JEE Main April 2023

Question Paper With Text Solution

11 April | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN APRIL 2023 | 11TH APRIL SHIFT-1

SECTION – A

Question ID : 3666944176

1. Let (α, β, γ) be the image of the point P(2, 3, 5) in the plane $2x + y - 3z = 6$. Then $\alpha + \beta + \gamma$ is equal to:

माना बिंदु P(2, 3, 5) का समतल $2x + y - 3z = 6$ में प्रतिबिंब (α, β, γ) है। तो $\alpha + \beta + \gamma$ बराबर है :

(1) 5 (2) 12 (3) 9 (4) 10

Ans. Official Answer NTA (4)

Sol. $\frac{\alpha-2}{2}=\frac{\beta-3}{1}=\frac{\gamma-5}{-3}=-2\frac{(4+3-15-6)}{14}=2$

$\Rightarrow \frac{\alpha-2}{2}=2 \Rightarrow \alpha=6$

$\frac{\beta-3}{1}=2 \Rightarrow \beta=5$

$\frac{\gamma-5}{-3}=2 \Rightarrow \gamma=-1$

$\therefore \quad \alpha+\beta+\gamma=6+5-1=10$

Question ID : 3666944192

2. Let sets A and B have 5 elements each. Let the mean of the elements in sets A and B be 5 and 8 respectively and the variance of the elements in sets A and B be 12 and 20 respectively. A new set C of 10 elements is formed by subtracting 3 from each element of A and adding 2 to each element of B. Then the sum of the mean and variance of the elements of C is :

माना समुच्चय A में 5 अवयव है तथा समुच्चय B में भी 5 अवयव हैं। माना समुच्चयों A तथा B के अवयवों में माध्य क्रमशः 5 तथा 8 है और समुच्चयों A तथा B के अवयवों के प्रसरण क्रमशः 12 तथा 20 है। A के प्रत्येक अवयव में से 3 घटा कर तथ्झा B में प्रत्येक अवयव में 2 जोड़ कर 10 अवयवों का एक नया समुच्चय C बनाया जाता है। तो C के अवयवों के माध्य तथा प्रसरण का योग है :

(1) 40 (2) 38 (3) 36 (4) 32

Ans. Official Answer NTA (2)

Sol. $\omega \quad A=\{a_1,a_2,a_3,a_4,a_5\}$

$B=\{b_1,b_2,b_3,b_4,b_5\}$

Given, $\sum_{i=1}^{5} ai = 25, \sum_{i=1}^{5} bi = 40$

$$\frac{\sum_{i=1}^{5} a_i^2}{5} - \left(\frac{\sum_{i=1}^{5} a_i}{5}\right)^2 = 12, \frac{\sum_{i=1}^{5} b_i^2}{5} - \left(\frac{\sum_{i-1}^{5} b_i}{5}\right)^2 = 20$$

$$\Rightarrow \sum_{i=1}^{5} a_i^2 = 185, \quad \sum_{i=1}^{5} b_i^2 = 420$$

Now, $C = \{C_1, C_2, \ldots..C_{10}\}$

s.f.$C_i = a_i = 3$ or $b_i + 2$

first fice elements   Last five elements

$$\therefore \text{Mean of C}, \overline{C} = \frac{\left(\sum a_i - 15\right) + \left(\sum b_i + 10\right)}{10}$$

$$\overline{C} = \frac{10 + 50}{10} = 6$$

$$\therefore \sigma^2 = \frac{\sum_{i=1}^{10} C_i^2}{10} = (\overline{C})^2$$

$$= \frac{\sum (a_i - 3)^2 + \sum (b_i + 2)^2}{10} - (6)^2$$

$$= \frac{\sum a_i^2 + \sum b_i^2 - 6\sum a_i + 4\sum b_i + 65}{10} - 36$$

$$= \frac{185 + 420 - 150 + 160 + 65}{10} - 36$$

$= 32$

$\therefore$ Mean + Variance $= \overline{C} + \sigma^2 = 6 + 32 = 38$

Question ID : 3666944180

3. Let A be a 2×2 matrix with real entries such that $A' = \alpha A + I$, where $\alpha \in \mathbb{R} - \{-1, 1\}$. If $\det\left(A^2 - A\right) = 4$, then the sum of all possible values of α is equal to :

माना A वास्तविक अवयवों का एक 2×2 आव्यूह है जिसके लिए $A' = \alpha A + I$ है, $\alpha \in \mathbb{R} - \{-1, 1\}$ है। यदि $\det\left(A^2 - A\right) = 4$ है, तो α के सभी संभव मानों का योग बराबर है :

(1) $\frac{3}{2}$ (2) 2 (3) $\frac{5}{2}$ (4) 0

Ans. Official Answer NTA (3)

Sol. $A^T = \alpha A + I$

$A = \alpha A^T + I$

$A = \alpha(\alpha A + I) + I$

$A = \alpha^2 A + (\alpha + 1)I$

$A\left(1-\alpha^2\right) = (\alpha+1)I$

$A = \frac{I}{1-\alpha} \ldots(1)$

$|A| = \frac{1}{(1-\alpha)^2} \ldots(2)$

$\left|A^2 - A\right| = |A||A-1| \ldots(3)$

$A - I = \frac{I}{I-\alpha} - I = \frac{\alpha}{1-\alpha} I$

$|A-I| = \left(\frac{\alpha}{1-\alpha}\right)^2 \ldots(4)$

Now $\left|A^2 - A\right| = 4$

$|A||A-I| = 4$

$\Rightarrow \frac{1}{(1-\alpha)^2} \frac{\alpha^2}{\left(1-\alpha^2\right)} = 4$

$\Rightarrow \frac{\alpha}{(1-\alpha)^2} = \pm 2$

$\Rightarrow 2(1-\alpha)^2 = \pm\alpha$

$(C_1) 2(1-\alpha)^2 = \alpha$

$2\alpha^2 - 5\alpha + 2 = 0 \; \langle \; \alpha_1, \alpha_2$

$\alpha_1 + \alpha_2 = \frac{5}{2}$

$(C_2) 2(1-\alpha)^3 = -\alpha$

$2\alpha^2 - 3\alpha + 2 = 0$

$\alpha \notin R$

Question ID : 3666944186

4. For any vector $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, with $10|a_i|<1$, i = 1, 2, 3, consider the following statements :

(A) : $\max\{|a_1|,|a_2|,|a_3|\}\le|\vec{a}|$

(B) : $|\vec{a}|\le 3\max\{|a_1|,|a_2|,|a_3|\}$

(1) Both (A) and (B) are true
(2) Only (B) is true
(3) Only (A) is true
(4) Neither (A) nor (B) is true

किसी सदिश $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $10|a_i|<1$, i = 1, 2, 3 के लिए, निम्नलिखित कथनों पर विचार कीजिए :

(A) : $\max\{|a_1|,|a_2|,|a_3|\}\le|\vec{a}|$

(B) : $|\vec{a}|\le 3\max\{|a_1|,|a_2|,|a_3|\}$

(1) (A) तथा (B) दोनों सत्य है
(2) केवल (B) सत्य है
(3) केवल (A) सत्य है
(4) न (A) और न (B) सत्य है

Ans. Official Answer NTA (1)

Sol. $\vec{a}=a_1 i+a_2\hat{j}+a_3\hat{k},|a_i|<\frac{1}{10},i=1,2,3$

$\because|\vec{a}|=\sqrt{a_1^2+a_2^2+a_3^2}\Rightarrow|\vec{a}|=\sqrt{|a_1|^2+|a_2|^2+|a_3|^2}\ge\max(|a_1|,|a_2|,|a_3|)$

Similarly $|\vec{a}|=\sqrt{a_1^2+a_2^2+a_3^2}<\sqrt{\frac{1}{100}+\frac{1}{100}+\frac{1}{100}}$

$|\vec{a}|<\frac{\sqrt{3}}{10}<\frac{3}{10}$

$\Rightarrow|\vec{a}|<3\max(|a_1|,|a_2|,|a_3|)\Rightarrow$ A and B both are correct

Question ID : 3666944177

5. Consider ellipses $E_k : kx^2+k^2y^2=1, k=1,2,\ldots,20$. Let C_k be the circle which touches the four chords joining the end points (one on minor axis and another on major axis) of the ellipse E_k. If r_k is the radius of the circle C_k, then the value of $\sum_{k=1}^{20}\frac{1}{r_k^2}$ is :

दीर्घवृत्तों $E_k : kx^2+k^2y^2=1, k=1,2,\ldots,20$ का विचार कीजिए। माना C_k वह वृत्त है, जो दीर्घवृत्त E_k के अन्त्य बिंदुओं (एक

लघु अक्ष पर तथा दूसरा दीर्घ अक्ष पर) को मिलाने वाली चार जीवाओं को स्पर्श करता है। यदि वृत्त C_k की त्रिज्या r_k है, तो $\sum_{k=1}^{20} \frac{1}{r_k^2}$ का मान है :

(1) 3320 (2) 2870 (3) 3210 (4) 3080

Ans. Official Answer NTA (4)

Sol. $E_k = \frac{x^2}{\left(\frac{1}{\sqrt{k}}\right)^2} + \frac{y^2}{\left(\frac{1}{k}\right)^2} = 1$

Chord

$L_k : \frac{x}{\left(\frac{1}{\sqrt{k}}\right)} + \frac{y}{\left(\frac{1}{k}\right)} = 1$

$\Rightarrow \sqrt{k}x + ky - 1 = 0$

r_k = Perpendicular distance of L_k from (0, 0),

$r_k = \left|\frac{-1}{\sqrt{k+k^2}}\right|$

$\Rightarrow r_k^2 = \frac{1}{k+k^2}$

$\sum_{k=1}^{20} \frac{1}{r_k^2} = \sum_{k=1}^{20} k + k^2 = \frac{20 \times 21}{2} + \frac{20 \times 21 \times 41}{6}$

= 210 + 2870

= 3080

Question ID : 3666944191

6. Let $S = \left\{M = \left[a_{ij}\right], a_{ij} \in \{0,1,2\}, 1 \le i, j \le 2\right\}$ be a sample space and $A = \{M \in S : M \text{ is invertible}\}$ be an event. Then P(A) is equal to :

माना $S = \left\{M = \left[a_{ij}\right], a_{ij} \in \{0,1,2\}, 1 \le i, j \le 2\right\}$ एक प्रतिदर्श समष्टि है तथा $A = \{M \in S : M$ व्युत्क्रमणीय है$\}$ एक घटना है। तो P(A) बराबर है :

(1) $\frac{50}{81}$ (2) $\frac{47}{81}$ (3) $\frac{49}{81}$ (4) $\frac{16}{27}$

Ans. Official Answer NTA (1)

Sol. $M\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $a, b, c, d, \in \{0,1,2\}$

$n(s) = 3^4 = 81$

we first bound $p(\overline{A})$

$|m| = 0 \Rightarrow ad = bc$

$ad = bc = 0 \Rightarrow no\cdot\ of\ (a,b,c,d) = (3^2 - 2^2)^2 = 25$

$ad = bc = 1 \Rightarrow no.\ of\ (a,b,c,d) = 1^2 = 1$

$ad = bc = 2 \Rightarrow no\cdot\ of\ (a,b,c,d) = 2^2 = 4$

$ad = bc = 4 \Rightarrow\ no.\ of\ (a,b,c,d) = 1^2 = 1$

$: P(\overline{A}) = \frac{31}{81} \Rightarrow p(A) = \frac{50}{81}$

Question ID : 3666944179

7. The number of integral solutions x of $\log_{\left(x+\frac{7}{2}\right)}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$ is :

$\log_{\left(x+\frac{7}{2}\right)}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$ के पूर्णांक हलों x की संख्या है :

(1) 5 (2) 7 (3) 6 (4) 8

Ans. Official Answer NTA (3)

Sol. $\log_{x+\frac{7}{2}}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$

Feasible region : $x + \frac{7}{2} > 0 \Rightarrow x > -\frac{7}{2}$

And $x + \frac{7}{2} \neq 1 \Rightarrow x \neq \frac{-5}{2}$

And $\frac{x-7}{2x-3} \neq 0$ and $2x-3 \neq 0$

$\Downarrow$ $\Downarrow$

$x \neq 7$ $x \neq \frac{3}{2}$

Taking intersection : $x \in \left(\frac{-7}{2}, \infty\right) - \left\{-\frac{5}{2}, \frac{3}{2}, 7\right\}$

Now $\log_a b \geq 0$ if $a > 1$ and $b \geq 1$ OR

$a \in (0,1)$ and $b \in (0,1)$

$C-I; x + \frac{7}{2} > 1$ and $\left(\frac{x-7}{2x-3}\right)^2 \geq 1$

$x > -\frac{5}{2}; (2x-3)^2 - (x-7)^2 \leq 0$

$(2x-3+x-7)(2x-3-x+7) \leq 0$

$(3x-10)(x+4) \leq 0$

$x \in \left[-4, \frac{10}{3}\right]$

Intersection : $x \in \left(\frac{-5}{2}, \frac{10}{3}\right]$

$C-II : x + \frac{7}{2} \in (0,1)$ and $\left(\frac{x-7}{2x-3}\right)^2 \in (0,1)$

$0 < x + \frac{7}{2} < 1; \left(\frac{x-7}{2x-3}\right)^2 < 1$

$-\frac{7}{2} < x < \frac{-5}{2}; (x-7)^2 < (2x-3)^2$

$x \in (-\infty, -4) \cup \left(\frac{10}{3}, \infty\right)$

No common values of x.

Hence intersection with feasible region

We get $x \in \left(\frac{-5}{2}, \frac{10}{3}\right] - \left\{\frac{3}{2}\right\}$

Integral value of x are $\{-2, -1, 0, 1, 2, 3\}$

No. of integral values = 6

Question ID : 3666944187

8. Let $x_1, x_2, \ldots\ldots, x_{100}$ be in an arithmetic progression, with $x_1 = 2$ and their mean equal to 200. If $y_i = i(x_i - i), 1 \le i \le 100$, then the mean of $y_1, y_2, \ldots\ldots, y_{100}$ is :

माना $x_1, x_2, \ldots\ldots, x_{100}$ एक समांतर श्रेणी में हैं, जिनका माध्य 200 है तथा $x_1 = 2$ है। यदि $y_i = i(x_i - i), 1 \le i \le 100$ हैं, तो $y_1, y_2, \ldots\ldots, y_{100}$ का माध्य है :

(1) 10051.50 (2) 10049.50 (3) 10101.50 (4) 10100

Ans. Official Answer NTA (2)

Sol. $2 + (2+d) + \ldots\ldots\ldots\ldots + (2+99d) = 20000$

$\Rightarrow 200 + \frac{99}{2} \times 100d = 20000$

$\Rightarrow \quad 99d = 396$

$d = 4$

$y_1 + y_2 + \ldots\ldots\ldots + y_{100} = \sum_{i=1}^{100} i(2+(i-1)4-i) = \sum_{i=1}^{100}\left(3i^2 - 2i\right) = \frac{3.100.101.201}{6} - 100.101$

$= 10100\left(\frac{201}{2} - 1\right) = (10100).\left(\frac{199}{2}\right)$

So mean $= \frac{(10100)(99.5)}{100} = 10049.5$

Question ID : 3666944183

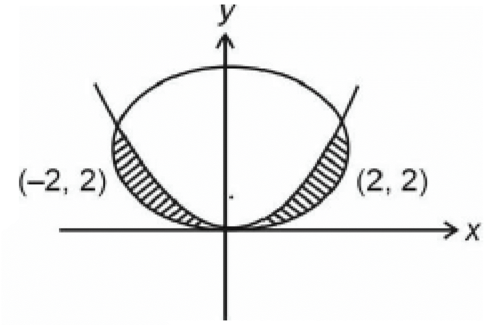
9. Area of the region $\{(x,y): x^2 + (y-2)^2 \le 4, x^2 \ge 2y\}$ is :

क्षेत्र $\{(x,y): x^2 + (y-2)^2 \le 4, x^2 \ge 2y\}$ का क्षेत्रफल :

(1) $2\pi - \frac{16}{3}$ (2) $\pi + \frac{8}{3}$ (3) $\pi - \frac{8}{3}$ (4) $2\pi + \frac{16}{3}$

Ans. Official Answer NTA (1)

Sol. $x^2 + (y-2)^2 \le 4, x^2 \ge 2y$



Area of required region $= 2\left[\frac{1}{4}\pi(4) - \int_0^2 \sqrt{2}\cdot\sqrt{y}dy\right]$

$$\Rightarrow 2\left[\pi - \left.\frac{\sqrt{2}\cdot y^{3/2}}{3/2}\right|_0^2\right]$$

$$\Rightarrow 2\left[\pi - \frac{2\sqrt{2}}{3}\cdot 2\sqrt{2}\right] = 2\left[\pi - \frac{8}{3}\right] = 2\pi - \frac{16}{3}$$

Question ID : 3666944190

10. The number of elements in the set $S = \left\{\theta \in [0, 2\pi] : 3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0\right\}$ is :

समुच्चय $S = \left\{\theta \in [0, 2\pi] : 3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0\right\}$ में अवयवों की संख्या है :

(1) 9 (2) 10 (3) 8 (4) 12

Ans. Official Answer NTA (1)

Sol. $3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0$

$\Rightarrow 3\cos^4\theta - 3\cos^2\theta - 2\cos^2\theta - 2\sin^6\theta + 2 = 0$

$\Rightarrow 3\cos^4\theta - 3\cos^2\theta + 2\sin^2\theta - 2\sin^6\theta = 0$

$\Rightarrow 3\cos^2\theta\left(\cos^2\theta - 1\right) + 2\sin^2\theta\left(\sin^4\theta - 1\right) = 0$

$\Rightarrow -3\cos^2\theta\sin^2\theta + 2\sin^2\theta\left(1 + \sin^2\theta\right)\cos^2\theta - 1$

$\Rightarrow \sin^2\theta\cos^2\theta\left(2 + 2\sin^2\theta - 3\right) = 0$

$\Rightarrow \sin^2\theta\cos^2\theta\left(2\sin^2\theta - 1\right) = 0$

(C1) $\sin^2\theta = 0 \rightarrow 3$ solution; $\theta = \{0, \pi, 2\pi\}$

(C2) $\cos^2\theta = 0 \rightarrow 2$ solution; $\theta = \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

(C3) $\sin^2\theta = \frac{1}{2} \rightarrow 4$ solution; $\theta = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

No. of solution = 9

Question ID : 3666944185

11. Let $\vec{a}$ be a non-zero vector parallel to the line of intersection of the two planes described by $\hat{i}+\hat{j}, \hat{i}+\hat{k}$ and $\hat{i}-\hat{j}, \hat{j}-\hat{k}$. If θ is the angle between the vector $\vec{a}$ and the vector $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{a}\cdot\vec{b} = 6$, then the ordered pair $(\theta, |\vec{a}\times\vec{b}|)$ is equal to :

माना $\vec{a}$; $\hat{i}+\hat{j}, \hat{i}+\hat{k}$ तथा $\hat{i}-\hat{j}, \hat{j}-\hat{k}$ द्वारा वर्णित दो समतलो के दो प्रतिच्छेदन रेखा के समान्तर एक अशून्य सदिश है। यदि सदिश $\vec{a}$ तथा सदिश $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ के बीच का कोण θ है तो क्रमित युग्म $(\theta, |\vec{a}\times\vec{b}|)$ का मान ज्ञात कीजिए :

(1) $\left(\frac{\pi}{4}, 6\right)$ (2) $\left(\frac{\pi}{3}, 6\right)$ (3) $\left(\frac{\pi}{4}, 3\sqrt{6}\right)$ (4) $\left(\frac{\pi}{3}, 3\sqrt{6}\right)$

Ans. Official Answer NTA (1)

Sol. $\vec{n}_1$ and $\vec{n}_2$ are normal vector to the plane $\hat{i}+\hat{j}, \hat{i}+\hat{k}$ and $\hat{i}-\hat{j}, \hat{i}-\hat{k}$ respectively

$$\vec{n}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = \hat{i} - \hat{j} - \hat{k}$$

$$\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{a} = \lambda|\vec{n}_2 \times \vec{n}_2|$$

$$= \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{vmatrix} = \lambda(-2\hat{j} + 2\hat{k})$$

$$\vec{a}\cdot\vec{b} = \lambda|0+4+2| = 6$$

$$\Rightarrow \lambda = 1$$

$\vec{\alpha} = -2\hat{j} + 2\hat{k}$

$\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|a||b|}$

$\cos\theta = \dfrac{6}{2\sqrt{2}\times 3} = \dfrac{1}{\sqrt{2}}$

$\theta = \dfrac{\pi}{4}$

Now $|\vec{a}.\vec{b}|^2 + |\vec{a}\times\vec{b}|^2 = |a|^2|\vec{b}|^2$

$36 + \left|\vec{a}\times\mathbf{b}^2\right| = 8\times 9 = 72$

$|\vec{\mathbf{a}}\times\mathbf{b}|^2 = 36$

$|\vec{\mathbf{a}}\times\vec{b}| = 6$

Question ID : 3666944194

12. Let $y = y(x)$ be a solution curve of the differential equation $(1 - x^2y^2)dx = ydx + xdy$. If the line $x = 1$ intersects the curve $y = y(x)$ at $y = 2$ and the line $x = 2$ intersects the curve $y = y(x)$ at $y = \alpha$, then a value of α is :

माना अवकल समीकरण $(1 - x^2y^2)dx = ydx + xdy$ का हल वक्र $y = y(x)$ है। यदि रेखा $x = 1$, वक्र $y = y(x)$ को $y = 2$ पर काटती है तथा रेखा $x = 2$, वक्र $y = y(x)$ को $y = \alpha$ पर काटती है, तो α का एक मान है

(1) $\dfrac{3e^2}{2(3e^2+1)}$ (2) $\dfrac{3e^2}{2(3e^2-1)}$ (3) $\dfrac{1+3e^2}{2(3e^2-1)}$ (4) $\dfrac{1-3e^2}{2(3e^2+1)}$

Ans. Official Answer NTA (3)

Sol. $dx = \dfrac{d(xy)}{1-(xy)^2} \Rightarrow 2dx = \dfrac{d(xy)}{1-xy} + \dfrac{d(xy)}{1+xy}$

$2x + c = \ell n\left|\dfrac{1+xy}{1-xy}\right| \Rightarrow \left|\dfrac{xy+1}{xy-1}\right| = e^c\cdot e^{2x}$

$\because y(1) = 2$

$\Rightarrow 3 = e^c\cdot e^2$

$\Rightarrow e^c = \dfrac{3}{e^2}e^{2x}$

So at $x = 2$

$\Rightarrow \left|\dfrac{2\alpha+1}{2\alpha-1}\right| = 3e^2$

Case : 1 for +ve sign

$\Rightarrow \frac{2\alpha+1}{2\alpha-1}=\frac{3e^2}{1}$

$\Rightarrow \frac{4\alpha}{2}=\frac{3e^2+1}{3e^2-1}$

$\Rightarrow \alpha=\frac{1}{2}\left(\frac{3e^2+1}{3e^2-1}\right)$

Case: 2 for (–ve)sign not required

Question ID : 3666944193

13. Let R be a rectangle given by the lines $x = 0, x = 2, y = 0$ and $y = 5$. Let $A(\alpha, 0)$ and $B(0, \beta)$, $\alpha \in [0, 2]$ and $\beta \in [0, 5]$, be such that the line segment AB divides the area of the rectangle R in the ratio 4 : 1. Then, the mid-point of AB lies on a :

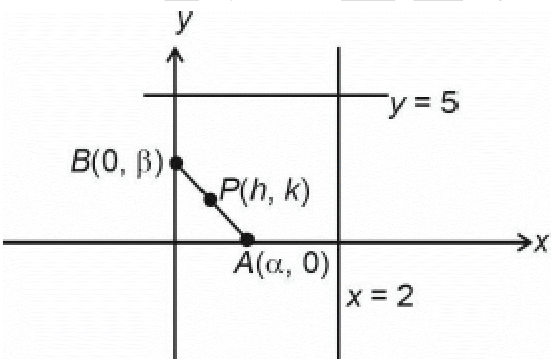
(1) circle (2) straight line (3) hyberbola (4) parabola

माना रेखाओं $x = 0, x = 2, y = 0$ तथा $y = 5$ द्वारा निर्मित आयत R है। माना $A(\alpha, 0)$ तथा $B(0, \beta)$, $\alpha \in [0, 2]$, $\beta \in [0, 5]$, इस प्रकार हैं कि रेखाखंड AB आयत R क्षेत्रफल को 4 : 1 के अनुपात में बाँटती है। तो AB का मध्य बिंदु एक

(1) वृत्त (2) सरल रेखा (3) अतिपरवलय (4) परवलय

Ans. Official Answer NTA (3)

Sol.



$\frac{10-\frac{1}{2}\alpha\beta}{\frac{1}{2}\alpha\beta}=\frac{4}{1} \Rightarrow 20-\alpha\beta=4\alpha\beta$

$\Rightarrow \alpha\beta = 4$

Let $h=\frac{\alpha}{2}, \beta=\frac{k}{2}$

$\therefore \quad 4hk = 4$

$\therefore xy = 1$

Question ID : 3666944178

14. The number of triplets (x, y, z) where, x, y, z are distinct non negative integers satisfying $x + y + z = 15$, is :

त्रिकों (x, y, z) जहाँ x, y, z भिन्न ऋणेत्तर पूर्णांक हैं तथा $x + y + z = 15$ को संतुष्ट करते है :

(1) 136 (2) 80 (3) 114 (4) 92

Ans. Official Answer NTA (3)

Sol. $x + y + z = 15$

Total no. solution $= {}^{15+3-1}C_3 = 136$..... (1)

Let $x = y \neq z$

$2x + z = 15 \Rightarrow z = 15 - 2t$

$\Rightarrow r \in \{0, 1, 2, ...7\} - \{5\}$

$\therefore$ 7 solutions

$\therefore$ there are 21 solutions in which exactly

Two of x, y, z are equal … (2)

There is one solution in which $x = y = z$ … (3)

Required answer = 136 – 21 – 1= 114

Question ID : 3666944188

15. Let w_1 be the point obtained by the rotation of $z_1 = 5 + 4i$ about the origin through a right angle in the anticlockwise direction, and w_2 be the point obtained by the rotation of $z_2 = 3 + 5i$ about the origin through a right angle in the clockwise direction. Then the principal argument of $w_1 - w_2$ is equal to :

माना $z_1 = 5 + 4i$ को मूल बिंदु के सापेक्ष घड़ी की विपरीत दिशा में एक समकोण तक घुमाने पर बिंदु w_1 प्राप्त होता है तथा $z_2 = 3 + 5i$ को मूलबिंदु के सापेक्ष घड़ी की दिशा में एक समकोण तक घुमाने पर बिंदु w_2 प्राप्त होता है। तो $w_1 - w_2$ का मुख्य आयाम बराबर है :

(1) $-\pi + \tan^{-1}\frac{8}{9}$ (2) $\pi - \tan^{-1}\frac{33}{5}$ (3) $-\pi + \tan^{-1}\frac{33}{5}$ (4) $\pi - \tan^{-1}\frac{8}{9}$

Ans. Official Answer NTA (4)

Sol. $W_1 = z_i i = (5+4i)i = -4+5i\ldots$ (i)

$W_1 = z_2(-i) = (3+5i)(-i) = 5-3i\ldots(2)$

$W_1 - W_2 = -9 + 8i$

Principal argument $= \pi - \tan^{-1}\left(\frac{8}{9}\right)$

Question ID : 3666944175

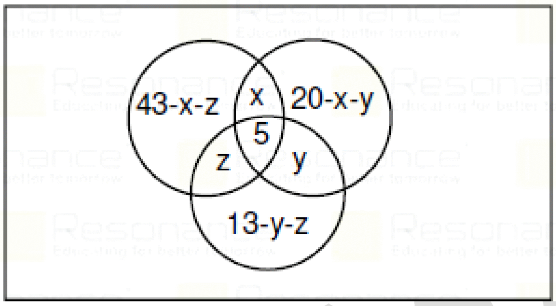
16. An organization awarded 48 medals in event 'A', 25 in event 'B' and 18 in event 'C'. If these medals went to total 60 men and only five men got medals in all the three events, then, how many received medals in exactly two of three events :

एक संस्था ने प्रतियोगिता A में 48 पदक, प्रतियोगिता B में 25 पदक तथा प्रतियोगिता C में 18 पदक दिए। यदि यह पदक कुल 60 पुरूषों को मिले तथा केवल पाँच पुरूषों को तीनों प्रतियोगिताओं में पदक मिले, तो कितने पुरूषों को ठीक दो प्रतियोगिताओं में पदक मिलें :

(1) 9 (2) 15 (3) 10 (4) 21

Ans. Official Answer NTA (21)

Sol.



$(20 - x - y) + (43 - x - z) + (13 - y - z) + x + y + z + 5 = 60$

$21 = x + y + z$

Question ID : 3666944189

17. The value of the integral $\int_{-\log_e 2}^{\log_e 2} e^x \left(\log_e\left(e^x + \sqrt{1+e^{2x}}\right)\right)dx$ is equal to :

निम्न $\int_{-\log_e 2}^{\log_e 2} e^x \left(\log_e\left(e^x + \sqrt{1+e^{2x}}\right)\right)dx$ का मान ज्ञात कीजिए :

(1) $\log_e\left(\frac{2(2+\sqrt{5})}{\sqrt{1+\sqrt{5}}}\right) - \frac{\sqrt{5}}{2}$

(2) $\log_e\left(\frac{\sqrt{2}(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right) - \frac{\sqrt{5}}{2}$

(3) $\log_e\left(\frac{(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right) + \frac{\sqrt{5}}{2}$

(4) $\log_e\left(\frac{\sqrt{2}(3-\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right) + \frac{\sqrt{5}}{2}$

Ans. Official Answer NTA (2)

Sol. $I = \int_{-\log_e 2}^{\log_e 2} e^x \left[\log_e\left(e^x + \sqrt{1+e^{2x}}\right)\right]dx$

Put $e^x = t$

$e^x dx = dt$

$$I = \int_{\frac{1}{2}}^{2} 1 \times \log_e\left[t + \sqrt{1+t^2}\right]dt$$

$$= \left[t\ln\left(\sqrt{t^2+1}+t\right)\right]_{\frac{1}{2}}^{2} - \int_{1/2}^{2}\frac{t}{\sqrt{t^2+1}}dt$$

$$= \left[t\ln\sqrt{t^2+1}+t-\sqrt{t^2+1}\right]_{\frac{1}{2}}^{2}$$

$$= \left[2\ln\sqrt{5}+2-\sqrt{5}\right]-\left[\frac{1}{2}\ln\left(\sqrt{\frac{5}{2}}+\frac{1}{2}\right)-\sqrt{\frac{5}{2}}\right]$$

$$= \ln\left(\frac{\sqrt{2}(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right)-\frac{\sqrt{5}}{2}$$

Question ID : 3666944181

18. Let $f(x) = \left[x^2 - x\right] + |-x + [x]|$, where $x \in \mathbb{R}$ and [t] denotes the greatest integer less than or equal to t. Then, f is :

(1) continuous at x = 0 and x = 1

(2) not continuous at x = 0 and x = 1

(3) continuous at x = 0, but not continuous at x = 1

(4) continuous at x = 1, but not continuous at x = 0

माना $f(x) = \left[x^2 - x\right] + |-x + [x]|$ है, जहाँ $x \in \mathbb{R}$ है तथा [t] महत्तम पूर्णांक $\le t$ है। तो f :

(1) x = 0 तथा x = 1 पर संतत है

(2) x = 0 तथा x = 1 पर संतत नहीं है

(3) x = 0 पर संतत है, परन्तु x = 1 पर संतत नहीं है

(4) x = 1 पर संतत है, परन्तु x = 0 संतत नहीं है

Ans. Official Answer NTA (4)

Sol. Here $f(x) = [x(x-1)] + \{x\}$

$f(o^+) = -1 + 0 = -1$ $\quad$ $f(1^+) = 0 + 0 = 0$

$f(o) = 0$ $\quad$ $f(1) = 0$

$f(1^-) = -1 + 1 = 0$

$\therefore$ f(x) is continuous at x = 1, discontinuous at x = 0

Question ID : 3666944182

19. If equation of the plane that contains the point (–2, 3, 5) and is perpendicular to each of the planes 2x + 4y + 5z = 8 and 3x –2y + 3z = 5 is $\alpha x + \beta y + \gamma z + 97 = 0$ then $\alpha + \beta + \gamma$:

यदि समतल, जिसमें बिंदु (–2, 3, 5) स्थित है तथा जो दो समतलों 2x + 4y + 5z = 8 तथा 3x –2y + 3z = 5 के लंबवत् है, का समीकरण $\alpha x + \beta y + \gamma z + 97 = 0$ है, तो $\alpha + \beta + \gamma$ =

(1) 18 (2) 17 (3) 15 (4) 16

Ans. Official Answer NTA (3)

Sol. The equation of plane through (–2,3,5) is

$a(x+2)+b(y-3)+c(z-5)=0$

It is perpendicular to $2x+4y+5z=8$ & $3x-2y+3z=5$

$\therefore 2a+4b+5c=0$

$3a-2b+3c=0$

$$\therefore \frac{a}{\begin{vmatrix} 4 & 5 \\ -2 & 3 \end{vmatrix}} = \frac{-b}{\begin{vmatrix} 2 & 5 \\ 3 & 3 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 2 & 4 \\ 3 & -2 \end{vmatrix}}$$

$$\Rightarrow \frac{a}{22} = \frac{b}{9} = \frac{c}{-16}$$

$\therefore$ Equation of plane is

$22(x+2)+9(y-3)-16(z-5)=0$

$\Rightarrow 22x+9y-16z+97=0$

Comparing with $\alpha x+\beta y+\gamma x+97=0$

We get $\alpha+\beta+\gamma = 22+9-16=15$

Question ID : 3666944184

20. Let $f:[2,4]\to\mathbb{R}$ be a differentiable function such that

$(x\log_e x)f'(x)+(\log_e x)f(x)+f(x)\ge 1, x\in[2,4]$ with $f(2)=\frac{1}{2}$ and $f(4)=\frac{1}{4}$.

Consider the following two statements :

(A) : $f(x)\le 1$, for all $x\in[2,4]$

(B) : $f(x) \geq \frac{1}{8}$, for all $x \in [2, 4]$

Then :

(1) Only statement (A) is true

(2) Both the statements (A) and (B) are true

(3) Neither statement (A) nor statement (B) is true

(4) Only statement (B) is true

माना $f:[2,4] \to \mathbb{R}$ एक अवकलनीय फलन है, जिसके लिए

$(x\log_e x)f'(x)+(\log_e x)f(x)+f(x)\geq 1, x\in[2,4]$, $f(2)=\frac{1}{2}$ तथा $f(4)=\frac{1}{4}$ है।

निम्न में दो कथनों का विचार कीजिए :

(A) : सभी $x \in [2, 4]$ के लिए $f(x) \leq 1$ है।

(B) : सभी $x \in [2, 4]$ के लिए $f(x) \geq \frac{1}{8}$ है।

तो :

(1) केवल कथन (A) सत्य है

(2) (A) तथा (B) दोनों कथन सत्य हैं

(3) न तो कथन (A) न ही कथन (B) सत्य है

(4) केवल कथन (B) सत्य है

Ans. Official Answer NTA (2)

Sol. $\frac{d}{dx}\left(x\,\ell n(x).f(x)-x\right)\geq 0, \forall x\in[2,4]$

let $g(x)=x\,\ell n(x)f(x)-x, \quad \forall x\in[2,4]$

$\Rightarrow g'(x)\geq 0, \forall x\in[2,4]$

$\Rightarrow g'(x)$ is increasing funtion in [2,4]

$\Rightarrow g(2)\leq g(x)\leq g(4)$

$\Rightarrow 2\cdot \ell n-\frac{1}{2}-2\le x\ \ell n(x)f(x)-x\le 4\cdot \ell n(4)\cdot\frac{1}{4}-4$

$\Rightarrow \ln 2-2+x\le x.\ell nx.f(x)\le 2(\ell n2-2)+x$

$\Rightarrow \frac{\ell n2-2}{x/na}+\frac{1}{\ell nx}\le f(x)\le\frac{2(\ln 2-2)}{x\ell nx}+\frac{1}{\ell nx}$

Since numerator is constant

$\Rightarrow \frac{\ell n2-2}{4\ell n4}+\frac{1}{\ell ln4}\le f(x)\le\frac{2(\ell n2-2)}{2\ell n2}+\frac{1}{\ell n2}$

$\Rightarrow \frac{1}{8}+\frac{1}{4\ln 2}\le f(x)\le 1-\frac{1}{\ell n2}$

$2\Rightarrow\frac{1}{8}<f(x)<1,\forall x\in[2,4]$

SECTION – B

Question ID : 3666944203

21. Let $A=\begin{bmatrix}0&1&2\\a&0&3\\1&c&0\end{bmatrix}$, where $a,c\in\mathbb{R}$. If $A^3=A$ and the positive value of a belongs to the interval $(n-1, n]$, where $n\in\mathbb{N}$, then n is equal to ________.

माना $A=\begin{bmatrix}0&1&2\\a&0&3\\1&c&0\end{bmatrix}$, $a,c\in\mathbb{R}$ है। यदि $A^3=A$ है तथा a का घनात्मक मान अंतराल $(n-1, n]$ में है, जहाँ $n\in\mathbb{N}$ है, तो n बराबर है।

Ans. Official Answer NTA (2.00)

Sol. $A^2=\begin{bmatrix}0&1&2\\a&0&3\\1&c&0\end{bmatrix}\begin{bmatrix}0&1&2\\a&0&3\\1&c&0\end{bmatrix}$

$=\begin{bmatrix}a+2&2c&3\\3&a+3c&2a\\ca&1&2+3c\end{bmatrix}$

$$A^3 = \begin{bmatrix} a+2 & 2c & 3 \\ 3 & a+3c & 2a \\ ca & 1 & 2+3c \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2ca+3 & a+2+3c & 2a+4+6c \\ a^2+3ca+2a & 3+2ac & 6+3a+9c \\ a+2+3c & ca+2c+3c^2 & 2ca+3 \end{bmatrix}$$

$2ca+3=0, a+2+3c=1$

$a+1+3\left(\frac{-3}{2a}\right)=1$

$a+1-\frac{9}{2a}=0$

$2a^2+2a-9=0$

$a \in (1, 2]$

Question ID : 3666944201

22. The number of integral terms in the expansion of $\left(3^{\frac{1}{2}}+5^{\frac{1}{4}}\right)^{680}$ is equal to ________.

$\left(3^{\frac{1}{2}}+5^{\frac{1}{4}}\right)^{680}$ के प्रसार में पूर्णांक पदों की संख्या है ________

Ans. Official Answer NTA (171)

Sol. The number of integral term in the expression of $\left(3^{\frac{1}{2}}+5^{\frac{1}{4}}\right)^{680}$ is equal to

General term $= {}^{680}C_r \left(3^{\frac{1}{2}}\right)^{680-r}\left(5^{\frac{1}{4}}\right)^r$

$= {}^{680}C_r 3^{\frac{680-r}{2}} 5^{\frac{r}{4}}$

Value's of r, where $\frac{r}{4}$ goes to integer

r = 0, 4, 8, 12, ...680

All value of r are accepted for $\frac{680-r}{2}$ as well so

No of integral terms = 171.

Question ID : 3666944200

23. For m, n > 0, let $\alpha(m,n)=\int_0^2 t^m(1+3t)^n\,dt$. If $11\alpha(10,6)+18\alpha(11,5)=p(14)^6$, then p is equal to ________.

m, n > 0 के लिए माना $\alpha(m,n)=\int_0^2 t^m(1+3t)^n\,dt$ है। यदि $11\alpha(10,6)+18\alpha(11,5)=p(14)^6$ तो p बराबर ________ है।

Ans. Official Answer NTA (32)

Sol. $\alpha(m,n)=\int_0^2 t^m(1+3t)^n\,dt$

If $11\alpha(10,6)+18\alpha(11,5)=p(14)^6$ then P

$=11\int_0^2 \underset{II}{t^{10}}\,\underset{I}{(1+3t)^6}+10\int_0^2 t^{11}(1+3t)^5 dt$

$=11\left[(1+3t)^6\cdot\frac{t^{11}}{11}-\int 6(1+3t)^5\cdot 3\frac{t^{11}}{11}\right]_0^2+18\int_0^2 t^{11}(1+3t)^5 dt$

$=\left(t^{11}(1+3t)^6\right)_0^2$

$=2^{11}(7)^6$

$=2^5(14)^6$

$=32(14)^6$

Question ID : 3666944196

24. Let a line ℓ pass through the origin and be perpendicular to the lines

$\ell_1:\vec{r}=(\hat{i}-11\hat{j}-7\hat{k})+\lambda(\hat{i}+2\hat{j}+3\hat{k}),\lambda\in\mathbb{R}$ and

$\ell_2:\vec{r}=(-\hat{i}+\hat{k})+\mu(2\hat{i}+2\hat{j}+\hat{k}),\mu\in\mathbb{R}$.

If P is the point of intersection of ℓ and ℓ_1, and $Q(\alpha,\beta,\gamma)$ is the foot of perpendicular from P on ℓ_2, then $9(\alpha+\beta+\gamma)$ is equal to ________.

माना एक रेखा ℓ मूल बिंदु से होकर जाती है तथा रेखाओं

$\ell_1:\vec{r}=(\hat{i}-11\hat{j}-7\hat{k})+\lambda(\hat{i}+2\hat{j}+3\hat{k}),\lambda\in\mathbb{R}$,

$\ell_2 : \vec{r} = (-\hat{i}+\hat{k})+\mu(2\hat{i}+2\hat{j}+\hat{k}), \mu \in \mathbb{R}$

के लंबवत् है। यदि ℓ तथा ℓ_1 का प्रतिच्छेदन बिंदु P है तथा P से ℓ_2 पर लंब का पाद Q(α, β, γ) है, तो 9(α + β + γ) बराबर है।

Ans. Official Answer NTA (5)

Sol. Since $\ell \perp \ell_1$ and $\ell \perp \ell_2$

Let dr's of ℓ be a, b, c

$\Rightarrow a+2b+3c=0$

$2a+2b+c=0$

$\Rightarrow \frac{a}{-4}=\frac{b}{5}=\frac{c}{-2}$

hence dr's of ℓ can be 4, –5, 2

So any point P on ℓ can be taken as $(4\alpha, -5\alpha, 2\alpha)$

$\Rightarrow 4a = \lambda+1$ and $2\lambda-11=-5\alpha, 3\lambda-7=2\alpha$

$\Rightarrow 2(3\lambda-7)=\lambda+1$

$\Rightarrow 5\lambda=15 \Rightarrow \lambda=3$

so $P(4,-5,2)$

dr's of $PQ : 2\mu\text{-}5, 2\mu\text{+}5, \mu\text{-}1$

$\because PQ \perp \ell_2$

$= 2(2\mu-5)+2(2\mu+5)+\mu-1=0$

$\Rightarrow 9\mu=1 \Rightarrow \mu=\frac{1}{9}$

$\Rightarrow Q\left(\frac{-7}{9},\frac{2}{9},\frac{10}{9}\right)$

hence $9(\alpha+\beta+\gamma)=5$

Question ID : 3666944202

25. Let $H_n : \frac{x^2}{1+n}-\frac{y^2}{3+n}=1, n \in N$, Let k be the smallest even value of n such that the eccentricity of H_k is a rational number. If l is the length of the latus rectum of H_k, then $21l$ is equal to ________.

माना $H_n : \frac{x^2}{1+n}-\frac{y^2}{3+n}=1, n \in N$ है। माना k, n का वह न्यूनतम सम मान है जिसके लिए H_k की उत्केन्द्रता एक परिमेय संख्या है। यदि H_k की नाभिलंब जीवा की लंबाई l है, तो $21l$ बराबर है।

Ans. Official Answer NTA (306)

Sol. $(3+n)=(1+n)(e^2-1)$

$$e^2 = \frac{2n+4}{n+1} = \frac{2(n+2)}{n+1}$$

Check when (n + 1) = 9, 25, 49, ……

$$n = 8, e^2 = \frac{20}{9}$$

$$n = 24, e^2 = \frac{52}{25}$$

$$n = 48, e^2 = \frac{100}{49} \Rightarrow e = \frac{10}{7}$$

$$\Rightarrow n = 48$$

$$\Rightarrow 21/ = 21\times\frac{2b^2}{a} = 42\times\frac{n+3}{\sqrt{n+1}} = \frac{42\times 51}{7} = 306$$

Question ID : 3666944198

26. If a and b are the roots of the equation $x^2 - 7x - 1 = 0$, then the value of $\frac{a^{21}+b^{21}+a^{17}+b^{17}}{a^{19}+b^{19}}$ is equal to ________.

यदि समीकरण $x^2 - 7x - 1 = 0$ के मूल a तथा b हैं, तो $\frac{a^{21}+b^{21}+a^{17}+b^{17}}{a^{19}+b^{19}}$ का मान बराबर है।

Ans. Official Answer NTA (51)

Sol. $x^2 - 7x - 1 = 0 <^{a}_{b}$

By newton's theorem

$$S_{n+2} - 7S_{n+1} - S_n = 0$$

$$S_{21} - 7S_{20} - S_{19} = 0$$

$$S_{20} - 7S_{19} - S_{18} = 0$$

$$S_{19} - 7S_{18} - S_{17} = 0$$

$$\frac{S_{21}+S_{17}}{S_{19}} = \frac{S_{21}+(S_{19}-7S_{18})}{S_{19}}$$

$$= \frac{S_{21}+S_{19}-7(S_{20}-7S_{19})}{S_{19}}$$

$$= \frac{50S_{19}+(S_{21}-7S_{20})}{S_{19}}$$

$= 51 \cdot \frac{S_{19}}{S_{19}} = 51$

Question ID : 3666944197

27. The number of ordered triplets of the truth values of p, q and r such that the truth value of the statement $(p \vee q) \wedge (p \vee r) \Rightarrow (q \vee r)$ is true, is equal to________.

p, q तथा r के सत्यमानों के क्रमित त्रिकों, जिनके लिए कथन $(p \vee q) \wedge (p \vee r) \Rightarrow (q \vee r)$ का सत्यमान True है, की संख्या बराबर है।

Ans. Official Answer NTA (7)

Sol.

p	q	r	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$	$q \vee r$	$(p \vee q) \wedge (p \vee r) \rightarrow q \vee r$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	F	T	F	T	T
F	F	F	F	F	F	F	T

Hence total no of ordered triplets are 7

Question ID : 3666944204

28. In an examination, 5 students have been allotted their seats as per their roll numbers. The number of ways, in which none of the students sits on the allotted seat, is ________.

एक परीक्षा में 5 छात्रों को उनके रोल नंबर के अनुसार सीट दी गई है। उन तरीकों, जिनमें कोई भी छात्र दी गई सीट पर नहीं बैठता है, की संख्या है।

Ans. Official Answer NTA (44)

Sol. Clearly Derangement of 5 persons $= 5!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right)$

$= 5!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right)$

$= 60 - 20 + 5 - 1 = 40 + 4 = 44$

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29. Let $S=109+\frac{108}{5}+\frac{107}{5^2}+....+\frac{2}{5^{107}}+\frac{1}{5^{108}}$. Then the value of $\left(16S-(25)^{-54}\right)$ is equal to ________.

माना $S=109+\frac{108}{5}+\frac{107}{5^2}+....+\frac{2}{5^{107}}+\frac{1}{5^{108}}$ है। तो $\left(16S-(25)^{-54}\right)$ का मान बराबर है।

Ans. Official Answer NTA (2175)

Sol. $S=109+\frac{108}{5}+\frac{107}{5^2}+\ldots+\frac{2}{5^{107}}+\frac{1}{5^{108}}$

$\frac{S}{5}=\frac{109}{5}+\frac{108}{5^2}+\ldots+\ldots+\frac{2}{5^{108}}+\frac{1}{5^{109}}\cdots$

Equation (ii) – (i) gives

$$\frac{-4S}{5}=-109+\left(\frac{1}{5}+\frac{1}{5^2}+\ldots+\frac{1}{5^{108}}+\frac{1}{5^{109}}\right)$$

$$\frac{-4S}{5}=-109+\frac{1}{5}\frac{\left(1-\left(\frac{1}{5}\right)^{109}\right)}{1-\frac{1}{5}}$$

$$\frac{4S}{5}=-109+\frac{1}{4}\left(1-\frac{1}{5^{109}}\right)$$

$$\Rightarrow 16S-(25)^{-54}=2175$$

Question ID : 3666944195

30. The mean of the coefficients of $x, x^2,...,x^7$ in the binomial expansion of $(2+x)^9$ is ________.

$(2 + x)^9$ के द्विपद प्रसार में $x, x^2,...,x^7$ के गुणांकों का माध्य है।

Ans. Official Answer NTA (2736)

Sol. Coefficient of $x={}^9C_1 2^8$

Of $x^2={}^9C_2 2^7$

Of $x^7={}^9C_7\cdot 2^2$

$$\text{Mean}=\frac{{}^9C_1\cdot 2^8+{}^9C_2\cdot 2^7\ldots.+{}^9C_7\cdot 2^2}{7}$$

$$= \frac{(1+2)^9 - {}^9C_0 \cdot 2^9 - {}^9C_8 \cdot 2^1 - {}^9C_9}{7}$$

$$= \frac{3^9 - 2^9 - 18 - 1}{7}$$

$$= \frac{19152}{7} = 2736$$