MATHS

12 APRIL 2019 [Phase : I]

JEE MAIN PAPER ONLINE

Matrices

1. If A is a symmetric matrix and B is a skew-symmetric matrix such that $A+B=\begin{bmatrix}2 & 3\\5 & -1\end{bmatrix}$, then AB is equal to:

यदि एक सममित (symmetric) आव्यूह A तथा एक विषम सममित (skew-symmetric) आव्यूह B इस प्रकार हैं कि $A+B=\begin{bmatrix}2 & 3\\5 & -1\end{bmatrix}$, तो AB बराबर है :

(1) $\begin{bmatrix}4 & -2\\1 & -4\end{bmatrix}$ (2) $\begin{bmatrix}4 & -2\\-1 & -4\end{bmatrix}$ (3) $\begin{bmatrix}-4 & -2\\-1 & 4\end{bmatrix}$ (4) $\begin{bmatrix}-4 & 2\\1 & 4\end{bmatrix}$

A. 2

sol. Let $A=\begin{bmatrix}a & c\\c & b\end{bmatrix}$ and $B=\begin{bmatrix}0 & d\\-d & 0\end{bmatrix}$

$\Rightarrow \quad A+B=\begin{bmatrix}a & c+d\\c-d & b\end{bmatrix}=\begin{bmatrix}2 & 3\\5 & -1\end{bmatrix}$

$\Rightarrow \quad a = 2, b = -1, c - d = 5, c + d = 3$

$\Rightarrow \quad a = 2, b = -1, c = 4, d = -1$

$\Rightarrow \quad A+B=\begin{bmatrix}2 & 4\\4 & -1\end{bmatrix}\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$

$\begin{bmatrix}4 & -2\\-1 & -4\end{bmatrix}$

Sequence & Progression

2. If α and β are the roots of the equation $375x^2-25x-2=0$, then $\lim_{n\to\infty}\sum_{r=1}^{n}\alpha^r+\lim_{n\to\infty}\sum_{r=1}^{n}\beta^r$ is equal to :

यदि समीकरण $375x^2-25x-2=0$ के मूल α तथा β हैं, तो $\lim_{n\to\infty}\sum_{r=1}^{n}\alpha^r+\lim_{n\to\infty}\sum_{r=1}^{n}\beta^r$ बराबर है :

(1) $\frac{21}{346}$ (2) $\frac{7}{116}$ (3) $\frac{29}{358}$ (4) $\frac{1}{12}$

A. 4

sol. $375x^2-25x-2=0$

$\alpha+\beta=\frac{25}{375}, \alpha\beta=\frac{-2}{375}$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\left(\alpha^r+\beta^r\right)=\left(\alpha+\alpha^2+\alpha^3+....\infty\right)+\left(\beta+\beta^2+\beta^3+...\infty\right)$$

$$=\frac{\alpha}{1-\alpha}+\frac{\beta}{1-\beta}$$

$$=\frac{\alpha+\beta-2\alpha\beta}{1-(\alpha+\beta)+\alpha\beta}$$

$$=\frac{\frac{25}{375}+\frac{4}{375}}{1-\frac{25}{375}-\frac{2}{375}}=\frac{29}{375-25-2}$$

$$=\frac{29}{348}=\frac{1}{12}$$

Matrices

3. If $B=\begin{bmatrix}5 & 2\alpha & 1\\ 0 & 2 & 1\\ \alpha & 3 & -1\end{bmatrix}$ is the inverse of a 3×3 matrix A, then the sum of all value of α for which $\det(A)+1=0$, is:

यदि एक 3 × 3 के आव्यूह A का व्युत्क्रम (inverse) $B=\begin{bmatrix}5 & 2\alpha & 1\\ 0 & 2 & 1\\ \alpha & 3 & -1\end{bmatrix}$ है, तो α के उन सभी मानों का योग, जिनके लिए det (A) + 1 = 0 है, है :

(1) –1 (2) 2 (3) 0 (4) 1

A. 4

sol. As $B=A^{-1}$

$$|B|=\frac{1}{|A|}$$

Now $|B|=\begin{vmatrix}5 & 2\alpha & 1\\ 0 & 2 & 1\\ \alpha & 3 & -1\end{vmatrix}=2\alpha^2-2\alpha-25$

Given $|A|+1=0$

$$\frac{1}{2\alpha^2-2\alpha-25}+1=0$$

$$\Rightarrow \quad \frac{2\alpha^2-2\alpha-24}{2\alpha^2-2\alpha-25}=0$$

$\alpha=4,-3$

Sum of values = 1

Function

4. For $x \in R$, let [x] denote the greatest integer $\leq x$, then the sum of the series

$$\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+\ldots+\left[-\frac{1}{3}-\frac{99}{100}\right] \text{ is :}$$

$x \in R$ के लिए माना [x], x के समान या उससे कम महत्तम पूर्णांक को दर्शाता है, तो श्रेणी

$$\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+\ldots+\left[-\frac{1}{3}-\frac{99}{100}\right] \text{ का योग है :}$$

(1) –135 (2) –153 (3) –133 (4) –131

A. 3

sol. As $[x]+\left[x+\frac{1}{n}\right]+\left[x+\frac{2}{n}\right]......\left[x+\frac{n-1}{n}\right]=[nx]$

As $[x]+[-x]=-1\ (x \notin z)$

Required value

$$=-100-\left\{\left[\frac{1}{3}\right]+\left[\frac{1}{3}+\frac{1}{100}\right]+....\left[\frac{1}{3}+\frac{99}{100}\right]\right\}$$

$$=-100-\left[\frac{100}{3}\right]$$

$= -133$

Monotonocity

5. If m is the minimum value of k for which the function $f(x)=x\sqrt{kx-x^2}$ is increasing in the interval [0, 3] and M is the maximum value of f in [0, 3] when k = m, then the ordered pair (m, M) is equal to :

यदि k का न्यूनतम मान, जिसके लिए फलन $f(x)=x\sqrt{kx-x^2}$ अंतराल [0, 3] में वर्धमान है, m है तथा [0, 3] में f का अधिकतम मान जब k = m है, M है, तो क्रमित युग्म (m, M) बराबर है :

(1) $(4,3\sqrt{2})$ (2) $(3,3\sqrt{3})$ (3) $(5,3\sqrt{6})$ (4) $(4,3\sqrt{3})$

A. 4

sol. $f(x)=x\sqrt{kx-x^2}=\sqrt{kx^2-x^4}$

$$f'(x)=\frac{\left(3kx^2-4x^3\right)}{2\sqrt{kx^3-x^4}}\geq 0 \text{ for } x\in[0,3]$$

$\Rightarrow \quad 3k-4x\geq 0$

$\quad 2k\geq 4x$

$3k \geq 4x$ for $x \in [0, 3]$

Hence $k\geq 4$

i.e., m = 4

For k = 4,

$f(x) = x\sqrt{4x - x^2}$

For max. value, $f'(x) = 0$

$\Rightarrow \quad x = 3$

i.e., $y = 3\sqrt{3}$

Hence $M = 3\sqrt{3}$

3 D

6. If the line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$ intersects the plane 2x + 3y – z + 13 = 0 at a point P and the plane 3x + y + 4z = 16 at a point Q, then PQ is equal to:

यदि रेखा $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$, समतल 2x + 3y – z + 13 = 0 को बिन्दु P पर काटती है तथा समतल 3x + y + 4z = 16 को बिन्दु Q पर काटती है, तो PQ बराबर है :

(1) $2\sqrt{14}$ (2) 14 (3) $2\sqrt{7}$ (4) $\sqrt{14}$

A. 1

sol. Let P(3λ + 2, 2λ – 1, –λ + 1) and

Q(3μ + 2, 2μ – 1, –μ + 1)

As P lies on 2x + 3y – z + 13 = 0

$6\lambda + 4 + 6\lambda - 3 + \lambda - 1 + 13 = 0$

$\Rightarrow \quad 13\lambda = -13$

$\Rightarrow \quad \lambda = -1$

$\therefore \quad P(-1, -3, 2)$

Q lies on 3x + y + 4z = 16

$9\mu + 6 + 2\mu - 1 - 4\mu + 4 = 16$

$\Rightarrow \quad 7\mu = 7$

$\Rightarrow \quad \mu = 1$

Q is (5, 1, 0)

$PQ = \sqrt{36+16+4} = \sqrt{56} = 2\sqrt{14}$

Probability

7. If three of the six vertices of a regular hexagon are chosen at random, then the probability that the triangle formed with these chosen vertices is equilateral is:

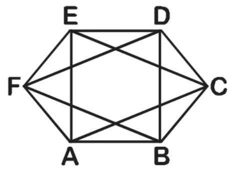
यदि एक नियमित षड्भुज के छः शीर्षों में से तीन यादृच्छिक चुने जाते हैं, तो इन चुने गए शीर्षों द्वारा बने त्रिभुज के समबाहु होने की प्रायिकता है :

(1) $\frac{3}{20}$ (2) $\frac{1}{5}$ (3) $\frac{3}{10}$ (4) $\frac{1}{10}$

A. 4

sol. Only two equilateral triangles are possible i.e.

ΔAEC and ΔBDF.



Hence, required probability

$= \frac{2}{{}^6C_3} = \frac{1}{10}$

Circle

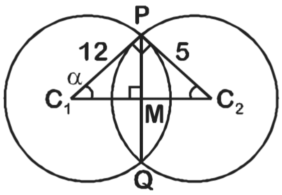
8. If the angle of intersection at a point where the two circles with radii 5 cm and 12 cm intersect is 90°, then the length (in cm) of their common chord is:

यदि एक बिन्दु, जहाँ 5 cm तथा 12 cm त्रिज्या के दो वृत्त एक दूसरे को काटते हैं, पर प्रतिच्छेदन कोण 90° है तो उनकी उभयनिष्ठ जीवा की लम्बाई (cm में) है :

(1) $\frac{120}{13}$ (2) $\frac{13}{2}$ (3) $\frac{13}{5}$ (4) $\frac{60}{13}$

A. 1

sol.



In ΔPC_1C_2,

$\tan\alpha = \frac{5}{12}$

$\Rightarrow \sin\alpha = \frac{5}{13}$

In ΔPC_1M, $\sin\alpha = \frac{PM}{12}$

$\Rightarrow \frac{5}{13} = \frac{PM}{12}$

$\Rightarrow PM = \frac{60}{13}$

Length of common chord (PQ) $= \frac{120}{13}$

Vector

9. If the volume of parallelopiped formed by the vectors $\hat{i}+\lambda\hat{j}+\hat{k}, \hat{j}+\lambda\hat{k}$ and $\lambda\hat{i}+\hat{k}$ is minimum, then λ is equal to :

यदि सदिशों $\hat{i}+\lambda\hat{j}+\hat{k}, \hat{j}+\lambda\hat{k}$ तथा $\lambda\hat{i}+\hat{k}$ द्वारा बनाये गये समान्तर षट्फलक का आयतन न्यूनतम है, तो λ बराबर है :

(1) $\frac{1}{\sqrt{3}}$ (2) $-\sqrt{3}$ (3) $\sqrt{3}$ (4) $-\frac{1}{\sqrt{3}}$

A. 1

sol. $V = \left\|\begin{matrix} 1 & \lambda & 1 \\ 0 & 1 & \lambda \\ \lambda & 0 & 1 \end{matrix}\right\|$

$= |1(1) + \lambda(\lambda^2) + 1(-\lambda)|$

$= |\lambda^3 - \lambda + 1|$

Let $f(x) = x^3 - x + 1$

$f'(x) = 3x^2 - 1$

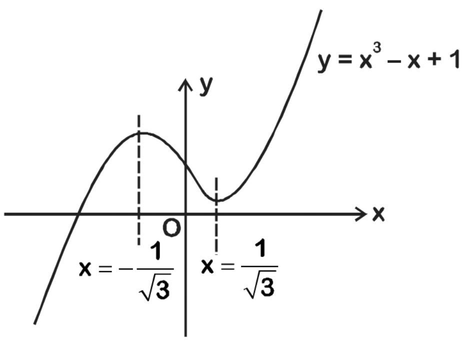
For maxima/minima, $f'(x) = 0$

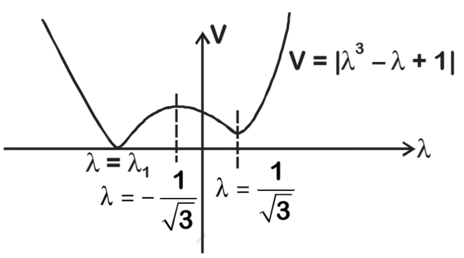
$x = \pm\frac{1}{\sqrt{3}}$

$f''(x) = 6x$

$\because \quad f''\left(\frac{1}{\sqrt{3}}\right) > 0$

$x = \frac{1}{\sqrt{3}}$ is point of local minima





When $\lambda = \lambda_1$, volume of paralleopiped is zero (vectors are coplanar)

Indefinite Integration

10. The integral $\int \frac{2x^3-1}{x^4+x}dx$ is equal to :

(Here C is a constant of integration)

समाकल $\int \frac{2x^3-1}{x^4+x}dx$ बराबर है :

(यहाँ C समाकलन अचर है)

(1) $\log_e\left|\frac{x^3+x}{x}\right|+C$

(2) $\frac{1}{2}\log_e\frac{\left(x^3+1\right)^2}{|x^3|}+C$

(3) $\frac{1}{2}\log_e\frac{|x^3+1|}{x^2}+C$

(4) $\log_e\frac{|x^3+1|}{x^2}+C$

A. 1

sol. $I=\int\frac{(2x^3-1)dx}{x^4+x}=\int\frac{(2x-x^{-2})dx}{x^2+x^{-1}}$

Put $x^2 + x^{-1} = t$

$(2x - x^{-2})dx = dt$

$I=\int\frac{dt}{t}=ln|t|+c$

$= ln|x^2 + x^{-1}| + c$

$=ln\left|\frac{x^3+1}{x}\right|+c$

Ellipse

11. If the normal to the ellipse $3x^2 + 4y^2 = 12$ at a point P on it is parallel to the line, $2x + y = 4$ and the tangent to the ellipse at P passes through Q(4, 4) then PQ is equal to :

यदि दीर्घवृत्त $3x^2 + 4y^2 = 12$ के एक बिन्दु P पर अभिलम्ब, रेखा $2x + y = 4$ के समान्तर है तथा P पर दीर्घवृत्त की स्पर्श रेखा Q(4, 4) से होकर जाती है, तो PQ बराबर है :

(1) $\frac{\sqrt{61}}{2}$ (2) $\frac{5\sqrt{5}}{2}$ (3) $\frac{\sqrt{157}}{2}$ (4) $\frac{\sqrt{221}}{2}$

A. 2

sol. Slope of tangent at point P is $\frac{1}{2}$

$3x^2 + 4y^2 = 12 \quad\Rightarrow\quad \frac{x^2}{2^2}+\frac{y^2}{(\sqrt{3})^2}=1$

Let point $P\left(2\cos\theta,\sqrt{3}\sin\theta\right)$

$\frac{x}{2}\cos\theta+\frac{y}{\sqrt{3}}\sin\theta=1$

$\Rightarrow \quad m_T = -\frac{\sqrt{3}}{2}\cot\theta = \frac{1}{2}$

$\tan\theta = -\sqrt{3} \quad \Rightarrow \quad \theta = \pi - \frac{\pi}{3} \quad$ or $\quad \theta = 2\pi - \frac{\pi}{3}$

If $\theta = \frac{2\pi}{3}$, then $P\left(-1, \frac{3}{2}\right)$ and $PQ = \frac{5\sqrt{5}}{2}$

$\theta = \frac{5\pi}{3}$, then tangent does not pass through Q(4, 4)

Sequence & Progression

12. Let S_n denote the sum of the first n terms of an A.P. If $S_4 = 16$ and $S_6 = -48$, then S_{10} is equal to :

माना S_n एक समान्तर श्रेढ़ी के प्रथम n पदों के योग को दर्शाता है। यदि $S_4 = 16$ तथा $S_6 = -48$ है, तो S_{10} बराबर है :

(1) –260 (2) –380 (3) –320 (4) –410

A. 3

sol. $S_4 = 16, S_6 = -48$

$2(2a + 3d) = 16$

$\Rightarrow \quad 2a + 3d = 8$

Also $3[2a + 5d] = -48$

$\Rightarrow \quad 2a + 5d = -16$

$2d = -24$

$d = -12 \quad \Rightarrow \quad a = 22$

$S_{10} = 5(44 + 9(-12))$

$= -320$

Tangent & Normal

13. A 2 m ladder leans against a vertical wall. If the top of the ladder begins to slide down the wall at the rate 25 cm/sec., then the rate (in cm/sec.) at which the bottom of the ladder slides away from the wall on the horizontal ground when the top of the ladder is 1 m above the ground is :

2 m लम्बी एक सीढ़ी एक ऊर्ध्वाधर दीवार के साथ झुकी हुई है। यदि सीढ़ी का शिखर 25 cm/sec. की दर से दीवार के साथ नीचे की ओर फिसलना शुरू करता है, तो वह दर (cm/sec में), जिससे सीढ़ी का पाद, क्षैतिज धरातल पर, दीवार से दूर फिसलता है जब सीढ़ी का शिखर धरातल से 1 m की ऊँचाई पर है, है :

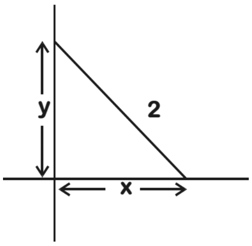
(1) $\frac{25}{3}$ (2) $25\sqrt{3}$ (3) $\frac{25}{\sqrt{3}}$ (4) 25

A. 3

sol. Given $\frac{dy}{dt} = -25$ at $y = 1$

$x^2 + y^2 = 4$

when $y = 1$, $x = \sqrt{3}$



$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\Rightarrow \quad x\frac{dx}{dt} + y\frac{dy}{dt} = 0$$

$$\Rightarrow \quad \sqrt{3}\frac{dx}{dt} + (-25) = 0$$

$$\Rightarrow \quad \frac{dx}{dt} + \frac{25}{\sqrt{3}} \text{cm / s}$$

P & C

14. The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct, is :

31 वस्तुओं, जिनमें 10 समरूप (identical) हैं तथा 21 भिन्न हैं, में से 10 वस्तुओं के चुने जाने के तरीकों की संख्या है :

(1) $2^{20} + 1$ (2) 2^{21} (3) $2^{20} - 1$ (4) 2^{20}

A. 4

sol. Number of ways of selecting 10 objects

= (10I, 0D) or (9I, 1D) or (8I, 1D) or ... (0I, 10D)

where D signifies distinct object and I indicates identical object

$= 1 + {}^{21}C_1 + {}^{21}C_2 + ... + {}^{21}C_{10}$

$$= \frac{2^{21}}{2} = 2^{20}$$

Hyperbola

15. Let P be the point of intersection of the common tangents to the parabola $y^2 = 12x$ and the hyperbola $8x^2 - y^2 = 8$. If S and S' denote the foci of the hyperbola where S lies on the positive x-axis then P divides SS' in a ratio :

माना परवलय $y^2 = 12x$ तथा अतिपरवलय $8x^2 - y^2 = 8$ की उभयनिष्ठ स्पर्श रेखाओं का प्रतिच्छेदन बिन्दु P है। यदि S तथा S' अतिपरवलय की नाभियाँ हैं, जहाँ S धनात्मक x–अक्ष पर स्थित है, तो P, SS' को निम्न में से किस अनुपात में विभाजित करता है?

(1) 13 : 11 (2) 14 : 13 (3) 5 : 4 (4) 2 : 1

A. 3

sol. Equation of tangent to $y^2 = 12x$ is $y = mx + \frac{3}{m}$

Equation of tangent $\frac{x^2}{1} - \frac{y^2}{8} = 1$ is $y = mx \pm \sqrt{m^2 - 8}$

for common tangent,

$$\frac{3}{m} = \pm\sqrt{m^2 - 8} \quad \Rightarrow \quad \frac{9}{m^2} = m^2 - 8$$

Put $m^2 = t$

$t^2 - 8t - 9 = 0 \quad \Rightarrow \quad t^2 - 9t + t - 9 = 0$

$\Rightarrow \quad (t + 1)(t - 9) = 0$

$\because \quad t = m^2 \geq 0 \quad \Rightarrow \quad t = m^2 = 9$

$\Rightarrow \quad m = \pm 3$

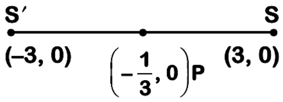
$\Rightarrow$ Equation of tangent is $y = 3x + 1$

or $y = -3x - 1$

Intersection point $P\left(-\frac{1}{3}, 0\right)$

$8 = 1\,(e^2 - 1) \quad \Rightarrow \quad e = 3$

foci $(\pm 3, 0) \quad \Rightarrow$



$$\frac{S'P}{SP} = \frac{3 - \frac{1}{3}}{3 + \frac{1}{3}} = \frac{8}{10} = \frac{4}{5}$$

MOD

16. If $e^y + xy = e$, the ordered pair $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}\right)$ at $x = 0$ is equal to :

यदि $e^y + xy = e$, तो $x = 0$ पर क्रमित युग्म $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}\right)$ बराबर है :

(1) $\left(\frac{1}{e}, -\frac{1}{e^2}\right)$ (2) $\left(-\frac{1}{e}, -\frac{1}{e^2}\right)$ (3) $\left(-\frac{1}{e}, \frac{1}{e^2}\right)$ (4) $\left(\frac{1}{e}, \frac{1}{e^2}\right)$

A. 3

sol. $e^y + xy = e$ ...(i)

Put $x = 0$ in (i)

$\Rightarrow \quad e^y = e \quad \Rightarrow \quad y = 1$

Differentiate (i) w.r. to x

$$e^y \frac{dy}{dx} + x\frac{dy}{dx} + y = 0 \qquad ...(ii)$$

Put $y = 1$ in (ii)

$$e\frac{dy}{dx} + 0 + 1 = 0 \quad \Rightarrow \quad \frac{dy}{dx} = -\frac{1}{e}$$

Differentitate (ii) w.r.t. to x

$$e^y \frac{d^2y}{dx^2} + \frac{dy}{dx}.e^y\frac{dy}{dx} + x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} = 0 \qquad ....(iii)$$

Put y = 1, x = 0, $\frac{dy}{dx} = -\frac{1}{e}$

$e\frac{d^2y}{dx^2} + \frac{1}{e} - \frac{2}{e} = 0 \quad \Rightarrow \quad \frac{d^2y}{dx^2} = \frac{1}{e^2}$

$\Rightarrow \quad \left(\frac{dy}{dx}, \frac{d^2y}{dx^2}\right) \equiv \left(-\frac{1}{e}, \frac{1}{e^2}\right)$

Statistics

17. If the data $x_1, x_2, ..., x_{10}$ is such that the mean of first four of these is 11, the mean of the remaining six is 16 and the sum of squares of all of these is 2,000; then the standard deviation of this data is :

यदि आँकड़ें $x_1, x_2, ..., x_{10}$ इस प्रकार हैं कि इनमें से प्रथम चार का माध्य 11 है, बाकी छः का माध्य 16 है तथा इन सभी के वर्गों का योग 2,000 है, तो इन आँकड़ों का मानक विचलन है :

(1) $2\sqrt{2}$ (2) 4 (3) 3 (4) $\sqrt{2}$

A. 3

sol. $\frac{x_1 + x_2 + x_3 + x_4}{4} = 11$ and $x_1 + x_2 + x_3 + x_4 = 44$

$\frac{x_5 + x_6 + ... + x_{10}}{6} = 16 \Rightarrow x_5 + x_6 + ... + x_{10} = 96$

$x_1^2 + x_2^2 + ... + x_{10}^2 = 2000$

$\sigma^2 = \frac{\Sigma x_i^2}{N} - (\overline{x})^2$

$= \frac{2000}{10} - \left(\frac{140}{10}\right)^2 = 4$

$\Rightarrow \quad \sigma = 2$

Binomial Theorem

18. The coefficient of x^{18} in the product $(1 + x)(1 - x)^{10}(1 + x + x^2)^9$ is :

गुणनफल $(1 + x)(1 - x)^{10}(1 + x + x^2)^9$ में x^{18} का गुणांक है :

(1) 84 (2) –126 (3) –84 (4) 126

A. 1

sol. $(1 - x)^{10}(1 + x + x^2)^9(1 \times x)$

$= (1 - x^3)^9(1 - x^2)$

$= (1 - x^3)^9 - x^2(1 - x^3)^9$

$\Rightarrow$ Coefficient of x^{18} in $(1 - x^3)^9$ – coeff. of x^{16} in $(1 - x^3)^9$

$$^9C_6 = \frac{9!}{6!3!} = \frac{7\times8\times9}{6} = 84$$

Mathematical Reasoning

19. If the truth value of the statement $p \rightarrow (\sim q \vee r)$ is false(F), then the truth values of the statements p, q, r are respectively :

यदि कथन $p \rightarrow (\sim q \vee r)$ का सत्य मान असत्य (F) है, तो कथनों p, q, r के सत्यमान क्रमशः हैं :

(1) F, T, T (2) T, T, F (3) T, F, F (4) T, F, T

A. 2

sol. $P \rightarrow (\sim q \vee r)$ is a fallacy

$\Rightarrow$ P is True and $\sim q \vee r$ is False

$\Rightarrow$ P is True and $\sim q$ is False and r is False

$\Rightarrow$ Truth values of p, q, r are T, T, F respectively

Probability

20. Let a random variable X have a binomial distribution with mean 8 and variance 4. If $P(X \le 2) = \frac{k}{2^{16}}$, then k is equal to :

माना एक यादृच्छिक चर X के द्विपद बंटन का माध्य 8 तथा प्रसरण 4 है। यदि $P(X \le 2) = \frac{k}{2^{16}}$ है, तो k बराबर है :

(1) 121 (2) 1 (3) 17 (4) 137

A. 4

sol. $\mu = 8, \sigma^2 = 4$

$\Rightarrow$ $\mu = np = 8, \sigma^2 = npq = 4, p + q = 1$

$\Rightarrow$ $q = \frac{1}{2}, p = \frac{1}{2}, n = 16$

$$P(X \le 2) = \frac{k}{2^{16}}$$

$$^{16}C_0\left(\frac{1}{2}\right)^{16} + {}^{16}C_1\left(\frac{1}{2}\right)^{16} + {}^{16}C_2\left(\frac{1}{2}\right)^{16} = \frac{k}{2^{16}}$$

$\Rightarrow$ $k = (1 + 16 + 120) = 137$

Vector

21. Let $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ be two vectors. If a vector perpendicular to both the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ has the magnitute 12 then one such vector is :

माना $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ तथा $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ दो सदिश हैं। यदि दोनों सदिशों $\vec{a} + \vec{b}$ तथा $\vec{a} - \vec{b}$ के लम्बवत् एक सदिश का

परिमाण 12 है, तो एक ऐसा सदिश है :

(1) $4(-2\hat{i}-2\hat{j}+\hat{k})$ (2) $4(2\hat{i}+2\hat{j}-\hat{k})$ (3) $4(2\hat{i}+2\hat{j}+\hat{k})$ (4) $4(2\hat{i}-2\hat{j}-\hat{k})$

A. 4

sol. Let vector be $\lambda\left[\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)\right]$

$\vec{a}+\vec{b}=4\hat{i}+4\hat{j}$

$\vec{a}-\vec{b}=2\hat{i}+4\hat{k}$

vector $=\lambda\left[\left(4\hat{i}+4\hat{j}\right)\times\left(2\hat{i}+4\hat{k}\right)\right]$

$=\lambda\left[16\hat{i}-16\hat{j}-8\hat{k}\right]$

$=8\lambda\left[2\hat{i}-2\hat{j}-\hat{k}\right]$

$\Rightarrow \quad 12=8|\lambda|\sqrt{4+4+1}$

$|\lambda|=\frac{1}{2}$

Hence required vector is $\pm4(2\hat{i}-2\hat{j}-\hat{k})$

Differential Equation

22. Consider the differential equation, $y^2dx+\left(x-\frac{1}{y}\right)dy=0$. If value of y is 1 when x = 1, then the value of x for with y = 2, is:

अवकल समीकरण $y^2dx+\left(x-\frac{1}{y}\right)dy=0$ पर विचार कीजिए। यदि x = 1 पर y का मान 1 है, तो x का मान, जिसके लिए y = 2 है, है:

(1) $\frac{5}{2}+\frac{1}{\sqrt{e}}$ (2) $\frac{3}{2}-\sqrt{e}$ (3) $\frac{3}{2}-\frac{1}{\sqrt{e}}$ (4) $\frac{1}{2}+\frac{1}{\sqrt{e}}$

A. 3

sol. $y^2dx+\left(x-\frac{1}{y}\right)dy=0$

$\frac{dx}{dy}+\left(\frac{1}{y^2}\right)x=\frac{1}{y^3}$

$\text{I.F.}=e^{\int\frac{1}{y^2}dy}=e^{-\frac{1}{y}}$

its solution is

$x.e^{-\frac{1}{y}}=\int e^{-\frac{1}{y}}\frac{1}{y^3}dy+c$

put $-\frac{1}{y} = t \Rightarrow \frac{1}{y^2}dy = dt$

$\Rightarrow \quad x.e^{-\frac{1}{y}} = -\int te^t dt + c = -te^t + e^t + c$

$x.e^{-\frac{1}{y}} = e^{-\frac{1}{y}}\left(\frac{1}{y}+1\right)+c$ passes through (1, 1)

$\Rightarrow \quad 1 = 2 + ce \quad \Rightarrow \quad c = -\frac{1}{e}$

$\Rightarrow \quad x\left(1+\frac{1}{y}\right)-\frac{1}{e}e^{\frac{1}{y}}$ passes through (k, 2)

$\Rightarrow \quad k = \frac{3}{2}-\frac{1}{\sqrt{e}}$

Definite Integration

23. Let $f : R \to R$ be a continuously differentiable function such that $f(2) = 6$ and $f'(2) = \frac{1}{48}$. If $\int_6^{f(x)} 4t^3\, dt = (x-2)g(x)$, then $\lim_{x\to 2} g(x)$ is equal to :

माना $f : R \to R$ एक संततत: अवकलनीय (continuously differentiable) फलन इस प्रकार है कि $f(2) = 6$ तथा $f'(2) = \frac{1}{48}$.

यदि $\int_6^{f(x)} 4t^3\, dt = (x-2)g(x)$, तो $\lim_{x\to 2} g(x)$ बराबर है :

(1) 18 (2) 36 (3) 24 (4) 12

A. 1

sol. $\int_6^{f(x)} 4t^3 dt = (x-2)\,g(x)$

$4(f(x)).\, f'(x) = g'(x)(x-2) + g(x)$

put $x = 2$,

$\frac{4(6)^3.1}{48}g(2)$

$\lim_{x\to 2} g(x) = 18$

Straight Line

24. The equation $y = \sin x \sin(x + 2) - \sin^2(x + 1)$ represents a straight line lying in :

(1) Third and fourth quadrants only (2) First, third and fourth quadrants

(3) First, second and fourth quadrants (4) Second and third quadrants only

समीकरण $y = \sin x \sin(x + 2) - \sin^2(x + 1)$ एक सरल रेखा को निरूपित करता है, जो स्थित है :

(1) मात्र तीसरे तथा चौथे चतुर्थांश में। (2) पहले, तीसरे तथा चौथे चतुर्थांश में।

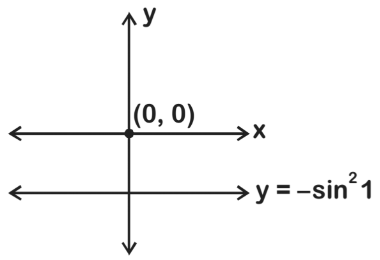
(3) पहले, दूसरे तथा चौथे चतुर्थांश में।　　(4) मात्र दूसरे तथा तीसरे चतुर्थांश में।

A. 1

sol. $y = \sin x \cdot \sin(x+2) - \sin^2(x+1)$

$= \frac{1}{2}\cos(-2) - \frac{\cos(2x+2)}{2} - \left[\frac{1-\cos(2x+2)}{2}\right]$

$= \frac{(\cos 2) - 1}{2} = -\sin^2 1$



Graph of y lies in
III and IV Quadrant

Area Under Curve

25. If the area (in sq. units) of the region $\{(x, y) : y^2 \le 4x, x + y \le 1, x \ge 0, y \ge 0\}$ is $a\sqrt{2} + b$, then $a - b$ is equal to :

यदि क्षेत्र $\{(x, y) : y^2 \le 4x, x + y \le 1, x \ge 0, y \ge 0\}$ का क्षेत्रफल (वर्ग इकाइयों में) $a\sqrt{2} + b$ है, तो $a - b$ बराबर है :

(1) $-\frac{2}{3}$　(2) 6　(3) $\frac{10}{3}$　(4) $\frac{8}{3}$

A. 2

sol. $y^2 = 4x$

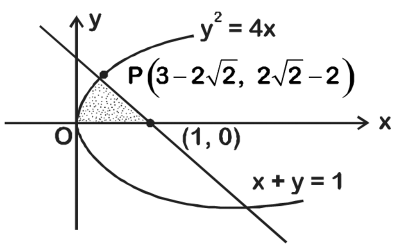
$x + y = 1$

$y^2 = 4(1 - y)$

$y^2 + 4y - 4 = 0$

$(y + 2)^2 = 8$

$y + 2 = \pm 2\sqrt{2}$



required area

$= \int_0^{3-2\sqrt{2}} 2\sqrt{x}\,dx + \frac{1}{2} \times (2\sqrt{2} - 2) \times (2\sqrt{2} - 2)$

$= \left[2 \times \frac{2}{3} x^{3/2}\right]_0^{3-2\sqrt{2}} + \frac{1}{2}(8 + 4 - 8\sqrt{2})$

$= \frac{4}{3} \times (3 - 2\sqrt{2})\sqrt{3 - 2\sqrt{2}} + 6 - 4\sqrt{2}$

$=\frac{4}{3}\times\left(3-2\sqrt{2}\right)\left(\sqrt{2}-1\right)+6-4\sqrt{2}$

$=\frac{4}{3}\times\left(3\sqrt{2}-3-4+2\sqrt{2}\right)+6-4\sqrt{2}$

$=\left(6-\frac{28}{3}\right)+\left(\frac{20}{3}-4\right)\sqrt{2}$

$=-\frac{10}{3}+\frac{8}{3}\sqrt{2}$

$\Rightarrow \quad a-b=\frac{10}{3}+\frac{8}{3}=6$

ITF

26. The value of $\sin^{-1}\left(\frac{12}{13}\right)-\sin^{-1}\left(\frac{3}{5}\right)$ is equal to :

$\sin^{-1}\left(\frac{12}{13}\right)-\sin^{-1}\left(\frac{3}{5}\right)$ का मान है :

(1) $\frac{\pi}{2}-\sin^{-1}\left(\frac{56}{65}\right)$ (2) $\pi-\sin^{-1}\left(\frac{63}{65}\right)$ (3) $\pi-\cos^{-1}\left(\frac{33}{65}\right)$ (4) $\frac{\pi}{2}-\cos^{-1}\left(\frac{9}{65}\right)$

A. 1

sol. $-\sin^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{12}{13}\right)=-\sin^{-1}\left(\frac{3}{5}\times\frac{5}{13}-\frac{12}{13}\times\frac{4}{5}\right)$

$(\because\ xy\geq 0 \text{ and } x^2+y^2\leq 1)$

$=-\sin^{-1}\left(\frac{-33}{65}\right)$

$=-\sin^{-1}\left(\frac{33}{65}\right)$

$=\cos^{-1}\left(\frac{56}{65}\right)$

$=\frac{\pi}{2}-\sin^{-1}\left(\frac{56}{65}\right)$

Definite Integration

27. If $\int_0^{\pi/2}\frac{\cot x}{\cot x+\operatorname{cosec} x}dx=m(\pi+n)$, then m . n is equal to :

यदि $\int_0^{\pi/2}\frac{\cot x}{\cot x+\operatorname{cosec} x}dx=m(\pi+n)$, तो m . n बराबर है :

(1) $\frac{1}{2}$ (2) 1 (3) -1 (4) $-\frac{1}{2}$

A. 3

sol. $\int_0^{\pi/2} \frac{\cot x dx}{\cot x + \operatorname{cosec} x}$

$\int_0^{\pi/2} \frac{\cos x\, dx}{1+\cos x} = \int_0^{\pi/2}\left(1 - \frac{1}{1+\cos x}\right)dx$

$= [x]_0^{\pi/2} - \int_0^{\pi/2} \frac{1}{2\cos^2 \frac{x}{2}} dx$

$= \frac{\pi}{2} - \frac{1}{2}\int_0^{\pi/2} \sec^2 \frac{x}{2} dx$

$= \frac{\pi}{2} - \left[\tan\frac{x}{2}\right]_0^{\pi/2}$

$= \frac{\pi}{2} - [1] = \left(\frac{\pi}{2} - 1\right)$

$m = \frac{1}{2}, n = -2$

$\Rightarrow \quad mn = -1$

Function

28. For $x \in (0, 3/2)$, let $f(x) = \sqrt{x}$, $g(x) = \tan x$ and $h(x) = \frac{1-x^2}{1+x^2}$. If $\phi(x) = ((hof)og)(x)$, then $\phi\left(\frac{\pi}{3}\right)$ is equal to

$x \in (0, 3/2)$ के लिए माना $f(x) = \sqrt{x}$, $g(x) = \tan x$ तथा $h(x) = \frac{1-x^2}{1+x^2}$. यदि $\phi(x) = ((hof)og)(x)$, तो $\phi\left(\frac{\pi}{3}\right)$ बराबर है:

(1) $\tan\frac{5\pi}{12}$ (2) $\tan\frac{\pi}{12}$ (3) $\tan\frac{11\pi}{12}$ (4) $\tan\frac{7\pi}{12}$

A. 3

sol. $\phi\left(\frac{\pi}{3}\right) = h\left(f\left(g\left(\frac{\pi}{3}\right)\right)\right)$

$= h\left(f\left(\sqrt{3}\right)\right) = h(3^{1/4})$

$= \frac{1-\sqrt{3}}{1+\sqrt{3}} = -\frac{1}{2}(1+3-2\sqrt{3}) = \sqrt{3}-2 = -(-\sqrt{3}+2)$

$= \tan 15^\circ = \tan(180^\circ - 15^\circ) = \tan\left(\pi - \frac{\pi}{12}\right)$

$= \tan\dfrac{11\pi}{12}$

Complex Number

29. The equation $|z-i| = |z-1|$, $i = \sqrt{-1}$, represents :

(1) The line through the origin with slope –1 (2) A circle of radius $\frac{1}{2}$

(3) A circle of radius 1 (4) The line through the origin with slope 1

समीकरण $|z-i| = |z-1|$, $i = \sqrt{-1}$, निम्न में से किसको निरूपित करती है?

(1) मूलबिन्दु से होकर जाने वाली रेखा जिसका ढाल –1 है।

(2) त्रिज्या $\frac{1}{2}$ का एक वृत्त।

(3) त्रिज्या 1 का एक वृत्त।

(4) मूलबिन्दु से होकर जाने वाली रेखा जिसका ढाल 1 है।

A. 4

sol. $|z-1| = |z-i|$

Let $z = x + iy$

$(x-1)^2 + y^2 = x^2 + (y-1)^2$

$1 - 2x = 1 - 2y$

$\Rightarrow \quad x - y = 0$

Locus is straight line with slope 1

Trig Equation

30. The number of solutions of the equation $1 + \sin^4 x = \cos^2 3x$, $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ is :

समीकरण $1 + \sin^4 x = \cos^2 3x$, $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ के हलों की संख्या है :

(1) 3 (2) 5 (3) 4 (4) 7

A. 2

sol. $1 + \sin^4 x = \cos^2 3x$

L.H.S $= 1 + \sin^4 x$, R.H.S $= \cos^2 3x$

L.H.S ≥ 1 R.H.S ≤ 1

$\Rightarrow \quad$ L.H.S. = R.H.S. = 1

$\sin^4 x = 0$ and $\cos^2 3x = 1$

$\sin x = 0$ and $(4\cos^2 x - 3)^2 \cos^2 x = 1$

$\Rightarrow \quad \sin x = 0$ and $\cos^2 x = 1$

$\Rightarrow \quad x = 0, \pm\pi, \pm 2\pi$

$\Rightarrow \quad$ Total number of solutions is 5