JEE Main April 2023

Question Paper With Text Solution

11 April | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN APRIL 2023 | 11TH APRIL SHIFT-2

SECTION – A

Question ID : 7155054055

1. If the radius of the largest circle with centre (2, 0) inscribed in the ellipse $x^2 + 4y^2 = 36$ is r, then $12r^2$ is equal to :

यदि दीर्घवृत्त $x^2 + 4y^2 = 36$ के अंतर्गत, केन्द्र (2,0) के सबसे बड़े वृत्त की त्रिज्या r है, तो $12r^2$ बराबर है :

(1) 115 (2) 72 (3) 92 (4) 69

Ans. Official Answer NTA (3)

Sol. Equation of normal at $P(6\cos\theta, 3\sin\theta)$ is

$6\sec\theta x - 3\operatorname{cosec}\theta y = 27$

If passes through (2, 0)

$\therefore\ 12\sec\theta = 27$

$\therefore\ \cos\theta = \frac{4}{9}, \sin\theta = \frac{\sqrt{65}}{9}$

$P\left(\frac{8}{3}, \frac{\sqrt{65}}{3}\right)$

r = OP (O = (2, 0))

$= \sqrt{\left(\frac{8}{3} - 2\right)^2 + \left(\frac{\sqrt{65}}{3}\right)^2}$

$= \frac{\sqrt{69}}{3}$

$\therefore\ 12r^2 = 12 \times \frac{69}{9} = 92$

Question ID : 7155054053

2. If $f : \mathbb{R} \to \mathbb{R}$ be a continuous function satisfying $\int_0^{\frac{\pi}{2}} f(\sin 2x)\sin x dx + \alpha\int_0^{\frac{\pi}{4}} f(\cos 2x)\cos x dx = 0$, then the value of α is :

यदि $f : \mathbb{R} \to \mathbb{R}$ एक संतत फलन है तथा $\int_0^{\frac{\pi}{2}} f(\sin 2x)\sin x dx + \alpha\int_0^{\frac{\pi}{4}} f(\cos 2x)\cos x dx = 0$ है, तो α का मान है :

(1) $-\sqrt{3}$ (2) $\sqrt{2}$ (3) $-\sqrt{2}$ (4) $\sqrt{3}$

Ans. Official Answer NTA (3)

Sol. $I=\int_0^{\frac{\pi}{4}} f(\sin 2x)\sin x dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} f(\sin 2x)\sin x dx$

$+\alpha\int_0^{\frac{\pi}{4}} f(\cos 2x)\cos x dx=0$

Apply king in first part and put $x-\frac{\pi}{4}=t$ in second part.

$I=\int_0^{\frac{\pi}{4}} f(\cos 2x)\sin\left(\frac{\pi}{4}-x\right)dx+\int_0^{\frac{\pi}{4}} f(\cos 2t)\sin\left(\frac{\pi}{4}+t\right)dt$

$+\alpha\int_0^{\frac{\pi}{4}} f(\cos 2x)\cos x dx=0$

$I=\int_0^{\frac{\pi}{4}} f(\cos 2x)\left[2\sin\frac{\pi}{4}\cdot\cos x+\alpha\cos x\right]dx=0$

$I=(\alpha+\sqrt{2})\int_0^{\frac{\pi}{4}} f(\cos 2x)\cos x dx=0$

$\therefore \alpha=-\sqrt{2}$

Question ID : 7155054054

3. Let $y=y(x)$ be the solution of the differential equation $\frac{dy}{dx}+\frac{5}{x\left(x^5+1\right)}y=\frac{\left(x^5+1\right)^2}{x^7}, x>0$. If $y(1)=2$, then $y(2)$ is equal to :

माना अवकल समीकरण $\frac{dy}{dx}+\frac{5}{x\left(x^5+1\right)}y=\frac{\left(x^5+1\right)^2}{x^7}, x>0$ का हल $y=y(x)$ है। यदि $y(1)=2$ है, तो $y(2)$ बराबर है :

(1) $\frac{679}{128}$ (2) $\frac{637}{128}$ (3) $\frac{697}{128}$ (4) $\frac{693}{128}$

Ans. Official Answer NTA (4)

Sol. $I.F=e^{\int\frac{5dx}{x\left(x^5+1\right)}}=e^{\int\frac{5x^{-6}}{\left(x^{-5}+1\right)}}$

$\text{Put}, 1+x^{-5}=t\Rightarrow -5x^{-6}dx=dt$

$$\Rightarrow e^{\int \frac{-dt}{t}} = \frac{1}{t} = \frac{x^5}{1+x^5}$$

$$y \cdot \frac{x^5}{1+x^5} = \int \frac{x^5}{(1+x^5)} \times \frac{(1+x^5)^2}{x^7} dx$$

$$= \int x^3 dx + \int x^{-2} dx$$

$$y \cdot \frac{x^5}{1+x^5} = \frac{x^4}{4} - \frac{1}{x} + c$$

Given than : $x = 1 \Rightarrow y = 2$

$$2 \cdot \frac{1}{2} = \frac{1}{4} - 1 + c$$

$$c = \frac{7}{4}$$

$$y \cdot \frac{x^5}{1+x^5} = \frac{x^4}{4} - \frac{1}{x} + \frac{7}{4}$$

Now put, $x = 2$

$$y \cdot \left(\frac{32}{33}\right) = \frac{21}{4}$$

$$y = \frac{693}{128}$$

Question ID : 7155054052

4. Let the function $f:[0,2] \to \mathbb{R}$ be defined as $f(x) = \begin{cases} e^{\min\{x^2, x-[x]\}}, & x \in [0,1) \\ e^{[x - \log_e x]}, & x \in [1,2] \end{cases}$ where [t] denotes the greatest integer less than or equal to t. Then the value of the integral $\int_0^2 xf(x)dx$ is :

माना $f:[0,2] \to \mathbb{R}$, $f(x) = \begin{cases} e^{\min\{x^2, x-[x]\}}, & x \in [0,1) \\ e^{[x - \log_e x]}, & x \in [1,2] \end{cases}$ द्वारा परिभाषित है, जहाँ [t] का महत्तम पूर्णांक $\le$ t है। तो समाकलन $\int_0^2 xf(x)dx$ का मान है :

(1) $2e - 1$ (2) $1 + \frac{3e}{2}$ (3) $2e - \frac{1}{2}$ (4) $(e-1)\left(e^2 + \frac{1}{2}\right)$

Ans. Official Answer NTA (3)

Sol. $f(x)=\begin{cases} e^{x^2} & x\in(0,1) \\ e & x\in[1,2) \end{cases}$

Now $\int_0^2 xf(x)dx=\int_0^1 xe^{x^2}dx+\int_1^2 x\cdot e dx=2e-\frac{1}{2}$

Question ID : 7155054043

5. The domain of the function $f(x)=\frac{1}{\sqrt{[x]^2-3[x]-10}}$ is : (where [x] denotes the greatest integer less than or equal to x) :

फलन $f(x)=\frac{1}{\sqrt{[x]^2-3[x]-10}}$, जहाँ [x] महत्तम पूर्णांक ≤ x है, का प्रांत है :

(1) $(-\infty,-3]\bigcup(5,\infty)$ (2) $(-\infty,-2)\cup[6,\infty)$ (3) $(-\infty,-2)\cup(5,\infty)$ (4) $(-\infty,-3]\cup[6,\infty)$

Ans. Official Answer NTA (2)

Sol. $f(x)=\frac{1}{\sqrt{[x]^2-3[x]-10}}$

For Domain $[x]^2-3[x]-10>0$

$\Rightarrow$ $([x]-5)([x]+2)>0$

$\Rightarrow$ $[x]\in(-\infty,-2)\cup(5,\infty)$

$\therefore$ $x\in(-\infty,-2)\cup[6,\infty)$

Question ID : 7155054044

6. For $a\in\mathbb{C}$, let $A=\{z\in\mathbb{C}:\text{Re}(a+\bar{z})>\text{Im}(\bar{a}+z)\}$ and $B=\{z\in\mathbb{C}:\text{Re}(a+\bar{z})<\text{Im}(\bar{a}+z)\}$. Then among the two statements :

(S1) : If Re (a), Im (a) > 0, then the set A contains all the real numbers

(S2) : If Re (a), Im (a) < 0, then the set B contains all the real numbers

(1) only (S1) is true (2) both are false (3) only (S2) is true (4) both are true

$a\in\mathbb{C}$ के लिए, माना $A=\{z\in\mathbb{C}:\text{Re}(a+\bar{z})>\text{Im}(\bar{a}+z)\}$ तथा $B=\{z\in\mathbb{C}:\text{Re}(a+\bar{z})<\text{Im}(\bar{a}+z)\}$ हैं। तो दो कथनों :

(S1) : यदि Re (a), Im (a) > 0 है, तो सभी वास्तविक संख्याएँ A में हैं

(S2) : यदि Re (a), Im (a) < 0 है, तो सभी वास्तविक संख्याएँ B में हैं

(1) केवल (S1) सही है (2) दोनो गलत है (3) केवल (S2) सही है (4) दोनो सही है

Ans. Official Answer NTA (2)

Sol. Let $a = x_1 + iy_1, z = x + iy$

Now $\text{Re}(a + \bar{z}) > \text{Im}(\bar{a} + z)$

$\therefore x_1 + x > -y_1 + y$

$x_1 = 2, y_1 = 10, x = -12, y = 0$

Given inequality is not valid for these values.

S1 is false.

Now $\text{Re}(a + \bar{z}) < \text{Im}(\bar{a} + z)$

$x_1 + x < -y_1 + y$

$x_1 = -2, y_1 = -10, x = 12, y = 0$

Given inequality is not valid for these values.

S2 is false.

Question ID : 7155054048

7. If the 1011[th] term from the end in the binomial expansion of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^{2022}$ is 1024 times 1011[th] term from the beginning, then |x| is equal to :

यदि $\left(\frac{4x}{5} - \frac{5}{2x}\right)^{2022}$ के द्विपद प्रसार में अंत से 1011वाँ पद, आरंभ से 1011वें पद का 1024 गुना है, तो |x| बराबर है :

(1) 8 (2) 12 (3) 10 (4) 15

Ans. Official Answer NTA (10) **Bonus**

Sol. T_{1011} from beginning $= T_{1010+1}$

$= {}^{2022}C_{1010}\left(\frac{4x}{5}\right)^{1012}\left(\frac{-5}{2x}\right)^{1010}$

T_{1011} from end

$= {}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010}$

Given : $= {}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010}$

$= 2^{10} \cdot {}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1010}\left(\frac{4x}{5}\right)^{1012}$

$\left(\frac{-5}{2x}\right)^2 = 2^{10}\left(\frac{4x}{5}\right)^2$

$x^4 = \frac{5^4}{2^{16}}$

$|x| = \frac{5}{16}$

Question ID : 7155054049

8. The sum of the coefficients of three consecutive terms in the binomial expansion of $(1+x)^{n+2}$, which are in the ratio 1 : 3 : 5, is equal to :

$(1 + x)^{n+2}$ के द्विपद प्रसार में तीन क्रमागत पदों के गुणांकों, जो 1 : 3 : 5 के अनुपात में है, का योग बराबर है :

(1) 25 (2) 92 (3) 41 (4) 63

Ans. Official Answer NTA (4)

Sol. Coefficients are ${}^{n+2}C_{r-1}, {}^{n+2}C_r, {}^{n+2}C_{r+1}$

Now $\frac{{}^{n+2}c_{r-1}}{{}^{n+2}c_r} = \frac{1}{3} \Rightarrow n+3 = 4r$ ....(1)

$\frac{{}^{n+2}c_r}{{}^{n+2}c_{r+1}} = \frac{3}{5} \Rightarrow 3n+1 = 8r$ ....(2)

By(1) & (2) $n = 5, r = 2$

Now coefficients are ${}^7C_1, {}^7C_2, {}^7C_3$

sum = 63

Question ID : 7155054046

9. If $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{8}(103x+81)$, then $\lambda, \frac{\lambda}{3}$ are the roots of the equation :

यदि $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{8}(103x+81)$ है, तो $\lambda, \frac{\lambda}{3}$ किस समीकरण के मूल है :

(1) $4x^2 - 24x + 27 = 0$ (2) $4x^2 - 24x - 27 = 0$

(3) $4x^2 + 24x + 27 = 0$ (4) $4x^2 + 24x - 27 = 0$

Ans. Official Answer NTA (1)

Sol. $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{8}(103x+81)$

Put x = 0

$\begin{vmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^2 \end{vmatrix} = \frac{9}{8} \times 81$

$\lambda^3 = \frac{3^6}{2^3}$

$\lambda = \frac{9}{2}$

$\frac{\lambda}{3} = \frac{3}{2}$

Equation : $x^2 - \left(\frac{9}{2} + \frac{3}{2}\right)x + \frac{9}{2} \times \frac{3}{2} = 0$

$4x^2 - 24x + 27 = 0$

Question ID : 7155054061

10. The converse of $((\sim p) \vee q) \Rightarrow r$ is :

$((\sim p) \vee q) \Rightarrow r$ का विलोम है :

(1) $(p \vee (\sim q)) \Rightarrow (\sim r)$ (2) $(\sim r) \Rightarrow p \wedge q$

(3) $(\sim r) \Rightarrow ((\sim p) \wedge q)$ (4) $((\sim p) \vee q) \Rightarrow r$

Ans. Official Answer NTA (1)

Sol. Converse of $((\sim p) \wedge q) \Rightarrow r$

$\equiv r \Rightarrow (\sim p \wedge q)$

$\equiv \sim r \vee (\sim p \wedge q)$

$\equiv \sim r \vee (p \vee \sim q) \equiv (p \vee \sim q) \Rightarrow \sim r$

Question ID : 7155054051

11. Let f and g be two functions defined by $f(x)=\begin{cases} x+1, & x<0 \\ |x-1|, & x\geq 0 \end{cases}$ and $g(x)=\begin{cases} x+1, & x<0 \\ 1, & x\geq 0 \end{cases}$. Then (gof) (x) is:

(1) continuous everywhere but not differentiable at x = 1

(2) not continuous at x = –1

(3) differentiable everywhere

(4) continuous everywhere but not differentiable exactly at one point

माना दो फलन f तथा g, $f(x)=\begin{cases} x+1, & x<0 \\ |x-1|, & x\geq 0 \end{cases}$ तथा $g(x)=\begin{cases} x+1, & x<0 \\ 1, & x\geq 0 \end{cases}$ द्वारा परिभाषित है। तो (gof) (x) :

(1) R पर संतत है परन्तु x = 1 पर अवकलनीय नहीं है

(2) x = –1 पर संतत नहीं है

(3) R पर अवकलनीय है

(4) R पर संतत है परन्तु मात्र एक बिंदु पर अवकलनीय नहीं है

Ans. Official Answer NTA (4)

Sol. $f(x)=\begin{cases} x+1, x<0 \\ 1-x, 0\leq x<1 \\ x-1, 1\leq x \end{cases}$

$g(x)=\begin{cases} x+1, x<0 \\ 1, x\geq 0 \end{cases}$

$g(f(x))=\begin{cases} x+2, x<-1 \\ 1, x\geq -1 \end{cases}$

$\therefore g(f(x))$ is continuous everywhere

$g(f(x))$ is not differentiable at $x = -1$

Differentiable everywhere else

Question ID : 7155054058

12. If four distinct points with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are coplanar, then $[\vec{a}\ \vec{b}\ \vec{c}]$ is equal to :

यदि चार भिन्न बिंदु, जिनके स्थिति सदिश $\vec{a}$, $\vec{b}$, $\vec{c}$ तथा $\vec{d}$ हैं, सहतलीय हैं, तो $[\vec{a}\,\vec{b}\,\vec{c}]$ बराबर है :

(1) $[\vec{a}\,\vec{d}\,\vec{b}]+[\vec{d}\,\vec{c}\,\vec{a}]+[\vec{d}\,\vec{b}\,\vec{c}]$
(2) $[\vec{d}\,\vec{b}\,\vec{a}]+[\vec{a}\,\vec{c}\,\vec{d}]+[\vec{d}\,\vec{b}\,\vec{c}]$
(3) $[\vec{d}\,\vec{c}\,\vec{a}]+[\vec{b}\,\vec{d}\,\vec{a}]+[\vec{c}\,\vec{d}\,\vec{b}]$
(4) $[\vec{b}\,\vec{c}\,\vec{d}]+[\vec{d}\,\vec{a}\,\vec{c}]+[\vec{d}\,\vec{b}\,\vec{a}]$

Ans. Official Answer NTA (3)

Sol. $\vec{a},\vec{b},\vec{c},\vec{d}$ are coplanar point then

vectors $\vec{b}-\vec{a},\vec{c}-\vec{a},\vec{d}-\vec{a}$ are also coplanar

$$[\vec{b}-\vec{a}\;\;\vec{c}-\vec{a}\;\;\vec{d}-\vec{a}]=0$$

$$[\vec{b}\;\vec{c}\;\vec{d}]-[\vec{b}\;\vec{c}\;\vec{a}]-[\vec{b}\;\vec{a}\;\vec{d}]-[\vec{a}\;\vec{c}\;\vec{d}]=0$$

$$[\vec{a}\;\vec{b}\;\vec{c}]=[\vec{b}\;\vec{c}\;\vec{d}]+[\vec{a}\;\vec{b}\;\vec{d}]+[\vec{a}\;\vec{d}\;\vec{c}]$$

Question ID : 7155054057

13. Let the line passing through the points P(2, –1, 2) and Q(5, 3, 4) meet the plane $x-y+z=4$ at the point R. Then the distance of the point R from the plane x + 2y + 3z + 2 = 0 measured parallel to the line $\frac{x-7}{2}=\frac{y+3}{2}=\frac{z-2}{1}$ is equal to :

माना बिंदुओं P(2, –1, 2) तथा Q(5, 3, 4) से होकर जाने वाली रेखा, समतल $x-y+z=4$ को बिंदु R पर मिलती है। तो बिंदु R की समतल x + 2y + 3z + 2 = 0 से, रेखा $\frac{x-7}{2}=\frac{y+3}{2}=\frac{z-2}{1}$ के समांतर मापी गई, दूरी है :

(1) $\sqrt{89}$ (2) 3 (3) $\sqrt{31}$ (4) $\sqrt{61}$

Ans. Official Answer NTA (2)

Sol. Equation of line PQ.

$$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}=\lambda$$

Let R be $(3\lambda+2,4\lambda-1,2\lambda+2)$

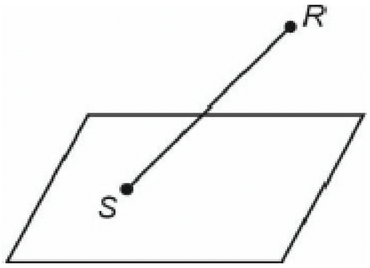
R lies on plane $x-y+z=4$

$\therefore 3\lambda+2-4\lambda+1+2\lambda+2=4$

$\Rightarrow \lambda=-1$

$\therefore R(-1,-5,0)$

Let SR be : $\frac{x+1}{2}=\frac{y+5}{2}=\frac{z}{1}=k$



$S:(2k-1, 2k-5, k)$

S lies on plane : $x+2y+3z+2=0$

$\Rightarrow (2k-1)+(4k-10)+3k+2=0$

$\Rightarrow 9k-9=0 \Rightarrow k=1$

$S(1,-3,1) \quad \therefore SR=\sqrt{4+4+1}=3$

Question ID : 7155054059

14. Let the mean of 6 observations 1, 2, 4, 5, x and y be 5 and their variance be 10 . Then their mean deviation about the mean is equal to :

माना 6 प्रेक्षणों 1, 2, 4, 5, x तथा y का माध्य 5 है तथा इनका प्रसरण 10 है। तो इनका माध्य के सापेक्ष माध्य विचलन है :

(1) $\frac{10}{3}$ (2) 3 (3) $\frac{8}{3}$ (4) $\frac{7}{3}$

Ans. Official Answer NTA (3)

Sol. $x+y=18\{\because \text{ mean}=5\}$ ....(i)

$10=\frac{1+4+16+25+x^2+y^2}{6}-25$

$x^2+y^2=164$ ....(ii)

By solving (i) and (ii)

$x=8, y=10$

$\text{M.D. }(\bar{x})=\frac{\sum|x_i-\bar{x}|}{6}=\frac{8}{3}$

Question ID : 7155054050

15. Let a, b, c and d be positive real numbers such that a + b + c + d = 11. If the maximum value of $a^5b^3c^2d$ is 3750β, then the value of β is :

माना a, b, c तथा d धनात्मक वास्तविक संख्याएँ हैं तथा a + b + c + d = 11 है। यदि $a^5b^3c^2d$ का उच्चतम मान 3750β है, तो

β का मान है :

(1) 108 (2) 110 (3) 90 (4) 55

Ans. Official Answer NTA (3)

Sol. Given a + b + c + d = 11 (a, b, c, d > 0}

$(a^5b^3c^2d)$max. = ?

Let assume Numbers –

$$\frac{a}{5},\frac{a}{5},\frac{a}{5},\frac{a}{5},\frac{a}{5},\frac{b}{3},\frac{b}{3},\frac{b}{3},\frac{c}{2},\frac{c}{2},$$

We know A.M. ≥ G.M.

$$\frac{\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{2}+\frac{c}{2}+d}{11}\geq\left(\frac{a^5b^3c^2d}{5^5\cdot3^3\cdot2^2\cdot1}\right)^{\frac{1}{11}}$$

$$\frac{11}{11}\geq\left(\frac{a^5b^3c^2d}{5^5\cdot3^3\cdot2^2\cdot1}\right)^{\frac{1}{11}}$$

$$a^5\cdot b^3\cdot c^2\cdot d\leq5^5\cdot3^3\cdot2^2$$

$$\max\left(a^5b^3c^2d\right)=5^5\cdot3^3\cdot2^2=337500$$

$$=90\times3750=\beta\times3750$$

$$\beta=90$$

Question ID : 7155054042

16. Let A = {1, 3, 4, 6, 9} and B = {2, 4, 5, 8, 10}. Let R be a relation defined on A × B such that $R=\{((a_1,b_1),(a_2,b_2)):a_1\leq b_2 \text{ and } b_1\leq a_2\}$. Then the number of elements in the set R is :

माना A = {1, 3, 4, 6, 9} तथा B = {2, 4, 5, 8, 10} हैं। मान लो A × B पर एक संबंध $R=\{((a_1,b_1),(a_2,b_2)):a_1\leq b_2$ तथा $b_1\leq a_2\}$ है। तो R में अवयवों की संख्या है :

(1) 180 (2) 52 (3) 160 (4) 26

Ans. Official Answer NTA (3)

Sol. If a_1 = 1 then b_2 can be 2, 4, 5, 8, 10 = 5 cases

a_1 = 3 then b_2 = 4 cases

a_1 = 4 then b_2 = 4

a_1 = 6 then b_2 = 2 case

$a_1 = 9$ then $b_2 = 1$ case
total = 16 cases
also with
$a_2 = 3$ then $b_1 = 1$ case
$a_2 = 4$ then $b_1 = 2$ cases
$a_2 = 6$ then $b_1 = 3$cases
$a_2 = 9$ then $b_1 = 4$ cases
toal 10 cases
so total relations = 160

Question ID : 7155054047

17. If the letters of the word MATHS are permuted and all possible words so formed are arranged as in a dictionary with serial numbers, then the serial number of the word THAMS is :

यदि MATHS शब्द के अक्षरों के क्रमचयों से बने शब्दों को एक शब्दकोश की तरह क्रम संख्या के साथ व्यवस्थित किया जाता है, तो शब्द THAMS की क्रम संख्या है :

(1) 101 (2) 104 (3) 102 (4) 103

Ans. Official Answer NTA (4)

5 2 1 3 4
T H A M S
Sol. 4 1 0 0 0
4! 3! 2! 1! 0!

Rank = $4 \times 4! + 1 \times 3! + 1$
$= 96 + 6 + 1$
$= 103$

Question ID : 7155054045

18. If the system of linear equations
$7x + 11y + \alpha z = 13$
$5x + 4y + 7z = \beta$
$175x + 194y + 57z = 361$
has infinitely many solutions, then $\alpha + \beta + 2$ is equal to :

यदि रैखिक समीकरण निकाय

$7x + 11y + \alpha z = 13$
$5x + 4y + 7z = \beta$
$175x + 194y + 57z = 361$

के अनंत हल हैं तो $\alpha + \beta + 2$ बराबर है :

(1) 5 (2) 6 (3) 3 (4) 4

Ans. Official Answer NTA (4)

Sol. $7x+11y+\alpha z=13$ ....(i)

$5x+4y+7z=\beta$ ....(ii)

$175x+194y+57z=361$ ....(iii)

$(i)\times10+(ii)\times21-(iii)$

$z(10\alpha+147-57)=130+21\beta-361$

$\therefore 10\alpha+90=0$

$\alpha=-9$

$130-361+21\beta=0$

$\beta=11$

$\alpha+\beta+2=4$

Question ID : 7155054056

19. Let P be the plane passing through the points (5, 3, 0), (13, 3, –2) and (1, 6, 2). For $\alpha \in \mathbb{N}$, if the distances of the points $A(3, 4, \alpha)$ and $B(2, \alpha, a)$ from the plane P are 2 and 3 respectively, then the positive value of a is :

माना समतल P, बिंदुओं (5, 3, 0), (13, 3, –2) तथा (1, 6, 2) से होकर जाता है। $\alpha \in \mathbb{N}$ के लिए यदि बिंदुओं $A(3, 4, \alpha)$ तथा $B(2, \alpha, a)$ की समतल P से दूरियाँ क्रमशः 2 तथा 3 है, तो a का धनात्मक मान है :

(1) 5 (2) 3 (3) 4 (4) 6

Ans. Official Answer NTA (3)

Sol. $$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & 0 & -2 \\ 4 & -3 & -2 \end{vmatrix} = \hat{i}(-6)+8\hat{j}-24\hat{k}$$

Normal of the plane $=3\hat{i}-4\hat{j}+12\hat{k}$

Plane : $3x - 4y + 12z = 3$

Distance from $A(3,4,\alpha)$

$$\left|\frac{9-16+12\alpha-3}{13}\right|=2$$

$\alpha=3$

$\alpha = -8$ (rejected)

Distance from B(2,3,a)

$$\left|\frac{6-12+12a-3}{13}\right| = 3$$

a = 4

Question ID : 7155054060

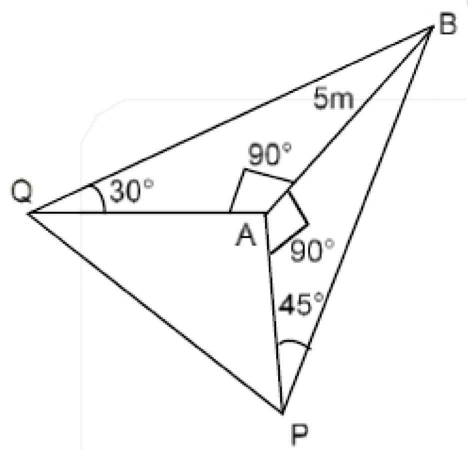
20. The angle of elevation of the top P of a tower from the feet of one person standing due South of the tower is 45° and from the feet of another person standing due west of the tower is 30°. If the height of the tower is 5 meters, then distance (in meters) between the two persons is equal to :

एक व्यक्ति, जो मीनार के दक्षिण की ओर खड़ा है, के पैर से मीनार के शिखर P का उन्नयन कोण 45° है तथा एक अन्य व्यक्ति, जो मीनार के पश्चिम की ओर खड़ा है, के पैर से P का उन्नयन कोण 30° है। यदि मीनार की ऊँचाई 5 मीटर है, तो इन दो व्यक्तियों के बीच की दूरी (मीटर में) है :

(1) 10 (2) $5\sqrt{5}$ (3) 5 (4) $\frac{5}{2}\sqrt{5}$

Ans. Official Answer NTA (1)

Sol.



Let tower be AB

A is foot of tower

Then AP = 5cot 45°

= 5

& $AQ = 5\cot 30^\circ = 5\sqrt{3}m$

Now $PQ = \sqrt{AP^2 + AQ^2} = \sqrt{25 + 25\times 3} = 10$

SECTION – B

Question ID : 7155054068

21. Let the tangent to the parabola $y^2 = 12x$ at the point $(3, \alpha)$ be perpendicular to the line $2x + 2y = 3$. Then the square of distance of the point $(6, -4)$ from the normal to the hyperbola $\alpha^2x^2 - 9y^2 = 9\alpha^2$ at its point $(\alpha - 1, \alpha + 2)$ is equal to ________.

माना परवलय $y^2 = 12x$ के बिंदु $(3, \alpha)$ पर स्पर्श रेखा, रेखा $2x + 2y = 3$ के लंबवत् है। तो बिंदु $(6, -4)$ की, अतिपरवलय $\alpha^2x^2 - 9y^2 = 9\alpha^2$ के बिंदु $(\alpha - 1, \alpha + 2)$ पर अभिलंब से दूरी का वर्ग है ________

Ans. Official Answer NTA (116)

Sol. Slope of tangent $= 1 = \frac{6}{\alpha} \Rightarrow \alpha = 6$

$$36x^2 - 9y^2 = 324$$

$$\frac{x^2}{9} - \frac{y^2}{36} = 1$$

Tangent at (5, 8)

$$\frac{5x}{9} - \frac{8y}{36} = 1$$

$5x - 2y = 9$

Slope of normal $= \frac{-2}{5}$

Equation of normal

$$y - 8 = \frac{-2}{5}(x - 5)$$

$5y - 40 = -2x + 10$

$5y + 2x = 50$

Distance from (6, –4)

$$= \left|\frac{12 - 20 - 50}{\sqrt{29}}\right| = \frac{58}{\sqrt{29}}$$

$$= 2\sqrt{29}$$

Question ID : 7155054070

22. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c} = 11$, $\vec{b}\cdot(\vec{a}\times\vec{c}) = 27$ and $\vec{b}\cdot\vec{c} = -\sqrt{3}|\vec{b}|$, then $|\vec{a}\times\vec{c}|^2$ is equal to ________.

माना $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ तथा $\vec{b} = \hat{i} + \hat{j} - \hat{k}$ हैं। यदि एक सदिश $\vec{c}$ के लिए $\vec{a}\cdot\vec{c} = 11$, $\vec{b}\cdot(\vec{a}\times\vec{c}) = 27$ तथा

$\vec{b}\cdot\vec{c} = -\sqrt{3}|\vec{b}|$ है, तो $|\vec{a}\times\vec{c}|^2$ बराबर है ________

Ans. Official Answer NTA (285)

Sol. $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} + \hat{j} - \hat{k}$

$\vec{b}.(\vec{a}\times\vec{c}) = 27,\ \vec{a}.\vec{b} = 0$

$\vec{b}\times(\vec{a}\times\vec{c}) = -3\vec{a}$

Let θ be angle between $\vec{b}, \vec{a}\times\vec{c}$

Then $|\vec{b}|.|\vec{a}\times\vec{c}|\sin\theta = 3\sqrt{14}$

$|\vec{b}|.|\vec{a}\times\vec{c}|\cos\theta = 27$

$\Rightarrow \sin\theta = \frac{\sqrt{14}}{\sqrt{95}}$

$\therefore |\vec{b}|.|\vec{a}\times\vec{c}| = 3\sqrt{95}$

$\Rightarrow |\vec{a}\times\vec{c}| = \sqrt{3}\times\sqrt{95}$

Question ID : 7155054064

23. For $k \in \mathbb{N}$, if the sum of the series $1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \ldots$ is 10, then the value of k is ________.

$k \in \mathbb{N}$ के लिए, यदि श्रेणी $1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \ldots$ का योग 10 है, तो k का मान है ________

Ans. Official Answer NTA (2)

Sol. $10 = 1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \ldots\text{upto } \infty$

$9 = \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \ldots \text{ upto } \infty$

$\frac{9}{k} = \frac{4}{k^2} + \frac{8}{k^3} + \frac{13}{k^4} + \ldots \text{ upto } \infty$

$S = 9\left(1 - \frac{1}{k}\right) = \frac{4}{k} + \frac{4}{k^2} + \frac{5}{k^3} + \frac{6}{k^4} + \ldots\ldots \text{upto } \infty$

$\frac{S}{k} = \frac{4}{k^2} + \frac{4}{k^3} + \frac{5}{k^4} + \ldots\ldots\text{upto } \infty$

$\left(1 - \frac{1}{k}\right)S = \frac{4}{k} + \frac{1}{k^3} + \frac{1}{k^4} + \frac{1}{k^5} + \ldots\infty$

$$9\left(1-\frac{1}{k}\right)^2=\frac{4}{k}+\frac{\frac{1}{k^3}}{\left(1-\frac{1}{k}\right)}$$

$9(k-1)^3=4k(k-1)+1$

$k=2$

Question ID : 7155054069

24. Let the line $\ell : x=\frac{1-y}{-2}=\frac{z-3}{\lambda},\lambda\in\mathbb{R}$ meet the plane $P : x+2y+3z=4$ at the point (α, β, γ). If the angle between the line l and the plane P is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then $\alpha+2\beta+6\gamma$ is equal to ________.

माना रेखा $\ell : x=\frac{1-y}{-2}=\frac{z-3}{\lambda},\lambda\in\mathbb{R}$ समतल $P : x + 2y + 3z = 4$ को बिंदु (α, β, γ) पर मिलती है। यदि रेखा l तथा समतल P के बीच का कोण $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$ है, तो $\alpha+2\beta+6\gamma$ बराबर है ________

Ans. Official Answer NTA (11)

Sol. $\ell : x=\frac{y-1}{2}=\frac{z-3}{\lambda},\lambda\in\mathbb{R}$

Dr's of line $\ell(1,2,\lambda)$

Dr's of normal vector of plane $P : x+2y+3z=4$

are (1, 2, 3)

Now, angle between line ℓ and plane P is given by

$$\sin\theta=\left|\frac{1+4+3\lambda}{\sqrt{5+\lambda^2}\cdot\sqrt{14}}\right|=\frac{3}{\sqrt{14}}\left(\text{given } \cos\theta=\sqrt{\frac{5}{14}}\right)$$

$\Rightarrow \lambda=\frac{2}{3}$

Let variable point of line ℓ is $\left(t, 2t+1, \frac{2}{3}t+3\right)$

line of plane P.

$\Rightarrow t=-1$

$\Rightarrow\left(-1,-1,\frac{7}{3}\right)\equiv(\alpha,\beta,\gamma)$

$\Rightarrow \alpha+2\beta+6\gamma=11$

Question ID : 7155054071

25. Let the probability of getting head for a biased coin be $\frac{1}{4}$. It is tossed repeatedly until a head appears. Let N be the number of tosses required. If the probability that the equation $64x^2+5Nx+1=0$ has no real root is $\frac{p}{q}$, where p and q are coprime, then q – p is equal to ________.

माना एक अभिनत सिक्के के लिए चित आने की प्रायिकता $\frac{1}{4}$ है। इसे बार बार उछाला जाता है जब तक कि चित प्राप्त न हो जाऐ। माना सिक्के को उछालने की आवश्यक संख्या N है। यदि समीकरण $64x^2+5Nx+1=0$ के वास्तविक हल न होने की प्रायिकता $\frac{p}{q}$ है, जहाँ p तथा q असहभाज्य हैं, तो q – p बराबर है ________

Ans. Official Answer NTA (27)

Sol. $64x^2+5nx+1=0$

$D<0$

$25N^2-256<0$

$-\frac{16}{5}<N<\frac{16}{5}$

$N=1,2,3$

$P=\frac{1}{4}+\frac{3}{4}\cdot\frac{1}{4}+\left(\frac{3}{4}\right)^2\frac{1}{4}$

$=\frac{16+12+9}{64}=\frac{37}{64}$

$q-p=27$

Question ID : 7155054063

26. Let A = {1, 2, 3, 4, 5} and B = {1, 2, 3, 4, 5, 6}. Then the number of functions $f:A\to B$ satisfying $f(1)+f(2)=f(4)-1$ is equal to ________.

माना A = {1, 2, 3, 4, 5} तथा B = {1, 2, 3, 4, 5, 6} हैं। तो $f(1)+f(2)=f(4)-1$ को संतुष्ट करने वाले फलनों $f:A\to B$ की संख्या है ________

Ans. Official Answer NTA (360)

Sol. $f(1) + f(2) + 1 = f(4)$

$\Rightarrow f(1) + f(2) \le 5$

Let $f(4) = 6$, then $f(1) + f(2) = 5$

So (f_1, f_2) may be (1, 4), (4, 1), (2, 3), (3, 2)

Total ways = 4 . 1 . 6 . 6 = 144

Let f(4) = 5, f(1) + f(2) = 4

so (f_1, f_2) may be (1, 3), (3, 1), (2, 2)

Total ways = 3 . 1 . 6 . 6 = 108

Let f(4) = 4, f(1) + f(2) = 3

so (f_1, f_2) may be (1, 2), (2, 1)

Total ways = 2 . 1 . 6 . 6 = 72

Let f(4) = 3, f(1) + f(2) = 2

Only way (1, 1)

Total ways = 1 . 1 . 36 = 36

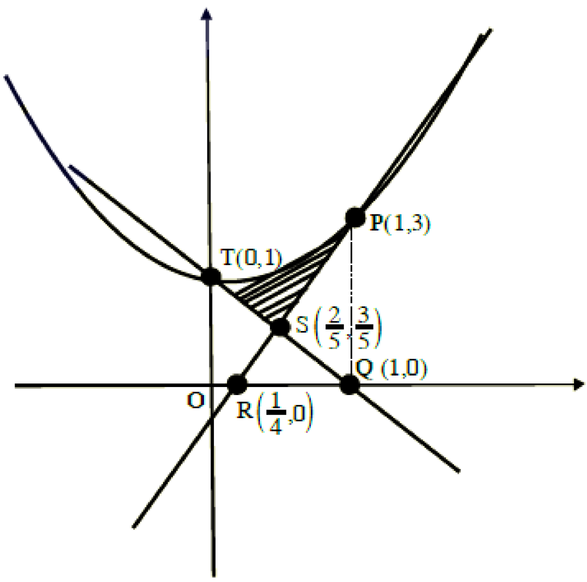
Question ID : 7155054066

27. If A is the area in the first quadrant enclosed by the curve $C : 2x^2 - y + 1 = 0$, the tangent to C at the point (1, 3) and the line $x + y = 1$, then the value of 60A is ________.

यदि वक्र $C : 2x^2 - y + 1 = 0$, C के बिंदु (1, 3) पर स्पर्श रेखा तथा रेखा $x + y = 1$ से घिरे, प्रथम चतुर्थांश में, क्षेत्र का क्षेत्रफल A है, तो 60A बराबर है ________

Ans. Official Answer NTA (16)

Sol.



$y = 2x^2 + 1$

Tangent at (1, 3)

$y = 4x - 1$

$A = \int_0^1 (2x^2+1)dx$ – area of (ΔQOT) – are of (ΔPQR) + area of (ΔQRS)

$A = \left(\frac{2}{3}+1\right) - \frac{1}{2} - \frac{9}{8} + \frac{9}{40} = \frac{16}{60}$

Question ID : 7155054067

28. If the line $l_1 : 3y - 2x = 3$ is the angular bisector of the lines $l_2 : x - y + 1 = 0$ and $l_3 : \alpha x + \beta y + 17 = 0$, then $\alpha^2 + \beta^2 - \alpha - \beta$ is equal to ________.

यदि रेखा $l_1 : 3y - 2x = 3$ रेखाओं $l_2 : x - y + 1 = 0$ तथा $l_3 : \alpha x + \beta y + 17 = 0$ की कोण समद्विभाजक है, तो $\alpha^2 + \beta^2 - \alpha - \beta$ बराबर है ________

Ans. Official Answer NTA (348)

Sol. Point of intersection of $\ell_1 : 3y - 2x = 3$

$\ell_2 : x - y + 1 = 0$ is $P \equiv (0,1)$

Which lies on $\ell_3 : \alpha x + \beta y + 17 = 0$

$\Rightarrow \beta = -17$

Consider a random point $Q \equiv (-1, 0)$

on $\ell_2 : x - y + 1 = 0$, image of Q about

$\ell_2 : x - y + 1 = 0$, is $Q' \equiv \left(\frac{-17}{13}, \frac{6}{13}\right)$ which is calculated by formulae

$\frac{x-(-1)}{2} = \frac{y-0}{-3} = 2\left(\frac{-2+3}{13}\right)$

Now, Q' lies in $\ell_3 : \alpha x + \beta y + 17 = 0$

$\Rightarrow \alpha = 7$

Now, $\alpha^2 + \beta^2 - \alpha - \beta = 348$

Question ID : 7155054062

29. Let $S = \left\{ z \in \mathbb{C} - \{i, 2i\} : \frac{z^2 + 8iz - 15}{z^2 - 3iz - 2} \in \mathbb{R} \right\}$. If $\alpha - \frac{13}{11} i \in S, \alpha \in \mathbb{R} - \{0\}$, then $242\alpha^2$ is equal to ________.

माना $S = \left\{ z \in \mathbb{C} - \{i, 2i\} : \frac{z^2 + 8iz - 15}{z^2 - 3iz - 2} \in \mathbb{R} \right\}$ है। यदि $\alpha - \frac{13}{11} i \in S, \alpha \in \mathbb{R} - \{0\}$ है, तो $242\alpha^2$ बराबर है ________

Ans. Official Answer NTA (1680)

Sol. $\frac{z^2+8iz-15}{z^2-3iz-2}=\frac{\bar{z}^2-8i\bar{z}-15}{\bar{z}^2+3i\bar{z}-2}$

$z^2\bar{z}^2+3\bar{z}z^2i-2z^2+8iz\bar{z}^2-24z\bar{z}-16iz$

$-15\bar{z}^2-45i\bar{z}+30$

$=z^2\bar{z}^2-8i\bar{z}^2-15z^2$

$-3i\bar{z}^2 24z\bar{z}+45iz$

$-2\bar{z}^2+16i\bar{z}+30$

$11\bar{z}z^2+13z^2+11iz\bar{z}^2-61iz-13\bar{z}^2-61\bar{z}=0$

$11i\left(z^2\bar{z}+\bar{z}^2z\right)+13(z-\bar{z})-61i=0$

$z+\bar{z}\pm 0$ So $11i(z\bar{z})+13(z-\bar{z})-61i=0$

$11i\left(x^2+y^2\right)+13(2iy)-61i=0$

$11x^2+11y^2+26y-61=0$

$y=\frac{-13}{11}$ then $11x^2+\frac{169}{11}-\frac{338}{11}-61=0$

$\Rightarrow \alpha^2=\frac{840}{121}$

Question ID : 7155054065

30. The number of points, where the curve $f(x)=e^{8x}-e^{6x}-3e^{4x}-e^{2x}+1, x\in\mathbb{R}$ cuts x-axis, is equal to ________.

उन बिंदुओं, जहाँ वक्र $f(x)=e^{8x}-e^{6x}-3e^{4x}-e^{2x}+1, x\in\mathbb{R}$, x-अक्ष को काटता है, की संख्या है ________

Ans. Official Answer NTA (2)

Sol. $f(x)=e^{8x}-e^{6x}-3e^{4x}-e^{2x}+1, x\in\mathbb{R}$

$\lim_{x\to-\infty} f(x)=1,\quad f(0)=-3,\quad \lim_{x\to\infty} f(x)\to\infty$

One root of f(x) = 0 is negative and other root of f(x) = 0 is positive

Curve y = f(x) cuts x-axis at two points