : 41652938464

sol. No. of ways

will be ${}^{10}C_3$

= 120

29. Let C_1 and C_2 be the centres of the circles $x^2+y^2-2x-2y-2=0$ and $x^2+y^2-6x-6y+14=0$ respectively. If P and Q are the points of intersection of these circles, then the area (in sq. units) of the quadrilateral PC_1QC_2 is :

माना C_1 तथा C_2 क्रमशः वृत्तों $x^2+y^2-2x-2y-2=0$ तथा $x^2+y^2-6x-6y+14=0$ के केंद्र है। यदि P तथा Q इन वृत्तों के प्रतिच्छेदन बिन्दु है, तो चतुर्भुज PC_1QC_2 का क्षेत्रफल (वर्ग इकाइयों में) है –

(1) 6 (2) 9 (3) 8 (4) 4

A. 4

Question ID : 4165299765

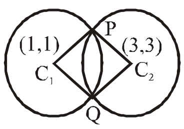
Option 1 ID : 41652938519 Option 2 ID : 41652938521

Option 3 ID : 41652938520 Option 4 ID : 41652938518

sol. $r_1 = 2,$ $r_2 = 2$

$C_1 = x^2+y^2-2x-2y-2=0$

$C_2 = x^2+y^2-6x-6y+14=0$



$(C_1C_2)^2 = r_1^2 + r_2^2$

$4 + 4 = 4 + 4$

So circles are orthogonal

Area of quadrilateral = 2 × ½ × 4

= 4

30. The perpendicular distance from the origin to the plane containing the two lines, $\frac{x+2}{3} = \frac{y-2}{5} = \frac{z+5}{7}$ and

$\frac{x-1}{1} = \frac{y-4}{4} = \frac{z+4}{7}$ is :

दो रेखाओं $\frac{x+2}{3}=\frac{y-2}{5}=\frac{z+5}{7}$ तथा $\frac{x-1}{1}=\frac{y-4}{4}=\frac{z+4}{7}$ को अंतर्विष्ट करने वाले समतल की मूलबिन्दु से लंबवत दूरी है –

(1) $\frac{11}{\sqrt{6}}$ (2) 11 (3) $6\sqrt{11}$ (4) $11\sqrt{6}$

A. 1

Question ID : 4165299768

Option 1 ID : 41652938532 Option 2 ID : 41652938530

Option 3 ID : 41652938533 Option 4 ID : 41652938531

sol. $\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 5 & 7\\ 1 & 4 & 7\end{vmatrix}$

$\vec{n}=\hat{i}-2\hat{j}+\hat{k}$

$\vec{a}=(-2,2,-5)$

$\vec{r}.\vec{n}=\vec{a}.\vec{n}$

$x-2y+z+11=0$

Distance from origin

$=\frac{11}{\sqrt{6}}$

MATRIX