MATHEMATICS
12 Jan. 2019 [Session : 2 : 30 PM to 5 : 30 PM]
JEE MAIN PAPER ONLINE

1. The integral $\int_1^e \left\{\left(\frac{x}{e}\right)^{2x} - \left(\frac{e}{x}\right)^{x}\right\} \log_e x \, dx$ is equal to :

समाकल $\int_1^e \left\{\left(\frac{x}{e}\right)^{2x} - \left(\frac{e}{x}\right)^{x}\right\} \log_e x \, dx$ बराबर है –

(1) $-\frac{1}{2}+\frac{1}{e}-\frac{1}{2e^2}$ (2) $\frac{3}{2}-e-\frac{1}{2e^2}$ (3) $\frac{1}{2}-e-\frac{1}{e^2}$ (4) $\frac{3}{2}-\frac{1}{e}-\frac{1}{2e^2}$

A. 2

Question ID : 4165299940

Option 1 ID : 41652939220 Option 2 ID : 41652939219

Option 3 ID : 41652939221 Option 4 ID : 41652939218

sol. $\int_1^e \left\{\left(\frac{x}{e}\right)^{2x} - \left(\frac{e}{x}\right)^{x}\right\} \log_e x$

$\int_1^e \left(\frac{x}{e}\right)^{2x} \log x - \int_1^e \left(\frac{e}{x}\right)^{x} \log x$

$\left(\frac{x}{e}\right)^{2x} = t$ $\left(\frac{e}{x}\right)^{x} = v$

$= \frac{1}{2}\int_{\left(\frac{1}{e}\right)^2}^{1} dt + \int_{(e)}^{1} dv$

$= \frac{3}{2} - e - \frac{1}{2e^2}$

2. Let z_1 and z_2 be two complex numbers satisfying $|z_1| = 9$ and $|z_2 - 3 - 4i| = 4$. Then the minimum value of $|z_1 - z_2|$ is :

माना z_1 तथा z_2 दो सम्मिश्र संख्यायें है, जो $|z_1| = 9$ तथा $|z_2 - 3 - 4i| = 4$ को सन्तुष्ट करती है, तो $|z_1 - z_2|$ का न्यूनतम मान है–

(1) 1 (2) 0 (3) $\sqrt{2}$ (4) 2

A. 2

Question ID : 4165299928

Option 1 ID : 41652939171 Option 2 ID : 41652939170

Option 3 ID : 41652939172 Option 4 ID : 41652939173

sol. $|z_1| = 9$ and $|z_2 - 3 - 4i| = 4$

Circle centre (0, 0) Centre (3, 4)

$r = 3$ $r_2 = 2$

$c_1c_2 = (r_1+r_2)$ so both circle touch each other

$|z_1 - z_2|$ minimum value = 0

3. If the function f given by $f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$, for some $a \in R$ is increasing in $(0, 1]$ and decreasing in $[1, 5)$, then a root of the equation, $\frac{f(x)-14}{(x-1)^2} = 0 (x \neq 1)$ is :

यदि फलन $f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$, किसी $a \in R$ के लिए $(0, 1]$ में वर्धमान है तथा $[1, 5)$ में ह्रासमान है तो समीकरण $\frac{f(x)-14}{(x-1)^2} = 0 (x \neq 1)$ का एक हल है –

(1) 7 (2) –7 (3) 6 (4) 5

A. 1

Question ID : 4165299938

Option 1 ID : 41652939213 Option 2 ID : 41652939210

Option 3 ID : 41652939212 Option 4 ID : 41652939211

sol. $f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$

$f'(x) = 3x^2 - 6(a-2)x + 3a$

For increasing fuction

$f'(x) > 0$

$3x^2 - 6(a-2)x + 3a > 0$ in $(0, 1]$

in root of this equation

$3 - 6a + 12 + 3a = 0$

$15 - 3a = 0$

$a = 5$

$$f = \frac{x^3 - 9x^2 + 15x + 7 - 14}{(x-1)^2}$$

$$= \frac{x^3 - 9x^2 + 15x - 7}{(x-1)^2}$$

$$\frac{(x-1)^2(x-7)}{(x-1)^2} \Rightarrow (x-7)$$

one solution $x = 7$

4. The mean and the variance of five observations are 4 and 5.20, respectively. If three of the observations are 3, 4 and 4; then the absolute value of the difference of the other two observations, is :

पाँच प्रेक्षणों का माध्य तथा प्रसरण क्रमशः 4 तथा 5.20 है। यदि इन प्रेक्षणों में से तीन 3, 4 तथा 4 हैं, तो अन्य दो प्रेक्षणों के अन्तर का निरपेक्ष मान है –

(1) 3 (2) 7 (3) 1 (4) 5

A. 2

Question ID : 4165299950

Option 1 ID : 41652939259 Option 2 ID : 41652939261

Option 3 ID : 41652939258 Option 4 ID : 41652939260

sol. 5 observation

Mean = 4

$\sigma^2 = 5.20$

3, 4, 4, a, b

$$4 = \frac{11 + a + b}{5}$$

$a + b = 9$

$\theta = \text{angle}$

$$5.20 = \frac{1 + (4-a)^2 + (4-b)^2}{5}$$

$26 = 1 + (4 - a)^2 + (4 - b)^2$

$25 = 32 + a^2 + b^2 - 8(a + b)$

$65 = a^2 + b^2$

$(a + b)^2 - 2ab = 65$

$2ab = 81 - 65$

$2ab = 16$

$(a - b)^2 = (a + b)^2 - 4ab = 81 - 32$

$|a - b| = \sqrt{49} = 7$

5. $\lim\limits_{x\to 1-} \frac{\sqrt{\pi} - \sqrt{2\sin^{-1} x}}{\sqrt{1-x}}$ is equal to :

$\lim\limits_{x\to 1-} \frac{\sqrt{\pi} - \sqrt{2\sin^{-1} x}}{\sqrt{1-x}}$ बराबर है –

(1) $\frac{1}{\sqrt{2\pi}}$ (2) $\sqrt{\pi}$ (3) $\sqrt{\frac{\pi}{2}}$ (4) $\sqrt{\frac{2}{\pi}}$

A. 4

Question ID : 4165299935

Option 1 ID : 41652939200 Option 2 ID : 41652939201

Option 3 ID : 41652939199 Option 4 ID : 41652939198

sol. $\lim\limits_{x\to 1-} \frac{\sqrt{\pi} - \sqrt{2\sin^{-1} x}}{\sqrt{1-x}}$

Rationalization

$$\lim_{x\to 1^-} \frac{\pi - 2\sin^{-1} x}{\sqrt{1-x}\left(\sqrt{\pi} + \sqrt{2\sin^{-1} x}\right)}$$

$$\lim_{x\to 1^-} \frac{2\left(\frac{\pi}{2} - \sin^{-1} x\right)}{\sqrt{1-x}\left(\sqrt{\pi} + \sqrt{\pi}\right)}$$

$$\lim_{x\to 1^-}\frac{2\cos^{-1}x}{\sqrt{1-x}}\frac{1}{2\sqrt{\pi}}$$

$$\frac{1}{\sqrt{\pi}}\lim_{x\to 1^-}\frac{\cos^{-1}x}{\sqrt{1-x}}$$

$x=\cos\theta$

$$\frac{1}{\sqrt{\pi}}\lim_{x\to 0}\frac{2.\theta/2}{\sqrt{2}\sin\frac{\theta}{2}}$$

$$\frac{1}{\sqrt{\pi}}.\sqrt{2}$$

$$\Rightarrow\sqrt{\frac{2}{\pi}}$$

6. In a class of 60 students, 40 opted for NCC, 30 opted for NSS and 20 opted for both NCC and NSS. If one of these students is selected at random, then the probability that the student selected has opted neither for NCC nor for NSS is :

60 छात्रों की एक कक्षा में, 40 ने NCC ली, 30 ने NSS ली तथा 20 ने NCC और NSS दोनों ली। यदि इनमें से एक छात्र यादृच्छिक चुना गया है, तो चुने गए छात्र के न तो NCC, न ही NSS लेने की प्रायिकता है –

(1) $\frac{5}{6}$ (2) $\frac{1}{6}$ (3) $\frac{1}{3}$ (4) $\frac{2}{3}$

A. 2

Question ID : 4165299951

Option 1 ID : 41652939264 Option 2 ID : 41652939262

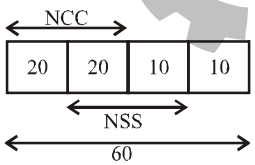
Option 3 ID : 41652939265 Option 4 ID : 41652939263

sol. Total students = 60

NCC taken by student = 40

NSS taken by student = 30

both taken by student = 20



= 10/60

$=\frac{1}{6}$

7. Let S be the set of all real values of λ such that a plane passing through the points $(-\lambda^2, 1, 1)$, $(1, -\lambda^2, 1)$ and $(1, 1, -\lambda^2)$ also passes through the point $(-1, -1, 1)$. Then S is equal to :

यदि λ के उन सभी वास्तविक मानों, जिनके लिए बिन्दुओं $(-\lambda^2, 1, 1)$, $(1, -\lambda^2, 1)$ तथा $(1, 1, -\lambda^2)$ से होकर जाने वाला एक समतल, बिन्दु $(-1, -1, 1)$ से भी होकर जाता है, का समुच्चय S है, तो S बराबर है –

(1) $\{3, -3\}$ (2) $\{\sqrt{3}, -\sqrt{3}\}$ (3) $\{\sqrt{3}\}$ (4) $\{1, -1\}$

A. 2

Question ID : 4165299948

Option 1 ID : 41652939252 Option 2 ID : 41652939253

Option 3 ID : 41652939250 Option 4 ID : 41652939251

sol. All four points are coplanar

$$\begin{vmatrix} -\lambda^2+1 & 2 & 0 \\ 2 & -\lambda^2+1 & 0 \\ 2 & 2 & -\lambda^2-1 \end{vmatrix} = 0$$

$$-(\lambda^2+1)((1-\lambda^2)-4)=0$$

$$(\lambda^4-2\lambda^2+1-4)=0$$

$$\lambda^4-2\lambda^2-3=0$$

$$(\lambda^2-3)(\lambda^2+1)=0$$

$$(\lambda^2-3)(\lambda^2+1)=0$$

$$\lambda=\pm\sqrt{3}$$

8. The equation of a tangent to the parabola, $x^2 = 8y$, which makes an angle θ with the positive direction of x-axis, is :

परवलय $x^2 = 8y$ पर एक स्पर्श रेखा जो x-अक्ष की धनात्मक दिशा के साथ कोण θ बनाती है, का समीकरण है –

(1) $x = y\ \cot\theta + 2\tan\theta$ (2) $x = y\cot\theta - 2\tan\theta$

(3) $y = x\tan\theta - 2\cot\theta$ (4) $y = x\tan\theta + 2\cot\theta$

A. 1

Question ID : 4165299945

Option 1 ID : 41652939239 Option 2 ID : 41652939240

Option 3 ID : 41652939238 Option 4 ID : 41652939241

sol. $x^2 = 8y$

for $y^2 = 4ax$

$y = mx + a/m$

for $x^2 = 4ay$ $\quad x = \frac{1}{m}y + am$

$mx = y + am^2$

$y = mx - am^2$

$a = 2 \quad m = \tan\theta$

$y = x\tan\theta - 2\tan^2\theta$

$x = y\cot\theta + 2\tan\theta$

9. If a curve passes throuth the point (1, –2) and has slope of the tangent at any point (x, y) on it as $\frac{x^2-2y}{x}$, then the curve also passes through the point :

यदि एक वक्र बिन्दु (1, –2) से होकर जाता है तथा इस पर किसी बिन्दु (x, y) पर स्पर्श रेखा का ढाल $\frac{x^2-2y}{x}$ है, तो यह वक्र निम्न में से किस बिन्दु से होकर जाता है ?

(1) $\left(-\sqrt{2},1\right)$ (2) (–1, 2) (3) $\left(\sqrt{3},0\right)$ (4) (3, 0)

A. 3

Question ID : 4165299942

Option 1 ID : 41652939228 Option 2 ID : 41652939227

Option 3 ID : 41652939229 Option 4 ID : 41652939226

sol. $\frac{dy}{dx}=\frac{x^2-2y}{x}$

$\frac{dy}{dx}+\frac{2}{x}y=x$

y. I.F = $\int$ Q.I.F

y. $x^2=\int x.x^2$

$y.x^2=\frac{x^4}{4}+c$

Now curve passing through (1, –2)

$(-2)\ 1=\frac{1}{4}+c$

$c=-2-\frac{1}{4}=-\frac{9}{4}$

Equation of curve

$y.x^2=\frac{x^4}{4}-\frac{9}{4}$

10. If a straight line passing through the point P(–3, 4) is such that its intercepted portion between the coordinate axes is bisected at P, then its equation is :

यदि बिन्दु P(–3, 4) से होकर जाने वाली एक सरल रेखा इस प्रकार है कि इसका निर्देशांक अक्षों के बीच अंतः खण्डित भाग का मध्य बिन्दु P है, तो इसका समीकरण है –

(1) $x - y + 7 = 0$ (2) $4x + 3y = 0$ (3) $3x - 4y + 25 = 0$ (4) $4x - 3y + 24 = 0$

A. 4

Question ID : 4165299943

Option 1 ID : 41652939231 Option 2 ID : 41652939230

Option 3 ID : 41652939232 Option 4 ID : 41652939233

sol. P(–3, 4)

Line passing through (–3, 4)

x-intercept $\left(\frac{-4}{m} - 3, 0\right)$

y -intercept $(0, 4 + 3m)$

mid point of x and y intercept is P

$\frac{-4}{m} - 3 = -6$

$\frac{-4}{m} = -3$

$m = 4/3$

$y - 4 = \frac{4}{3}(x+3)$

$3y - 12 = 4x + 12$

$4x - 3y + 24 = 0$

11. $\lim\limits_{n\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+\ldots+\frac{1}{5n}\right)$ is equal to :

$\lim\limits_{n\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+\ldots+\frac{1}{5n}\right)$ बराबर है –

(1) $\frac{\pi}{2}$ (2) $\tan^{-1}(3)$ (3) $\frac{\pi}{4}$ (4) $\tan^{-1}(2)$

A. 4

Question ID : 4165299941

Option 1 ID : 41652939225 Option 2 ID : 41652939223

Option 3 ID : 41652939224 Option 4 ID : 41652939222

sol. $\lim\limits_{n\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+\ldots+\frac{1}{5n}\right)$

$\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{1}{n\left(1+\frac{r^2}{n^2}\right)}$

$= \int_0^2 \frac{dx}{1+x^2}$

$= \tan^{-1}(2)$

12. Let Z be the set of integers. If $A = \left\{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\right\}$ and $B = \{x \in Z : -3 < 2x-1 < 9\}$, then the number of subsets of the set $A \times B$, is :

माना Z पूर्णांकों का समुच्चय है। यदि $A = \left\{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\right\}$ तथा $B = \{x \in Z : -3 < 2x-1 < 9\}$ तो $A \times B$ के उपसमुच्चयों की संख्या है –

(1) 2^{15} (2) 2^{10} (3) 2^{18} (4) 2^{12}

A. 1

Question ID : 4165299926

Option 1 ID : 41652939164 Option 2 ID : 41652939162

Option 3 ID : 41652939165 Option 4 ID : 41652939163

sol. $A = \left\{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\right\}$

$(x+2)(x^2 - 5x + 6) = 0$

$(x+2)(x-2)(x-3) = 0$

$x = -2,\ x = 2$ and $x = 3$

$B = \{x \in Z : -3 < 2x-1 < 9\}$

$-3 < 2x - 1 < 9$

$-2 < 2x < 10$

$-1 < x < 5$

x (1, 2, 3, 4)

A ∈ (2, 2, 3)

B ∈ (0, 1, 2, 3, 4)

To no. of element in $A \times B = 15$

So subset = 2^{15}

13. If the sum of the first 15 terms of the series $\left(\frac{3}{4}\right)^3 + \left(1\frac{1}{2}\right)^3 + \left(2\frac{1}{4}\right)^3 + 3^3 + \left(3\frac{3}{4}\right)^3 + \ldots\ldots$ is equal to 225k, then k is equal to :

यदि श्रेणी $\left(\frac{3}{4}\right)^3 + \left(1\frac{1}{2}\right)^3 + \left(2\frac{1}{4}\right)^3 + 3^3 + \left(3\frac{3}{4}\right)^3 + \ldots\ldots$ के प्रथम 15 पदों का योग 225k के बराबर है, तो k बराबर है –

(1) 9 (2) 27 (3) 108 (4) 54

A. 2

Question ID : 4165299934

Option 1 ID : 41652939194 Option 2 ID : 41652939195

Option 3 ID : 41652939197 Option 4 ID : 41652939196

sol. $\left(\frac{3}{4}\right)^3 + \left(1\frac{1}{2}\right)^3 + \left(2\frac{1}{4}\right)^3 + 3^3 + \left(3\frac{3}{4}\right)^3 + \ldots\ldots$ 15 terms

$$\frac{27}{64}\sum_{r=1}^{15} r^3$$

$$\frac{27}{64}\left(\frac{(15)16}{2}\right)^2$$

$$=\frac{27}{64}\times 64\times 225$$

$= 27 \times 225 = 225\ k$

$k = 27$

14. The number of integral values of m for which the quadratic expression, $(1+2m)x^2 - 2(1+3m)x + 4(1+m)$, $x \in R$, is always positive, is :

m के उन पूर्णांक मानों, जिनके लिए व्यंजक $(1+2m)x^2 - 2(1+3m)x + 4(1+m)$, $x \in R$, सदा धनात्मक है, की संख्या है –

(1) 7 (2) 6 (3) 3 (4) 8

A. 1

Question ID : 4165299927

Option 1 ID : 41652939167 Option 2 ID : 41652939168

Option 3 ID : 41652939169 Option 4 ID : 41652939166

sol. $(1+2m)x^2 - 2(1+3m)x + 4(1+m)$,

is positive for all $x \in R$

$a > 0 \quad \Delta < 0$

$1 + 2m > 0$

$m > -½$

$4(1+3m)^2 - 16(1+m)(1+2m) < 0$

$(1+3m)^2 - 4(1+m)(1+2m) < 0$

$1 + 9m^2 + 6m - 4(1 + 2m^2 + 3m) < 0$

$m^2 - 6m - 3 < 0$

$$m = \frac{6 \pm \sqrt{36+12}}{2}$$

$$= \frac{6 \pm 4\sqrt{3}}{2}$$

$$= 3 \pm 2\sqrt{3}$$

$$(3-2\sqrt{3}, 3+2\sqrt{3})$$

$= 0, 1, 2, 3, 4, 5, 6$

15. Let f be a differentiable function such that $f(1) = 2$ and $f'(x) = f(x)$ for all $x \in R$. If $h(x) = f(f(x))$, then $h'(1)$ is equal to :

माना f एक अवकलनीय फलन इस प्रकार है कि $f(1) = 2$ तथा सभी $x \in R$ के लिए $f'(x) = f(x)$ यदि $h(x) = f(f(x))$ तो $h'(1)$ बराबर है –

(1) $2e$ (2) $2e^2$ (3) $4e$ (4) $4e^2$

A. NOT

Question ID : 4165299936

Option 1 ID : 41652939204 Option 2 ID : 41652939202

Option 3 ID : 41652939203 Option 4 ID : 41652939205

sol. $f'(x) = f(x)$

$f(1) = 2$

$f(x) = a.e^{bx}$

$a.e^{bx} = a.be^{bx}$

$b = 1$

$f(x) = a.e^{x}$

$f(1) = 2$

$2 = ae$

$a = 2/e$

$$f(x) = \frac{2}{e}e^{x}$$

$h(x) = f(f(x))$

$h'(x) = f'(f(x)).\ f'(x)$

$h'(1) = f'(f(1)).\ f'(1)$

$2e.2 \Rightarrow 4e$

16. The set of all values of λ for which the system of linear equations

$x - 2y - 2z = \lambda x$

$x + 2y + z = \lambda y$

$-x - y = \lambda z$

has a non-trivial solution :

(1) is a singleton (2) contains exactly two elements

(3) is an empty set (4) contains more than two elements

λ के उन सभी मानों, जिनके लिए रैखिक समीकरण निकाय

$x - 2y - 2z = \lambda x$

$x + 2y + z = \lambda y$

$-x - y = \lambda z$

का एक अतुच्छ हल है –

(1) का समुच्चय एकल है (2) के समुच्चय में मात्र दो अवयव है

(3) का समुच्चय रिक्त है (4) के समुच्चय में दो से अधिक अवयव है

A. 1

Question ID : 4165299930

Option 1 ID : 41652939178 Option 2 ID : 41652939180

Option 3 ID : 41652939179 Option 4 ID : 41652939181

Sol. Non-trivial solution

$\Delta = 0$

$$\begin{vmatrix} 1-\lambda & -2 & -2 \\ 1 & 2-\lambda & 1 \\ -1 & -1 & -\lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} 1-\lambda & -2 & -2 \\ 0 & 1-\lambda & 1-\lambda \\ -1 & -1 & -\lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} 1-\lambda & 0 & -2 \\ 0 & 0 & 1-\lambda \\ -1 & -1+\lambda & -\lambda \end{vmatrix} = 0$$

$(\lambda-1)(\lambda-1)^2 = 0$

$\lambda = 1$

17. There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of m is :

एक शतरंज प्रतियोगिता में m पुरूष तथा दो महिलाएं भाग ले रही है। प्रत्येक भागी दूसरे प्रत्येक भागी के साथ दो गेम खेलता है। यदि पुरूषों के बीच आपस में खेले गये गेमों की संख्या, पुरूषों तथा महिलाओं के बीच खेले गये गेमों की संख्या से 84 अधिक है, तो m का मान है –

(1) 11 (2) 7 (3) 12 (4) 9

A. 3

Question ID : 4165299931

Option 1 ID : 41652939184 Option 2 ID : 41652939182

Option 3 ID : 41652939185 Option 4 ID : 41652939183

sol. On paper

18. In a game, a man wins Rs. 100 if he gets 5 or 6 on a throw of a fair die and loses Rs. 50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws, then his expected gain/loss (in rupees) is :

एक खेल में एक अनभिनम पासा फेंकने पर 5 या 6 आने पर एक व्यक्ति 100 रू जीतता है तथा पासे पर कोई अन्य संख्या आने पर 50 रू हारता है। यदि वह यह तय करता है कि वह या तब तक पासा फेंकेगा जब तक 5 या 6 न आ जाए अथवा अधिक से अधिक तीन बार पाया फेंकेगा, तो उसकी संभवित लाभ/हानि (रूपयों में) है –

(1) $\frac{400}{3}$ gain (2) $\frac{400}{3}$ loss (3) 0 (4) $\frac{400}{9}$ loss

A. 3

Question ID : 4165299952

Option 1 ID : 41652939268 Option 2 ID : 41652939267

Option 3 ID : 41652939269 Option 4 ID : 41652939266

sol. Rs. 100 when he gets 5 or 6

probability to come 5 or 6 = $\frac{1}{3}$

For one $\frac{\text{Throw}}{\text{Gain}} \Rightarrow \frac{1}{2}\times 100$

For two throw $\Rightarrow$ one is loss and one is gain

$$\Rightarrow \frac{2}{3}\times 50 \text{ loss} \qquad \frac{1}{3}\times 100$$

For three throw $\Rightarrow$ two loss + 1 gain

$$\Rightarrow 2\left(\frac{2}{3}\times 50\right) \qquad \frac{1}{3}\times 100$$

Overall gain/loss = 0

19. The total number of irrational terms in the binomial expansion of $\left(7^{\frac{1}{5}}-3^{\frac{1}{10}}\right)^{60}$ is :

$\left(7^{\frac{1}{5}}-3^{\frac{1}{10}}\right)^{60}$ के द्विपद प्रसार में अपरिमेय पदों की कुल संख्या है –

(1) 55 (2) 48 (3) 49 (4) 54

A. 4

Question ID : 4165299932

Option 1 ID : 41652939189 Option 2 ID : 41652939186

Option 3 ID : 41652939187 Option 4 ID : 41652939188

sol.

20. If $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\ \sin\alpha\cos\beta$; $\alpha, \beta \in [0, \pi]$, then $\cos(\alpha+\beta) - \cos(\alpha-\beta)$ is equal to :

यदि $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\ \sin\alpha\cos\beta$; $\alpha, \beta \in [0, \pi]$, तो $\cos(\alpha+\beta) - \cos(\alpha-\beta)$ बराबर है –

(1) 0 (2) $-\sqrt{2}$ (3) $\sqrt{2}$ (4) –1

A. 2

Question ID : 4165299953

Option 1 ID : 41652939272 Option 2 ID : 41652939271

Option 3 ID : 41652939270 Option 4 ID : 41652939273

sol. $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\ \sin\alpha\cos\beta$; $\alpha, \beta \in [0, \pi]$

AM ≥ GM

$$\frac{\sin^4\alpha + 4\cos^4\beta + 1 + 1}{4} \geq \sqrt[4]{\sin^4\alpha\cos^4\beta.1.1.4}$$

$\sin^4\alpha + 4\cos^4\beta + 2 \geq 4\sqrt{2}\ \sin\alpha\cos\beta$

AM = GM

So

$\sin^4\alpha = 4\cos^4\beta = 1$

$\sin\alpha = 1 \quad \cos\beta = \frac{\pm 1}{\sqrt{2}}$

$\sin\beta = \frac{1}{\sqrt{2}} \quad \beta \in [0, \pi]$

$\cos(\alpha+\beta) - \cos(\alpha-\beta)$

$= -2\sin\alpha\sin\beta$

$= -2\frac{1}{\sqrt{2}}$

$-2\sqrt{2}$

21. If the angle of elevation of a cloud from a point P which is 25m above a lake be 30° and the angle of depression of reflection of the cloud in the lake from P be 60°. then the height of the cloud (in meters) from the surface of

the lake is :

एक झील से 25 m ऊपर एक बिन्दु P से एक बादल का उन्नयन कोण 30° है तथा P से झील में बादल के प्रतिबिम्ब का अवनमन कोण 60° है, तो झील की सतह से बादल की ऊँचाई (मीटर में) है –

(1) 50 (2) 60 (3) 42 (4) 45

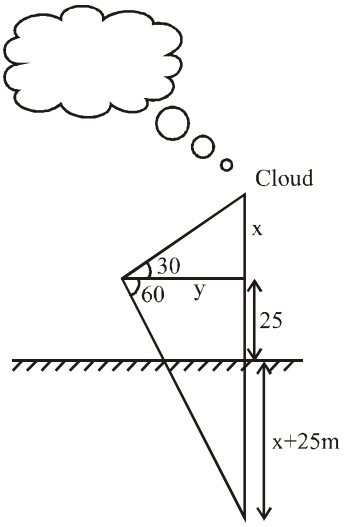
A. 1

Question ID : 4165299954

Option 1 ID : 41652939275 Option 2 ID : 41652939274

Option 3 ID : 41652939277 Option 4 ID : 41652939276

sol.



$\tan 30^\circ = \frac{x}{y}$

$y = \sqrt{3}x$

$\tan 60^\circ = \frac{25 + x + 25}{y}$

$x = 25$ m

height = 50 m

22. If a circle of radius R passes through the origin O and intersects the coordinate axes at A and B, then the locus of the foot of perpendicular from O on AB is :

यदि R त्रिज्या का एक वृत मूल बिन्दु O से होकर जाता है तथा निर्देशांक अक्षों को A और B पर काटता है, तो O से रेखा AB परडाले गये लम्ब के पाद का बिन्दुपथ है –

(1) $(x^2 + y^2)^2 = 4R^2x^2y^2$ (2) $(x^2 + y^2)^2 = 4Rx^2y^2$ (3) $(x^2 + y^2)^3 = 4R^2x^2y^2$ (4) $(x^2 + y^2)(x + y) = R^2xy$

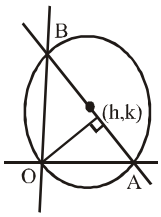
A. 3

Question ID : 4165299944

Option 1 ID : 41652939235 Option 2 ID : 41652939236

Option 3 ID : 41652939234 Option 4 ID : 41652939237

sol.



Length of AB = 2R

Equation of line AB

$hx + ky = h^2 + x^2$

$$A\left(\frac{h^2+k^2}{h},0\right) \qquad B\left(0,\frac{h^2+k^2}{k}\right)$$

$$AB = 2R = \sqrt{\frac{\left(h^2+k^2\right)^2}{h^2}+\frac{\left(h^2+k^2\right)^2}{k^2}}$$

$\Rightarrow 4R^2.(h^2k^2) = (h^2+k^2)^2 (h^2+k^2)$

$(x+y^2)^3 = 4R^2x^2y^2$

23. Let S and S' be the foci of an ellipse and B be any one of the extremities of its minor axis. If $\Delta S'BS$ is a right angled triangle with right angle at B and area ($\Delta S'BS$) = 8 sq. units, then the length of a latus rectum of the ellipse is :

माना एक दीर्घवृत की नाभियाँ S तथा S' है तथा इसके लघु अक्ष का कोई एक शीर्ष B है। यदि $\Delta S'BS$ एक समकोण त्रिभुज है जिसका समकोण B पर है तथा $\Delta S'BS$ का क्षेत्रफल = 8 वर्ग इकाई है, तो दीर्घवृत की एक नाभिलम्ब जीवा की लम्बाई है –

(1) 4 (2) $4\sqrt{2}$ (3) 2 (4) $2\sqrt{2}$

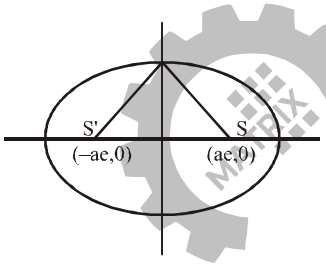
A. 1

Question ID : 4165299946

Option 1 ID : 41652939245 Option 2 ID : 41652939244

Option 3 ID : 41652939242 Option 4 ID : 41652939243

sol.



$$\frac{b}{ae}.\frac{b}{-ae} = -1$$

$b^2 = a^2e^2$

$$\frac{2ae \times b}{2} = 8$$

$ae.b = 8$

$b^2 = 8$

$b^2 = a^2 - a^2e^2$

$2b^2 = a^2$

$$\text{Latus rectum} = \frac{2b^2}{a}$$

$\frac{a^2}{a} = a$

$= 4$

24. If $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$; then for all $\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$, det (A) lies in the interval :

यदि $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$ तो सभी $\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$ के लिए, det (A) निम्न में से किस अंतराल में स्थित है।

(1) $\left(0, \frac{3}{2}\right]$ (2) $\left(\frac{3}{2}, 3\right]$ (3) $\left(1, \frac{5}{2}\right]$ (4) $\left[\frac{5}{2}, 4\right)$

A. 2

Question ID : 4165299929

Option 1 ID : 41652939174 Option 2 ID : 41652939176

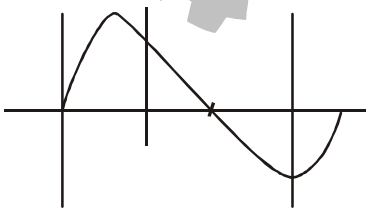
Option 3 ID : 41652939175 Option 4 ID : 41652939177

sol. $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$

$C_1 \Rightarrow C_1 - C_3$

$\begin{bmatrix} 0 & \sin\theta & 1 \\ -2\sin\theta & 1 & \sin\theta \\ -2 & -\sin\theta & 1 \end{bmatrix}$

$\det(A) = -\sin\theta(-2\sin\theta + 2\sin\theta)$
$+ 1\,(2\sin^2\theta + 2)$
$= 2(1 + \sin^2\theta)$



Minimum value = 2
Maximum value = 3
and it will lie in interval

$\left(\frac{3}{2}, 3\right]$

25. The integral $\int \frac{3x^{13}+2x^{11}}{\left(2x^4+3x^2+1\right)^4}dx$ is

equal to : (where C is a constant of integration)

समाकल $\int \frac{3x^{13}+2x^{11}}{\left(2x^4+3x^2+1\right)^4}dx$ बराबर है।

(जहाँ C समाकलन का एक अचर है)

(1) $\frac{x^4}{6\left(2x^4+3x^2+1\right)^3}+C$

(2) $\frac{x^{12}}{\left(2x^4+3x^2+1\right)^3}+C$

(3) $\frac{x^4}{\left(2x^4+3x^2+1\right)^3}+C$

(4) $\frac{x^4}{6\left(2x^4+3x^2+1\right)^3}+C$

A. 2

Question ID : 4165299939

Option 1 ID : 41652939214

Option 2 ID : 41652939217

Option 3 ID : 41652939216

Option 4 ID : 41652939215

Sol. $-2\left(\frac{3}{x^2}+\frac{1}{x^4}\right)=t$

$$\int \frac{3x^{13}+2x^{11}}{x^{16}\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^4}dx$$

$$\int \frac{\frac{3}{x^3}+\frac{2}{x^5}}{\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^4}dx$$

Now put $2+\frac{3}{x^2}+\frac{1}{x^4}=t$

$$-2\left(\frac{2}{x^3}+\frac{2}{x^5}\right)dx=dt$$

$$\frac{-1}{2}\int \frac{1}{t^4}dt=\frac{-1}{2}\frac{(-4)}{t^3}$$

$$=\frac{2}{\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^3}$$

$$= \frac{2.x^{12}}{\left(2x^4+3x^2+1\right)^3}$$

26. The tangent to the curve $y = x^2 - 5x + 5$, parallel to the line $2y = 4x + 1$, also passes through the point :
वक्र $y = x^2 - 5x + 5$ की स्पर्श रेखा, जो रेखा $2y = 4x + 1$ के समान्तर है, निम्न में से किस बिन्दु से होकर जाती है ?

(1) $\left(\frac{7}{2}, \frac{1}{4}\right)$ (2) $\left(-\frac{1}{8}, 7\right)$ (3) $\left(\frac{1}{4}, \frac{7}{2}\right)$ (4) $\left(\frac{1}{8}, -7\right)$

A. 4

Question ID : 4165299937

Option 1 ID : 41652939206 Option 2 ID : 41652939208

Option 3 ID : 41652939207 Option 4 ID : 41652939209

sol. $y = x^2 - 5x + 5$

$$\frac{dy}{dx} = 2x - 5 = 2$$

$x = 7/2$

$$y = \frac{49}{4} - \frac{35}{2} + 5$$

$$= \frac{49 - 70 + 20}{4}$$

$$= \frac{-1}{4}$$

$$y + \frac{1}{4} = 2\left(x - \frac{7}{2}\right)$$

$$\frac{4y+1}{4} = 2x - 7$$

$4y + 1 = 8x - 28$

$8x - 4y = 29$

27. If an angle between the line, $\frac{x+1}{2} = \frac{y-2}{1} = \frac{z-3}{-2}$ and the plane, $x - 2y - kz = 3$ is $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$, then a value of k is :

यदि रेखा $\frac{x+1}{2} = \frac{y-2}{1} = \frac{z-3}{-2}$ तथा समतल $x - 2y - kz = 3$ के बीच का कोण $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ है, तो k का एक मान है –

(1) $\sqrt{\frac{3}{5}}$ (2) $-\frac{3}{5}$ (3) $\sqrt{\frac{5}{3}}$ (4) $-\frac{5}{3}$

A. 3

Question ID : 4165299947

Option 1 ID : 41652939247 Option 2 ID : 41652939248

Option 3 ID : 41652939246 Option 4 ID : 41652939249

sol. $\frac{x+1}{2}=\frac{y-2}{1}=\frac{z-3}{-2}$

$x-2y-kz=3$

$\frac{\pi}{2}-\cos^{-1}\theta=\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

$\cos\theta=\frac{2-2+2x}{3\sqrt{5+x^2}}$

$\frac{1}{3}=\frac{2x}{3\sqrt{5+x^2}}$

$5+x^2=(2x)^2$

$3x^2=5$

$x=\sqrt{\frac{5}{3}}$

28. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vectors, out of which vectors $\vec{b}$ and $\vec{c}$ are non-parallel. If α and β are the angles which vector $\vec{a}$ makes with vectors $\vec{b}$ and $\vec{c}$ respectively and $\vec{a}\times(\vec{b}\times\vec{c})=\frac{1}{2}\vec{b}$, then $|\alpha-\beta|$ is equal to :

माना $\vec{a}$, $\vec{b}$ तथा $\vec{c}$ तीन एकक सदिश हैं, जिनमें से सदिश $\vec{b}$ तथा $\vec{c}$ असमान्तर हैं यदि सदिश $\vec{a}$, सदिशों $\vec{b}$ तथा $\vec{c}$ से क्रमशः कोण α तथा β बनाता है और $\vec{a}\times(\vec{b}\times\vec{c})=\frac{1}{2}\vec{b}$ तो $|\alpha-\beta|$ बराबर है –

(1) 30° (2) 60° (3) 90° (4) 45°

A. 1

Question ID : 4165299949

Option 1 ID : 41652939256 Option 2 ID : 41652939254

Option 3 ID : 41652939257 Option 4 ID : 41652939255

sol. $\vec{a}$, $\vec{b}$ and $\vec{c}$ three unit vector

$\alpha\Rightarrow$ Angle between $\vec{a}$ and $\vec{b}$

$\beta\Rightarrow$ Angle between $\vec{a}$ and $\vec{c}$

$\vec{a}\times(\vec{b}\times\vec{c})=\frac{1}{2}\vec{b}$

$(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}=\frac{1}{2}\vec{b}$

$\vec{a}.\vec{b}=0$

$\vec{a}.\vec{c}=\frac{1}{2}$

$\alpha\Rightarrow 90^o$

$\beta\Rightarrow 60^o$

29. The expression $\sim(\sim p\rightarrow q)$ is logically equivalent to :

व्यंजक $\sim(\sim p \rightarrow q)$ निम्न में से किसके तर्क संगत तुल्य है –

(1) $\sim p \wedge q$ (2) $\sim p \wedge \sim q$ (3) $p \wedge \sim q$ (4) $p \wedge q$

A. 2

Question ID : 4165299955

Option 1 ID : 41652939280 Option 2 ID : 41652939281

Option 3 ID : 41652939279 Option 4 ID : 41652939278

sol. $\sim(\sim p \rightarrow q)$

p	q	~p	~p → q	~(~ p → q)	~p ∧ ~ q
T	T	F	T	F	F
F	T	T	T	F	F
T	F	F	T	F	F
F	F	T	F	T	T

30. If ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P., then n can be :

यदि ${}^nC_4$, ${}^nC_5$ तथा ${}^nC_6$ समान्तर श्रेढ़ी में हैं, तो n हो सकता है –

(1) 11 (2) 12 (3) 14 (4) 9

A. 3

Question ID : 4165299933

Option 1 ID : 41652939191 Option 2 ID : 41652939192

Option 3 ID : 41652939193 Option 4 ID : 41652939190

Sol. ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ in AP

$${}^nC_4 + {}^nC_6 = {}^{2n}C_5$$

$$\frac{n!}{n-4!4!}+\frac{n!}{n-6!6!}=\frac{2n!}{n-5!5!}$$

$$\frac{1}{(n-4)(n-5)}+\frac{1}{30}=\frac{2}{5(n-5)}$$

$n = 14$