MATHS

10 APRIL 2019 [Phase : I]

JEE MAIN PAPER ONLINE

Trig Equation

1. All the pairs (x, y) that satisfy the inequality $2^{\sqrt{\sin^2 x-2\sin x+5}}\cdot\frac{1}{4^{\sin^2 y}}\le 1$ also satisfy the equation :

वह सभी युग्म (x, y) जो असमिका $2^{\sqrt{\sin^2 x-2\sin x+5}}\cdot\frac{1}{4^{\sin^2 y}}\le 1$ को संतुष्ट करते हैं, निम्न में से किस समीकरण को भी संतुष्ट करते हैं?

(1) $\sin x=|\sin y|$ (2) $\sin x=2\sin y$ (3) $2\sin x=\sin y$ (4) $2|\sin x|=3\sin y$

A. 1

sol. $2^{\sqrt{\sin^2 x-2\sin x+5}}\le 2^{2\sin^2 y}$

$\Rightarrow \sqrt{\sin^2 x-2\sin x+5}\le 2\sin^2 y$

$\Rightarrow \sqrt{(\sin x-1)^2+4}\le 2\sin^2 y$

LHS ≥ 2 and RHS ≤ 2

So, this relation will hold only when LHS = 2 = RHS

it is true when $\sin x = 1$

$|\sin y| = 1$

so $\sin x = |\sin y|$

Complex Number

2. If a > 0 and $z=\frac{(1+i)^2}{a-i}$, has magnitude $\sqrt{\frac{2}{5}}$, then $\overline{z}$ is equal to :

यदि a > 0 तथा $z=\frac{(1+i)^2}{a-i}$ का परिमाण (magnitude) $\sqrt{\frac{2}{5}}$ है, तो $\overline{z}$ बराबर है :

(1) $-\frac{1}{5}+\frac{3}{5}i$ (2) $-\frac{3}{5}-\frac{1}{5}i$ (3) $\frac{1}{5}-\frac{3}{5}i$ (4) $-\frac{1}{5}-\frac{3}{5}i$

A. 4

sol. $z=\frac{(1+i)^2}{a-i}\times\frac{a+i}{a+i}$

$z=\frac{(1-1+2i)(a+i)}{a^2+i}=\frac{2ai-2}{a^2+1}$ ......(i)

$|z|=\sqrt{\left(\frac{-2}{a^2+1}\right)^2+\left(\frac{2a}{a^2+1}\right)^2}=\sqrt{\frac{4+4a^2}{(a^2+1)^2}}$

$= \sqrt{\frac{4(1+a^2)}{(1+a^2)^2}} = \frac{2}{\sqrt{1+a^2}}$

given $|z| = \sqrt{\frac{2}{5}}$

so $\sqrt{\frac{2}{5}} = \frac{2}{\sqrt{1+a^2}}$ from equation (i)

(square both side)

$\Rightarrow \frac{2}{5} = \frac{4}{1+a^2}$

$\Rightarrow 1 + a^2 = 10$

$a^2 = 9$

$\Rightarrow a \pm 3 \quad \because (a > 0) \therefore a = 3$

Hence $z = \frac{1+i^2+2i}{3-i} = \frac{2i}{3-i} = \frac{2i(3+i)}{10} = \frac{-1+3i}{5}$

$\bar{z} = \frac{-1}{5} - \frac{3}{5}i$

P & C

3. The number of 6 digit numbers that can be formed using the digits 0, 1, 2, 5, 7 and 9 which are divisible by 11 and no digit is repeated, is :

अंकों (digits) 0, 1, 2, 5, 7 तथा 9 के प्रयोग से छः अंकों वाली ऐसी संख्याओं, जो 11 से भाज्य हों तथा जिनमें कोई भी अंक दोबारा न आए, की संख्या है :

(1) 72 (2) 48 (3) 60 (4) 36

A. 3

sol.

a_1	a_2	a_3	a_4	a_5	a_6

digit 0, 1, 2, 5, 7, 9

$(a_1 + a_3 + a_5) - (a_2 + a_4 + a_6) = 11\,K$

so (1, 2, 9) (0, 5, 7)

Now number of ways to arranging them

$= 3! \times 3! + 3! \times 2 \times 2$

$= 6 \times 6 + 6 \times 4$

$= 6 \times 10$

$= 60$

Definite Integration

4. The value of $\int_0^{2\pi} [\sin 2x(1 + \cos 3x)]\, dx$, where [t] denotes the greatest integer function, is :

$\int_0^{2\pi} [\sin 2x(1 + \cos 3x)]\, dx$ का मान, जहाँ [t] महत्तम पूर्णांक फलन है, है:

(1) π (2) $-\pi$ (3) -2π (4) 2π

A. 2

sol. $I = \int_0^{2\pi} \left[\sin 2x\left(1+\cos 3x\right)\right] dx$ ....(i)

$\because \int_0^a f(x) = \int_0^a f(a-x)\, dx$

$\therefore\ I = \int_0^{2\pi} \left[-\sin 2x(1+\cos 3x)\right] dx$ ....(ii)

By (i) + (ii)

$2I = \int_0^{2\pi} (-1)\, dx$

$2I = -(x)_0^{2\pi}$

$\Rightarrow\ I = -\pi$

Determinant

5. If $\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}$, $x \neq 0$; then for all $\theta \in \left(0, \frac{\pi}{2}\right)$:

यदि $\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ तथा $\Delta_2 = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}$, $x \neq 0$; तो सभी $\theta \in \left(0, \frac{\pi}{2}\right)$ के लिए :

(1) $\Delta_1 + \Delta_2 = -2x^3$ (2) $\Delta_1 - \Delta_2 = -2x^3$

(3) $\Delta_1 + \Delta_2 = -2(x^3 + x - 1)$ (4) $\Delta_1 - \Delta_2 = x(\cos 2\theta - \cos 4\theta)$

A. 1

sol. $\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$

$= x(-x^2 - 1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$

$= -x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \cos\theta\sin\theta + x\cos^2\theta$

$= -x^3 - x + x = -x^3$

Similarly $\Delta_2 = -x^3$

$\Delta_1 + \Delta_2 = -2x^3$

Continuity & Diff

6. If $f(x) = \begin{cases} \dfrac{\sin(p+1)x + \sin x}{x} & , \quad x < 0 \\ q & , \quad x = 0 \\ \dfrac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}} & , \quad x > 0 \end{cases}$

is continuous at x = 0, then the ordered pair (p, q) is equal to :

यदि $f(x) = \begin{cases} \dfrac{\sin(p+1)x + \sin x}{x} & , \quad x < 0 \\ q & , \quad x = 0 \\ \dfrac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}} & , \quad x > 0 \end{cases}$

x = 0 पर संतत है, तो क्रमित युग्म (p, q) बराबर है :

(1) $\left(\frac{5}{2}, \frac{1}{2}\right)$ (2) $\left(-\frac{3}{2}, \frac{1}{2}\right)$ (3) $\left(-\frac{3}{2}, -\frac{1}{2}\right)$ (4) $\left(-\frac{1}{2}, \frac{3}{2}\right)$

A. 2

sol. $f(x) = \begin{cases} \dfrac{\sin(p+1)x + \sin x}{x} & x < 0 \\ q & x = 0 \\ \dfrac{\sqrt{x^2 + x} - \sqrt{x}}{x^{\frac{3}{2}}} & x > 0 \end{cases}$

is continuous at x = 0

So $f(0^-) = f(0) = f(0^+)$ ...(1)

$f(0^-) = \lim\limits_{h \to 0} f(0 - h)$

$$= \lim_{h\to 0} \frac{\sin(p+1)(-h)+\sin(-h)}{-h}$$

$$= \lim_{h\to 0} \left[\frac{-\sin(p+1)h}{-h} + \frac{\sinh}{h}\right]$$

$$= \lim_{h\to 0} \frac{\sin(p+1)h}{h(p+1)} \times (p+1) + \lim_{h\to 0} \frac{\sinh}{h}$$

$$= (p+1) + 1 = p + 2 \qquad ...(2)$$

Now $f(0^+) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} \frac{\sqrt{h^2+h}-\sqrt{h}}{h^{3/2}}$

$$= \lim_{h\to 0} \frac{(h)^{\frac{1}{2}}\left[\sqrt{h+1}-1\right]}{h\left(h^{\frac{1}{2}}\right)}$$

$$= \lim_{h\to 0} \frac{\sqrt{h+1}-1}{h} \times \frac{\sqrt{h+1}+1}{\sqrt{h+1}+1}$$

$$= \lim_{h\to 0} \frac{h+1-1}{h\left(\sqrt{h+1}+1\right)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{h+1}+1} = \frac{1}{1+1} = \frac{1}{2} \qquad ...(3)$$

Now, from equation (1)

$f(0^-) = f(0) = f(0^+)$

$p + 2 = q = \frac{1}{2}$

So, $q = \frac{1}{2}$ and $p = \frac{1}{2} - 2 = \frac{-3}{2}$

$(p, q) \equiv \left(-\frac{3}{2}, \frac{1}{2}\right)$

3 D

7. If the length of the perpendicular from the point $(\beta, 0, \beta)$ $(\beta \neq 0)$ to the line, $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then β is equal to :

यदि बिन्दु $(\beta, 0, \beta)$ $(\beta \neq 0)$ से रेखा $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1}$ पर खींचे गए लम्ब की लम्बाई $\sqrt{\frac{3}{2}}$ है, तो β बराबर है :

(1) –1 (2) –2 (3) 1 (4) 2

A. 1

sol. $\frac{x}{1}=\frac{y-1}{0}=\frac{z+1}{-1}=p$ P(β, 0, β)

any point on line A = (p, 1, – p – 1)

Now, DR of AP ≡ $< p-\beta,\ 1-0,\ -p-1-\beta >$

Which is perpendicular to line so

$(p-\beta).\ 1+0.1-1(-p-1-\beta)=0$

$\Rightarrow\ p-\beta+p+1+\beta=0$

$p=\frac{-1}{2}$

Point $A\left(\frac{-1}{2},1-\frac{1}{2}\right)$

Now, distance $AP=\sqrt{\frac{3}{2}}$

$\Rightarrow\ AP^2=\frac{3}{2}$

$\Rightarrow\ \left(\beta+\frac{1}{2}\right)^2+1+\left(\beta+\frac{1}{2}\right)^2=\frac{3}{2}$

$2\left(\beta+\frac{1}{2}\right)^2=\frac{1}{2}$

$\Rightarrow\ \left(\beta+\frac{1}{2}\right)^2=\frac{1}{4}$

$\Rightarrow\ \beta=0,\ \ -1,\ (\beta\neq 0)$

$\therefore\ \boxed{\beta=-1}$

Definite Integration

8. $\lim\limits_{n\to\infty}\left(\frac{(n+1)^{1/3}}{n^{4/3}}+\frac{(n+2)^{1/3}}{n^{4/3}}+....+\frac{(2n)^{1/3}}{n^{4/3}}\right)$ is equal to :

$\lim\limits_{n\to\infty}\left(\frac{(n+1)^{1/3}}{n^{4/3}}+\frac{(n+2)^{1/3}}{n^{4/3}}+....+\frac{(2n)^{1/3}}{n^{4/3}}\right)$ बराबर है :

(1) $\frac{4}{2}(2)^{4/3}$ (2) $\frac{3}{4}(2)^{4/3}-\frac{3}{4}$ (3) $\frac{4}{3}(2)^{3/4}$ (4) $\frac{3}{4}(2)^{4/3}-\frac{4}{3}$

A. 2

sol. $\lim_{n\to\infty}\dfrac{(n+1)^{\frac{1}{3}}+(n+2)^{\frac{1}{3}}+......+(n+n)^{\frac{1}{3}}}{n(n)^{\frac{1}{3}}}$

$\lim_{n\to\infty}\sum_{r=1}^{n}\dfrac{(n+r)^{\frac{1}{3}}}{n.n^{\frac{1}{3}}}$ $\quad\quad \dfrac{r}{n}\to x$ and $\dfrac{1}{n}\to dx$

$\int_0^1 (1+x)^{\frac{1}{3}}\,dx$

$=\left[\dfrac{3}{4}(1+x)^{\frac{4}{3}}\right]_0^1=\dfrac{3}{4}(2)^{\frac{4}{3}}-\dfrac{3}{4}$

Circle

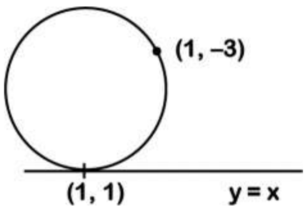
9. The line x = y touches a circle at the point (1, 1). If the circle also passes through the point (1, –3), then its radius is :

रेखा x = y एक वृत्त को बिन्दु (1, 1) पर स्पर्श करती है। यदि यह वृत्त बिन्दु (1, –3) से भी होकर जाता है, तो इसकी त्रिज्या है :

(1) $3\sqrt{2}$ (2) 2 (3) $2\sqrt{2}$ (4) 3

A. 3

sol. Equation of circle = $(x-1)^2+(y-1)^2+\lambda(y-x)=0$
Which passes through (1, –3)



So, $0+16+\lambda(-3-1)=0$
$16+\lambda(-4)=0$
$\boxed{\lambda=4}$
Now equation of circle
$(x-1)^2+(y-1)^2+4y-4x=0$
$\Rightarrow x^2+y^2-6x+2y+2=0$
radius $=\sqrt{9+1-2}=2\sqrt{2}$

Mathematical Reasoning

10. Which one of the following Boolean expressions is a tautology ?

बूले के निम्न व्यंजकों में से कौनसा एक, एक पुनरूक्ति है?

(1) $(p \vee q) \vee (p \vee \sim q)$ (2) $(p \wedge q) \vee (p \wedge \sim q)$

(3) $(p \vee q) \wedge (p \vee \sim q)$ (4) $(p \vee q) \wedge (\sim p \vee \sim q)$

A. 1

sol. $(p \vee q) \vee (p \vee \sim q)$

$= p \vee (q \vee p) \vee \sim q$

$= (p \vee p) \vee (q \vee \sim q)$

$= p \vee T$

$= T$ so first statement is tautology

Limit

11. If $\lim\limits_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim\limits_{x \to k} \frac{x^3 - k^3}{x^2 - k^2}$, then k is :

यदि $\lim\limits_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim\limits_{x \to k} \frac{x^3 - k^3}{x^2 - k^2}$ है, तो k बराबर है :

(1) $\frac{4}{3}$ (2) $\frac{3}{2}$ (3) $\frac{8}{3}$ (4) $\frac{3}{8}$

A. 3

sol. If $\lim\limits_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim\limits_{x \to K} \left(\frac{x^3 - k^3}{x^2 - k^2}\right)$

L.H.S.

$\underset{x \to 1}{\text{Lt}} \frac{x^4 - 1}{x - 1} = \left(\frac{0}{0}\text{form}\right)$

$\underset{x \to 1}{\text{Lt}} \frac{4x^3}{1} = 4$

Now, $\lim\limits_{x \to K} \frac{x^3 - k^3}{x^2 - k^2} = 4$

$\Rightarrow \lim\limits_{x \to K} \frac{3x^2}{2x} = 4$

$\Rightarrow \frac{3}{2}k = 4$

$k = \frac{8}{3}$

Hyperbola

12. If a directrix of a hyperbola centred at the origin and passing through the point $\left(4,-2\sqrt{3}\right)$ is $5x=4\sqrt{5}$ and its eccentricity is e, then :

एक अतिपरवलय का केन्द्र मूलबिन्दु पर है तथा यह बिन्दु $\left(4,-2\sqrt{3}\right)$ से होकर जाता है। यदि इसकी नियता (directrix) $5x=4\sqrt{5}$ है तथा इसकी उत्केन्द्रता e है, तो :

(1) $4e^4+8e^2-35=0$

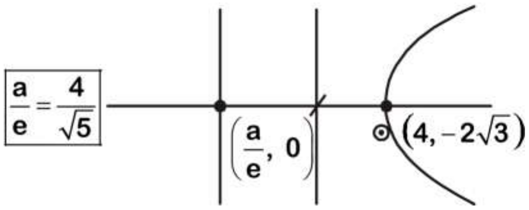
(2) $4e^4-24e^2+35=0$

(3) $4e^4-12e^2-27=0$

(4) $4e^4-24e^2+27=0$

A. 2

sol. $x=\dfrac{4}{\sqrt{5}}$

$\because$ 

Equation of hyperbola

$\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ it passes through $\left(4,-2\sqrt{3}\right)$

$\because\ e^2=1+\dfrac{b^2}{a^2}$

$\Rightarrow \boxed{a^2e^2-a^2=b^2}$

$\Rightarrow \dfrac{16}{a^2}-\dfrac{12}{a^2e^2-a^2}=1$

$\Rightarrow \dfrac{4}{a^2}\left[\dfrac{4}{1}-\dfrac{3}{e^2-1}\right]=1$

$\Rightarrow 4e^2-4-3=\left(e^2-1\right)\left(\dfrac{a^2}{4}\right)$

$\Rightarrow 4(4e^3-7)=(e^2-1)\left(\dfrac{4e}{\sqrt{5}}\right)^2$

$\Rightarrow 4e^4-24e^2+35=0$

Ellipse

13. If the line $x - 2y = 12$ is tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $\left(3,\frac{-9}{2}\right)$, then the length of the latus rectum of the ellipse is :

यदि रेखा, $x - 2y = 12$ दीर्घवृत्त, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ को बिन्दु $\left(3,\frac{-9}{2}\right)$ पर स्पर्श करती है, तो इसके नाभिलम्ब की लम्बाई है :

(1) 5 (2) $8\sqrt{3}$ (3) $12\sqrt{2}$ (4) 9

A. 4

sol. Equation of tangent at $\left(3,-\frac{9}{2}\right)$ to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

$\frac{3x}{a^2}-\frac{y9}{2b^2}=1$ which is equivalent to $x - 2y = 12$

$\frac{3}{a^2}=\frac{-9}{2b^2.(-2)}=\frac{1}{12}$ (On comparing)

$a^2 = 3 \times 12$ and $b^2 = \frac{9\times12}{4}$

$\boxed{a=6} \Rightarrow\ b=3\sqrt{3}$

So latus rectum $=\frac{2b^2}{a}=\frac{2\times27}{6}=9$

Vector

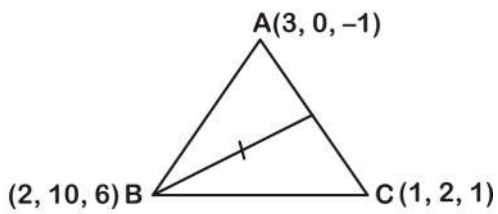
14. Let A(3, 0, –1), B(2, 10, 6) and C(1, 2, 1) be the vertices of a triangle and M be the mid point of AC. If G divides BM in the ratio, 2 : 1 then cos(∠GOA) (O being the origin) is equal to :

माना एक त्रिभुज के शीर्ष बिन्दु A(3, 0, –1), B(2, 10, 6) तथा C(1, 2, 1) हैं तथा AC का मध्यबिन्दु M है। यदि G, BM को 2 : 1 के अनुपात में विभाजित करता है, तो cos(∠GOA) (O मूलबिन्दु है) बराबर है :

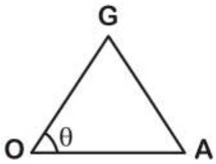
(1) $\frac{1}{6\sqrt{10}}$ (2) $\frac{1}{\sqrt{30}}$ (3) $\frac{1}{2\sqrt{15}}$ (4) $\frac{1}{\sqrt{15}}$

A. 4

sol. G is the centroid of ΔABC



G (2, 4, 2)



$OG = \sqrt{4+16+4}, \quad OA = \sqrt{9+1}$

$AG = \sqrt{1+16+9}$

$$\cos\theta = \frac{24+10-26}{2\sqrt{24}\sqrt{10}}$$

$$= \frac{8}{2\sqrt{8\times3\times2\times5}}$$

$$= \frac{4}{4\sqrt{15}} = \frac{1}{\sqrt{15}}$$

Determinant

15. If the system of linear equations

$x + y + z = 5$

$x + 2y + 2z = 6$

$x + 3y + \lambda z = \mu, (\lambda, \mu \in R)$, has infinitely many solutions, then the value of $\lambda + \mu$ is :

यदि रैखिक समीकरण निकाय

$x + y + z = 5$

$x + 2y + 2z = 6$

$x + 3y + \lambda z = \mu, (\lambda, \mu \in R)$ के अनन्त हल है, तो $\lambda + \mu$ का मान है :

(1) 10 (2) 12 (3) 7 (4) 9

A. 1

sol. $x + y + z = 5$

$x + 2y + 2z = 6$

$x + 3y + \lambda z = \mu$ have infinite solution

$\Delta = 0, \Delta_x = \Delta_y = \Delta_z = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & \lambda \end{vmatrix} = 0$$

$\Rightarrow \quad 1(2\lambda - 6) - 1(\lambda - 2) + 1\,(3 - 2) = 0$

$\Rightarrow \quad 2\lambda - 6 - \lambda + 2 + 1 = 0$

$\boxed{\lambda = 3}$

Now, $\Delta x = \begin{vmatrix} 5 & 1 & 1 \\ 6 & 2 & 2 \\ \mu & 3 & 3 \end{vmatrix} = 0,\ \Delta y = \begin{vmatrix} 1 & 5 & 1 \\ 1 & 6 & 2 \\ 1 & \mu & 3 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 1 & 5 & 1 \\ 0 & 1 & 1 \\ 0 & \mu-5 & 2 \end{vmatrix} = 0$

$\Rightarrow\ 1(2 - \mu + 5) = 0$

$\boxed{\mu = 7}$

$\Delta z = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 6 \\ 1 & 3 & \mu \end{vmatrix} = \begin{vmatrix} 1 & 1 & 5 \\ 0 & -1 & -1 \\ 0 & 2 & \mu-5 \end{vmatrix}$

$\Rightarrow\ 1(5 - \mu + 2) = 0$

$\Rightarrow\ \mu = 7$

So, $\lambda + \mu = 10$

Probability

16. Assume that each born child is equally likely to be a boy or a girl. If two families have two children each, then the conditional probability that all children are girls given that at least two are girls is :

माना प्रत्येक जन्म लेने वाले बच्चे का लड़का अथवा लड़की होना समसंभाव्य है। माना दो परिवारों में प्रत्येक में दो बच्चे हैं। यदि यह दिया गया है कि कम से कम दो बच्चे लड़कियाँ है, तो सभी बच्चों के लड़की होने की सप्रतिबंध प्रायिकता है :

(1) $\frac{1}{11}$ (2) $\frac{1}{12}$ (3) $\frac{1}{10}$ (4) $\frac{1}{17}$

A. 1

sol. A = At least two girls

B = All girls

$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$

$= \frac{P(B)}{P(A)} = \frac{\left(\frac{1}{4}\right)^4}{1 - {}^4C_0\left(\frac{1}{2}\right)^4 - {}^4C_1\left(\frac{1}{2}\right)^4}$

$= \frac{1}{16 - 1 - 4} = \frac{1}{11}$

Statistics

17. If for some $x \in R$, the frequency distribution of the marks obtained by 20 students in a test is :

यदि किसी $x \in R$ के लिए, 20 विद्यार्थियों द्वारा एक परीक्षा में प्राप्त अंकों का बारंबारता बंटन है,

Marks	2	3	5	7
Frequency	$(x+1)^2$	$2x-5$	x^2-3x	x

Then the mean of the marks is :

तो अंकों का माध्य है :

(1) 3.2 (2) 3.0 (3) 2.5 (4) 2.8

A. 4

sol. Number of students

$\Rightarrow (x+1)^2 + (2x-5) + (x^2-3x) + x = 20$

$\Rightarrow 2x^2 + 2x - 4 = 20$

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$x = 3$

So,

Marks	2	3	5	7
No. of students	16	1	0	3

Average marks $= \dfrac{32+3+21}{20} = \dfrac{56}{20} = 2.8$

Circle

18. If the circles $x^2 + y^2 + 5Kx + 2y + K = 0$ and $2(x^2 + y^2) + 2Kx + 3y - 1 = 0$, $(K \in R)$, intersect at the points P and Q, then the line $4x + 5y - K = 0$ passes through P and Q, for :

(1) Exactly one value of K (2) Infinitely many values of K

(3) Exactly two values of K (4) No value of K

यदि वृत्तों $x^2 + y^2 + 5Kx + 2y + K = 0$ तथा $2(x^2 + y^2) + 2Kx + 3y - 1 = 0$, $(K \in R)$, के प्रतिच्छेदन बिन्दु P तथा Q हैं, तो रेखा $4x + 5y - K = 0$ के बिन्दुओं P तथा Q से होकर जाने के लिए :

(1) K का मात्र एक मान है। (2) K के अनन्त मान हैं।

(3) K के मात्र दो मान है। (4) K का कोई भी मान नहीं है।

A. 4

sol. $S_1 \equiv x^2 + y^2 + 5Kx + 2y + K = 0$

$S_2 \equiv x^2 + y^2 + Kx + \dfrac{3}{2}y - \dfrac{1}{2} = 0$

Equation of common chord is

$S_1 - S_2 = 0$

$\Rightarrow\ 4Kx + \frac{y}{2} + K + \frac{1}{2} = 0$ ...(1)

$4x + 5y - K = 0$ ...(2) (given)

On comparing (1) and (2)

$$\frac{4K}{4} = \frac{1}{10} = \frac{2K+1}{-2K}$$

$\Rightarrow\ \boxed{K = \frac{1}{10}}$ and $-2K = 20K + 10$

$\Rightarrow\ 22K = -10$

$\boxed{K = \frac{-5}{11}}$

$\therefore$ No value of K exists

Area Under Curve

19. The region represented by $|x - y| \le 2$ and $|x + y| \le 2$ is bounded by a :

(1) Square of side length $2\sqrt{2}$ units (2) Square of area 16 sq. units

(3) Rhombus of side length 2 units (4) Rhombus of area $8\sqrt{2}$ sq. units

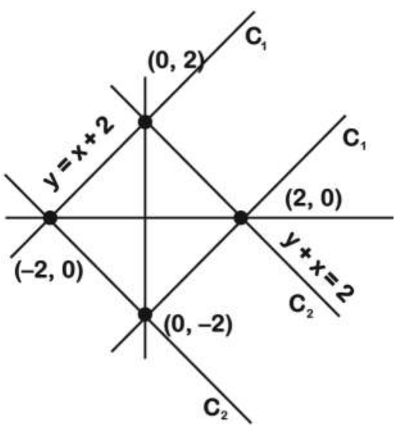
$|x - y| \le 2$ तथा $|x + y| \le 2$ द्वारा प्रदर्शित क्षेत्र जिसके द्वारा प्रतिबद्ध (bounded) है, वह है :

(1) एक वर्ग जिसकी भुजा की लम्बाई $2\sqrt{2}$ इकाई है। (2) एक वर्ग जिसका क्षेत्रफल 16 वर्ग इकाई है।

(3) एक समचतुर्भुज जिसकी भुजा की लम्बाई 2 इकाई है। (4) एक समचतुर्भुज जिसका क्षेत्रफल $8\sqrt{2}$ वर्ग इकाई है।

A. 1

sol. $C_1 : |y - x| \le 2$
$C_2 : |y + x| \le 2$
Now region is square



Length of side = $\sqrt{2^2 + 2^2} = 2\sqrt{2}$

Sequence & Progression

20. If $a_1, a_2, a_3, ..., a_n$ are in A.P. and $a_1 + a_4 + a_7 + ... + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to :

यदि $a_1, a_2, a_3, ..., a_n$ एक समान्तर श्रेढ़ी में हैं तथा $a_1 + a_4 + a_7 + ... + a_{16} = 114$ है, तो $a_1 + a_6 + a_{11} + a_{16}$ बराबर है :

(1) 98 (2) 38 (3) 64 (4) 76

A. 4

sol. $3(a_1 + a_{16}) = 114$

$\boxed{a_1 + a_{16} = 38}$

Now $a_1 + a_6 + a_{11} + a_{16} = 2(a_1 + a_{16}) = 2 \times 38 = 76$

3 D

21. If Q(0, –1, –3) is the image of the point P in the plane $3x - y + 4z = 2$ and R is the point (3, –1, –2), then the area (in sq. units) of ΔPQR is :

यदि बिन्दु P का समतल $3x - y + 4z = 2$ में प्रतिबिम्ब Q(0, –1, –3) है तथा R (3, –1, –2) एक अन्य बिन्दु है, तो ΔPQR का क्षेत्रफल (वर्ग इकाइयों में) है :

(1) $\frac{\sqrt{65}}{2}$ (2) $\frac{\sqrt{91}}{4}$ (3) $2\sqrt{13}$ (4) $\frac{\sqrt{91}}{2}$

A. 4

sol. Image of Q in plane

$$\frac{(x-0)}{3} = \frac{(y+1)}{-1} = \frac{z+3}{+4} = \frac{-2(1-12-2)}{9+1+16} = 1$$

x = 3, y = –2, z = 1

P(3, –2, 1), Q(0, –1, –3), R(3, –1, –2)

Now area of ΔPQR is

$$\frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{QR}| = \frac{1}{2}\left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 4 \\ 3 & 0 & 1 \end{matrix}\right\|$$

$$= \frac{1}{2}\left|\left\{\hat{i}(-1) - \hat{j}(3-12) + \hat{k}(3)\right\}\right|$$

$$= \frac{1}{2}\sqrt{(1+81+9)}$$

$$= \frac{\sqrt{91}}{2}$$

Function

22. Let $f(x) = x^2, x \in R$. For any $A \subseteq R$, define $g(A) = \{x \in R : f(x) \in A\}$. If S = [0, 4], then which one of the following statements is not true ?

माना $f(x) = x^2, x \in R$ । किसी भी $A \subseteq R$, के लिए $g(A) = \{x \in R : f(x) \in A\}$ है। यदि $S = [0, 4]$ है, तो निम्न में से कौनसा एक कथन सही नहीं है?

(1) $f(g(S)) = S$ (2) $g(f(S)) = g(S)$ (3) $g(f(S)) \neq S$ (4) $f(g(S)) \neq f(S)$

A. 2

sol. $f(x) = x^2, x \in R$

$g(A) = \{x \in R : f(x) \in A\}$ $S \equiv [0, 4]$

$g(S) = \{x \in R : f(x) \in S\}$

$= \{x \in R : 0 \leq x^2 \leq 4\}$

$= \{x \in R : -2 \leq x \leq 2\}$

$\therefore$ $g(S) \neq S$

$\therefore$ $f(g(S)) \neq f(S)$

$g(f(S)) = \{x \in R : f(x) \in f(S)\}$

$= \{x \in R : x^2 \in S^2\}$

$= \{x \in R : 0 \leq x^2 \leq 16\}$

$= \{x \in R : -4 \leq x \leq 4\}$

$\therefore$ $g(f(S)) \neq g(S)$

$\therefore$ $g(f(S)) = g(S)$ is incorrect

Binomial Theorem

23. If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $(1 + ax + bx^2) (1 - 3x)^{15}$ in powers of x, then the ordered pair (a, b) is equal to :

यदि x की घातों (powers) में, व्यंजक $(1 + ax + bx^2) (1 - 3x)^{15}$ के प्रसार में x^2 तथा x^3 दोनों के गुणांक शून्य के बराबर हैं, तो क्रमित युग्म (a, b) बराबर है :

(1) (– 54, 315) (2) (28, 861) (3) (–21, 714) (4) 28, 315

A. 4

sol. $(1 + ax + bx^2)(1 - 3x)^{15}$

Co-eff. of $x^2 = 1.{}^{15}C_2(-3)^2 + a.{}^{15}C_1(-3) + b.{}^{15}C_0$

$= \frac{15 \times 14}{2} \times 9 - 15 \times 3a + b = 0$ (Given)

$\Rightarrow$ $945 - 45a + b = 0$ ...(i)

Now co-eff. of $x^3 = 0$

$\Rightarrow$ ${}^{15}C_3(-3)^3 + a.{}^{15}C_2(-3)^2 + b.{}^{15}C_1(-3) = 0$

$\Rightarrow \quad \frac{15\times14\times13}{3\times2}\times(-3\times3\times3)+a\times\frac{15\times14\times9}{2}-b\times3\times15=0$

$\Rightarrow \quad 15\times3[-3\times7\times13+a\times7\times3-b]=0$

$\Rightarrow \quad 21a-b=273 \qquad ...(ii)$

From (i) and (ii)

$a=+28,\ b=315 \equiv (a,b)\equiv(28,315)$

Sequence & Progression

24. The sum $\frac{3\times1^3}{1^2}+\frac{5\times(1^3+2^3)}{1^2+2^2}+\frac{7\times(1^3+2^3+3^3)}{1^2+2^2+3^2}+....$ upto 10^{th} term, is :

$\frac{3\times1^3}{1^2}+\frac{5\times(1^3+2^3)}{1^2+2^2}+\frac{7\times(1^3+2^3+3^3)}{1^2+2^2+3^2}+....$ के प्रथम दस पदों का योगफल है :

(1) 620 (2) 600 (3) 680 (4) 660

A. 4

sol. $T_r=\frac{(2r+1)(1^3+2^3+3^3+......+r^3)}{1^2+2^2+3^2+.....+r^2}$

$T_r=(2r+1)\left(\frac{r(r+1)}{2}\right)^2\times\frac{6}{r(r+1)(2r+1)}$

$T_r=\frac{3r(r+1)}{2}$

Now,

$S=\sum_{r=1}^{10}T_r=\frac{3}{2}\sum_{r=1}^{10}(r^2+r)$

$=\frac{3}{2}\left\{\frac{10\times(10+1)(2\times10+1)}{6}+\frac{10\times11}{2}\right\}$

$=\frac{3}{2}\left\{\frac{10\times11\times21}{6}+5\times11\right\}$

$=\frac{3}{2}\times5\times11\times8=660$

Monotonocity

25. Let $f(x)=e^x-x$ and $g(x)=x^2-x$, $\forall x\in R$. Then the set of all $x\in R$, where the function $h(x)=(fog)(x)$ is increasing, is :

माना $f(x)=e^x-x$ तथा $g(x)=x^2-x$, $\forall x\in R$, तो सभी $x\in R$, जिनके लिए फलन $h(x)=(fog)(x)$ वर्धमान है, का समुच्चय

है :

(1) $\left[-1,\frac{-1}{2}\right]\cup\left[\frac{1}{2},\infty\right)$ (2) $[0,\infty)$

(3) $\left[0,\frac{1}{2}\right]\cup[1,\infty)$ (4) $\left[\frac{-1}{2},0\right]\cup[1,\infty)$

A. 3

sol. $f(x) = e^x - x,\ g(x) = x^2 - x$

$f(g(x)) = e^{(x^2-x)} - (x^2 - x)$

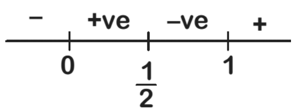
If f(g(x)) is increasing function

$(f(g(x)))' = e^{(x^2-x)} \times (2x-1) - 2x + 1$

$= (2x-1)\,e^{(x^2-x)} + 1 - 2x$

$= (2x-1)[e^{(x^2-x)} - 1]$
A B

A & B are either both positive or negative



for $(f(g(x)))' \geq 0$,

$x \in \left[0,\frac{1}{2}\right]\cup[1,\infty)$

Quadratic Equation

26. If α and β are the roots of the quadratic equation, $x^2 + x\sin\theta - 2\sin\theta = 0,\ \theta \in \left(0,\frac{\pi}{2}\right)$, then

$\frac{\alpha^{12}+\beta^{12}}{(\alpha^{-12}+\beta^{-12})\cdot(\alpha-\beta)^{24}}$ is equal to :

यदि द्विघात समीकरण, $x^2 + x\sin\theta - 2\sin\theta = 0,\ \theta \in \left(0,\frac{\pi}{2}\right)$ के मूल α तथा β हैं, तो

$\frac{\alpha^{12}+\beta^{12}}{(\alpha^{-12}+\beta^{-12})\cdot(\alpha-\beta)^{24}}$ बराबर है :

(1) $\frac{2^{12}}{(\sin\theta-8)^6}$ (2) $\frac{2^{12}}{(\sin\theta-4)^{12}}$ (3) $\frac{2^6}{(\sin\theta+8)^{12}}$ (4) $\frac{2^{12}}{(\sin\theta+8)^{12}}$

A. 4

sol. Given $\alpha + \beta = -\sin\theta$ and $\alpha\beta = -2\sin\theta$

$$\frac{\left(\alpha^{12}+\beta^{12}\right)\alpha^{12}\beta^{12}}{\left(\alpha^{12}+\beta^{12}\right)(\alpha-\beta)^{24}}=\frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}$$

$$|\alpha-\beta|=\sqrt{(\alpha+\beta)^2-4\alpha\beta}=\sqrt{\sin^2\theta+8\sin\theta}$$

Hence required quantity

$$\frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}=\frac{(2\sin\theta)^{12}}{\sin^{12}\theta(\sin\theta+8)^{12}}=\frac{2^{12}}{(\sin\theta+8)^{12}}$$

Differential Equation

27. If $y=y(x)$ is the solution of the differential equation $\frac{dy}{dx}=(\tan x-y)\sec^2 x,\ x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, such that $y(0)=0$, then $y\left(-\frac{\pi}{4}\right)$ is equal to :

यदि $y=y(x)$, अवकल समीकरण $\frac{dy}{dx}=(\tan x-y)\sec^2 x,\ x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ जबकि $y(0)=0$ का हल है, तो $y\left(-\frac{\pi}{4}\right)$ बराबर है:

(1) $\frac{1}{e}-2$ (2) $\frac{1}{2}-e$ (3) $e-2$ (4) $2+\frac{1}{e}$

A. 3

sol. $\frac{dy}{dx}+y\sec^2 x=\sec^2 x\tan x \rightarrow$ This is linear differential equation

$IF=e^{\int\sec^2 x dx}=e^{\tan x}$

Now solution is

$y.e^{\tan x}=\int e^{\tan x}\sec^2 x\tan x dx$

$\because$ Let $\tan x=t$

$\sec^2 x dx=dt$

$ye^{\tan x}=\int \underset{II}{e^t}\,\underset{I}{t}\,dt$

$ye^{\tan x}=te^t-e^t+c$

$ye^{\tan x}=(\tan x-1)e^{\tan x}+c$

$y=(\tan x-1)+c\cdot e^{-\tan x}$

Given $y(0)=0$

$\Rightarrow$ $0=-1+c$

$\Rightarrow$ $c=1$

$$y\left(-\frac{\pi}{4}\right)=-1-1+e=-2+e$$

Continuity & Diff

28. Let $f : R \to R$ be differentiable at $c \in R$ and $f(c) = 0$. If $g(x) = |f(x)|$, then at $x = c$, g is :

(1) not differentiable if $f'(c) = 0$ (2) differentiable if $f'(c) = 0$

(3) not differentiable (4) differentiable if $f'(c) \neq 0$

माना $f : R \to R$, $c \in R$ पर अवकलनीय है तथा $f(c) = 0$ है। यदि $g(x) = |f(x)|$, तो $x = c$ पर, g :

(1) अवकलनीय नहीं है, यदि $f'(c) = 0$ (2) अवकलनीय है, यदि $f'(c) = 0$

(3) अवकलनीय नहीं है। (4) अवकलनीय है, यदि $f'(c) \neq 0$

A. 2

sol. $\because$ $g'(c) = \lim_{x \to c} \frac{g(x) - g(c)}{x - c}$

$\Rightarrow$ $g'(c) = \lim_{x \to c} \frac{|f(x)| - |f(c)|}{x - c}$

$\because$ $f(c) = 0$

$\Rightarrow$ $g'(c) = \lim_{x \to c} \frac{|f(x)|}{x - c}$

$\Rightarrow$ $g'(c) = \lim_{x \to c} \frac{|f(x)|}{x - c}$ if $f(x) > 0$

and $g'(c) = \lim_{x \to c} \frac{-f(x)}{x - c}$ if $f(x) < 0$

$\Rightarrow$ $g'(c) = f'(c) = -f'(c)$

$\Rightarrow$ $2f'(c) = 0$

$\Rightarrow$ $f'(c) = 0$

Height & Distance

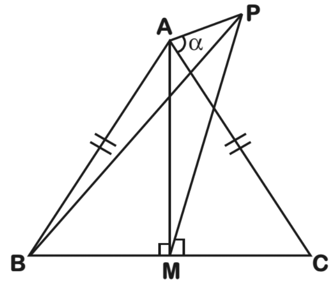
29. ABC is a triangular park with AB = AC = 100 metres. A vertical tower is situated at the mid-point of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}(3\sqrt{2})$ and $\text{cosec}^{-1}(2\sqrt{2})$ respectively, then the height of the tower (in metres) is :

ABC एक त्रिभुजाकार पार्क है जिसमें AB = AC = 100 मीटर है। BC के मध्य बिन्दु पर एक सीधी मीनार खड़ी है। यदि मीनार के शिखर के बिन्दुओं A तथा B पर उन्नयन कोण क्रमशः $\cot^{-1}(3\sqrt{2})$ तथा $\text{cosec}^{-1}(2\sqrt{2})$ हैं, तो मीनार की ऊँचाई (मीटर में) है :

(1) 20 (2) $10\sqrt{5}$ (3) 25 (4) $\frac{100}{3\sqrt{3}}$

A. 1

sol.



ΔAPM

$\frac{h}{AM} = \frac{1}{3\sqrt{2}}$

ΔBPM

$\frac{h}{AM} = \frac{1}{\sqrt{7}}$

ΔABM

$\therefore \quad AM^2 + MB^2 = (100)^2$

$\Rightarrow \quad 18h^2 + 7h^2 = 100 \times 100$

$\Rightarrow \quad h^2 = 4 \times 100$

$\Rightarrow \quad h = 20$

Indefinite Integration

30. If $\int \frac{dx}{(x^2-2x+10)^2} = A\left(\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{f(x)}{x^2-2x+10}\right) + C$

where C is a constant of integration, then :

यदि $\int \frac{dx}{(x^2-2x+10)^2} = A\left(\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{f(x)}{x^2-2x+10}\right) + C$

जहाँ C एक समाकलन अचर है, तो :

(1) $A = \frac{1}{81}$ and $f(x) = 3(x-1)$

(2) $A = \frac{1}{54}$ and $f(x) = 3(x-1)$

(3) $A = \frac{1}{27}$ and $f(x) = 9(x-1)$

(4) $A = \frac{1}{54}$ and $f(x) = 9(x-1)^2$

A. 2

sol. $\int \frac{dx}{(x^2-2x+10)^2} = \int \frac{dx}{((x-1)^2+9)^2}$

Let $(x-1)^2 = 9\tan^2\theta$ ....(i)

$\Rightarrow \qquad \tan\theta = \dfrac{x-1}{3}$

On differentiating ....(i)

$2(x-1)dx = 18\tan\theta \sec^2\theta\, d\theta$

$\therefore \qquad I = \int \dfrac{18\tan\theta\sec^2\theta d\theta}{2\times 3\tan\theta \times 81\sec^2\theta}$

$$I = \frac{1}{27}\int \cos^2\theta d\theta = \frac{1}{27}\times\frac{1}{2}\int (1+\cos 2\theta)\, d\theta$$

$$I = \frac{1}{54}\left\{\theta + \frac{\sin 2\theta}{2}\right\} + c$$

$$I = \frac{1}{54}\left[\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{1}{2}\times\frac{2\left(\frac{x-1}{3}\right)}{1+\left(\frac{x-1}{3}\right)^2}\right] + c$$

$$I = \frac{1}{54}\left[\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{3(x-1)}{x^2-2x+10}\right] + c$$

So $A = \dfrac{1}{54}$

$f(x) = 3(x-1)$