MATHS
12 APRIL 2019 [Phase : II]
JEE MAIN PAPER ONLINE

Ellipse

1. An ellipse, with foci at (0, 2) and (0, –2) and minor axis of length 4, passes through which of the following points?
एक दीर्घवृत्त, जिसकी नाभियाँ (0, 2) तथा (0, –2) पर हैं तथा जिसके लघु अक्ष की लम्बाई 4 है, निम्न में से किस बिन्दु से होकर जाता है?

(1) $\left(\sqrt{2},2\right)$ (2) $\left(2,2\sqrt{2}\right)$ (3) $\left(1,2\sqrt{2}\right)$ (4) $\left(2,\sqrt{2}\right)$

A. 1

sol. Let the equation of ellipse : $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\because$ be = 2 and a = 2 (Given)
$\because$ $a^2 = b^2 (1 - e^2)$
$\Rightarrow$ $4 = b^2 - 4$
$\Rightarrow$ $b = 2\sqrt{2}$

Equation of ellipse will be $\frac{x^2}{4}+\frac{y^2}{8}=1$

Only $(\sqrt{2},2)$ satisfies this equation

Probability

2. For an initial screening of an admission test a candidate is given fifty problems to solve.If the probability that the candidate can solve any problems is $\frac{4}{5}$ then the probability that he is unable to solve less than two problems is:
प्रारंभिक जाँच के लिए एक प्रवेश परीक्षा में एक परीक्षार्थी को पचास प्रश्न हल करने के लिए दिए गए हैं। यदि परीक्षार्थी के किसी एक प्रश्न को हल कर सकने की प्रायिकता $\frac{4}{5}$ है, तो उसके दो से कम प्रश्नों को हल करने में असमर्थ होने की प्रायिकता है :

(1) $\frac{316}{25}\left(\frac{4}{5}\right)^{48}$ (2) $\frac{54}{5}\left(\frac{4}{5}\right)^{49}$ (3) $\frac{201}{5}\left(\frac{1}{5}\right)^{49}$ (4) $\frac{164}{25}\left(\frac{1}{5}\right)^{48}$

A. 2

sol. Let p is the probability that candidate can solve a problem.

$p=\frac{4}{5};\ q=\frac{1}{5}$ $(\because\ p+q=1)$

Probability that candidate is able to solve either 50 or 49 problems

$= {}^{50}C_{50}\,p^{50}.\,q^0 + {}^{50}C_{49}\,.\,p^{49}.\,q^1$

$= p^{49}\,[p + 50q]$

$=\left(\frac{4}{5}\right)^{49}.\left(\frac{4}{5}+\frac{50}{5}\right)$

$=\frac{54}{5}.\left(\frac{4}{5}\right)^{49}$

MOD

3. The derivative of $\tan^{-1}\left(\frac{\sin x - \cos x}{\sin x + \cos x}\right)$ with repect to $\frac{x}{2}$ where $\left(x \in \left(0, \frac{\pi}{2}\right)\right)$ is

$\frac{x}{2}$ के सापेक्ष $\tan^{-1}\left(\frac{\sin x - \cos x}{\sin x + \cos x}\right)$, जहाँ $\left(x \in \left(0, \frac{\pi}{2}\right)\right)$ का अवकलज है :

(1) $\frac{1}{2}$ (2) $\frac{2}{3}$ (3) 2 (4) 1

A. 3

sol. $f(x) = \tan^{-1}\left(\frac{\tan x - 1}{\tan x + 1}\right) = -\tan^{-1}\left(\tan\left(\frac{\pi}{4} - x\right)\right)$

$\because \quad \frac{\pi}{4} - x \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

So, $f(x) = -\left(\frac{\pi}{4} - x\right) = x - \frac{\pi}{4}$

Let $y = \frac{x}{2}$

$$\frac{d}{dy} f(x) = \frac{\frac{d}{dx} f(x)}{\frac{d}{dx} y} = \frac{1}{½} = 2$$

Tangent & Normal

4. The tangents to the curve $y = (x-2)^2 - 1$ at its points of intersection with the line $x - y = 3$, intersect at the point:
वक्र $y = (x - 2)^2 - 1$ के रेखा $x - y = 3$ से प्रतिच्छेदन बिन्दुओं पर वक्र की स्पर्शरेखायें निम्न में से किस बिन्दु पर मिलती हैं?

(1) $\left(\frac{5}{2}, 1\right)$ (2) $\left(-\frac{5}{2}, 1\right)$ (3) $\left(\frac{5}{2}, -1\right)$ (4) $\left(-\frac{5}{2}, -1\right)$

A. 3

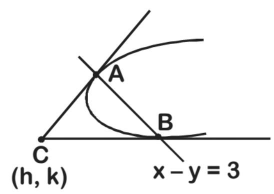
sol. Equation of chord is $T = 0$

$\Rightarrow \quad \frac{1}{2}(y + k) = (x - 2)(h - 2) - 1$

$\Rightarrow \quad \frac{y + k}{2} = xh - 2x - 2h + 3$

$\Rightarrow \quad (2h - 4)x - y - 4h + 6 - k = 0$

Given $x - y - 3 = 0$



$\Rightarrow \quad \frac{2h - 4}{1} = \frac{4h - 6 + k}{3} = 1$

$\Rightarrow \quad h = \frac{5}{2}, k = -1$

Area Under Curve

5. If the area (in sq units) bounded by the parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$, $\lambda > 0$, is $\frac{1}{9}$ then λ is equal to

यदि परवलय $y^2 = 4\lambda x$ तथा रेखा $y = \lambda x$, $\lambda > 0$, से घिरे क्षेत्र का क्षेत्रफल (वर्ग इकाइयों में) $\frac{1}{9}$ है, तो λ बरबार है :

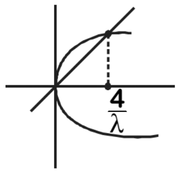
(1) 48 (2) 24 (3) $4\sqrt{3}$ (4) $2\sqrt{6}$

A. 2

sol. $y^2 = 4\lambda x$ and $y = \lambda x$

On solving ; $(\lambda x)^2 = 4\lambda x$

$x = 0, \frac{4}{\lambda}$



Required area $= \int_0^{\frac{4}{\lambda}} \left(2\sqrt{\lambda x} - \lambda x\right) dx$

$$= \frac{2\sqrt{\lambda}.x^{3/2}}{3/2} - \frac{\lambda x^2}{2}\Bigg|_0^{4/\lambda}$$

$$= \frac{32}{3\lambda} - \frac{8}{\lambda} = \frac{8}{3\lambda} = \frac{1}{9}$$

$\lambda = 24$

P & C

6. A group of students comprises of 5 boys and n girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then n is equal to :

विद्यार्थियों के एक समूह में 5 लड़के तथा n लड़कियाँ हैं। यदि इस समूह में से तीन विद्यार्थियों की टीम यादृच्छिक इस प्रकार चुनने के तरीके, कि प्रत्येक टीम में कम से कम एक लड़का तथा कम से कम एक लड़की हो, 1750 है, तो n बराबर है :

(1) 24 (2) 27 (3) 25 (4) 28

A. 3

sol. Number of ways of selecting three persons such that there is atleast one boy and atleast one girl in the selected persons

$$= {}^{n+5}C_3 - {}^{n}C_3 - {}^{5}C_3 = 1750$$

$$\Rightarrow \quad \frac{(n+5)(n+4)(n+3)}{6} - \frac{n(n-1)(n-2)}{6} = 1760$$

$$\Rightarrow \quad n^2 + 3n - 700 = 0$$

$\Rightarrow$ n = – 28 (rejected) or n = 25

Sequence & Progression

7. If $a_1, a_2, a_3, ...$ are in A.P. such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this A.P. is :

यदि $a_1, a_2, a_3, ...$ एक समान्तर श्रेढ़ी में इस प्रकार हैं कि $a_1 + a_7 + a_{16} = 40$ है, तो इस समान्तर श्रेढ़ी के प्रथम 15 पदों का योगफल है :

(1) 150 (2) 280 (3) 200 (4) 120

A. 3

sol. Let the common difference is 'd'.

$$a_1 + a_7 + a_{16} = 40$$

$\Rightarrow \quad a_1 + a_1 + 6d + a_1 + 15d = 40$

$\Rightarrow \quad 3a_1 + 21d = 40$

$\Rightarrow \quad a_1 + 7d = \dfrac{40}{3}$

$$S_{15} = \frac{15}{2}[2a_1 + 14d]$$
$$= 15(a_1 + 7d)$$
$$= 15\left(\frac{40}{3}\right)$$
$$= 200$$

Height & Distance

8. The angle of elevation of the top of a vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30 m vertically above the point A. If the angle of elevation of the top of the tower from B be 30°, then the distance (in m) of the foot of the tower from the point A is :

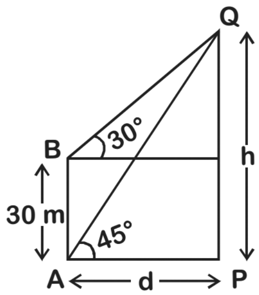
क्षैतिज तल पर खड़ी एक ऊर्ध्वाधर मीनार के शिखर का तल पर एक बिन्दु A से उन्नयन कोण 45° है। माना बिन्दु A से 30 m ऊर्ध्वाधर ऊपर बिन्दु B है। यदि B से मीनार के शिखर का उन्नयन कोण 30° है, तो मीनार के पाद की बिन्दु A से दूरी (मीटर में) है :

(1) $15(3+\sqrt{3})$ (2) $15(1+\sqrt{3})$ (3) $15(3-\sqrt{3})$ (4) $15(5-\sqrt{3})$

A. 1

sol. Let the height of the tower be h.

Refer to diagram ;



$\tan 45^\circ = \dfrac{h}{d} = 1$

$h = d$ ....(i)

$\tan 30^\circ = \dfrac{h-30}{d}$

$\sqrt{3}(h-30) = d$ ....(ii)

from (i) and (ii)

$\sqrt{3}d = d + 30\sqrt{3}$

$$d = \frac{30\sqrt{3}}{\sqrt{3}-1} = 15\sqrt{3}\left(\sqrt{3}+1\right) = 15\left(3+\sqrt{3}\right)$$

Function

9. If[x] denotes the greatest integer $\le$ x, then the system of linear equation

$[\sin\theta]x + [-\cos\theta]y = 0$ $[\cot\theta]x + y = 0$ $[\cot\theta]x + y = 0$

(1) Has a unique solution if $\theta \in \left(\dfrac{\pi}{2}, \dfrac{2\pi}{3}\right)$ and have infinitely many solutions if $\theta \in \left(\pi, \dfrac{7\pi}{6}\right)$

(2) Has a unique solution if $\theta \in \left(\dfrac{\pi}{2}, \dfrac{2\pi}{3}\right) \cup \left(\pi, \dfrac{7\pi}{6}\right)$

(3) Have infinitely many solutions if $\theta \in \left(\dfrac{\pi}{2}, \dfrac{2\pi}{3}\right)$ and has a unique solutions if $\theta \in \left(\pi, \dfrac{7\pi}{6}\right)$

(4) Have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

यदि [x] महत्तम पूर्णांक ≤ x है, तो रैखिक समीकरण निकाय $[\sin\theta]x + [-\cos\theta]y = 0$ $[\cot\theta]\, x + y = 0$ $[\cot\theta]\, x + y = 0$

(1) का मात्र एक हल है यदि $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ तथा अनन्त हल हैं यदि $\theta \in \left(\pi, \frac{7\pi}{6}\right)$

(2) का मात्र एक हल है यदि $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

(3) के अनन्त हल हैं यदि $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ तथा मात्र एक हल है यदि $\theta \in \left(\pi, \frac{7\pi}{6}\right)$

(4) के अनन्त हल हैं यदि $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

A. 3

sol. There are two cases.

Case 1 : $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

So ; $[\sin\theta] = 0, [-\cos\theta] = 0, [\cot\theta] = -1$

The system of equations will be ;

$0 \cdot x + 0 \cdot y = 0$ and $-x + y = 0$

(Infinitely many solutions)

Case 2 : $\theta \in \left(\pi, \frac{7\pi}{6}\right)$

So ; $[\sin\theta] = -1, [-\cos\theta] = 0,$

The system of equations will be ;

$-x + 0 \cdot y = 0$ and $[\cot\theta]\, x + y = 0$

Clearly $x = 0$ and $y = 0$ (unique solution)

Determinant

10. A value of $\theta \in (0, \pi/3)$, for which

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0 \text{, is :}$$

$\theta \in (0, \pi/3)$ का एक मान, जिसके लिए

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$$ है, है –

(1) $7\pi/36$ (2) $7\pi/24$ (3*) $\pi/9$ (4) $\pi/18$

Sol. $C_1 \to C_1 \to + C_2$

$$\begin{vmatrix} 2 & \sin^2\theta & 4\cos 6\theta \\ 2 & 1\sin^2\theta & 4\cos 6\theta \\ 1 & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & \sin^2\theta & (1+4\cos 6\theta) \end{vmatrix} = 0$$

$\Rightarrow \quad 2 + 4\cos 6\theta = 0$

$\cos 6\theta = \frac{-1}{2}$

$\because \quad 6\theta \in (0, 2\pi)$

So, $\quad 6\theta = \frac{2\pi}{3} \text{ or } \frac{4\pi}{3}$

$\Rightarrow \quad \theta = \frac{\pi}{9} \text{ or } \frac{2\pi}{9}$

Mathematical Reasoning

11. The boolean expression $\sim(p \Rightarrow (\sim q))$ is equivalent to

बूले का व्यंजक $\sim(p \Rightarrow (\sim q))$ निम्न में से किसके समतुल्य है ?

(1) $(\sim p) \Rightarrow q$ (2) $p \vee q$ (3) $p \wedge q$ (4) $q \Rightarrow -p$

A. 3

sol. $\sim(p \Rightarrow (\sim q))$ $\quad \{\because\ p \Rightarrow q$ is same as $\sim p \vee q\}$

$\equiv \quad \sim((\sim p) \vee (\sim q))$

$\equiv \quad p \wedge q$

Hyperbola

12. The equation of a common tangent to the curves, $y^2 = 16x$ and $xy = -4$, is :

वक्रों $y^2 = 16x$ तथा $xy = -4$ की एक उभयनिष्ठ स्पर्शरेखा का समीकरण है :

(1) $x + y + 4 = 0$ (2) $2x - y + 2 = 0$ (3) $x - 2y + 16 = 0$ (4) $x - y + 4 = 0$

A. 4

sol. $y^2 = 16x$ and $xy = -4$

Equation of tangent to the given parabola ;

$y = mx + \frac{4}{m}$

If this is common tangent, then

$x\left(mx + \frac{4}{m}\right) + 4 = 0$

$\Rightarrow \quad mx^2 + \frac{4}{m}x + 4 = 0$

$D = 0$

$\frac{16}{m^2} = 16m$

$\Rightarrow \quad m^3 = 1 \qquad \Rightarrow m = 1$

Equation of common tangent is $y = x + 4$

Differential Equation

13. The general solution of the differential equation $(y^2 - x^3)\,dx - xydy = 0\ (x \neq 0)$ is :
(where c is a constant of integration)
अवकल समीकरण $(y^2 - x^3)\,dx - xydy = 0\ (x \neq 0)$ का व्यापक हल है :
(जहाँ c एक समाकलन अचर है)

(1) $y^2 + 2x^3 + cx^2 = 0$ (2) $y^2 - 2x^2 + cx^3 = 0$
(3) $y^2 - 2x^3 + cx^2 = 0$ (4) $y^2 + 2x^2 + cx^3 = 0$

A. 1

sol. $y^2dx - xydy = x^3dx$

$\Rightarrow \quad \frac{(ydx - xdy)\,y}{x^2} = xdx$

$\Rightarrow \quad -yd\left(\frac{y}{x}\right) = xdx$

$\Rightarrow \quad -\frac{y}{x}.d\left(\frac{y}{x}\right) = dx$

$\Rightarrow \quad -\frac{1}{2}\left(\frac{y}{x}\right)^2 = x + c_1$

$\Rightarrow \quad 2x^3 + cx^2 + y^2 = 0$

3 D

14. A plane which bisects the angle between the two given planes $2x - y + 2z - 4 = 0$ and $x + 2y + 2z - 2 = 0$, passes through the point :
दो दिए गए समतलों $2x - y + 2z - 4 = 0$ तथा $x + 2y + 2z - 2 = 0$ के बीच के कोण को समद्विभाजित करता एक समतल, निम्न में से किस बिन्दु से होकर जाता है?

(1) (1, –4, 1) (2) (2, –4, 1) (3) (1, 4, –1) (4) (2, 4, 1)

A. 2

sol. Equation of angle bisectors ;

$\frac{x + 2y + 2z - 2}{3} = \pm\frac{2x - y + 2z - 4}{3}$

$\Rightarrow \quad x - 3y - 2 = 0 \quad$ or $\quad 3x + y + 4z - 6 = 0$

Only (2, –4, 1) lies on the second plane

Trig Equation

15. Let S be the set of all $\alpha \in R$ such that the equation, $\cos 2x + \alpha \sin x = 2\alpha - 7$ has a solution. Then S is equal to:
माना सभी $\alpha \in R$, जिसके लिए समीकरण $\cos 2x + \alpha \sin x = 2\alpha - 7$ का एक हल है, का समुच्चय S है, तो S बराबर है:

(1) [1, 4] (2) R (3) [2, 6] (4) [3, 7]

A. 3

sol. $\cos 2x + \alpha \sin x = 2\alpha - 7$

$1 - 2\sin^2 x - \alpha \sin x = 2\alpha - 7$

$\Rightarrow$ $2\sin^2 x - \alpha \sin x + (2\alpha - 8) = 0$

$\Rightarrow$ $\sin x = \dfrac{\alpha \pm \sqrt{\alpha^2 - 8(2\alpha - 8)}}{4}$

$\Rightarrow$ $\sin x = \dfrac{\alpha \pm (\alpha - 8)}{4}$

$\Rightarrow$ $\sin x = 2$ (rejected) or $\sin x = \dfrac{\alpha - 4}{2}$

$\because$ Equation has solution, then $\dfrac{\alpha - 4}{2} \in [1, 1]$

$\Rightarrow$ $\alpha \in [2, 6]$

Vector

16. Let $\alpha \in R$ and the three vectors $\vec{a} = \alpha\hat{i} + \hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}$ and $\vec{c} = \alpha\hat{i} - 2\hat{j} + 3\hat{k}$ then the set

S = {α : $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar}

(1) contains exactly two numbers only one of which is positive
(2) is singleton
(3) contains exactly two positive numbers
(4) is empty

माना $\alpha \in R$ तथा तीन सदिश $\vec{a} = \alpha\hat{i} + \hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}$ और $\vec{c} = \alpha\hat{i} - 2\hat{j} + 3\hat{k}$ हैं, तो समुच्चय

S = {α : $\vec{a}, \vec{b}$ और $\vec{c}$ समतलीय हैं}

(1) में तथ्यतः दो संख्यायें हैं जिनमें केवल एक धनात्मक है।
(2) एकल है।
(3) में तथ्यतः दो धनात्मक संख्यायें हैं।
(4) रिक्त है।

A. 4

Sol. If $\vec{a}, \vec{b}, \vec{c}$ are coplanar, then

$[\vec{a}, \vec{b}, \vec{c}] = 0$

$\Rightarrow$ $\begin{vmatrix} \alpha & 1 & 3 \\ 2 & 1 & -\alpha \\ \alpha & -2 & 3 \end{vmatrix} = 0$

$\Rightarrow$ $\alpha^2 + 6 = 0$

No value of 'α' exist

Set S is an empty set.

3 D

17. The length of the perpendicular drawn from the point (2,1,4) to the plane containing the lines

$\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + 2\hat{j} - 2\hat{k})$ is

रेखाओं $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ तथा $\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + 2\hat{j} - 2\hat{k})$ को अंतर्विष्ट करते समतल पर बिन्दु (2, 1, 4)

से डाले गये लम्ब की लम्बाई है :

(1) $\frac{1}{3}$ (2) 3 (3) $\frac{1}{\sqrt{3}}$ (4) $\sqrt{3}$

A. 4

sol. Equation of plane containing two given lines ;

$$\begin{vmatrix} x-1 & y-1 & z \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix} = 0$$

$\Rightarrow \quad x - y - z = 0$

The length of perpendicular from (2, 1, 4) to this plane $= \left|\frac{2-1-4}{\sqrt{1^2+1^2+1^2}}\right| = \sqrt{3}$

Indefinite Integration

18. Let $\alpha \in (0, \pi/2)$ be fixed. If the integral $\int \frac{\tan x + \tan\alpha}{\tan x - \tan\alpha} dx =$ $A(x)\cos 2\alpha + B(x)\sin 2\alpha + C$, where C is a constant of integration then the functions A(x) and B(x) are respectively

माना $\alpha \in (0, \pi/2)$ दिया है। यदि समाकल $\int \frac{\tan x + \tan\alpha}{\tan x - \tan\alpha} dx =$ $A(x)\cos 2\alpha + B(x)\sin 2\alpha + C$, जहाँ C एक समाकलन अचर है, तो फलन A(x) तथा B(x) क्रमशः हैं:

(1) $x - \alpha$ and $\log_e|\cos(x-\alpha)|$ (2) $x + \alpha$ and $\log_e|\sin(x-\alpha)|$

(3) $x - \alpha$ and $\log_e|\sin(x-\alpha)|$ (4) $x + \alpha$ and $\log_e|\sin(x+\alpha)|$

A. 3

sol. $\int \frac{\tan x + \tan\alpha}{\tan x - \tan\alpha} dx$

$= \int \frac{\sin(x+\alpha)}{\tan(x-\alpha)} dx$ let $x - \alpha = t$

$\Rightarrow$ dx = dt

$= \int \frac{\sin(t+2\alpha)}{\sin t} dt$

$= [\cos 2\alpha + \sin 2\alpha \cdot \cot t]dt$

$= \cos 2\alpha \cdot t + \sin 2\alpha \cdot \ln|\sin t| + c$

$= (x-\alpha)\cdot\cos 2\alpha + \sin 2\alpha \cdot \ln|\sin(x-\alpha)| + c$

Binomial Theorem

19. If ${}^{20}C_1 + (2^2)\,{}^{20}C_2 + (3^2)\,{}^{20}C_3 + \ldots\ldots\ldots\ldots + (20^2)\,{}^{20}C_{20} = A(2^\beta)$, then the ordered pair (A, β) is equal to

यदि ${}^{20}C_1 + (2^2)\,{}^{20}C_2 + (3^2)\,{}^{20}C_3 + \ldots\ldots\ldots\ldots + (20^2)\,{}^{20}C_{20} = A(2^\beta)$, तो क्रमित युग्म (A, β) बराबर है :

(1) (420,19) (2) (380,19) (3) (420,18) (4) (380,18)

A. 3

sol. ${}^{20}C_1 + 2^2 \cdot {}^{20}C_2 + 3^2 \cdot {}^{20}C_3 + \ldots + 20^2 \cdot {}^{20}C_{20}$

$= \sum_{r=1}^{20} r^2 . {}^{20}C_r$

$= 20\sum_{r=1}^{20} r . {}^{19}C_{r-1}$

$= 20\left[\sum_{r=1}^{20}(r-1){}^{19}C_{r-1} + \sum_{r=1}^{20}{}^{19}C_{r-1}\right]$

$= 20\left[19\sum_{r=2}^{20}{}^{18}C_{r-2} + 2^{19}\right]$

$= 20[19 \cdot 2^{18} + 2^{19}]$

$= 20 \times 21 \times 2^{18}$

$= 420 \times 2^{18}$

So; $A = 420$ and $\beta = 18$

Straight Line

20. A triangle has a vertex at (1,2) and the mid points of the two sides through it are (–1,1) and (2,3). Then the centroid of this triangles is

एक त्रिभुज का एक शीर्ष (1,2) पर है तथा इससे होकर जाने वाली दो भुजाओं के मध्य–बिन्दु (–1,1) और (2,3) हैं, तो इस त्रिभुज का केन्द्रक है :

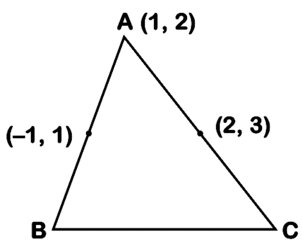
(1) $\left(\frac{1}{3}, 2\right)$ (2) $\left(\frac{1}{3}, \frac{5}{3}\right)$ (3) $\left(\frac{1}{3}, 1\right)$ (4) $\left(1, \frac{7}{3}\right)$

A. 1

sol. Co-ordinates of vertex B and C are B(–3, 0) and C(3, 4)

Centroid $G\left(\frac{3-3+1}{3}, \frac{0+4+2}{3}\right)$

$G\left(\frac{1}{3}, 2\right)$



Quadratic Equation

21. If α, β and γ are three consecutive terms of a non constant G.P. such that the equations $\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root then $\alpha(\beta + \gamma)$ is equal to

यदि एक भिन्न पदों वाली गुणोत्तर श्रेढ़ी के तीन क्रमागत पद α, β तथा γ इस प्रकार हैं कि समीकरणों $\alpha x^2 + 2\beta x + \gamma = 0$ तथा $x^2 + x - 1 = 0$ का एक मूल समान है, तो $\alpha(\beta + \gamma)$ बराबर है :

(1) 0 (2) αγ (3) βγ (4) αβ

A. 3

sol. $\beta^2 = \alpha\gamma$ so roots of the equation $\alpha x^2 + 2\beta x + \gamma = 0$

are $\frac{-2\beta \pm 2\sqrt{\beta^2 - \alpha\gamma}}{2\alpha} = -\frac{\beta}{\alpha}$

This root satisfy the equation $x^2 + x - 1 = 0$

$\beta^2 - \alpha\beta - \alpha^2 = 0$

$\Rightarrow \quad \alpha\gamma - \alpha\beta - \alpha^2 = 0$

$\Rightarrow \quad \alpha + \beta = \gamma$

Now, $\alpha(\beta + \gamma) = \alpha\beta + \alpha\gamma = \alpha\beta + \beta^2$

$= (\alpha + \beta)\beta = \beta\gamma$

Limit

22. $\lim_{x\to 0} \frac{x + 2\sin x}{\sqrt{x^2 + 2\sin x + 1} - \sqrt{\sin^2 x - x + 1}}$ is

$\lim_{x\to 0} \frac{x + 2\sin x}{\sqrt{x^2 + 2\sin x + 1} - \sqrt{\sin^2 x - x + 1}}$ बराबर है :

(1) 3 (2) 6 (3) 1 (4) 2

A. 4

sol. $\lim_{x\to 0} \frac{x + 2\sin x}{\sqrt{x^2 + 2\sin x + 1} - \sqrt{\sin^2 x - x + 1}}$

$$= \lim_{x\to 0} \frac{(x + 2\sin x)\left[\sqrt{x^2 + 2\sin x + 1} + \sqrt{\sin^2 x - x + 1}\right]}{(x^2 - \sin^2 x) + (x + 2\sin x)}$$

$$= \lim_{x\to 0} \frac{\left[1 + 2\left(\frac{\sin x}{x}\right)\right]\left[\sqrt{x^2 + 2\sin x + 1} + \sqrt{\sin^2 x - x + 1}\right]}{\left(x - \frac{\sin^2 x}{x}\right) + \left(1 + 2\left(\frac{\sin x}{x}\right)\right)}$$

$$= \frac{3\times 2}{3} = 2$$

Straight Line

23. A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinates axes and the paerpendicular from the origihn to this line makes an angle of 60° with the line $x + y = 0$. Then an equation of the line L is

मूलबिन्दु से 4 इकाई की दूरी पर एक सरल रेखा L निर्देशांक अक्षों पर धनात्मक अंतः खण्ड बनाती है तथा मूलबिन्दु से इस रेखा पर लम्ब, रेखा $x + y = 0$ के साथ 60° का कोण बनाता है, तो रेखा L का एक समीकरण है :

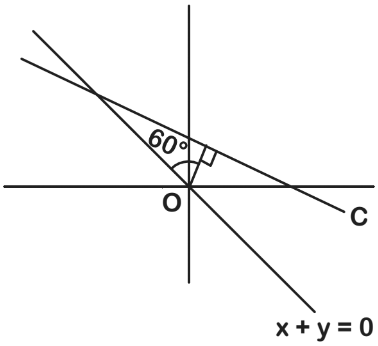
(1) $\left(\sqrt{3} + 1\right)x + \left(\sqrt{3} - 1\right)y = 8\sqrt{2}$ (2) $\left(\sqrt{3} - 1\right)x + \left(\sqrt{3} + 1\right)y = 8\sqrt{2}$

(3) $\sqrt{3}x + y = 8$ (4) $x + \sqrt{3y} = 8$

A. 1 OR 2

sol. If perpendicular makes an angle of 60° with the line $x + y = 0$.

Then the perpendicular makes an angle of 15° or 75° with x-axis. So the equation of line will be



$x\cos75^\circ + y\sin75^\circ = 4$ or $x\cos15^\circ + y\sin15^\circ = 4$

$\left(\sqrt{3}-1\right)x+\left(\sqrt{3}+1\right)y = 8\sqrt{2}$ or

$\left(\sqrt{3}+1\right)x+\left(\sqrt{3}-1\right)y = 8\sqrt{2}$

By rotating the normal towards the line x + y = 0 in anticlockwise sense we get the answer (2).

Binomial Theorem

24. The term independent of x in expansion of $\left(\frac{1}{60}-\frac{x^8}{81}\right)\cdot\left(2x^2-\frac{3}{x^2}\right)^6$ is equal to

$\left(\frac{1}{60}-\frac{x^8}{81}\right)\cdot\left(2x^2-\frac{3}{x^2}\right)^6$ के प्रसार में x से स्वतंत्र पद है :

(1) –108 (2) –36 (3) –72 (4) 36

A. 2

sol. $\left(\frac{1}{60}-\frac{x^8}{81}\right)\left(2x^2-\frac{3}{x^2}\right)^6$

$= \frac{1}{60}\left(2x^2-\frac{3}{x^2}\right)^6 - \frac{x^8}{81}\left(2x^2-\frac{3}{x^2}\right)^6$

Coefficient of x^o in $= \frac{1}{60}\left(2x^2-\frac{3}{x^2}\right)^6 - \frac{x^8}{81}$

coefficient of x^{-8} in $\left(2x^2-\frac{3}{x^2}\right)^6$

$= \frac{-1}{60}\,{}^6C_3(2)^3(3)^3 + \frac{1}{81}\,{}^6C_5(2)(3)^5$

$= -72 + 36$

$= -36$

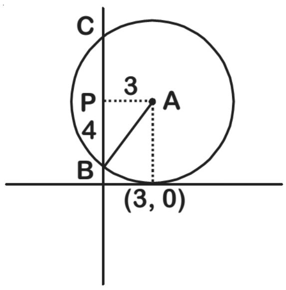
Circle

25. A circle touching the x-axis at (3, 0) and making an intercept of length 8 on the y-axis passes through the point:
x–अक्ष को (3, 0) पर स्पर्श करता हुआ तथा y–अक्ष पर 8 लम्बाई का अन्तःखण्ड बनाता हुआ एक वृत्त निम्न में से किस बिन्दु से होकर जाता है?

(1) (2, 3)   (2) (1, 5)   (3) (3, 5)   (4) (3, 10)

A. 4

sol. Let centre of circle is A and circle cuts the y axis at B and C. Let mid point of chord BC is P.



$AB = \sqrt{PA^2 + PB^2}$

$= 5 =$ radius of circle

Equation of circle is : $(x-3)^2 + (y-5)^2 = 5^2$

Only (3, 10) satisfies this equation.

Although there will be another circle satisfying the same conditions that will lie below the x-axis having equation

$(x-3)^2 + (y-5)^2 = 5^2$

Probability

26. A person throws two fair dice. He wins Rs. 15 for throwing a doublet (same numbers on the two dice), wins Rs. 12 when the throw results in the sum of 9, and loses Rs. 6 for any other outcome on the throw. Then the expected gain/ loss (in Rs.) of the person is

(1) 1/2 loss   (2) 3 gain   (3) 1/2 gain   (4) 1/4 loss

एक व्यक्ति दो न्याय्य (fair) पासे उछालता है। एक द्विक (दोनों पासों पर एक ही संख्या) आने पर वह रूपये 15 जीतता है, दोनों पासों पर आए अंकों का योग 9 होने पर रूपये 12 जीतता है तथा किसी अन्य परिणाम (outcome) पर रूपये 6 हारता है, तो उस व्यक्ति का प्रत्याशित (expected) लाभ/हानि (रूपयें में) है :

(1) 1/2 हानि   (2) 3 लाभ   (3) 1/2 लाभ   (4) 1/4 हानि

A. 1

sol. Let X be the random variable which denote the Rs gained by the person.

$$P(X=15) = \frac{6}{36} = \frac{1}{6} \quad \left\{\begin{array}{l}\text{Total cases} = 36\\ \text{favourable cases are}\\ (1,1),(2,2),(3,3),(4,4),\\ (5,5),(6,6)\end{array}\right\}$$

$$P(X=12) = \frac{4}{36} = \frac{1}{9} \quad \left\{\begin{array}{l}\text{Favourable cases are}\\ (6,3),(5,4),(4,5),(3,6)\end{array}\right\}$$

$$P(X=-6) = \frac{26}{36} = \frac{13}{18}$$

X	15	12	–6
P(X)	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{13}{18}$
X·P(X)	$\frac{5}{2}$	$\frac{4}{3}$	$\frac{-13}{3}$

$E(X) = \Sigma X \cdot P(X) = \frac{5}{2} + \frac{4}{3} - \frac{13}{3} = -\frac{1}{2}$

Sets & Relations

27. Let A, B and C be sets such that $\phi \neq A \cap B \subseteq C$. Then which of the following statements is not true

(1) $B \cap C \neq \phi$
(2) $(C \cup A) \cap (C \cup B) = C$
(3) If $(A - C) \subseteq B$, then $A \subseteq B$
(4) If $(A - C) \subseteq C$, then $A \subseteq C$

माना समुच्चय A, B तथा C इस प्रकार हैं कि $\phi \neq A \cap B \subseteq C$, तो निम्न में से कौनसा कथन सत्य नहीं है?

(1) $B \cap C \neq \phi$
(2) $(C \cup A) \cap (C \cup B) = C$
(3) यदि $(A - C) \subseteq B$, तो $A \subseteq B$
(4) यदि $(A - C) \subseteq C$, तो $A \subseteq C$

A. 3

sol. $\because A \cap B \subseteq C$ and $A \cap B \neq \phi$

(1) $B \cap C \neq \phi$ is correct

(2) $(C \cup A) \cap (C \cup B) = C \cup (A \cap B) = C$ (correct) (becasue $A \cap B \subseteq C$)

(3) If $A = C$ then $A - C = \phi$ Clearly $\phi \subseteq B$ but $A \subseteq B$ is not always true.

(4) $\because A - B \subseteq C$ and $A \cap B \subseteq C$ so $A \subseteq C$ (correct)

Definite Integration

28. A value of α such that

$$\int_{\alpha}^{\alpha+1} \frac{dx}{(x+\alpha)(x+\alpha+1)} = \log_e\left(\frac{9}{8}\right) \text{ is}$$

α का एक मान, जिसके लिए

$$\int_{\alpha}^{\alpha+1} \frac{dx}{(x+\alpha)(x+\alpha+1)} = \log_e\left(\frac{9}{8}\right) \text{ है, है :}$$

(1) $\frac{1}{2}$
(2) -2
(3) $-\frac{1}{2}$
(4) 2

A. 2

Sol. $$\int_{\alpha}^{\alpha+1} \frac{dx}{(x+\alpha)(x+\alpha+1)} = \int_{\alpha}^{\alpha+1} \left[\frac{1}{x+\alpha} - \frac{1}{x+\alpha+1}\right] dx$$

$$= ln\left(\frac{x+\alpha}{x+\alpha+1}\right)\Bigg|_{\alpha}^{\alpha+1}$$

$$= ln\left(\frac{2\alpha+1}{2\alpha+2} \cdot \frac{2\alpha+1}{2\alpha}\right) = ln\frac{9}{8}$$

So, $\frac{(2\alpha+1)^2}{\alpha(\alpha+1)} = \frac{9}{2}$

$\Rightarrow 8\alpha^2 + 8\alpha + 2 = 9\alpha^2 + 9\alpha$

$\Rightarrow \alpha^2 + \alpha - 2 = 0$

$\Rightarrow \alpha = 1, -2$

Complex Number

29. Let $z \in C$ with Im (z) = 10 and it satisfies $\frac{2z-n}{2z+n} = 2i-1$ for some natural numbers n then:

(1) n = 20 and Re(z) = 10 (2) n = 20 and Re(z) = –10
(3) n = 40 and Re (z) = –10 (4) n = 40 and Re(z) = 10

माना $z \in C$ जिसके लिए Im (z) = 10 तथा किसी प्राकृत संख्या n के लिए यह $\frac{2z-n}{2z+n} = 2i-1$ को संतुष्ट करता है, तो :

(1) n = 20 तथा Re(z) = 10 (2) n = 20 तथा Re(z) = –10
(3) n = 40 तथा Re (z) = –10 (4) n = 40 तथा Re(z) = 10

A. 3

sol. Let z = x + 10i
2z – n = (2i – 1) (2z + n)
(2x – n) + 20i = (2i – 1) ((2x + n) + 20i)
Comparing real and imaginary part
–(2x + n) – 40 = 2x – n and 20 = 4x + 2n – 20
⇒ 4x = –40 40 = –40 + 2n
⇒ x = –10 n = 40
⇒ Re(z) = –10

Limit

30. Let f(x) = 5– |x – 2| and g(x) = |x + 1|, $x \in R$. If f(x) attains maximum value at α and g(x) attains minimum value at β then is

$\lim_{x\to\alpha\beta} \frac{(x-1)(x^2-5x+6)}{x^2-6x+8}$ equal to

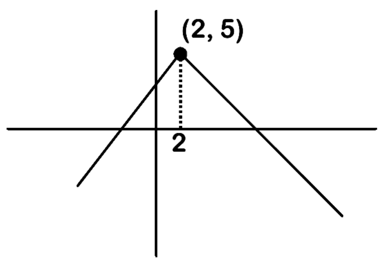
माना f(x) = 5– |x – 2| तथा g(x) = |x + 1|, $x \in R$. यदि f(x) का अधिकतम मान α पर है तथा g(x) का न्यूनतम मान β पर है,

तो $\lim_{x\to\alpha\beta} \frac{(x-1)(x^2-5x+6)}{x^2-6x+8}$ बराबर है :

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) $-\frac{3}{2}$ (4) $\frac{3}{2}$

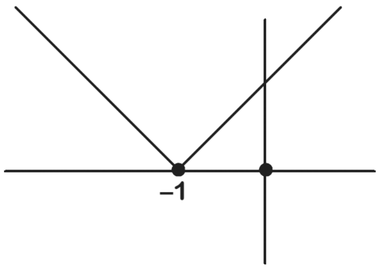
A. 1

sol. f(x) = 5 – |x – 2|
Graph of y = f(x)



f(x) is maximum at x = 2
α = 2
g(x) = |x + 1|

Graph of $y = g(x)$



$g(x)$ is minimum at $x = -1$

$\beta = -1$

$$\lim_{x\to 2}\frac{(x-1)(x-2)(x-3)}{(x-2)(x-4)} = \lim_{x\to 2}\frac{(x-1)(x-3)}{x-4}$$

$$= \frac{1}{2}$$