JEE Main April 2023

Question Paper With Text Solution

10 April | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911
Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in

JEE MAIN APRIL 2023 | 10TH APRIL SHIFT-2

SECTION – A

Question ID : 7155054141

1. For $\alpha, \beta, \gamma, \delta \in \mathbb{N}$, if $\int\left(\left(\frac{x}{e}\right)^{2x}+\left(\frac{e}{x}\right)^{2x}\right)\log_e x\,dx=\frac{1}{\alpha}\left(\frac{x}{e}\right)^{\beta x}-\frac{1}{\gamma}\left(\frac{e}{x}\right)^{\delta x}+C$, where $e=\sum_{n=0}^{\infty}\frac{1}{n!}$ and C is constant of integration, then $\alpha+2\beta+3\gamma-4\delta$ is equal to :

$\alpha, \beta, \gamma, \delta \in \mathbb{N}$ के लिए, यदि $\int\left(\left(\frac{x}{e}\right)^{2x}+\left(\frac{e}{x}\right)^{2x}\right)\log_e x\,dx=\frac{1}{\alpha}\left(\frac{x}{e}\right)^{\beta x}-\frac{1}{\gamma}\left(\frac{e}{x}\right)^{\delta x}+C$ है, जहाँ $e=\sum_{n=0}^{\infty}\frac{1}{n!}$ तथा C समाकलन अचर है, तो $\alpha + 2\beta + 3\gamma - 4\delta$ बराबर है :

(1) –4 (2) 1 (3) 4 (4) –8

Ans. Official Answer NTA (3)

Sol. Let $\left(\frac{x}{e}\right)^{2x}=t$

$2x(\ln x-1)=\ln t$

$\left(2(\ln x-1)+2x\times\frac{1}{x}\right)dx=\frac{1}{t}dt$

$\ln x dx=\frac{1}{2t}dt$

$I=\int\left(t+\frac{1}{t}\right)\times\frac{1}{2t}dt=\frac{1}{2}\int\left(1+\frac{1}{t^2}\right)dt$

$=\frac{1}{2}\left(t-\frac{1}{t}\right)+C$

$=\frac{1}{2}\left(\frac{x}{e}\right)^{2x}-\frac{1}{2}\left(\frac{e}{x}\right)^{2x}+C$

$\Rightarrow \alpha=2, \beta=2, \gamma=2, \delta=2$

$\therefore \quad \alpha+2\beta+3\gamma-4\delta=2+4+6-8=4$

Question ID : 7155054136

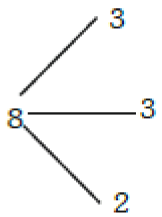
2. Eight persons are to be transported from city A to city B in three cars of different makes. If each car can accommodate at most three persons, then the number of ways, in which they can be transported, is :

भिन्न कंपनियों की तीन कारों में आठ व्यक्तियों को शहरों A से शहर B ले जाना है। यदि प्रत्येक कार में अधिकतम तीन व्यक्ति बैठ सकते हैं, तो इन व्यक्तियों को ले जाने के तरीकों की संख्या है :

(1) 1120 (2) 1680 (3) 3360 (4) 560

Ans. Official Answer NTA (2)

Sol.



$$\text{Ways} = \frac{8!}{3!3!2!2!} \times 3!$$

$$= \frac{8 \times 7 \times 6 \times 5 \times 4}{4}$$

$= 56 \times 30$

$= 1680$

Question ID : 7155054142

3. Let f be a continuous function satisfying $\int_0^{t^2} \left(f(x) + x^2\right) dx = \frac{4}{3} t^3, \forall t > 0$. Then $f\left(\frac{\pi^2}{4}\right)$ is equal to :

माना f एक सतत फलन है तथा $\int_0^{t^2} \left(f(x) + x^2\right) dx = \frac{4}{3} t^3, \forall t > 0$ है। तो $f\left(\frac{\pi^2}{4}\right)$ बराबर है :

(1) $\left(1 - \pi\left(1 + \frac{\pi^3}{16}\right)\right)$ (2) $-\pi\left(1 - \frac{\pi^2}{16}\right)$ (3) $\pi^2\left(1 - \frac{\pi^3}{16}\right)$ (4) $\pi\left(1 - \frac{\pi^2}{16}\right)$

Ans. Official Answer NTA (4)

Sol. $\int_0^{t^2} \left(f(x) + x^2\right) dx = \frac{4}{3} t^3, \forall t > 0$

$\left(f\left(t^2\right) + t^4\right) = 2t$

$f\left(t^2\right) = 2t - t^4$

$t = \frac{\pi}{2} \Rightarrow f\left(\frac{\pi^2}{4}\right) = \frac{2\pi}{2} - \frac{\pi^4}{16}$

$= \pi - \frac{\pi^4}{16} = \pi\left(1 - \frac{\pi^3}{16}\right)$

Question ID : 7155054132

4. Let A = {2, 3, 4} and B = {8, 9, 12}. Then the number of elements in the relation $R = \{((a_1, b_1), (a_2, b_2)) \in (A \times B, A \times B) : a_1$ divides b_2 and a_2 divides $b_1\}$ is :

माना A = {2, 3, 4} तथा B = {8, 9, 12} हैं। तो संबंध $R = \{((a_1, b_1), (a_2, b_2)) \in (A \times B, A \times B) : a_1, b_2$ को विभाजित करता है तथा a_2, b_1 को विभाजित करता है} में अवयवों की संख्या है :

(1) 24 (2) 12 (3) 18 (4) 36

Ans. Official Answer NTA (4)

Sol. $A = \{2, 3, 4\}, B = \{8, 9, 12\}$

$a_1 \in A, b_2 \in B$

a_1 divides b_2

$(a_1, b_2) \in \{(2,4), (2,12), (3,9), (3,12), (4,8), (4,12)\}$

$a_2 \in A, b_1 \in B$

a_2 divides b_1

.... same as above ....

$\therefore$ Number of relations $= 6 \times 6 = 36$

Question ID : 7155054144

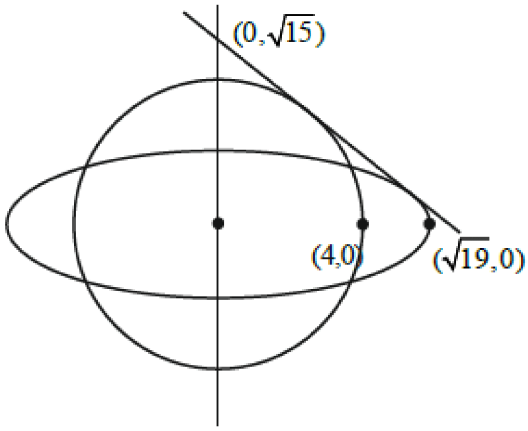
5. 5 Let a circle of radius 4 be concentric to the ellipse $15x^2 + 19y^2 = 285$. Then the common tangents are inclined to the minor axis of the ellipse at the angle :

माना त्रिज्या 4 का एक वृत्त तथा दीर्घवृत्त $15x^2 + 19y^2 = 285$ संकेन्द्री हैं। तो उभयनिष्ठ स्पर्श रेखाएँ दीर्घवृत्त के लघु अक्ष से कौन सा कोण बनाती है ?

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{12}$

Ans. Official Answer NTA (3)

Sol. $\frac{x^2}{19} + \frac{y^2}{15} = 1$



Let tangent be

$$y = mx \pm \sqrt{19m^2 + 15}$$

$$mx - y \pm \sqrt{19m^2 + 15} = 0$$

Distance from $(0, 0) = 4$

$$\left|\frac{\pm\sqrt{19m^2+15}}{\sqrt{m^2+1}}\right| = 4$$

$$19m^2 + 15 = 16m^2 + 16$$

$$3m^2 = 1$$

$$m = \pm\frac{1}{\sqrt{3}}$$

$\theta = \frac{\pi}{6}$ with x-axis

Required angle $\frac{\pi}{3}$.

Question ID : 7155054146

6. Let the line $\frac{x}{1} = \frac{6-y}{2} = \frac{z+8}{5}$ intersect the lines $\frac{x-5}{4} = \frac{y-7}{3} = \frac{z+2}{1}$ and $\frac{x+3}{6} = \frac{3-y}{3} = \frac{z-6}{1}$ at the points A and B respectively. Then the distance of the mid-point of the line segment AB from the plane $2x - 2y + z = 14$ is :

माना रेखा $\frac{x}{1} = \frac{6-y}{2} = \frac{z+8}{5}$ रेखाओं $\frac{x-5}{4} = \frac{y-7}{3} = \frac{z+2}{1}$ तथा $\frac{x+3}{6} = \frac{3-y}{3} = \frac{z-6}{1}$ को क्रमशः बिंदुओं A तथा B पर काटती है। तो रेखाखंड AB के मध्य बिंदु की समतल $2x - 2y + z = 14$ से दूरी है :

(1) $\frac{10}{3}$ (2) $\frac{11}{3}$ (3) 4 (4) 3

Ans. Official Answer NTA (3)

Sol. $\frac{x}{1}=\frac{y-6}{-2}=\frac{z+8}{5}=\lambda$ ....(1)

$\frac{x-5}{4}=\frac{y-7}{3}=\frac{z+2}{1}=\mu$ ....(2)

$\frac{x+3}{6}=\frac{y-3}{-3}=\frac{z-6}{1}=\gamma$ ....(3)

Intersection of (1) & (2) "A"

$(\lambda,-2\lambda+6,5\lambda-8)$ & $(4\mu+5,3\mu+7,\mu-2)$

$\lambda=1,\mu=-1$

$A(1,4,-3)$

Intersection (1) & (3) "B"

$(\lambda,-2\lambda+6,5\lambda-8)$ & $(4\mu+5,3\mu+7,\mu-2)$

$\lambda=1,\mu=-1$

$A(1,4,-3)$

Intersection (1) & (3) "B"

$(\lambda,-2\lambda+6,5\lambda-8)$ & $(6\gamma-3,-3\gamma+3,\gamma+6)$

$\lambda=3$

$\gamma=1$

$B(3,0,7)$

Mod point of A & B $\Rightarrow (2,2,2)$

Perpendicular distance from the plane

$2x-2y+z=14$

$$\left|\frac{2(2)-2(2)+2-14}{\sqrt{4+4+1}}\right|=4$$

Question ID : 7155054135

7. If $A=\frac{1}{5!6!7!}\begin{bmatrix}5! & 6! & 7!\\ 6! & 7! & 8!\\ 7! & 8! & 9!\end{bmatrix}$, then $|\text{adj}(\text{adj}(2A))|$ is equal to :

यदि $A = \frac{1}{5!6!7!}\begin{bmatrix} 5! & 6! & 7! \\ 6! & 7! & 8! \\ 7! & 8! & 9! \end{bmatrix}$ है, तो $|\text{adj}(\text{adj}(2A))|$ बराबर है :

(1) 2^{16} (2) 2^{8} (3) 2^{20} (4) 2^{12}

Ans. Official Answer NTA (1)

Sol. $|A| = \frac{1}{5!6!7!} \times 5!6!7! \begin{vmatrix} 1 & 6 & 42 \\ 1 & 7 & 56 \\ 1 & 8 & 72 \end{vmatrix}$

$R_2 \to R_2 - R_1 \ \& R_3 \to R_3 - R_2$

$|A| = \begin{vmatrix} 1 & 6 & 42 \\ 0 & 1 & 14 \\ 0 & 1 & 16 \end{vmatrix} \Rightarrow |A| = 2$

$|\text{Adj}\left(\text{adj}(2A)\right| = |2A|^{(n-1)^2} = |2A|^4 = 2^{12}|A|^4 = 2^{16}$

Question ID : 7155054134

8. Let $S = \left\{ z = x + iy : \frac{2z-3i}{4z+2i} \text{ is a real number} \right\}$. Then which of the following is NOT correct :

माना $S = \left\{ z = x + iy : \frac{2z-3i}{4z+2i} \text{ एक वास्तविक संख्या} \right\}$ । तो निम्न में कौन सा सही नहीं है ?

(1) $y + x^2 + y^2 \neq -\frac{1}{4}$

(2) $x = 0$

(3) $(x, y) = \left(0, -\frac{1}{2}\right)$

(4) $y \in \left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$

Ans. Official Answer NTA (3)

Sol. $\because$ $z = x + iy$ and $\frac{2z-3i}{4z+2i}$ is real number

$\therefore$ $\frac{2x+(2y-3)i}{4x+(4y+2)i}$ is real number.

$\therefore \dfrac{(2x+(2y-3)i)(4x-(4y+2)i)}{(4x)^2+(4y+2)^2}$ is real number

$\therefore \quad -2x(4y+2)+(2y-3)4x=0$

$\therefore \quad x=0$

Here $x=0$ and $y \in \mathbb{R}$ but $y \neq -\dfrac{1}{2}$ is not acceptable

in this case
Denominator will be zero.

Question ID : 7155054149

9. Let a die be rolled n times. Let the probability of getting odd numbers seven times be equal to the probability of getting odd numbers nine times. If the probability of getting even numbers twice is $\dfrac{k}{2^{15}}$, then k is equal to :

माना एक पासे को n बार फेंका जाता है। माना सात बार विषम संख्या प्राप्त करने की प्रायिकता, नो बार विषम संख्या प्राप्त करने की प्रायिकता के बराबर है। यदि दो बार सम संख्या प्रापत करने की प्रायिकता $\dfrac{k}{2^{15}}$ है, तो k बराबर है :

(1) 15 (2) 90 (3) 60 (4) 30

Ans. Official Answer NTA (3)

Sol. P(odd number 7 times) = P(odd number 9 times)

$${}^{n}C_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{n-7} = {}^{n}C_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{n-9}$$

$${}^{n}C_7 = {}^{n}C_9$$

$\Rightarrow n=16$

Required

$$P = {}^{16}C_2 \times \left(\frac{1}{2}\right)^{16}$$

$$= \frac{16\cdot 15}{2}\times\frac{1}{2^{16}} = \frac{15}{2^{13}}$$

$$\Rightarrow \frac{60}{2^{15}} \Rightarrow k=60$$

Question ID : 7155054148

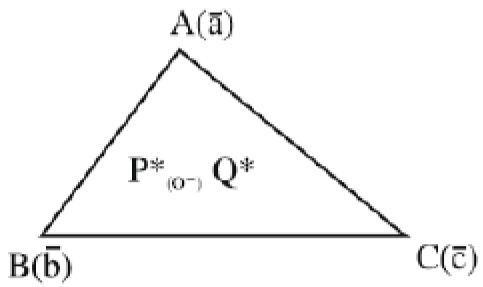
10. If the points P and Q are respectively the circumcenter and the orthocentre of a ΔABC, then $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}$ is :

यदि एक त्रिभुज ABC के परिकेन्द्र तथा लंबकेन्द्र क्रमशः P तथा Q है, तो $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}$ बराबर है :

(1) $2\overrightarrow{PQ}$ (2) $\overrightarrow{QP}$ (3) $2\overrightarrow{QP}$ (4) $\overrightarrow{PQ}$

Ans. Official Answer NTA (4)

Sol.



$$\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}=\vec{a}+\vec{b}+\vec{c}$$

$$\overrightarrow{PG}=\frac{\vec{a}+\vec{b}+\vec{c}}{3}$$

$$\Rightarrow \vec{a}+\vec{b}+\vec{c}=3\overrightarrow{PG}=\overrightarrow{PQ}$$

Question ID : 7155054133

11. Let $S=\left\{x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right):9^{1-\tan^2 x}+9^{\tan^2 x}=10\right\}$ and $\beta=\sum_{x\in S}\tan^2\left(\frac{x}{3}\right)$, then $\frac{1}{6}(\beta-14)^2$ is equal to :

माना $S=\left\{x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right):9^{1-\tan^2 x}+9^{\tan^2 x}=10\right\}$ तथा $\beta=\sum_{x\in S}\tan^2\left(\frac{x}{3}\right)$ हैं, तो $\frac{1}{6}(\beta-14)^2$ बराबर है :

(1) 32 (2) 64 (3) 16 (4) 8

Ans. Official Answer NTA (1)

Sol. $9^{1-\tan^2 x}+9^{\tan^2 x}=10$

$$\Rightarrow \frac{9}{y}+y=10 \ \left(\text{where } 9^{\tan^2 x}=y\right)$$

So, $9+y^2=10y$

$y^2-10y+9=0$

$(y-1)(y-9)=0$

So, $9^{\tan^2 x}=1$ & $9^{\tan^2 x}=9$

$\tan^2 x=0$ $\tan^2 x=1$

$\downarrow$ $x=0$ $\tan x=1$ $\tan x=-1$

$\downarrow$ $x=\frac{\pi}{4}$ $\downarrow$ $x=-\frac{\pi}{4}$

Now $\beta=\sum \tan^2\frac{x}{3}=\tan^2\frac{0}{3}+\tan^2\frac{\pi}{12}+\tan^2\frac{\pi}{12}$

$=2\tan^2\frac{\pi}{12}$

$=2(2-\sqrt{3})^2$

$=2(7-4\sqrt{3})$

$\beta=14-8\sqrt{3}\Rightarrow\frac{1}{6}(\beta-14)^2=32$

Question ID : 7155054139

12. If $S_n=4+11+21+34+50+\ldots$ to n term, then $\frac{1}{60}\left(S_{29}-S_9\right)$ is equal to :

यदि $S_n=4+11+21+34+50+\ldots$ n पदों तक है, तो $\frac{1}{60}\left(S_{29}-S_9\right)$ बराबर है :

(1) 226 (2) 227 (3) 220 (4) 223

Ans. Official Answer NTA (4)

Sol. $S=4+11+21+34+50+\ldots+T_n$

$$\frac{S=4+11+21+34+\ldots+T_{n-1}+T_n}{T_n=4+7+10+13+\ldots}$$

$T_n=4+\frac{1}{2}\left(3n^2+5n-8\right)$

$$\sum T_n = S_n = \frac{3}{2}\sum n^2 + \frac{5}{2}\sum n$$

$$= \frac{3}{2}\frac{n(n+1)(2n+1)}{6} + \frac{5}{2}\frac{n(n+1)}{2}$$

$$\frac{S_{29} - S_9}{60} = 223$$

Question ID : 7155054145

13. Let the image of the point P(1, 2, 6) in the plane passing through the points A(1, 2, 0), B(1, 4, 1) and C(0, 5, 1) be Q(α, β, γ). Then $\alpha^2 + \beta^2 + \gamma^2$) is equal to :

माना बिंदुओं A(1, 2, 0), B(1, 4, 1) तथा C(0, 5, 1) से होकर जाने वाले समतल में बिंदु P(1, 2, 6) का प्रतिबिंब Q(α, β, γ) है। तो ($\alpha^2 + \beta^2 + \gamma^2$) बराबर है :

(1) 62 (2) 70 (3) 65 (4) 76

Ans. Official Answer NTA (3)

Sol. Equation of plane A(x – 1) + B(y – 2) + C(z – 0) = 0

Put (1, 4, 1) ⇒ 2B + C = 0

Put (0, 5, 1) ⇒ –A + 3B + C = 0

Sub : B – A = 0 ⇒ A = B, C = –2B

1(x – 1) + 1(y – 2) – 2(z – 0) = 0

x + y – 2z – 3 = 0

Image is (α, β, γ)pt $\equiv (1,2,6)$

$$\frac{\alpha-1}{1} = \frac{\beta-2}{1} = \frac{\gamma-6}{-2} = \frac{-2(1+2-12-3)}{6}$$

$$\frac{\alpha-1}{1} = \frac{\beta-2}{1} = \frac{\gamma-6}{-2} = 4$$

$$\alpha = 5, \beta = 6, \gamma = -2 \Rightarrow \alpha^2 + \beta^2 + \gamma^2$$

= 25 + 36 + 4 = 65

Question ID : 7155054151

14. The statement $\sim[p \vee (\sim(p \wedge q))]$ is equivalent to :

कथन $\sim[p \vee (\sim(p \wedge q))]$ किस के तुल्य है :

(1) $(p \wedge q) \wedge (\sim p)$ (2) $\sim(p \vee q)$ (3) $\sim(p \wedge q)$ (4) $(\sim(p \wedge q)) \wedge q$

Ans. Official Answer NTA (1)

Sol. $\sim [p \vee (\sim (p \wedge q))]$

$\sim p \wedge (p \wedge q)$

Question ID : 7155054143

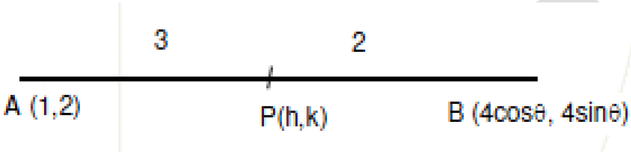
15. Let A be the point (1, 2) and B be any point on the curve $x^2 + y^2 = 16$. If the centre of the locus of the point P, which divides the line segment AB in the ratio 3 : 2 is the point $C(\alpha, \beta)$, then the length of the line segment AC is :

माना A बिंदु (1, 2) है तथा वक्र $x^2 + y^2 = 16$ पर कोई बिंदु B है। यदि रेखा खंड AB को 3 : 2 के अनुपात में विभाजित करने वाले बिंदु P के बिंदुपथ का केन्द्र $C(\alpha, \beta)$ है, तो रेखाखंड AC की लंबाई है :

(1) $\frac{4\sqrt{5}}{5}$ (2) $\frac{3\sqrt{5}}{5}$ (3) $\frac{6\sqrt{5}}{5}$ (4) $\frac{2\sqrt{5}}{5}$

Ans. Official Answer NTA (2)

Sol.



$$h = \frac{12\cos\theta + 2}{5}, \quad k = \frac{12\sin\theta + 4}{5}$$

$$5h - 2 = 12\cos\theta, \quad 5k - 4 = 12\sin\theta$$

locus of point P is

$$(5x-2)^2 + (5y-4)^2 = 144$$

its centre : C (2/5, 4/5)

$$AC = \sqrt{(1-2/5)^2 + (2-4/5)^2} = \sqrt{\frac{9}{25} + \frac{36}{25}} = \sqrt{\frac{45}{25}} = \frac{3}{\sqrt{5}}$$

Question ID : 7155054138

16. Let the number $(22)^{2022} + (2022)^{22}$ leave the remainder α when divided by 3 and β when divided by 7. Then $(\alpha^2 + \beta^2)$ is equal to :

माना $(22)^{2022} + (2022)^{22}$ को 3 से विभाजित करने पर शेषफल α है तथा 7 से विभाजित करने पर शेषफल β है। तो $(\alpha^2 + \beta^2)$ बराबर है :

(1) 13 (2) 5 (3) 20 (4) 10

Ans. Official Answer NTA (2)

Sol. For $\alpha : (21+1)^{2022} + \underbrace{(2022)^{22}}_{\text{divisible by 3}}$

$= 3K_1 + 1$

For $\beta : (21+1)^{2022} + (2023-1)^{22}$

$= 7\lambda + 1 + 7\mu + 1$

$= 7K_2 + 2$

So, $\alpha = 1, \beta = 2$

$\alpha^2 + \beta^2 = 5$

Question ID : 7155054147

17. Let $\vec{a} = 2\hat{i} + 7\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} + 5\hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + 2\hat{k}$. Let $\vec{d}$ be a vector which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}\cdot\vec{d} = 12$. Then $(-\hat{i} + \hat{j} - \hat{k})\cdot(\vec{c}\times\vec{d})$ is equal to :

माना $\vec{a} = 2\hat{i} + 7\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} + 5\hat{k}$ तथा $\vec{c} = \hat{i} - \hat{j} + 2\hat{k}$ है। माना सदिशों $\vec{a}$ तथा $\vec{b}$ के लंबवत् एक सदिश $\vec{d}$ है तथा $\vec{c}\cdot\vec{d} = 12$ है। तो $(-\hat{i} + \hat{j} - \hat{k})\cdot(\vec{c}\times\vec{d})$ बराबर है :

(1) 48 (2) 42 (3) 24 (4) 44

Ans. Official Answer NTA (4)

Sol. $\vec{a} = 2\hat{i} + 7\hat{j} - \hat{k}$

$\vec{b} = 3\hat{i} + 5\hat{k}$

$\vec{c} = \hat{i} - \hat{j} + 2\hat{k}$

$$\vec{d} = \lambda(\vec{a}\times\vec{b}) = \lambda\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 7 & -1 \\ 3 & 0 & 5 \end{vmatrix}$$

$\vec{d} = \lambda(35\hat{i} - 13\hat{j} - 21\hat{k})$

$\lambda(35 + 13 - 42) = 12$

$\lambda = 2$

$\vec{d} = 2(35\hat{i} - 13\hat{j} - 21\hat{k})$

$(\hat{i} + \hat{j} - \hat{k})(\vec{c}\times\vec{d})$

$$= \begin{vmatrix} -1 & 1 & -1 \\ 1 & -1 & 2 \\ 70 & -26 & -42 \end{vmatrix} = 44$$

Question ID : 7155054150

18. Let μ be the mean and σ be the standard deviation of the distribution

x_i	0	1	2	3	4	5
f_i	k + 2	2k	$k^2 - 1$	k^2 - 1	$k^2 + 1$	k – 1

where $\sum f_i = 62$. If [x] denotes the greatest integer ≤ x, then $\left[\mu^2 + \sigma^2\right]$ is equal to :

माना बंटन

x_i	0	1	2	3	4	5
f_i	k + 2	2k	$k^2 - 1$	k^2 - 1	$k^2 + 1$	k – 1

जहाँ $\sum f_i = 62$ है, का माध्य μ तथा मानक विचलन σ हैं। यदि [x] महत्तम पूर्णांक ≤ x है, तो $\left[\mu^2 + \sigma^2\right]$ बराबर है :

(1) 8 (2) 7 (3) 9 (4) 6

Ans. Official Answer NTA (1)

Sol. $\Sigma f_i = 62$

$3k^2 + 16k - 12k - 64 = 0$

$k = 4$ or $-\frac{16}{3}$ (rejected)

$$\mu = \frac{\sum f_i x_i}{\sum f_i}$$

$$\mu = \frac{8 + 2(15) + 3(15) + 4(17) + 5}{62} = \frac{156}{62}$$

$$\sigma^2 = \sum f_i x_i^2 - \left(\sum f_i x_i\right)^2$$

$$= \frac{8 \times 1^2 + 15 \times 13 + 17 \times 16 + 25}{62} - \left(\frac{156}{62}\right)^2$$

$$\sigma^2 = \frac{500}{62} - \left(\frac{156}{62}\right)^2$$

$$\sigma^2 + \mu^2 = \frac{500}{62}$$

$\left[\sigma^2+\mu^2\right]=8$

Question ID : 7155054140

19. Let $g(x)=f(x)+f(1-x)$ and $f''(x)>0, x\in(0,1)$. If g is decreasing in the interval $(0, \alpha)$ and increasing in the interval $(\alpha, 1)$, then $\tan^{-1}(2\alpha)+\tan^{-1}\left(\frac{1}{\alpha}\right)+\tan^{-1}\left(\frac{\alpha+1}{\alpha}\right)$ is equal to :

माना $g(x)=f(x)+f(1-x)$ तथा $f''(x)>0, x\in(0,1)$ है। यदि g अंतराल $(0, \alpha)$ में ह्रासमान है तथा अंतराल $(\alpha, 1)$ में वर्धमान है, तो $\tan^{-1}(2\alpha)+\tan^{-1}\left(\frac{1}{\alpha}\right)+\tan^{-1}\left(\frac{\alpha+1}{\alpha}\right)$ बराबर है :

(1) π (2) $\frac{3\pi}{4}$ (3) $\frac{5\pi}{4}$ (4) $\frac{3\pi}{2}$

Ans. Official Answer NTA (1)

Sol. $g'(x)=f'(x)-f'(1-x)$

If $x>1-x$

$\Rightarrow f'(x)>f'(1-x) \because f'(x)$ is strictly increasing

$\Rightarrow g'(x)>0 \Rightarrow$ so g(x) is increasing when $x>1-x \Rightarrow x>\frac{1}{2}$

$\Rightarrow \alpha=\frac{1}{2}$

similarly when $x<1-x \Rightarrow g(x)$ is decreeing

Now

$\tan^{-1}(2\alpha)+\tan^{-1}\left(\frac{1}{\alpha}\right)+\tan^{-1}\left(\frac{\alpha+1}{\alpha}\right)$

$=\tan^{-1}(1)+\tan^{-1}(2)+\tan^{-1}(3)=\pi(\because 1+2+3=1.2.3)$

Question ID : 7155054137

20. If the coefficients of x and x^2 in $(1+x)^p(1-x)^q$ are 4 and –5 respectively, then $2p+3q$ is equal to :

$(1+x)^p(1-x)^q$ में x तथा x^2 के गुणांक क्रमशः 4 तथा –5 हैं, तो $2p+3q$ बराबर है :

(1) 63 (2) 69 (3) 60 (4) 66

Ans. Official Answer NTA (1)

Sol. $(1+x)^p(1-x)^q$

$$=\left(1+px+\frac{p(p-1)}{2}x^2+....\right)\left(1-qx+\frac{q(q-1)}{2}x^2+....\right)$$

$$=1+(p-q)x+\left(\frac{p(p-1)}{2}+\frac{q(q-1)}{2}-pq\right)x^2+.....$$

$$p-q=4\Rightarrow p=q+4 \quad ...(i)$$

$$\frac{p(p-1)}{2}+\frac{q(q-1)}{2}-pq=-5$$

$$\frac{(q+4)(q+3)}{2}+\frac{q(q-1)}{2}-(q+4)q=-5$$

$$\Rightarrow \quad 6-q=-5$$

$$\Rightarrow \quad q=11$$

$$\therefore \quad p=15$$

$$2p+3q=63$$

SECTION – B

Question ID : 7155054153

21. Let S be the set of values of λ, for which the system of equations
$6\lambda x-3y+3z=4\lambda^2$,
$2x+6\lambda y+4z=1$,
$3x+2y+3\lambda z=\lambda$
has no solution. Then $12\sum_{\lambda\in S}|\lambda|$ is equal to ________.

माना λ के मानों, जिनके लिए समीकरण निकाय
$6\lambda x-3y+3z=4\lambda^2$,
$2x+6\lambda y+4z=1$,
$3x+2y+3\lambda z=\lambda$
का कोई हल नहीं है, का समुच्चय S है। तो $12\sum_{\lambda\in S}|\lambda|$ बराबर है ________

Ans. Official Answer NTA (24)

Sol. For no solution

$$\begin{vmatrix}6\lambda & -3 & 3\\ 2 & 6\lambda & 4\\ 3 & 2 & 3\lambda\end{vmatrix}=0$$

$$\Rightarrow 9\lambda^3-7\lambda-2=0$$

$$\Rightarrow (\lambda-1)(3\lambda+1)(3\lambda+2)=0$$

$$\Rightarrow \quad 12\sum_{\lambda\in S}|\lambda| = 12\times\left(1+\frac{1}{3}+\frac{2}{3}\right) = 24$$

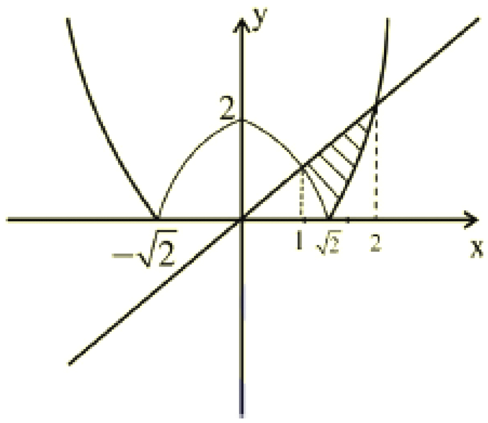
Question ID : 7155054157

22. If the area of the region $\{(x,y):|x^2-2|\le y\le x\}$ is A, then $6A+16\sqrt{2}$ is equal to ________.

यदि क्षेत्र $\{(x,y):|x^2-2|\le y\le x\}$ का क्षेत्रफल A है, तो $6A+16\sqrt{2}$ बराबर है ________

Ans. Official Answer NTA (27)

Sol. $|x^2-2|\le y\le x$



$$A = \int_1^{\sqrt{2}}\left(x-\left(2-x^2\right)\right)dx + \int_{\sqrt{2}}^{2}\left(x-\left(x^2-2\right)\right)dx$$

$$=\left(1-2\sqrt{2}+\frac{2\sqrt{2}}{3}\right)-\left(\frac{1}{2}-2+\frac{1}{3}\right)+\left(2-\frac{8}{3}+4\right)-\left(1-\frac{2\sqrt{2}}{3}+2\sqrt{2}\right)$$

$$=-4\sqrt{2}+\frac{4\sqrt{2}}{3}+\frac{7}{6}+\frac{10}{3}=\frac{-8\sqrt{2}}{3}+\frac{9}{2}$$

$$6A=-16\sqrt{2}+27 \therefore 6A+16\sqrt{2}=27$$

Question ID : 7155054155

23. Suppose $a_1, a_2, 2, a_3, a_4$ be in an arithmetico-geometric progression. If the common ratio of the corresponding geometric progression is 2 and the sum of all 5 terms of the arithmetico-geometric progression is $\frac{49}{2}$, then a_4 is equal to ________.

माना $a_1, a_2, 2, a_3, a_4$ एक समांतर–गुणोत्तर श्रेणी है। यदि संगत गुणोत्तर श्रेणी का सर्व अनुपात 2 है तथा समांतर–गुणोत्तर श्रेणी

के सभी 5 पदों का योग $\frac{49}{2}$ है, तो a_4 बराबर है ________

Ans. Official Answer NTA (16)

Sol. $\frac{(a-2d)}{4}, \frac{(a-d)}{2}, a, 2(a+d), 4(a+2d)$

$a = 2$

$\left(\frac{1}{4}+\frac{1}{2}+1+6\right)\times 2+(-1+2+8)d=\frac{49}{2}$

$2\left(\frac{3}{4}+7\right)+9d=\frac{49}{2}$

$9d=\frac{49}{2}-\frac{62}{4}=\frac{98-62}{4}=9$

$d = 1$

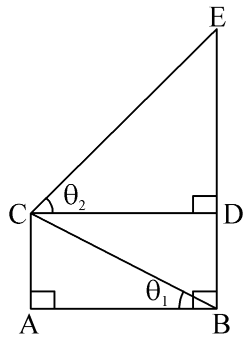
$\Rightarrow a_4 = 4(a+2d)$

$= 16$

Question ID : 7155054161

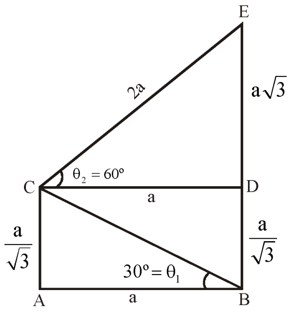
24. In the figure, $\theta_1+\theta_2=\frac{\pi}{2}$ and $\sqrt{3}(BE)=4(AB)$. If the area of ΔCAB is $2\sqrt{3}-3$ unit², when $\frac{\theta_2}{\theta_1}$ is the largest, then the perimeter (in unit) of ΔCED is equal to ________.

दी गई आकृति में $\theta_1+\theta_2=\frac{\pi}{2}$ तथा $\sqrt{3}(BE)=4(AB)$ है। यदि ΔCAB का क्षेत्रफल $2\sqrt{3}-3$ वर्ग इकाई है, जब $\frac{\theta_2}{\theta_1}$ अधिकतम है, तो ΔCED का परिमाप (इकाई में) बराबर है :



Ans. Official Answer NTA (6)

Sol.



$\tan\theta_2 = \frac{DE}{CD} = \frac{BE - AC}{AB}$

$\tan\theta_2 = \frac{4}{\sqrt{3}} - \tan\theta_1 \Rightarrow \tan\theta_1 + \tan\theta_2 = \frac{4}{\sqrt{3}}......$

Now $\theta_1 + \theta_2 = \frac{\pi}{2} \Rightarrow \tan\theta_1 \tan\theta_2 = 1...(2)$

Let $\tan\theta_1 = x$ and $\tan\theta_2 = y$ and $\frac{\theta_2}{\theta_1}$ is the largest

$\Rightarrow x + y = \frac{4}{\sqrt{3}}$ $x + \frac{1}{x} = \frac{4}{\sqrt{3}}$ $\sqrt{3}x^2 - 4x + \sqrt{3} = 0$

$xy = 1$ $\sqrt{3}x\left(x - \sqrt{3}\right) - 1\left(x - \sqrt{3}\right) = 0$

$x = \sqrt{3}, x = \frac{1}{\sqrt{3}}$

$y = \frac{1}{\sqrt{3}}, y = \sqrt{3}$

$x = 60^\circ, y = 30^\circ$

$\frac{x}{y} = 2$

Question ID : 7155054160

25. Let the foot of perpendicular from the point A(4, 3, 1) on the plane P : x – y + 2z + 3 = 0 be N. If B(5, α, β), α, β ∈ Z is a point on plane P such that the area of the triangle ABN is $3\sqrt{2}$, then $\alpha^2 + \beta^2 + \alpha\beta$ is equal to ________.

माना बिंदु A(4, 3, 1) से समतल P : x – y + 2z + 3 = 0 पर डाला गया लंबपाद N है। यदि समतल P पर एक बिंदु B(5, α, β), α, β ∈ Z इस प्रकार है कि त्रिभुज ABN का क्षेत्रफल $3\sqrt{2}$ है, तो $\alpha^2 + \beta^2 + \alpha\beta$ बराबर है :

Ans. Official Answer NTA (7.00)

Sol. $5-\alpha+2\beta+3=0$

$\alpha-2\beta=8$

$\frac{x_1-4}{1}=\frac{y_1-3}{-1}=\frac{z_1-1}{2}=\frac{-(6)}{6}$

$N\equiv(x_1,y_1,z_1)=(3,4,-1)$

$B\equiv(5,\alpha,\beta)$

$\frac{1}{2}AN\times BN=3\sqrt{2}$

$AN\cdot BN=6\sqrt{2}$

$\frac{6}{\sqrt{6}}\cdot BN=6\sqrt{2}$

$BN=2\sqrt{3}$

$4+(\alpha-4)^2+(\beta+1)^2=12$

$(\alpha-4)^2+(\beta+1)^2=8$

$(2\beta+4)^2+(\beta+1)^2=8$

$5\beta^2+18\beta+9=0$

$\beta=-3,-\frac{3}{5}$

$\beta=-3,\alpha=2$

$\alpha^2+\beta^2+\alpha\beta=9+4-6=7$

Question ID : 7155054158

26. Let the tangent at any point P on a curve passing through the points (1, 1) and $\left(\frac{1}{10},100\right)$, intersect positive x-axis and y-axis at the points A and B respectively. If PA : PB = 1: k and y = y(x) is the solution of the differential equation $e^{\frac{dy}{dx}}=kx+\frac{k}{2}, y(0)=k$, then $4y(1)-5\log_e 3$ is equal to ________.

माना बिंदुओं (1, 1) तथा $\left(\frac{1}{10},100\right)$ से होकर जाने वाले एक वक्र के किसी बिंदु P पर स्पर्श रेखा धनात्मक x-अक्ष तथा y-अक्षों को क्रमशः बिंदुओं A तथा B पर काटती है। यदि PA : PB = 1: k है तथा अवकल समीकरण $e^{\frac{dy}{dx}}=kx+\frac{k}{2}, y(0)=k$ का हल

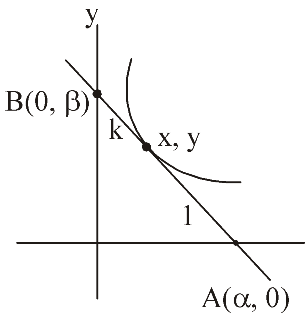
$y = y(x)$ है, तो $4y(1) - 5\log_e 3$ बराबर है :

Ans. Official Answer NTA (Bonus)

Sol. $Y - y = \frac{dy}{dx}(X - x)$

$Y = 0$

$X = \frac{-ydx}{dy} + x$



$\frac{k\alpha + 0}{k+1} = 0, \quad \alpha = \frac{k+1}{k}x$

$\frac{k+1}{k}x = -y\frac{dx}{dy} + x$

$x + \frac{x}{k} = -y\frac{dx}{dy} + x$

$x\frac{dy}{dx} + ky = 0 \quad \frac{dy}{dx} + \frac{k}{x}y = 0$

$y.\, x^k = C$

$C = 1$

$100 \cdot \left(\frac{1}{10}\right)^k = 1$

$K = 2$

$\frac{dy}{dx} = \ln(2x+1)$

$y = \frac{(2x+1)}{2}(\ln(2x+1) - 1) + c$

$2 = \frac{1}{2}(0-1) + C$

$C = 2 + \frac{1}{2} = \frac{5}{2}$

$y(1) = \frac{3}{2}(\ln 3 - 1) + \frac{5}{2}$

$= \frac{3}{2}\ln 3 + 1$

$4y(1) = 6\ln 3 + 4$

$4y(1) - 5\ln 3 = 4 + \ell \ln 3$

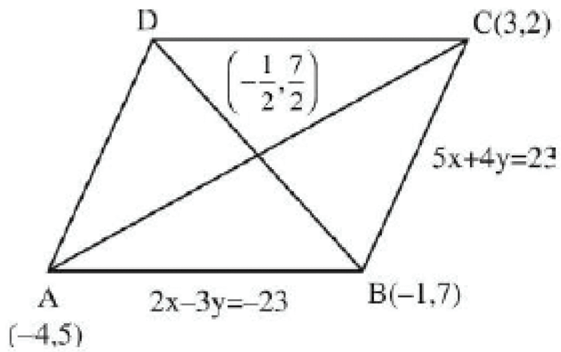
Question ID : 7155054159

27. Let the equations of two adjacent sides of a parallelogram ABCD be 2x – 3 y = –23 and 5x + 4y = 23. If the equation of its one diagonal AC is 3x + 7y = 23 and the distance of A from the other diagonal is d, then 50 d^2 is equal to ________.

माना एक समांतर चतुर्भुज की दो संलग्न भुजाओं के समीकरण 2x – 3 y = –23 तथा 5x + 4y = 23 हैं। यदि इसके एक विकर्ण AC का समीकरण 3x + 7y = 23 है तथा A की दूसरे विकर्ण से दूरी d है, तो 50d^2 बराबर है :

Ans. Official Answer NTA (529)

Sol.



A & C point will be (–4, 5) & (3, 2)

mid point of AC will be $\left(-\frac{1}{2},\frac{7}{2}\right)$

equation of diagonal BD is

$$y - \frac{7}{2} = \frac{\frac{7}{2}}{-\frac{1}{2}}\left(x + \frac{1}{2}\right)$$

$\Rightarrow 7x + y = 0$

Distance of A from diagonal BD

$= d = \frac{23}{\sqrt{50}}$

$\Rightarrow 50d^2 = (23)^2$

$50d^2 = 529$

Question ID : 7155054154

28. The sum of all the four-digit numbers that can be formed using all the digits 2, 1, 2, 3 is equal to ________.

सभी अंकों 2, 1, 2, 3 के प्रयोग से 4 अंकों की बनाई जा सकने वाली सभी संख्याओं का योग है :

Ans. Official Answer NTA (26664)

Sol. Sum of all 4 digit numbers

$= \text{sum of digit at unit place} \times 10^0$

$+ \text{sum of digit at term place} \times 10^1$

$+ \text{sum of digit at hundred place} \times 10^2$

$+ \text{sum of digit at thousand place} \times 10^3$

$= \left(1\times\frac{3!}{2!} + 2\times 3! + 3\times\frac{3!}{2!}\right)\left(10^0 + 10^1 + 10^2 + 10^3\right)$

$= 24 \times (1111)$

$= 26664$

Question ID : 7155054152

29. If the domain of the function $f(x) = \sec^{-1}\left(\frac{2x}{5x+3}\right)$ is $[\alpha, \beta) U (\gamma, \delta]$, then $|3\alpha + 10(\beta+\gamma) + 21\delta|$ is equal to ________.

यदि फलन $f(x) = \sec^{-1}\left(\frac{2x}{5x+3}\right)$ का प्रॉंत $[\alpha, \beta) U (\gamma, \delta]$ है, तो $|3\alpha + 10(\beta+\gamma) + 21\delta|$ बराबर है :

Ans. Official Answer NTA (24.00)

Sol. $\frac{2x}{5x+3} \geq 1$ OR ≤ -1

$\frac{-3x-3}{5x+3} \geq 0$ $\qquad$ $\frac{7x+3}{5x+3} \leq 0$

$\frac{x+1}{5x+3} \le 0$ $\quad\quad x \in \left(\frac{-3}{5}, \frac{-3}{7}\right]$

$x \in \left[-1, -\frac{3}{5}\right)$

$3\alpha = -3, 10(\beta + \gamma) = -12, 21\delta = -9$

Question ID : 7155054156

30. Let the quadratic curve passing through the point (–1, 0) and touching the line y = x at (1, 1) be y = f(x). Then the x-intercept of the normal to the curve at the point (α, α + 1) in the first quadrant is ________.

माना बिंदु (–1, 0) से होकर जाने वाला तथा रेखा y = x को (1, 1) पर स्पर्श करने वाला द्विघातीय वक्र y = f(x) है। तो प्रथम चतुर्थांश में बिंदु (α, α + 1) पर वक्र के अभिलंब का x-अंतःखंड है :

Ans. Official Answer NTA (11)

Sol. $f(x) = (x+1)(ax+b)$

$1 = 2a + 2b$ $\quad\quad$ ....(1)

$f'(x) = (ax+b) + a(x+1)$

$1 = (3a+b)$ $\quad\quad$ ....(2)

$\Rightarrow b = 1/4, a = 1/4$

$f(x) = \frac{(x+1)^2}{4}$

$f'(x) = \frac{x}{2} + \frac{1}{2}$ $\quad \alpha + 1 = \frac{(\alpha+1)^2}{4}, \alpha > -1$

$\alpha + 1 = 4$

$\alpha = 3$

normal at (3, 4)

$y - 4 = -\frac{1}{2}(x-3)$

$y = 0$ $\quad\quad$ $x = 8 + 3$