JEE Main April 2023

Question Paper With Text Solution

10 April | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN APRIL 2023 | 10TH APRIL SHIFT-1

SECTION – A

Question ID : 3666943126

1. Let O be the origin and the position vector of the point P be $-\hat{i}-2\hat{j}+3\hat{k}$. If the position vectors of the points A, B and C are $-2\hat{i}+\hat{j}-3\hat{k}$, $2\hat{i}+4\hat{j}-2\hat{k}$ and $-4\hat{i}+2\hat{j}-\hat{k}$ respectively, then the projection of the vector $\overrightarrow{OP}$ on a vector perpendicular to the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is :

माना O मूल बिंदु है तथा बिंदु P का स्थिति सदिश $-\hat{i}-2\hat{j}+3\hat{k}$ है। यदि बिंदुओं A, B तथा C के स्थिति सदिश क्रमशः $-2\hat{i}+\hat{j}-3\hat{k}$, $2\hat{i}+4\hat{j}-2\hat{k}$ तथा $-4\hat{i}+2\hat{j}-\hat{k}$ हैं, तो सदिशों $\overrightarrow{AB}$ तथा $\overrightarrow{AC}$ के लंबवत् एक सदिश पर, सदिश $\overrightarrow{OP}$ का प्रक्षेप है :

(1) $\frac{10}{3}$ (2) $\frac{8}{3}$ (3) 3 (4) $\frac{7}{3}$

Ans. Official Answer NTA (3)

Sol. $\overrightarrow{OP}=-\hat{i}-2\hat{j}+3k$

$\overrightarrow{AB}=4\hat{i}+3\hat{j}+k$

$\overrightarrow{AC}=-2\hat{i}+\hat{j}+2k$

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & k\\ 4 & 3 & 1\\ -2 & 1 & 2\end{vmatrix}$$

$=5\hat{i}-10\hat{j}+10k$

Projection of $\overrightarrow{OP}$ on

$$\overrightarrow{AB}\times\overrightarrow{AC}=\left|\frac{\overrightarrow{OP}\cdot(\overrightarrow{AB}\times\overrightarrow{AC})}{|\overrightarrow{AB}\times\overrightarrow{AC}|}\right|$$

$$=\frac{5(-\hat{i}-2\hat{j}+3k)(\hat{i}-2\hat{j}+2k)}{5\sqrt{1+4+4}}$$

$= 3$

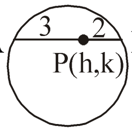
Question ID : 3666943122

2. A line segment AB of length λ moves such that the points A and B remain on the periphery of a circle of radius λ. Then the locus of the point, that divides the line segment AB in the ratio 2 : 3, is a circle of radius :

λ लंबाई का एक रेखा खंड AB इस प्रकार खिसकता है कि बिंदु A तथा B, λ त्रिज्या के एक वृत्त की परिधि पर रहते हैं। तो उस बिंदु, जो रेखाखंड AB को 2 : 3 के अनुपात में विभाजित करता है, का बिंदुपथ एक वृत्त है, जिसकी त्रिज्या है :

(1) $\frac{3}{5}\lambda$ (2) $\frac{\sqrt{19}}{5}\lambda$ (3) $\frac{2}{3}\lambda$ (4) $\frac{\sqrt{19}}{7}\lambda$

Ans. Official Answer NTA (2)

Sol. $\left(\frac{\lambda}{\sqrt{2}}\sin\theta, \frac{-\lambda}{\sqrt{2}}\cos\theta\right)$ A  B $\left(\frac{\lambda}{\sqrt{2}}\cos\theta, \frac{\lambda}{\sqrt{2}}\sin\theta\right)$

$$h = \frac{\frac{2\lambda}{\sqrt{2}\sin\theta} + 3\times\frac{\lambda}{\sqrt{2}}\cos\theta}{5}$$

$$k = \frac{\frac{-2\lambda}{\sqrt{2}}2\cos\theta + \frac{3\lambda}{\sqrt{2}}\sin\theta}{5}$$

$$h^2 + k^2 = \frac{19\lambda^2}{5}$$

$$r = \frac{\sqrt{19}\lambda}{5}$$

Question ID : 3666943112

3. Let the complex number $z = x + iy$ be such that $\frac{2z-3i}{2z+i}$ is purely imaginary. If $x + y^2 = 0$, then $y^4 + y^2 - y$ is equal to :

माना सम्मिश्र संख्या $z = x + iy$ के लिए $\frac{2z-3i}{2z+i}$ मात्र काल्पनिक है। यदि $x + y^2 = 0$ है, तो $y^4 + y^2 - y$ बराबर है :

(1) $\frac{4}{3}$ (2) $\frac{2}{3}$ (3) $\frac{3}{4}$ (4) $\frac{3}{2}$

Ans. Official Answer NTA (3)

Sol. $z = x + iy$

$$\frac{(2z-3i)}{2z+i} = \text{purely imaginary}$$

$$\text{means } \operatorname{Re}\left(\frac{2z-3i}{2z+i}\right) = 0$$

$$\Rightarrow \frac{(2z-3i)}{(2z+i)} = \frac{2(x+iy)-3i}{2(x+iy)+i}$$

$$= \frac{2x+2yi-3i}{2x+i2y+i}$$

$$= \frac{2x+i(2y-3)}{2x+i(2y+1)} \times \frac{2x-i(2y+1)}{2x-i(2y+1)}$$

$$\text{Re}\left[\frac{2z-3i}{2z+i}\right] = \frac{4x^2+(2y-3)(2y+1)}{4x^2+(2y+1)^2} = 0$$

$$\Rightarrow 4x^2+(2y-3)(2y+1)=0$$

$$\Rightarrow 4x^2+\left[4y^2+2y-6y-3\right]=0$$

$$\because x+y^2=0 \Rightarrow x=-y^2$$

$$\Rightarrow 4\left(-y^2\right)^2+4y^2-4y-3=0$$

$$\Rightarrow 4y^4+4y^2-4y=3$$

$$\Rightarrow y^4+y^2-y=\frac{3}{4}$$

Therefore, correct answer is option (3).

Question ID : 3666943129

4. $96\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}$ is equal to :

$96\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}$ बराबर है :

(1) 4 (2) 1 (3) 2 (4) 3

Ans. Official Answer NTA (4)

Sol. Let $\frac{\pi}{33} = A$

$96\cos A\cos 2A\cos 2^2A\cos 2^3A\cos 2^4A$

$$= 96\frac{\sin 2^5A}{2^5\sin A} = \frac{96.\sin\frac{32\pi}{33}}{32\sin\frac{\pi}{33}} = \frac{96.\sin\left(\pi-\frac{\pi}{33}\right)}{32\sin\frac{\pi}{33}} = \frac{96.\sin\frac{\pi}{33}}{32\sin\frac{\pi}{33}} = 3 \text{ Ans.}$$

Question ID : 3666943128

5. Let N denote the sum of the numbers obtained when two dice are rolled. If the probability that $2^N < N!$ is $\frac{m}{n}$, where m and n are coprime, then 4m – 3n is equal to :

माना दो पासे फेंकने पर प्राप्त संख्याओं का योग N है। यदि $2^N < N!$ होने की प्रायिकता $\frac{m}{n}$ है, जहाँ m तथा n असहभाज्य हैं, तो 4m – 3n बराबर है :

(1) 10 (2) 6 (3) 12 (4) 8

Ans. Official Answer NTA (4)

Sol. 2N < N!

N = 1 (not possible) $\to$ 0

N = 2 (not possible) $\to$ 1

N = 3 (not possible) $\to$ 2

N = 4 (possible)

$\therefore$ Required probability $= \frac{36-(1+2)}{36} = \frac{11}{12}$

$\therefore$ 4m – 3n

= 44 – 36

= 8

Question ID : 3666943121

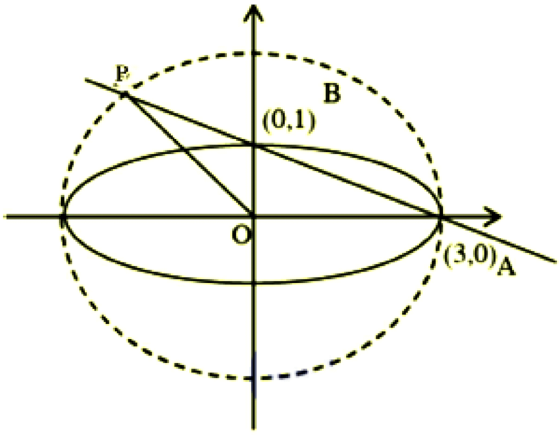
6. Let the ellipse E : $x^2 + 9y^2 = 9$ intersect the positive x- and y-axes at the points A and B respectively. Let the major axis of E be a diameter of the circle C. Let the line passing through A and B meet the circle C at the point P. If the area of the triangle with vertices A, P and the origin O is $\frac{m}{n}$, where m and n are coprime, then m – n is equal to :

माना दीर्घवृत्त E : $x^2 + 9y^2 = 9$ धनात्मक x तथा y अक्षों को क्रमशः बिंदुओं A तथा B पर काटता है। माना E का दीर्घ अक्ष, वृत्त C का एक व्यास है। माना बिंदुओं A तथा B से होकर जाने वाली रेखा, वृत्त C को बिंदु P पर मिलती है। यदि, त्रिभुज जिसके शीर्ष A, P तथा मूल बिंदु O हैं, का क्षेत्रफल $\frac{m}{n}$ है, जहाँ m तथा n असहभाज्य हैं, तो m – n बराबर है :

(1) 18 (2) 16 (3) 17 (4) 15

Ans. Official Answer NTA (3)

Sol.



For line AB $x + 3y = 3$ and circle is $x^2 + y^2 = 9$

$(3 - 3y)^2 + y^2 = 9$

$\Rightarrow 10y^2 - 18y = 0$

$\Rightarrow y = 0, \frac{9}{5}$

$\therefore$ Area $= \frac{1}{2} \times 3 \times \frac{9}{5} = \frac{27}{10}$

$m - n = 17$

Question ID : 3666943111

7. If $f(x) = \frac{(\tan 1^\circ)x + \log_e(123)}{x\log_e(1234) - (\tan 1^\circ)}, x > 0$, then the least value of $f(f(x)) + f\left(f\left(\frac{4}{x}\right)\right)$ is :

यदि $f(x) = \frac{(\tan 1^\circ)x + \log_e(123)}{x\log_e(1234) - (\tan 1^\circ)}, x > 0$ है, तो $f(f(x)) + f\left(f\left(\frac{4}{x}\right)\right)$ का निम्नतम मान है :

(1) 2 (2) 8 (3) 4 (4) 0

Ans. Official Answer NTA (3)

Sol. $f(x) = \frac{(\tan 1)x + \log 123}{x\log 1234 - \tan 1}$

Let $A = \tan 1, B = \log 123, C = \log 1234$

$f(x) = \frac{Ax + B}{xC - A}$

$$f(f(x)) = \frac{A\left(\frac{Ax+B}{xC-A}\right)+B}{C\left(\frac{Ax+B}{CX-A}\right)-A}$$

$$= \frac{A^2x + AB + xBC - AB}{ACx + BC - ACx + A^2}$$

$$= \frac{x\left(A^2+BC\right)}{\left(A^2+BC\right)} = x$$

$$f(f(x)) = x$$

$$f\left(f\left(\frac{4}{x}\right)\right) = \frac{4}{x}$$

$$f(f(x)) + f\left(f\left(\frac{4}{x}\right)\right)$$

$$AM \geq GM$$

$$x + \frac{4}{x} \geq 4$$

Question ID : 3666943130

8. The negation of the statement $(p \vee q) \wedge (q \vee (\sim r))$ is :

कथन $(p \vee q) \wedge (q \vee (\sim r))$ का निषेचन है :

(1) $((\sim p) \vee (\sim q)) \wedge (\sim r)$
(2) $((\sim p) \vee (\sim q)) \vee (\sim r)$
(3) $((\sim p) \vee r)) \wedge (\sim q)$
(4) $(p \vee r) \wedge (\sim q)$

Ans. Official Answer NTA (3)

Sol. $\sim ((p \vee q) \wedge (q \vee (\sim r)))$

$= \sim (p \vee q) \vee \sim (q \vee (\sim r))$

$= (\sim p \wedge \sim q) \vee (\sim q \wedge r)$

$= (\sim p \vee (\sim q \wedge r)) \wedge (\sim q \vee (\sim q \wedge r)$

$= (\sim p \vee (\sim q \wedge r)) \wedge \sim q$

$= (\sim p \vee \sim q) \wedge (\sim p \vee r) \wedge (\sim q)$

$= (\sim p \vee r) \wedge (\sim q)$

Question ID : 3666943115

9. If the coefficient of x^7 in $\left(ax-\frac{1}{bx^2}\right)^{13}$ and the coefficient of x^{-5} in $\left(ax+\frac{1}{bx^2}\right)^{13}$ are equal, then a^4b^4 is equal to :

यदि $\left(ax-\frac{1}{bx^2}\right)^{13}$ में x^7 का गुणांक तथा $\left(ax+\frac{1}{bx^2}\right)^{13}$ में x^{-5} का गुणांक बराबर हैं, तो a^4b^4 बराबर है :

(1) 22 (2) 33 (3) 44 (4) 11

Ans. Official Answer NTA (1)

Sol. Coefficient of x^7 in $\left(ax-\frac{1}{bx^2}\right)^{13}$

$T_{r+1} = {}^{13}C_r (9x)^{13-r}\left(-\frac{1}{bx^2}\right)^r$

$\Rightarrow r = 2$

$\therefore$ Coeff. $= {}^{13}C_2 \frac{a^{11}}{b^2}$

Similarly coeff. of x^{-5} in $\left(ax+\frac{1}{bx^2}\right)^{13}$

$\Rightarrow r = 6$

$\therefore$ Coeff. $= {}^{13}C_6 \frac{a^7}{b^6}$

Now, ${}^{13}C_2 \frac{a^{11}}{b^2} = {}^{13}C_6 \frac{a^7}{b^6}$

$\Rightarrow a^4b^4 = 22$

Question ID : 3666943114

10. If A is a 3 × 3 matrix and $|A| = 2$, then $\left|3\,\text{adj}\left(|3A|\,A^2\right)\right|$ is equal to :

यदि 3 × 3 का एक आव्यूह A है तथा |A| = 2 है, तो $\left|3\,\text{adj}\left(|3A|\,A^2\right)\right|$ बराबर है :

(1) $3^{11}\cdot 6^{10}$ (2) $3^{12}\cdot 6^{10}$ (3) $3^{10}\cdot 6^{11}$ (4) $3^{12}\cdot 6^{11}$

Ans. Official Answer NTA (1)

Sol. $\left|3\,\text{adj}\left(|3A|A^2\right)\right| = 3^3\left|\text{adj}\left(54A^2\right)\right| = 3^3\cdot\left|54A^2\right|^2$

$= 3^3\times 54^6\times |A|^4 = 3^{11}\times 6^{10}$

Question ID : 3666943125

11. The shortest distance between the lines $\frac{x+2}{1}=\frac{y}{-2}=\frac{z-5}{2}$ and $\frac{x-4}{1}=\frac{y-1}{2}=\frac{z+3}{0}$ is :

रेखाओं $\frac{x+2}{1}=\frac{y}{-2}=\frac{z-5}{2}$ तथा $\frac{x-4}{1}=\frac{y-1}{2}=\frac{z+3}{0}$ के बीच न्यूनतम दूरी है :

(1) 6 (2) 7 (3) 9 (4) 8

Ans. Official Answer NTA (3)

Sol. $\frac{x+2}{1}=\frac{y}{-2}=\frac{z-5}{2}$ and $\frac{x-4}{1}=\frac{y-1}{2}=\frac{z+3}{0}$

Shortest distance $(d) = \dfrac{\left\|\begin{matrix} a_2-a_1 & b_2-b_1 & c_2-c_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{matrix}\right\|}{\left\|\begin{matrix} \hat{i} & \hat{j} & k \\ l_1 & m_1 & n_1 \\ l_1 & m_2 & n_2 \end{matrix}\right\|}$

$= \dfrac{\left\|\begin{matrix} 4+2 & 1-0 & -3-5 \\ 1 & -2 & 2 \\ 1 & 2 & 0 \end{matrix}\right\|}{\left\|\begin{matrix} \hat{i} & \hat{j} & k \\ 1 & -2 & 2 \\ 1 & 2 & 0 \end{matrix}\right\|}$

$= \dfrac{\left|\begin{matrix} 6 & 1 & -8 \\ 1 & -2 & 2 \\ 1 & 2 & 0 \end{matrix}\right|}{|\hat{i}(-4)-\hat{j}(-2)+k(2+2)|}$

$$= \frac{|-54|}{|-4\hat{i}+2\hat{j}+4k|}$$

$$= \frac{54}{\sqrt{16+4+16}}$$

$$= \frac{54}{6}$$

$$= 9$$

Question ID : 3666943118

12. If $I(x) = \int e^{\sin^2 x}(\cos x \sin 2x - \sin x)dx$ and $I(0) = 1$, then $I\left(\frac{\pi}{3}\right)$ is equal to :

यदि $I(x) = \int e^{\sin^2 x}(\cos x \sin 2x - \sin x)dx$ तथा $I(0) = 1$ हैं, तो $I\left(\frac{\pi}{3}\right)$ बराबर है :

(1) $e^{\frac{3}{4}}$ (2) $-\frac{1}{2}e^{\frac{3}{4}}$ (3) $-e^{\frac{3}{4}}$ (4) $\frac{1}{2}e^{\frac{3}{4}}$

Ans. Official Answer NTA (4)

Sol. $\because I = \int e^{f(x)}\left(g'(x) + f'(x)\cdot g(x)\right)dx = e^{f(x)}\cdot g(x)$

$\therefore I = e^{\sin^2 x}\cdot \cos x + c$

but $I(0) = 1 \Rightarrow c = 0$

$I = e^{\sin^2 x}\cos x$

$I\left(\frac{\pi}{3}\right) = \frac{1}{2}e^{\frac{3}{4}}$

Question ID : 3666943124

13. Let P be the point of intersection of the line $\frac{x+3}{3} = \frac{y+2}{1} = \frac{1-z}{2}$ and the plane $x+y+z=2$. If the distance of the point P from the plane $3x - 4y + 12z = 32$ is q, then q and 2q are the roots of the equation:

माना रेखा $\frac{x+3}{3} = \frac{y+2}{1} = \frac{1-z}{2}$ तथा समतल $x + y + z = 2$ का प्रतिच्छेदन बिंदु P है। यदि बिंदु P की समतल $3x - 4y + 12z = 32$ से दूरी q है, तो q तथा 2q किस समीकरण के मूल है ?

(1) $x^2-18x-72=0$ (2) $x^2+18x-72=0$ (3) $x^2+18x+72=0$ (4) $x^2-18x+72=0$

Ans. Official Answer NTA (4)

Sol. $\frac{x+3}{3}=\frac{y+2}{1}=\frac{1-z}{2}=\lambda$

$A(3\lambda-3,\lambda-2,1-2\lambda)$

$3\lambda-3+\lambda-2+1-2\lambda=2$

$2\lambda=6$

$\lambda=3$

$P(6,1,-5)$

$q=\left|\frac{18-4-60-32}{\sqrt{9+16+144}}\right|=\frac{78}{13}=6$

Equation with roots q and 2q is

$x^2-3qx+2q^2=0$

$x^2-18x+72=0$

Question ID : 3666943119

14. Let f be a differentiable function such that $x^2f(x)-x=4\int_0^x tf(t)dt$, $f(1)=\frac{2}{3}$. Then 18f(3) is equal to :

माना एक अवकलनीय फलन f के लिए $x^2f(x)-x=4\int_0^x tf(t)dt$, $f(1)=\frac{2}{3}$ है। तो 18f(3) बराबर है :

(1) 180 (2) 210 (3) 160 (4) 150

Ans. Official Answer NTA (3)

Sol. Differentiate the given equation

$\Rightarrow 2xf(x)+x^2f'(x)-1=4xf(x)$

$\Rightarrow x^2\frac{dy}{dx}-2xy=1$

$\Rightarrow \frac{dy}{dx}+\left(-\frac{2}{x}\right)y=\frac{1}{x^2}$

$I.F.=e^{\int-\frac{2}{x}\ell nx}=\frac{1}{x^2}$

$\therefore y\left(\frac{1}{x^2}\right)=\int\frac{1}{x^4}dx$

$$\Rightarrow \frac{y}{x^2} = \frac{-1}{3x^3} + c$$

$$\Rightarrow y = -\frac{1}{3x^3} + c$$

$$\Rightarrow y = -\frac{1}{3x} + cx^2$$

$$\because f(1) = \frac{2}{3} = -\frac{1}{3} + c \Rightarrow c = 1$$

$$f(x) = -\frac{1}{3x} + x^2$$

$18f(3) = 160$

Question ID : 3666943117

15. A square piece of tin of side 30 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. If the volume of the box is maximum, then its surface area (in cm^2) is equal to :

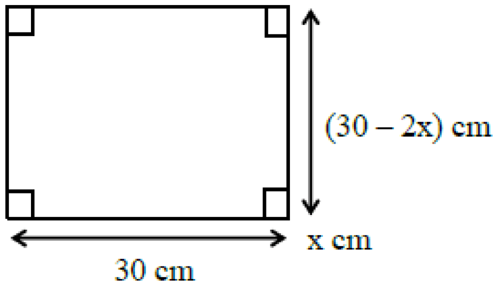
30 cm भुजा के टिन के एक एक वर्गाकार टुकड़े के प्रत्येक कोने पर एक वर्ग काटकर तथा इस प्रकार बनें टिन के फलकों को मोड़ कर ढक्कन रहित एक संदूक बनाना है। यदि संदूक का आयतन उच्चतम हैं, तो इसका पृष्ठीय क्षेत्रफल (वर्ग cm में) बराबर है :

(1) 675 (2) 1025 (3) 900 (4) 800

Ans. Official Answer NTA (4)

Sol. Let the side of the square to be cut off be x cm.

Then, the length and breadth of the box will be $(30 - 2x)$ cm each and the height of the box is x cm therefore,



The volume V(x) of the box is given by

$V(x) = x(30 - 2x)^2$

$$\frac{dv}{dx} = (30 - 2x)^2 + 2x(30 - 2x)(-2)$$

$0 = (30 - 2x)^2 - 4x(30 - 2x)$

$0 = (30 - 2x)[(30 - 2x) - 4x]$

$0 = (30 - 2x)(30 - 6x)$

$x = 15, 5$
$x \neq 15$ (Not possible)
$\{ V = 0\}$

Surface area without top of the box $= \ell b + 2(bh + h\ell)$
$= (30 - 2x)(30 - 2x) + 2[(30 - 2x)x + (30 - 2x)x]$
$= [(30 - 2x)^2 + 4\{(30 - 2x)x\}$
$= [(30 - 10)^2 + 4(5)(30 - 10)]$
$= 400 + 400$
$= 800\ cm^2$

Question ID : 3666943120

16. The slope of tangent at any point (x, y) on a curve $y = y(x)$ is $\frac{x^2+y^2}{2xy}, x > 0$. If $y(2) = 0$, then a value of $y(8)$ is :

वक्र $y = y(x)$ के किसी भी बिंदु (x, y) पर स्पर्श रेखा की प्रवणता $\frac{x^2+y^2}{2xy}, x > 0$ है। यदि $y(2) = 0$ है, तो $y(8)$ का एक मान है :

(1) $-4\sqrt{2}$ (2) $4\sqrt{3}$ (3) $2\sqrt{3}$ (4) $-2\sqrt{3}$

Ans. Official Answer NTA (2)

Sol. $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$

Let $y = \upsilon x \therefore \frac{dy}{dx} = \upsilon + x\frac{d\upsilon}{dx}$ $\therefore \upsilon + x\frac{d\upsilon}{dx} = \frac{x^2+\upsilon^2x^2}{2x.\upsilon x}$

$x\frac{d\upsilon}{dx} = \frac{1+\upsilon^2}{2\upsilon} - \upsilon = \frac{1-\upsilon^2}{2\upsilon} \Rightarrow \frac{2\upsilon}{1-\upsilon^2}d\upsilon = \frac{dx}{x} \Rightarrow \int\frac{2\upsilon d\upsilon}{\upsilon^2-1} + \int\frac{dx}{x} = 0$

$\Rightarrow \ell n(\upsilon^2 - 1) + \ell nx = \ell nc \Rightarrow x(\upsilon^2 - 1) = c \Rightarrow x\frac{y^2-x^2}{x^2} = c \Rightarrow y^2 - x^2 = cx$

Now $f(2) = 0 \therefore c = -2$

$\therefore y^2 - x^2 = -2x$ $\therefore x = 8$ then $y^2 - 64 = -16$

$y^2 = 48$

$y = f(8) = 4\sqrt{3}$

Question ID : 3666943116

17. Let the first term a and the common ratio r of a geometric progression be positive integers. If the sum of squares of its first three terms is 33033 , then the sum of these three terms is equal to :

माना एक गुणोतर श्रेढ़ी के प्रथम पद a तथा सार्व अनुपात r धनातम पूर्णांक है। यदि इसके प्रथम तीन पदों के वर्गों का योग 33033 है, तो इन तीन पदों का योग है :

(1) 231 (2) 220 (3) 210 (4) 241

Ans. Official Answer NTA (1)

Sol. $a^2 + a^2r^2 + a^2r^4 = 33033$

$a^2\left(1+r^2+r^4\right) = 33033$

$a^2\left(1+r^2+r^4\right) = 3\times 7\times (11)^2 \times 13$

$\Rightarrow \quad a^2 = (11)^2$

$a = 11$

$\Rightarrow \quad 1+r^2+r^4 = 273$

$r^4 + r^2 - 272 = 0$

$\Rightarrow \quad r^2 = 16$

$r = 4$

$a = 11$

$ar = 44$

$ar^2 = 176$

$a + ar + ar^2 = 231$

Question ID : 3666943127

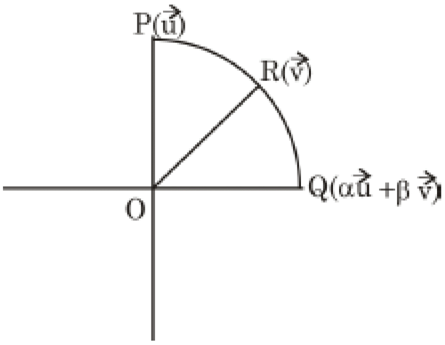
18. An arc PQ of a circle subtends a right angle at its centre O. The mid point of the arc PQ is R. If $\overrightarrow{OP} = \vec{u}, \overrightarrow{OR} = \vec{v}$ and $\overrightarrow{OQ} = \alpha\vec{u} + \beta\vec{v}$, then α, β^2 are the roots of the equation :

एक वृत्त का एक चाप PQ इसके केन्द्र पर समकोण बनाता है। चाप PQ का मध्य बिंदु R है। यदि $\overrightarrow{OP} = \vec{u}, \overrightarrow{OR} = \vec{v}$ तथा $\overrightarrow{OQ} = \alpha\vec{u} + \beta\vec{v}$ है, तो α, β^2 किस समीकरण के मूल है ?

(1) $x^2 + x - 2 = 0$ (2) $3x^2 - 2x - 1 = 0$ (3) $x^2 - x - 2 = 0$ (4) $3x^2 + 2x - 1 = 0$

Ans. Official Answer NTA (3)

Sol.



$|\vec{u}|=|\vec{v}|=|\alpha\vec{u}+\beta\vec{v}|$

$(\vec{u})\cdot(\alpha\vec{u}+\beta\vec{v})=0$

$\vec{u}\cdot\vec{v}=|u||v|\cos 45^{\circ}$

$\alpha=-\dfrac{\beta}{\sqrt{2}}$

$=|\alpha\vec{u}+\beta\vec{v}|=r$

$\alpha^2+\beta^2+\sqrt{2}\alpha\beta=1$

$\alpha=-1,\beta^2=2$

Question ID : 3666943123

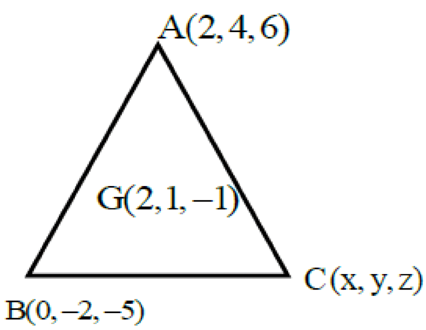
19. Let two vertices of a triangle ABC be (2, 4, 6) and (0, –2, –5), and its centroid be (2, 1, –1). If the image of the third vertex in the plane x + 2y + 4z = 11 is (α, β, γ), then $\alpha\beta + \beta\gamma + \gamma\alpha$ is equal to :

माना एक त्रिभुज ABC के दो शीर्ष (2, 4, 6) तथा (0, –2, –5) हैं तथा केन्द्रक (2, 1, –1) हैं यदि तीसरे शीर्ष का समतल x + 2y + 4z = 11 में प्रतिबिंब (α, β, γ) है, तो $\alpha\beta + \beta\gamma + \gamma\alpha$ बराबर है :

(1) 70 (2) 76 (3) 74 (4) 72

Ans. Official Answer NTA (3)

Sol.



Given Two vertices of Triangle A(2,4,6) and B(0,–2,–5) and if centroid G(2,1,–1)

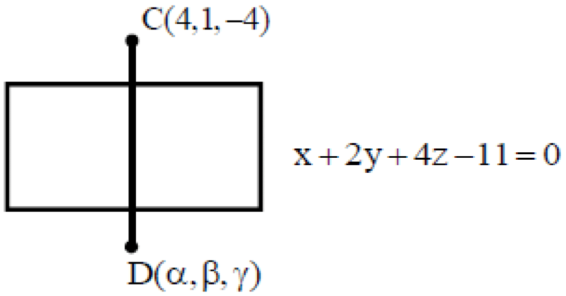
Let Third vertices be (x, y,z)

Now $\frac{2+0+x}{3}=2, \frac{4-2+y}{3}=1, \frac{6-5+z}{3}=-1$

$x = 4$, $y = 1$, $z = -1$

Third vertices C(4,1,–4)

Now, Image of vertices C(4,1,–4) in the given plane is D(α,β,γ)



Now

$$\frac{\alpha-4}{1}=\frac{\beta-1}{2}=\frac{\gamma+4}{4}=-2\frac{(4+2-16-11)}{1+4+16}$$

$$\frac{\alpha-4}{1}=\frac{\beta-1}{2}=\frac{\gamma+4}{4}=\frac{42}{21}\Rightarrow 2$$

$\alpha=6, \beta=5, \gamma=4$

Then $\alpha\beta+\beta\gamma+\gamma\alpha$

$= (6 \times 5) + (5 \times 4) + (4 \times 6)$

$= 30 + 20 + 24$

$= 74$

Question ID : 3666943113

20. For the system of linear equations

$2x - y + 3z = 5$

$3x + 2y - z = 7$

$4x + 5y + \alpha z = \beta$,

which of the following is NOT correct?

(1) The system has infinitely many solutions for $\alpha = -5$ and $\beta = 9$

(2) The system has infinitely many solutions for $\alpha = -6$ and $\beta = 9$

(3) The system has a unique solution for $\alpha \neq -5$ and $\beta = 8$

(4) The system is inconsistent for $\alpha = -5$ and $\beta = 8$

रैखिक समीकरण निकाय

$2x - y + 3z = 5$

$3x + 2y - z = 7$

$4x + 5y + \alpha z = \beta$,

के लिए निम्न में से कौन सा सही नहीं है ?

(1) $\alpha = -5$ तथा $\beta = 9$ के लिए समीकरण निकाय के अनंत हल हैं

(2) $\alpha = -6$ तथा $\beta = 9$ के लिए समीकरण निकाय के अनंत हल हैं

(3) $\alpha \neq -5$ तथा $\beta = 8$ के लिए समीकरण निकाय का एक अद्वितीय हल है

(4) $\alpha = -5$ तथा $\beta = 8$ के लिए समीकरण निकाय असंगत है

Ans. Official Answer NTA (2)

Sol. For infinite solution

$$\lambda_1 P_1 + \lambda_2 P_2 = P_3$$

$$\lambda_1(2x - y + 3z - 5) + (3x + 2y - z - 7)\lambda_2 = (4x + 5y + \alpha z - \beta)$$

$$\left.2\lambda_1 + 3\lambda_2 = 4\right] \quad \lambda_1 = -1$$

$$\left.-\lambda_1 + 2\lambda_2 = 5\right] \quad \lambda_2 = 2$$

$$3\lambda_1 - \lambda_2 = \alpha \rightarrow \alpha = -5$$

$$-5\lambda_1 - 7\lambda_2 = -\beta \rightarrow \beta = 9$$

SECTION – B

Question ID : 3666943139

21. Let a common tangent to the curves $y^2 = 4x$ and $(x - 4)^2 + y^2 = 16$ touch the curves at the points P and Q. Then $(PQ)^2$ is equal to ________.

माना वक्रों $y^2 = 4x$ तथा $(x - 4)^2 + y^2 = 16$ की उभयनिष्ठ स्पर्श रेखा, वक्रों को बिंदुओं P तथा Q पर स्पर्श करती है। तो $(PQ)^2$ बराबर है ________

Ans. Official Answer NTA (32.00)

Sol. $y = mx + \frac{1}{m}$

$\perp r \text{ from } (4,0) = 4$

$$\left|\frac{4m + \frac{1}{m}}{\sqrt{m^2+1}}\right| = 4$$

Or $16m^2 + \frac{1}{m^2} + 8 = 16m^2 + 16$

Or $m^2 = \frac{1}{8}$, if $m = \frac{1}{2\sqrt{2}}$ then

$P = \left(\frac{a}{m^2}, \frac{2a}{m}\right) = (8, 4\sqrt{2})$

$Q =$ foot of $\perp r$ from $(4,0)$ on tangent

$= \left(\frac{8}{3}, \frac{8\sqrt{2}}{3}\right)$

$(PQ)^2 = \left(\frac{16}{3}\right)^2 + \left(\frac{4\sqrt{2}}{3}\right)^2$

$= \frac{256+32}{9} = 32$

Question ID : 3666943131

22. Let a, b, c be three distinct positive real numbers such that $(2a)^{\log_e a} = (bc)^{\log_e b}$ and $b^{\log_e 2} = a^{\log_e c}$. Then 6a + 5bc is equal to ________.

माना तीन भिन्न धनात्मक वास्तविक संख्याओं a, b, c के लिए $(2a)^{\log_e a} = (bc)^{\log_e b}$ तथा $b^{\log_e 2} = a^{\log_e c}$ हें। तो 6a + 5bc बराबर है ________

Ans. Official Answer NTA (8) **Bonus**

Sol. $(2a)^{\ln a} = (bc)^{\ln b}$ $\quad 2a > 0, bc > 0 \quad b^{\ln 2} = a^{\ln c}$

$\ln a(\ln 2 + \ln a) = \ln b(\ln b + \ln c)$

$\ln 2 \cdot \ln b = \ln c \cdot \ln a$

$\ln 2 = \alpha, \ln a = x_1 \ln b = y, \ln c = z$

$\alpha y = yz$

$x(a + x) = y(y + 2)$

$\alpha = \frac{xz}{y}$

$(2a)^{\ln a} = (2a)^0$

$x\left(\frac{xz}{y} + x\right) = y(y + z)$

$x^2(z + y) = y^2(y + z)$

$y + z = 0$ or $x^2 = y^2 \Rightarrow x = -y$

$bc = 1$ or $ab = 1$

(1) if bc = 1 ⇒ $(2a)^{\ln a} = 1$ → a=1, a=1/2

$(a,b,c) = \left(\frac{1}{2}, \lambda, \frac{1}{\lambda}\right), \lambda \neq 1, 2, \frac{1}{2}$

then 6a + 5bc = 3 + 5 = 8

(2) $(a,b,c) = \left(\lambda, \frac{1}{\lambda}, \frac{1}{2}\right), \lambda \neq 1, 2, \frac{1}{2}$

In this situation infinite answer are possible

So, Bonus.

Question ID : 3666943138

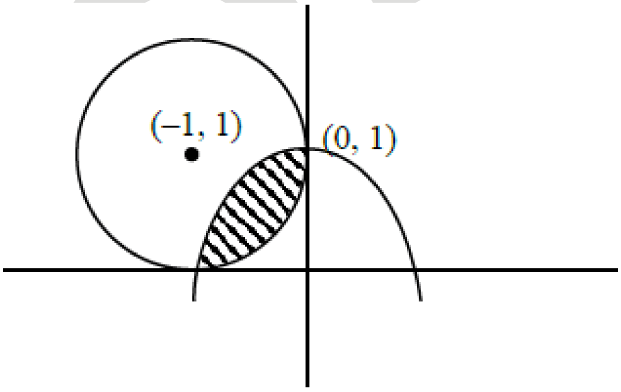
23. Let y = p(x) be the parabola passing through the points (–1, 0), (0, 1) and (1, 0). If the area of the region $\{(x,y) : (x+1)^2 + (y-1)^2 \le 1, y \le p(x)\}$ is A, then 12(π – 4A) is equal to ________.

माना बिंदुओं (–1, 0), (0, 1) तथा (1, 0) से होकर जाने वाला परवलय y = p(x) है। यदि क्षेत्र $\{(x,y) : (x+1)^2 + (y-1)^2 \le 1, y \le p(x)\}$ का क्षेत्रफल A है, तो 12(π – 4A) बराबर है ________

Ans. Official Answer NTA (16)

Sol. There can be infinitely many parabolas through given points.

Let parabola $x^2 = -4a(y-1)$



Passes through (1, 0)

$\therefore b = -4a(-1) \Rightarrow a = \frac{1}{4}$

$\therefore x^2 = -(y-1)$

Now area covered by parabola $= \int_{-1}^{0} (1 - x^2)dx$

$= \left(x - \frac{x^3}{3}\right)_1^b = (0-0) - \left\{-1 + \frac{1}{3}\right\}$

$= \frac{2}{3}$

Required Area = Area of sector – {Area of square – Area covered by Parabola}

$= \frac{\pi}{4} - \left\{1 - \frac{2}{3}\right\}$

$= \frac{\pi}{4} - \frac{1}{3}$

$\therefore 12(\pi - 4A) = 12\left[\pi - 4\left(\frac{\pi}{4} - \frac{1}{3}\right)\right]$

$= 12\left[\pi - \pi + \frac{4}{3}\right]$

= 16

Question ID : 3666943140

24. If the mean of the frequency distribution

Class :	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency :	2	3	x	5	4

is 28 , then its variance is ________.

यदि बारंबारता बंटन

वर्ग :	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
बारंबारता :	2	3	x	5	4

का माध्य 28 है, तो इसका प्रसरण है ________

Ans. Official Answer NTA (151)

Sol. Mean $= 28 = \frac{5\times 2 + 15\times 3 + 25\times x + 35\times 5 + 45\times 4}{14 + x}$

$392 + 28x = 410 + 25x$

$3x = 18$

$x = 6$

Now variance

$= \frac{\sum f_i (x_i)^2}{\sum f_i} - (\overline{x})^2$

$$= \frac{2(25)+3(15)^2+6(25)^2+5(35)^2+4(45)^5}{20} - 28^2$$

$$= \frac{50+675+3750+6125+8100}{20} - 784$$

$$= \frac{18700}{20} - 784 = 935 - 784$$

$= 151$

Question ID : 3666943134

25. Some couples participated in a mixed doubles badminton tournament. If the number of matches played, so that no couple played in a match, is 840 , then the total numbers of persons, who participated in the tournament, is ________.

कुछ दंपत्तियों ने मिश्रित युगल बैडमिंटन प्रतियोगिता में भाग लिया। यदि खेले गए मैचों, जबकि किसी भी दंपत्ति ने एक ही मैच में भाग नहीं लिया हो, की संख्या 840 हैं, तो प्रतियोगिता में भाग लेने वाले व्यक्तियों की कुल संख्या है ________

Ans. Official Answer NTA (16)

Sol. Let total number of persons = 2n

$\Rightarrow {}^nC_2 \cdot {}^{n-2}C_2 \cdot 2 = 840$

$\Rightarrow n(n-1)(n-2)(n-3) = 5\cdot6\cdot7\cdot8$

$n = 8$

$\Rightarrow 2n = 16$

Question ID : 3666943137

26. Let $f:(-2, 2) \to \mathbb{R}$ be defined by

$f(x) = \begin{cases} x[x] & , \quad -2 < x < 0 \\ (x-1)[x], & \quad 0 \le x < 2 \end{cases}$ where [x] denotes the greatest integer function. If m and n respectively are the points in (–2, 2) at which y = |f(x)| is not continuous and not differentiable, then m + n is equal to ________.

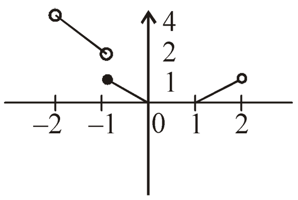
माना $f:(-2, 2) \to \mathbb{R}$

$$f(x) = \begin{cases} x[x] & , \quad -2 < x < 0 \\ (x-1)[x], & \quad 0 \le x < 2 \end{cases}$$

द्वारा परिभाषित है, जहाँ [x] महत्तम पूर्णांक फलन है। यदि अंतराल (–2, 2) में उन बिंदुओं, जिन पर y = |f(x)| संतत नहीं है तथा अवकलनीय नहीं है, की संख्या क्रमशः m तथा n है, तो m + n बराबर है ________

Ans. Official Answer NTA (4)

Sol. $f(x)=\begin{cases} x[x], & -2<x<0 \\ (x-1)[x], & 0\le x<2 \end{cases}$



$|f(x)|$ = Remain same

$m=1, n=3$

$m+n=4$

Question ID : 3666943132

27. The number of elements in the set $\{n\in\mathbb{Z}:|n^2-10n+19|<6\}$ is ________.

समुच्चय $\{n\in\mathbb{Z}:|n^2-10n+19|<6\}$ में अवयवों की संख्या है ________

Ans. Official Answer NTA (6)

Sol. $-6<n^2-10n+19<6$

$\Rightarrow n^2-10n+25>0$ and $n^2-10n+13<0$

$(n-5)^2>0$ $\quad$ $5-3\sqrt{2}<n<5+3\sqrt{2}$

$N\in Z-\{5\}$ $\quad$ $n=\{2,3,4,5,6,7,8\}$

.....(i) $\quad$ ....(ii)

From (i) $\cap$ (ii)

$N=\{2, 3, 4, 5, 6, 8,\}$

Number of values of $n=6$

Question ID : 3666943133

28. The number of permutations, of the digits 1, 2, 3, ... 7 without repetition, which neither contain the string 153 nor the string 2467, is ________.

बिना पुनरावृत्ति के, अंकों 1, 2, 3, ... 7 के क्रमचयों, जिनमें न तो श्रृंखला 153 हो नही श्रृंखला 2467 हो, की संख्या है ________

Ans. Official Answer NTA (4898)

Sol. $n(\bar{A}\cap\bar{B})=n(\cup)-n(A\cup B)$ $\quad$ $n(A)=5!;\boxed{153},2,4,6,7$

$7!-(5!+4!-2!)=4898$ $\quad$ $n(B)=4!;\boxed{2467},1,3,5$

Question ID : 3666943136

29. The sum of all those terms, of the arithmetic progression 3, 8, 13, ..., 373, which are not divisible by 3, is equal to ________.

समांतर श्रेढ़ी 3, 8, 13, ..., 373 के उन सभी पदों, जो 3 से विभाज्य नहीं है, का योग बराबर है ________

Ans. Official Answer NTA (9525)

Sol. $T_n = 3 + 5(n-1) = 5n - 2$

$\Rightarrow T_1, T_4, T_7 \ldots$ are divisible by 3

i.e. 3, 18, 33, 48, ..., 363

Sum of numbers divisible by 3.

$= \frac{25}{2}(3+363) = (25)(183)$

Sum of all numbers in A.P.

$= \frac{75}{2}(3+373) = (75)(188)$

Required sum = 9525

Question ID : 3666943135

30. The coefficient of x^7 in $\left(1 - x + 2x^3\right)^{10}$ is ________.

$\left(1 - x + 2x^3\right)^{10}$ में x^7 का गुणांक है ________

Ans. Official Answer NTA (960)

Sol. General term $= \frac{10!}{r_1!\cdot r_2!r_3!}(-1)^{r_2}\cdot(2)^{r_3}x^{r_2+3r_3}$

where $r_1 + r_2 + r_3 = 10$ and $r_2 + 3r_3 = 7$

r_1	r_2	r_3
3	7	0
5	4	1
7	1	2

Required coefficient

$= \frac{10!}{3!\cdot 7!}(-1)^7 + \frac{10!}{5!\cdot 4!}(-1)^4(2) + \frac{10!}{7!\cdot 2!}(-1)^1(2)^2$

$= -120 + 2520 - 1440 = 960$