MATHS

10 APRIL 2019 [Phase : II]

JEE MAIN PAPER ONLINE

MOD

1. Let $f(x) = \log_e(\sin x), (0 < x < \pi)$ and $g(x) = \sin^{-1}(e^{-x}), (x \geq 0)$. If α is a positive real number such that $a = (fog)'(\alpha)$ and $b = (fog)(\alpha)$, then :

माना $f(x) = \log_e(\sin x), (0 < x < \pi)$ तथा $g(x) = \sin^{-1}(e^{-x}), (x \geq 0)$ हैं। यदि एक धनात्मक वास्तविक संख्या α के लिए $a = (fog)'(\alpha)$ तथा $b = (fog)(\alpha)$ है, तो :

(1) $a\alpha^2 - b\alpha - a = 1$ (2) $a\alpha^2 + b\alpha + a = 0$

(3) $a\alpha^2 - b\alpha - a = 0$ (4) $a\alpha^2 + b\alpha - a = -2\alpha^2$

A. 1

sol. $f(x) = \ln(\sin x), g(x) = \sin^{-1}(e^{-x})$

$f(g(x)) = \ln(\sin(\sin^{-1} e^{-x}))$

$= -x$

$\Rightarrow -\alpha = b$

$f'(g(\alpha)) = a$

i.e.,$a = -1$

$\therefore a\alpha^2 - b\alpha + 1 = -\alpha^2 + \alpha^2 + 1 = -a$

Solution of Triangle

2. The angles A, B and C of a triangle ABC are in A.P. and $a : b = 1 : \sqrt{3}$. If c = 4 cm, then the area (in sq. cm) of this triangle is :

एक त्रिभुज ABC के कोण A, B तथा C समान्तर श्रेणी में हैं तथा $a : b = 1 : \sqrt{3}$ है। यदि c = 4 cm है, तो इस त्रिभुज का क्षेत्रफल (वर्ग सेमी में) है :

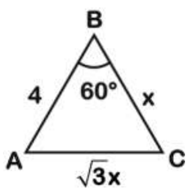
(1) $\frac{2}{\sqrt{3}}$ (2) $4\sqrt{3}$ (3) $2\sqrt{3}$ (4) $\frac{4}{\sqrt{3}}$

A. 3

sol. A, B, C, are in A.P

$\Rightarrow 2B = A + C$

$\Rightarrow B = \frac{\pi}{3}$



Area $= \frac{1}{2}(4x)\sin 60^\circ$

$= \sqrt{3}x$

Now $\cos 60^\circ = \frac{16 + x^2 - 3x^2}{8x}$

$\Rightarrow 4x = 16 - 2x^2$

x = 2 (as –4 is rejected)

Hence, area = $2\sqrt{3}$ sq. cm

Determinant

3. The sum of the real roots of the equation $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$, is equal to :

समीकरण $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$, के वास्तविक मूलों का योगफल है :

(1) 6 (2) 0 (3) –4 (4) 1

A. 2

sol. $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$

$\Rightarrow x(-3x^2 - 6x - 2x^2 + 6x) - 6(-3x + 9 - 2x - 4) - (4x - 9xA) = 0$

$\Rightarrow x(-5x^2) - 6(-5x + 5) - 4x + 9x = 0$

$\Rightarrow x^3 - 7x + 6 = 0$

All the roots are real

$\therefore$ Sum of real roots $= \frac{0}{1} = 0$

Complex Number

4. If z and w are two complex numbers such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$, then :

यदि z तथा w दो ऐसी सम्मिश्र संख्याएँ हैं कि $|zw| = 1$ तथा $\arg(z) - \arg(w) = \frac{\pi}{2}$, तो :

(1) $z\overline{w} = \frac{1-i}{\sqrt{2}}$ (2) $\overline{z}w = i$ (3) $z\overline{w} = \frac{-1+i}{\sqrt{2}}$ (4) $\overline{z}w = -i$

A. 4

sol. $|zw| = 1$ ...(i)

$\arg\left(\frac{z}{w}\right) = \frac{\pi}{2}$ ...(ii)

$\therefore \frac{z}{w} + \frac{\bar{z}}{\bar{w}} = 0 \Rightarrow z\bar{w} = -\bar{z}w$

from (i) $z\bar{z}\ w\bar{w} = 1$

$(\bar{z}w)^2 = -1 \Rightarrow \bar{z}w = \pm i$

from (ii) $-\arg(\bar{z}) - \arg w = \frac{\pi}{2}$

$\Rightarrow \arg(\bar{z}w) = \frac{-\pi}{2}$

Hence, $\bar{z}w = -i$

Circle

5. The locus of the centes of the circles, which touch the circle, $x^2 + y^2 = 1$ externally, also touch the y-axis and lie in the first quadrant, is :

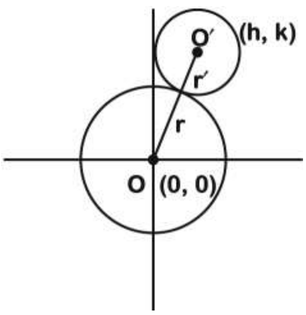
ऐसे वृत्तों, जो वृत्त $x^2 + y^2 = 1$ को बाह्य स्पर्श करते हैं, y-अक्ष को भी स्पर्श करते हैं तथा प्रथम चतुर्थांश में स्थित हैं, के केन्द्रों का बिन्दुपथ है :

(1) $y = \sqrt{1+2x}, x \geq 0$ (2) $x = \sqrt{1+4y}, y \geq 0$ (3) $x = \sqrt{1+2y}, y \geq 0$ (4) $y = \sqrt{1+4x}, x \geq 0$

A. 1

sol. Let centre of required circle is (h, k).

$\therefore$ OO' = r + r'



$\Rightarrow \sqrt{h^2 + k^2} = 1 + h$

$h^2 + k^2 = 1 + h^2 + 2h$

$k^2 = 1 + 2h$

Locus is $y = \sqrt{1+2x}$

Sequence & Progression

6. The sum $1 + \frac{1^3 + 2^3}{1+2} + \frac{1^3 + 2^3 + 3^3}{1+2+3} + ... + \frac{1^3 + 2^3 + 3^3 + ... + 15^3}{1+2+3+...+15} - \frac{1}{2}(1 + 2 + 3 + ... + 15)$ is equal to :

योगफल $1+\frac{1^3+2^3}{1+2}+\frac{1^3+2^3+3^3}{1+2+3}+...+\frac{1^3+2^3+3^3+...+15^3}{1+2+3+...+15}-\frac{1}{2}$ (1 + 2 + 3 + ... + 15) बराबर है :

(1) 1860 (2) 620 (3) 660 (4) 1240

A. 2

sol. $S=1+\frac{1^3+2^3}{1+2}+\frac{1^3+2^3+3^3}{1+2+3}+....15 \text{ terms}$

$$T_n=\frac{1^3+2^3+...n^3}{1+2+...n}=\frac{\frac{n^2(n+1)^2}{4}}{\frac{n(n+1)}{2}}=\frac{n(n+1)}{2}$$

$$S=\frac{1}{2}\left(\sum_{n=1}^{15}n^2+\sum_{n=1}^{15}n\right)=\frac{1}{2}\left(\frac{15(16)(31)}{6}+\frac{15(16)}{2}\right)=680$$

$$\Rightarrow 680-\frac{1}{2}\frac{15(16)}{2}=680-60=620$$

Binomial Theorem

7. The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2+\frac{1}{x^3}\right)^n$ is $^nC_{23}$, is :

वह न्यूनतम प्राकृत संख्या n, जिसके लिए $\left(x^2+\frac{1}{x^3}\right)^n$ के प्रसार में x का गुणांक $^nC_{23}$ है, है :

(1) 58 (2) 35 (3) 38 (4) 23

A. 3

sol. $\left(x^2+\frac{1}{x^3}\right)^n$

General term $T_{r+1}={}^nC_r(x^2)^{n-r}\left(\frac{1}{x^3}\right)^r$

$^nC_r \,.\, x^{2n-5r}$

for coefficiant of x, $2n-5r=1$

Given $^nC_r={}^nC_{23}$

$r = 23$ or $n-r=23$

$\Rightarrow n=58$ or $n=38$

Minimum value is $n=38$

Statistics

8. If both the mean and the standard deviation of 50 observations $x_1, x_2, ... x_{50}$ are equal to 16, then the mean of $(x_1-4)^2, (x_2-4)^2, ... (x_{50}-4)^2$ is :

यदि 50 प्रेक्षणों $x_1, x_2, ... x_{50}$ का माध्य तथा मानक विचलन दोनों 16 हैं, तो $(x_1 - 4)^2, (x_2 - 4)^2, ... (x_{50} - 4)^2$ का माध्य है:

(1) 380 (2) 480 (3) 400 (4) 525

A. 3

sol. $\dfrac{x_1 + x_2 + ...x_{50}}{50} = 16$

$16^2 = \dfrac{x_1^2 + x_2^2 + ...x_{50}^2}{50} - 16^2$

$2(16)^2 50 = x_1^2 + x_2^2 + ...x_{50}^2$

Required mean $= \dfrac{(x_1 - 4)^2 + (x_2 - 4)^2 + ...(x_{50} - 4)^2}{50}$

$\dfrac{16^2(100) + 4^2(50) - 8(16 \times 50)}{50}$

$= 16^2(2) + 16 - 8(16) = 400$

Indefinite Integration

9. If $\int x^5 e^{-x^2} dx = g(x) e^{-x^2} + c$, where c is a constant of integration, then g(–1) is equal to :

यदि $\int x^5 e^{-x^2} dx = g(x) e^{-x^2} + c$ है, जहाँ c एक समाकलन अचर है, तो g(–1) बराबर है :

(1) –1 (2) $-\dfrac{1}{2}$ (3) 1 (4) $-\dfrac{5}{2}$

A. 4

sol. $I = \int x^5 . e^{-x^2} dx$

Put $-x^2 = t$ $\Rightarrow -2xdx = dt$

$I = \int \dfrac{t^2 . e^t dt}{(-2)} = \dfrac{-1}{2} e^t (t^2 - 2t + 2) + c$

$\therefore \quad g(x) = \dfrac{-1}{2}(x^4 + 2x^2 + 2)$

$g(-1) = \dfrac{-5}{2}$

Sequence & Progression

10. Let a, b and c be in G.P. with common ratio r, where $a \neq 0$ and $0 < r \leq \dfrac{1}{2}$. If 3a, 7b and 15c are the first three terms of an A.P., then the 4th term of this A.P. is :

माना a, b तथा c गुणोत्तर श्रेढ़ी में हैं जिसका सार्वअनुपात r है, जहाँ $a \neq 0$ और $0 < r \leq \frac{1}{2}$ है। यदि 3a, 7b तथा 15c एक समान्तर श्रेढ़ी के प्रथम तीन पद हैं, तो इस समान्तर श्रेढ़ी का चौथा पद है :

(1) $\frac{2}{3}a$ (2) a (3) $\frac{7}{3}a$ (4) 5a

A. 2

sol. Let $b = ar$, $c = ar^2$

AP : 3a, 7ar, $15ar^2$

$14ar = 3a + 15ar^2$

$\Rightarrow 15r^2 - 14r + 3 = 0$

$\Rightarrow r = \frac{1}{3}$ or $r = \frac{3}{5}$ rejected

Fourth term $= 15ar^2 + 7ar - 3a$

$= a(15r^2 + 7r - 3)$

$= a\left(\frac{15}{9} + \frac{7}{3} - 3\right)$

$= a$

Area Under Curve

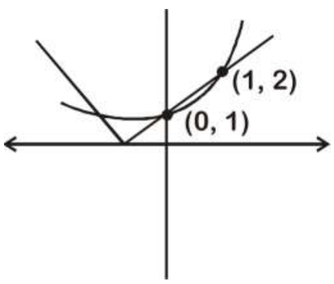
11. The area (in sq. units) of the region bounded by the curves $y = 2^x$ and $y = |x + 1|$, in the first quadrant is :

वक्रों $y = 2^x$ तथा $y = |x + 1|$ द्वारा प्रथम चतुर्थांश में परिबद्ध क्षेत्र का क्षेत्रफल (वर्ग इकाइयों में) है :

(1) $\frac{3}{2} - \frac{1}{\log_e 2}$ (2) $\frac{1}{2}$ (3) $\log_e 2 + \frac{3}{2}$ (4) $\frac{3}{2}$

A. 1

sol.



$$\text{Area} = \int_0^1 \left((x+1) - 2^x\right)dx$$

$$= \left[\frac{x^2}{2} + x - \frac{2^x}{\ln 2}\right]_0^1$$

$$= \left(\frac{1}{2} + 1 - \frac{2}{\ln 2}\right) - \left(\frac{-1}{\ln 2}\right)$$

$$= \frac{3}{2} - \frac{1}{\ln 2}$$

3 D

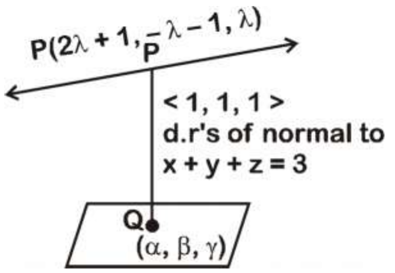
12. A perpendicular is drawn from a point on the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{1}$ to the plane $x+y+z=3$ such that the foot of the perpendicular Q also lies on the plane $x-y+z=3$. Then the co-ordinates of Q are :

रेखा $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{1}$ के एक बिन्दु से समतल $x+y+z=3$ पर एक लम्ब इस प्रकार डाला गया कि इसका लंबपाद Q, समतल $x-y+z=3$ पर भी स्थित है, तो Q के निर्देशांक हैं :

(1) (1, 0, 2) (2) (2, 0, 1) (3) (4, 0, –1) (4) (–1, 0, 4)

A. 2

sol.



Let Q be (α, β, γ)

$\alpha+\beta+\gamma=3$ ...(i)

$\alpha-\beta+\gamma=3$ ...(ii)

$\therefore\ \alpha+\gamma=3$ and $\beta=0$

Equating DR's of PQ :

$$\frac{\alpha-2\lambda-1}{1}=\frac{\lambda+1}{1}=\frac{3-\alpha-\lambda}{1}$$

$\Rightarrow \alpha=3\lambda+2, \gamma=2\lambda+1$

Substituting in equation (i) we get

$\Rightarrow 5\lambda+3=3$

$\lambda=0$

Point is Q(2, 0, 1)

Tangent & Normal

13. If the tangent to the curve $y=\frac{x}{x^2-3}$, $x\in R$, $\left(x\neq\pm\sqrt{3}\right)$, at a point $(\alpha,\beta)\neq(0,0)$ on it is parallel to the line $2x+6y-11=0$, then :

यदि वक्र $y=\frac{x}{x^2-3}$, $x\in R$, $\left(x\neq\pm\sqrt{3}\right)$ के एक बिन्दु $(\alpha,\beta)\neq(0,0)$ पर खींची गई स्पर्श रेखा, रेखा $2x+6y-11=0$ के समान्तर है, तो :

(1) $|6\alpha+2\beta|=19$ (2) $|2\alpha+6\beta|=19$ (3) $|6\alpha+2\beta|=9$ (4) $|2\alpha+6\beta|=11$

A. 1

sol. $y=\frac{x}{x^2-3}$

$$\frac{dy}{dx}=\frac{(x^2-3)-x(2x)}{(x^2-3)^2}=\frac{-x^2-3}{(x^2-3)^2}$$

$$\left.\frac{dy}{dx}\right|_{(\alpha,\beta)}=\frac{-\alpha^2-3}{(\alpha^2-3)^2}=-\frac{1}{3}$$

$3(\alpha^2+3)=(\alpha^2-3)^2$ ...(i)

i.e. $\alpha^2=9$

Also, $\beta=\frac{\alpha}{\alpha^2-3}\Rightarrow\alpha^2-3=\frac{\alpha}{\beta}\Rightarrow\frac{\alpha}{\beta}=6$

$\Rightarrow\alpha=\pm3,\ \beta=\pm\frac{1}{2}$

Which satisfies $|6\alpha+2\beta|=19$

Differential Equation

14. Let $y=y(x)$ be the solution of the differential equation, $\frac{dy}{dx}+y\tan x=2x+x^2\tan x,\ x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, such that $y(0)=1$. Then :

माना $y=y(x)$, अवकल समीकरण $\frac{dy}{dx}+y\tan x=2x+x^2\tan x,\ x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, जबकि $y(0)=1$ है, का हल है, तो:

(1) $y\left(\frac{\pi}{4}\right)+y\left(-\frac{\pi}{4}\right)=\frac{\pi^2}{2}+2$

(2) $y\left(\frac{\pi}{4}\right)-y\left(-\frac{\pi}{4}\right)=\sqrt{2}$

(3) $y'\left(\frac{\pi}{4}\right)+y'\left(-\frac{\pi}{4}\right)=-\sqrt{2}$

(4) $y'\left(\frac{\pi}{4}\right)-y'\left(-\frac{\pi}{4}\right)=\pi-\sqrt{2}$

A. 4

sol. $\frac{dy}{dx}+y\tan x=2x+x^2\tan x$

$P=\tan x,\ Q=2x+x^2\tan x$

$\text{I.F.}=e^{\int\tan x\,dx}=e^{\ln|\sec x|}=|\sec x|$

$$y(\sec x)=\int(2x+x^2\tan x)\sec x\,dx$$

$$=\int x^2\tan x\sec x\,dx+\int 2x\sec x\,dx$$

$$=x^2\sec x-\int 2x\sec x\,dx+\int 2x\sec x\,dx$$

$$=x^2\sec x+c$$

As $y(0)=1$, $c=1$

$\therefore\ y=x^2+\cos x$

$$\text{At } x = \frac{\pi}{4},\ y = \left(\frac{\pi}{4}\right) = \frac{\pi^2}{16} + \frac{1}{\sqrt{2}}$$

$$y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{16} + \frac{1}{\sqrt{2}}$$

$$y\left(\frac{\pi}{4}\right) - y\left(-\frac{\pi}{4}\right) = 0$$

$$\frac{dy}{dx} = 2x - \sin x$$

$$y'\left(\frac{\pi}{4}\right) = \frac{\pi}{2} - \frac{1}{\sqrt{2}},\ y'\left(-\frac{\pi}{4}\right) = -\frac{\pi}{2} + \frac{1}{\sqrt{2}}$$

$$y'\left(\frac{\pi}{4}\right) - y'\left(-\frac{\pi}{4}\right) = \pi - \sqrt{2}$$

Sequence & Progression

15. Let $a_1, a_2, a_3, ...$ be an A.P. with $a_6 = 2$. Then the common difference of this A.P., which maximises the product $a_1\, a_4\, a_5$, is :

माना $a_1, a_2, a_3, ...$ एक समान्तर श्रेढ़ी है जिसमें $a_6 = 2$ है, तो इस समान्तर श्रेढ़ी का वह सार्वअन्तर जो गुणनफल $a_1\, a_4\, a_5$ को अधिकतम करता है, है :

(1) $\frac{2}{3}$ (2) $\frac{8}{5}$ (3) $\frac{3}{2}$ (4) $\frac{6}{5}$

A. 1

sol. $a + 5d = 2$

Let $A = a_1\, a_4\, a_5 = a(a + 3d)(a + 4d)$

$= a(2 - 2d)(2 - d)$

$A = (2 - 5d)(4 - 6d + 2d^2)$

$$\frac{dA}{dd} = 0$$

$(2 - 5d)(-6 + 4d) + (4 - 6d + 2d^2)(-5) = 0$

$\Rightarrow 15d^2 - 34d + 16 = 0$

$$d = \frac{8}{5}, \frac{2}{3}$$

$$\text{For } d = \frac{2}{3}, \frac{d^2A}{dd^2} < 0$$

$$\text{Hence } d = \frac{2}{3}$$

3 D

16. If the plane $2x - y + 2z + 3 = 0$ has the distances $\frac{1}{3}$ and $\frac{2}{3}$ units from the planes $4x - 2y + 4z + \lambda = 0$ and $2x - y + 2z + \mu = 0$, respecitvely, then the maximum value of $\lambda + \mu$ is equal to :

यदि समतल $2x - y + 2z + 3 = 0$ की समतलों $4x - 2y + 4z + \lambda = 0$ तथा $2x - y + 2z + \mu = 0$ से दूरियाँ क्रमशः $\frac{1}{3}$ तथा $\frac{2}{3}$ इकाइयाँ हैं, तो $\lambda + \mu$ का अधिकतम मान है :

(1) 13 (2) 15 (3) 5 (4) 9

A. 1

sol. $P_1 : 2x - y + 2z + 3 = 0$

$P_2 : 2x - y + 2z + \frac{\lambda}{2} = 0$

$P_3 : 2x - y + 2z + \mu = 0$

Given $\frac{1}{3} = \frac{\left|3 - \frac{\lambda}{2}\right|}{\sqrt{9}} \Rightarrow \left|3 - \frac{\lambda}{2}\right| = 1$

$\lambda_{max} = 8$

Also, $\frac{2}{3} = \frac{|\mu - 3|}{\sqrt{9}} \Rightarrow \mu_{max} = 5$

$(\lambda + \mu)_{max} = 13$

Limit

17. If $\lim_{x \to 1} \frac{x^2 - ax + b}{x - 1} = 5$, then $a + b$ is equal to :

यदि $\lim_{x \to 1} \frac{x^2 - ax + b}{x - 1} = 5$ है, तो $a + b$ बराबर है –

(1) 5 (2) –4 (3) 1 (4) –7

A. 4

sol. $\lim_{x \to 1} \frac{x^2 - ax + b}{x - 1} = 5$

As limit is finite, $1 - a + b = 0$

$\Rightarrow \lim_{x \to 1} \frac{2x - a}{1} = 5 \quad \left(\frac{0}{0} \text{from}\right)$

i.e., $2 - a = 5$

or $a = -3$

$\therefore b = -4$

$a + b = -3 - 4 = -7$

Quadratic Equation

18. The number of real roots of the equation $5 + |2^x - 1| = 2^x(2^x - 2)$ is :

समीकरण $5 + |2^x - 1| = 2^x(2^x - 2)$ के वास्तविक मूलों की संख्या है :

(1) 4 (2) 2 (3) 1 (4) 3

A. 3

sol. Let $2^x - 1 = t$

$5 + |t| = (t + 1)(t - 1)$

$\Rightarrow |t| = t^2 - 6$

For $t > 0$, $t^2 - t - 6 = 0$

i.e., $t = 3$ or -2 (rejected)

For $t < 0$, $t^2 + t - 6 = 0$

i.e., $t = -3$ or 2 (both rejected)

$\therefore 2^x - 1 = 3$

$\Rightarrow x = 2$

Straight Line

19. Lines are drawn parallel to the line $4x - 3y + 2 = 0$, at a distance $\frac{3}{5}$ from the origin. Then which one of the following points lies on any of these lines ?

रेखा $4x - 3y + 2 = 0$ के समान्तर रेखाएँ खींची गई हैं जो मूलबिन्दु से $\frac{3}{5}$ की दूरी पर हैं, तो निम्न में से कौन–सा एक बिन्दु इनमें से किसी रेखा पर स्थित हैं?

(1) $\left(\frac{1}{4}, -\frac{1}{3}\right)$ (2) $\left(\frac{1}{4}, \frac{1}{3}\right)$ (3) $\left(-\frac{1}{4}, \frac{2}{3}\right)$ (4) $\left(-\frac{1}{4}, -\frac{2}{3}\right)$

A. 3

sol. Let straight line be $4x - 3y + \alpha = 0$

Given $\frac{3}{5} = \left|\frac{\alpha}{5}\right|$

$\Rightarrow \alpha = \pm 3$

Line is $4x - 3y + 3 = 0$ or $4x - 3y - 3 = 0$

Clearly $\left(-\frac{1}{4}, \frac{2}{3}\right)$ satisfies $4x - 3y + 3 = 0$

P & C

20. Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its nonadjacent pillars, then the total number of beams is :

माना एक वृत्तीय स्टेडियम की सीमा पर एक ही ऊँचाई के 20 खम्भे खड़े किए गए हैं। यदि प्रत्येक खम्भे के शिखर को सभी असंलग्न खम्भों के शिखरों से कड़ियों (beams) द्वारा जोड़ा गया है, तो ऐसी कड़ियों की कुल संख्या है :

(1) 210   (2) 180   (3) 170   (4) 190

A. 3

sol. Required number of beams = $^{20}C_2 - 20$

$= 190 - 20 = 170$

Tangent & Normal

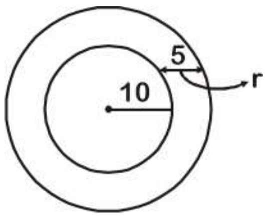
21. A spherical iron ball of radius 10 cm is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of the ice is 5 cm, then the rate at which the thickness (in cm/min) of the ice decreases, is :

10 cm त्रिज्या की लोहे की एक गोलाकार गेंद के चारों ओर समान मोटाई की बर्फ की तह चढ़ाई गई है, जो 50 cm^3/min की दर से पिघल रही है। जब बर्फ की मोटाई 5 cm है, तब बर्फ की मोटाई के घटने की दर (cm/min) में, है :

(1) $\frac{5}{6\pi}$ (2) $\frac{1}{36\pi}$ (3) $\frac{1}{9\pi}$ (4) $\frac{1}{18\pi}$

A. 4

sol. $\frac{dV_{ice}}{dt} = 50$



$V_{ice} = \frac{4}{3}\pi(10+r)^3 - \frac{4}{3}\pi(10)^3$

$\frac{dV}{dt} = \frac{4}{3}\pi 3(10+r)^2 \frac{dr}{dt}$

$= 4\pi(10+r)^2 \frac{dr}{dt}$

At r = 5, $50 = 4\pi(225)\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{50}{4\pi(225)}$

$= \frac{1}{18\pi}$ cm / min

Probability

22. Minimum number of times a fair coin must be tossed so that the probability of getting at least one head is more than 99% is :

एक न्याय्य सिक्के को न्यूनतम कितनी बार उछालें कि कम से कम एक चित्त आने की प्रायिकता 99% से अधिक हो?

(1) 8   (2) 6   (3) 5   (4) 7

A. 4

sol. $1 - \left(\frac{1}{2}\right)^n > \frac{99}{100}$

$\left(\frac{1}{2}\right)^n < \frac{1}{100}$

$\therefore\ n \geq 7$

Minimum value is 7.

Hyperbola

23. If $5x + 9 = 0$ is the directrix of the hyperbola $16x^2 - 9y^2 = 144$, then its corresponding focus is :

यदि अतिपरवलय $16x^2 - 9y^2 = 144$ की नियता (directrix) $5x + 9 = 0$ है, तो इसका संगत नाभिकेन्द्र है :

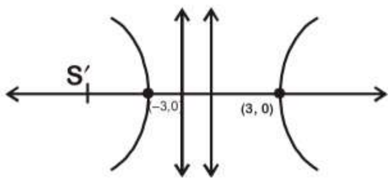
(1) $\left(\frac{5}{3}, 0\right)$ (2) $\left(-\frac{5}{3}, 0\right)$ (3) (–5, 0) (4) (5, 0)

A. 3

sol. $16x^2 - 9y^2 = 144$

i.e. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

Focus S'(–ae, 0)



$x = \frac{-9}{5}$

a = 3, b = 4

$e^2 = 1 + \frac{16}{9} = \frac{25}{9}$

$S' \equiv \left(-3 \times \frac{5}{3}, 0\right) \equiv (-5, 0)$

Determinant

24. Let λ be a real number for which the system of linear equations

$x + y + z = 6$

$4x + \lambda y - \lambda z = \lambda - 2$

$3x + 2y - 4z = -5$

has infinitely many solutions. Then λ is a root of the quadratic equation :

माना λ एक ऐसी वास्तविक संख्या है जिसके लिए रैखिक समीकरण निकाय

$x + y + z = 6$

$4x + \lambda y - \lambda z = \lambda - 2$

$3x + 2y - 4z = -5$

के अनन्त हल हैं, तो λ जिस द्विघात समीकरण का एक मूल है, वह है :

(1) $\lambda^2 + 3\lambda - 4 = 0$ (2) $\lambda^2 - \lambda - 6 = 0$ (3) $\lambda^2 + \lambda - 6 = 0$ (4) $\lambda^2 - 3\lambda - 4 = 0$

A. 2

sol. $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 4 & \lambda & -\lambda \\ 3 & 2 & -4 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ 4-\lambda & 2\lambda & -\lambda \\ 1 & 6 & -4 \end{vmatrix} = 0 \qquad \Rightarrow \lambda = 3$$

$$\Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ \lambda-2 & \lambda & -\lambda \\ -5 & 2 & -4 \end{vmatrix} = 0 \quad \text{for } \lambda = 3$$

$$\Delta_2 = \begin{vmatrix} 1 & 6 & 1 \\ 4 & \lambda-2 & -\lambda \\ 3 & -5 & -4 \end{vmatrix} = 0 \quad \text{for } \lambda = 3$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 4 & \lambda & \lambda-2 \\ 3 & 2 & -5 \end{vmatrix} = 0 \quad \text{for } \lambda = 3$$

$\therefore$ For $\lambda = 3$, infinitely many solutions is obtained.

Definite Integration

25. The integral $\int_{\pi/6}^{\pi/3} \sec^{2/3}x \operatorname{cosec}^{4/3}x \, dx$ is equal to :

समाकल $\int_{\pi/6}^{\pi/3} \sec^{2/3}x \operatorname{cosec}^{4/3}x \, dx$ बराबर है :

(1) $3^{7/6} - 3^{5/6}$ (2) $3^{5/3} - 3^{1/3}$ (3) $3^{5/6} - 3^{2/3}$ (4) $3^{4/3} - 3^{1/3}$

A. 1

sol. $I = \int_{\pi/6}^{\pi/3} \sec^{2/3} x . \operatorname{cosec}^{4/3} x \, dx$

$$= \int_{\pi/6}^{\pi/3} \frac{1.dx}{\cos^{2/3} x . \sin^{4/3} x}$$

$$= \int_{\pi/6}^{\pi/3} \frac{1 \quad dx}{\cos^2 x . \tan^{4/3} x} = \int_{\pi/6}^{\pi/3} \frac{\sec^2 x dx}{\tan^{4/3} x}$$

Let tan x = t

$$I = \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} t^{-4/3} dt = \frac{3\left[t^{-1/3}\right]_{1/\sqrt{3}}^{\sqrt{3}}}{-1}$$

$$= -3\left[3^{-1/6} - \frac{1}{3^{-1/6}}\right]$$

$$= -3\left(3^{-1/6} - 3^{-1/6}\right)$$

$$= 3\left(3^{1/6} - 3^{-1/6}\right)$$

$$= 3^{7/6} - 3^{5/6}$$

Parabola

26. If the line ax + y = c, touches both the curves $x^2 + y^2 = 1$ and $y^2 = 4\sqrt{2}x$, then |c| is equal to :

यदि रेखा ax + y = c, दोनों वक्रों $x^2 + y^2 = 1$ तथा $y^2 = 4\sqrt{2}x$, को स्पर्श करती है, तो |c| बराबर है :

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2}$ (3) 2 (4) $\sqrt{2}$

A. 4

sol. Tangent on $y^2 = 4\sqrt{2}x$ is $yt = x + \sqrt{2}t^2$

As it is tangent on circle also,

$$\left|\frac{\sqrt{2}t^2}{\sqrt{1+t^2}}\right| = 1$$

$2t^4 = 1 + t^2$ i.e. $t^2 = 1$

Equation is $\pm y = x + \sqrt{2}$

Hence $| c | = \sqrt{2}$

Ellipse

27. The tangent and normal to the ellipse $3x^2 + 5y^2 = 32$ at the point P(2, 2) meet the xaxis at Q and R, respectively. Then the area (in sq. units) of the triangle PQR is :

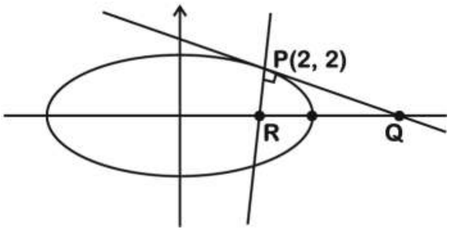
दीर्घवृत्त $3x^2 + 5y^2 = 32$ के बिन्दु P(2, 2) पर खींची गई स्पर्श रेखा तथा अभिलम्ब, x–अक्ष को क्रमशः Q तथा R पर काटते हैं, तो त्रिभुज PQR का क्षेत्रफल (वर्ग इकाइयों में) है:

(1) $\frac{16}{3}$ (2) $\frac{14}{3}$ (3) $\frac{34}{15}$ (4) $\frac{68}{15}$

A. 4

sol. For $\frac{3x^2}{32}+\frac{5y^2}{32}=1$

Tangent at P is



$\frac{3(2)x}{32}+\frac{5(2)y}{32}=1$

$\frac{3x}{16}+\frac{5y}{16}=1$

$Q\equiv\left(\frac{16}{3},0\right)$

Normal at P is $\frac{32x}{3(2)}-\frac{32y}{5(2)}=\frac{32}{3}-\frac{32}{5}$

$R\equiv\left(\frac{4}{5},0\right)$

area of $\Delta PQR = \frac{1}{2}$ (PQ) (PR) $= \frac{1}{2}\frac{\sqrt{136}}{3}.\frac{\sqrt{136}}{5}$

$=\frac{68}{15}$

Vector

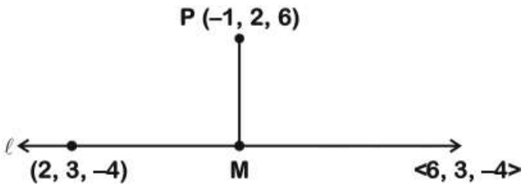
28. The distance of the point having position vector $-\hat{i}+2\hat{j}+6\hat{k}$ from the straight line passing through the point (2, 3, –4) and parallel to the vector, $6\hat{i}+3\hat{j}-4\hat{k}$ is :

एक बिन्दु जिसका स्थिति सदिश $-\hat{i}+2\hat{j}+6\hat{k}$ है, की एक सरल रेखा, जो बिन्दु (2, 3, –4) से होकर जाती है तथा सदिश $6\hat{i}+3\hat{j}-4\hat{k}$ के समान्तर है, से दूरी है :

(1) 7 (2) $4\sqrt{3}$ (3) 6 (4) $2\sqrt{13}$

A. 1

sol. Equation of I is $\frac{x-2}{6}=\frac{y-3}{3}=\frac{z+4}{-4}$



$(6\lambda + 2, 3\lambda + 3, -4\lambda - 4)$

Let M $(6\lambda + 2, 3\lambda + 3, -4\lambda - 4)$

DR's of PM is $< 6\lambda + 3, 3\lambda + 1, -4\lambda - 10 >$

$\Rightarrow (6\lambda + 3)(6) + (3\lambda + 1)(3) + (-4\lambda - 10)(-4) = 0$

$\Rightarrow \lambda = -1$

i.e. $M \equiv (-4, 0, 0)$

$\therefore$ PM $= \sqrt{9+4+36} = 7$

ITF

29. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, where $-1 \le x \le 1, -2 \le y \le 2, x \le \frac{y}{2}$, then for all x, y, $4x^2 - 4xy\cos + y^2$ is equal to :

यदि $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, जहाँ $-1 \le x \le 1, -2 \le y \le 2, x \le \frac{y}{2}$ है, तो सभी x, y, के लिए, $4x^2 - 4xy\cos + y^2$ बरबार है :

(1) $2\sin^2\alpha$ (2) $4\sin^2\alpha - 2x^2y^2$ (3) $4\sin^2\alpha + 2x^2y^2$ (4) $4\sin^2\alpha$

A. 4

sol. $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$

$\Rightarrow \cos^{-1}\left(\frac{xy}{2} + \sqrt{1-x^2}.\sqrt{1-\frac{y^2}{4}}\right) = \alpha$

$\Rightarrow \frac{xy}{2} + \frac{\sqrt{1-x^2}\sqrt{4-y^2}}{2} = \cos\alpha$

$\Rightarrow xy + \sqrt{1-x^2}\sqrt{4-y^2} = 2\cos\alpha$

$(xy - 2\cos\alpha)^2 = (1 - x^2)(4 - y^2)$

$x^2y^2 + 4\cos^2\alpha - 4xy\cos\alpha = 4 - y^2 - 4x^2 + x^2y^2$

$4x^2 - 4xy\cos\alpha + y^2 = 4\sin^2\alpha$

Mathematical Reasoning

30. The negation of the Boolean expression $\sim s \vee (\sim r \wedge s)$ is equivalent to :

बूले व्यंजक $\sim s \vee (\sim r \wedge s)$ का निषेधन निम्न में से किस के समतुल्य है?

(1) $s \wedge r$ (2) r (3) $\sim s \wedge \sim r$ (4) $s \vee r$

A. 1

sol. $\sim s \vee (\sim r \wedge s)$

$\equiv (\sim s \vee \sim r) \wedge (\sim s \vee s)$

$\equiv (\sim s \vee \sim r)$ ($\because$ $(\sim s \vee s)$ is tautology)

$\equiv \sim(s \wedge r)$

Hence its negation is $s \wedge r$