MATHS
09 Jan. 2019 [Session : 09.30 AM to 12.00 PM]
JEE MAIN PAPER ONLINE
RED COLOUR CONSIDER OFFICIAL ANSWER

1. For $x^2 \neq n\pi + 1, n \in N$ (the set of natural numbers), the integral

$$\int x\sqrt{\frac{2\sin(x^2-1)-\sin 2(x^2-1)}{2\sin(x^2-1)+\sin 2(x^2-1)}}dx$$

equal to:

(where c is a constant of integration)

$x^2 \neq n\pi + 1, n \in N$ (प्राकृत संख्याओं का समुच्चय), के लिए, समाकल

$$\int x\sqrt{\frac{2\sin(x^2-1)-\sin 2(x^2-1)}{2\sin(x^2-1)+\sin 2(x^2-1)}}dx$$

बराबर है :

(जहाँ c एक समाकलन अचर है)

(1) $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$

(2) $\log_e\left|\frac{1}{2}\sec^2(x^2-1)\right|+c$

(3) $\frac{1}{2}\log_e\left|\sec(x^2-1)\right|+c$

(4) $\log_e\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c$

A. 1,4

Sol. $$\int x\sqrt{\frac{2\sin(x^2-1)(1-\cos(x^2-1))}{2\sin(x^2-1)(1+\cos(x^2-1))}} = \int x\sqrt{\frac{2\sin^2\left(\frac{x^2-1}{2}\right)}{2\cos^2\left(\frac{x^2-1}{2}\right)}}$$

$$\int x\tan\left(\frac{x^2-1}{2}\right)dx = \frac{x^2-1}{2} = t$$

$$\int \tan t\,dt = \ln\left|\sec\left(\frac{x^2-1}{2}\right)\right| + C$$

Indefinite Integration

Question ID : 41652910119

Option 1 ID : 41652939937

Option 2 ID : 41652939936

Option 3 ID : 41652939934

Option 4 ID : 41652939935

2. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is:

5 लड़कियों तथा 7 लड़कों की एक कक्षा का विचार कीजिए। इस कक्षा की 2 लड़कियों तथा 3 लड़कों को लेकर बन सकने वाली भिन्न टीमों (teams) यदि दो विशेष लड़के A तथा B एक ही टीम के सदस्य बनने से मना करते हैं, की संख्या है :

(1) 300 (2) 200 (3) 350 (4) 500

A. 1

Sol. $A\bar{B}+\bar{A}B+\bar{A}\bar{B}$

$={}^5C_2\times{}^5C_2+{}^5C_2\times{}^5C_2+{}^5C_2\times{}^5C_3=300$

P & C

Question ID : 41652910111

Option 1 ID : 41652939903

Option 2 ID : 41652939902

Option 3 ID : 41652939904

Option 4 ID : 41652939905

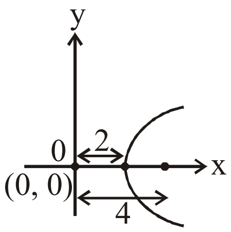
3. Axis of a parabola lies along *x*-axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive *x*-axis then which of the following points does not lie on it?

एक परवलय का अक्ष, x-अक्ष के अनुदिश है। यदि इसके शीर्ष तथा नाभि, x-अक्ष की धनात्मक दिशा में मूलबिंदु से क्रमशः 2 तथा 4 की दूरी पर हैं, तो इनमें से कौन–सा बिदुं इस परवलय पर स्थित नहीं है?

(1) (4, – 4) (2) $(5,2\sqrt{6})$ (3) (8, 6) (4) $(6,4\sqrt{2})$

A. 3

Sol.



$\boxed{y^2=8(x-2)}$ a = 2

PARABOLA

Question ID : 41652910126

Option 1 ID : 41652939962

Option 2 ID : 41652939964

Option 3 ID : 41652939965

Option 4 ID : 41652939963

4. Let $A=\left\{\theta\in\left(-\frac{\pi}{2},\pi\right):\frac{3+2i\sin\theta}{1-2i\sin\theta}\text{ is purely imaginary }\right\}$. Then the sum of the elements in A is:

मान $A=\left\{\theta\in\left(-\frac{\pi}{2},\pi\right):\frac{3+2i\sin\theta}{1-2i\sin\theta}\text{ पूर्णतः काल्पनिक है }\right\}$, तो A के अवयवों का योग है :

(1) π (2) $\frac{3\pi}{4}$ (3) $\frac{5\pi}{6}$ (4) $\frac{2\pi}{3}$

A. 4

Sol. $z=\frac{(3+2i\sin\theta)\times(1+2i\sin\theta)}{(1-2i\sin\theta)\times(1+2i\sin\theta)}=\frac{3-4\sin^2\theta+8i\sin\theta}{1+4\sin^2\theta}$, purely imaginary

So $\frac{3-4\sin^2\theta}{1+4\sin^2\theta}=0\Rightarrow\sin\theta=\pm\frac{\sqrt{3}}{2}\Rightarrow\theta=\frac{-\pi}{3},\frac{\pi}{3},\frac{2\pi}{3}$

Complex No.

Question ID : 41652910107

Option 1 ID : 41652939886

Option 2 ID : 41652939888

Option 3 ID : 41652939889

Option 4 ID : 41652939887

5. The plane through the intersection of the planes $x+y+z=1$ and $2x+3y-z+4=0$ and parallel to y - axis passes through the point :

y-अक्ष के समांतर तथा समतलों $x+y+z=1$ और $2x+3y-z+4=0$ के प्रतिच्छेदन से होकर जाने वाला समतल निम्न में से किस बिंदु से भी हो कर जाता है?

(1) (– 3, 0, – 1) (2) (– 3, 1, 1) (3) (3, 2, 1) (4) (3, 3, – 1)

A. 3

Sol. $P_1+\lambda P_2=0$

$(x+y+z-1)+\lambda(2x+3y-z+4)$ is parallel to y-axis (0, 1, 0)

$\left[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1-\lambda)\hat{k}\right].\hat{j}=0$

$\Rightarrow \boxed{\lambda=-\frac{1}{3}}$

$\Rightarrow(x+y+z-1)-\frac{1}{3}(2x+3y-z+4)=0$

$\Rightarrow x+4z-7=0$

VEctor 3D

Question ID : 41652910128

Option 1 ID : 41652939970

Option 2 ID : 41652939971

Option 3 ID : 41652939972

Option 4 ID : 41652939973

6. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to :

यदि संख्या $\frac{2^{403}}{15}$ का भिन्नात्मक भाग (fractional part) $\frac{k}{15}$ है, तो k बराबर है :

(1) 4 (2) 8 (3) 6 (4) 14

A. 2

Sol. $\frac{2^{403}}{15} = \frac{8.(16)^{100}}{15} = \frac{8(15+1)^{100}}{15}$

$= \frac{8.15k + 1 \times 8}{15}$

$= k + \frac{8}{15}$

$\frac{8}{15}$ is fractional part.

Bin. TH.

Question ID : 41652910112

Option 1 ID : 41652939906

Option 2 ID : 41652939907

Option 3 ID : 41652939909

Option 4 ID : 41652939908

7. Three circles of radii a, b, c (a < b < c) touch each other externally. If they have x-axis as a common tangent, then :

(1) a, b, c are in A.P. (2) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$ (3) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A.P. (4) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$

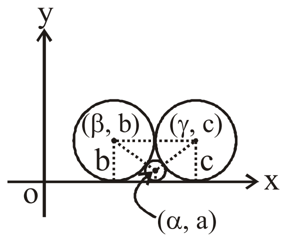
a, b, c (a < b < c) त्रिज्याओं वाले तीन वृत्त परस्पर बाह्य स्पर्श करते हैं। यदि x-अक्ष उनकी एक उभयनिष्ठ स्पर्श रेखा है, तो :

(1) a, b, c एक समांतर श्रेढ़ी में हैं। (2) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$

(3) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ एक समांतर श्रेढ़ी में हैं। (4) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$

A. 2

sol.



$$(\alpha-\beta)^2+(b-a)^2=(b+a)^2$$

$\Rightarrow \alpha-\beta=2\sqrt{ab}$ ...(1) $[\gamma>\alpha>\beta]$

similarly $\gamma-\beta=2\sqrt{bc}$

$\Rightarrow \beta-\gamma=2\sqrt{bc}$ ...(2)

and $\gamma-\alpha=2\sqrt{ac}$ ...(3)

(1) + (2) + (3) $\sqrt{bc}=\sqrt{ab}+\sqrt{ac}$

$$\Rightarrow \boxed{\frac{1}{\sqrt{a}}=\frac{1}{\sqrt{b}}+\frac{1}{\sqrt{c}}}$$

Question ID : 41652910125

Option 1 ID : 41652939958

Option 2 ID : 41652939960

Option 3 ID : 41652939959

Option 4 ID : 41652939961

8. If $\cos^{-1}\left(\frac{2}{3x}\right)+\cos^{-1}\left(\frac{3}{4x}\right)=\frac{\pi}{2}\left(x>\frac{3}{4}\right)$, then x is equal to:

यदि $\cos^{-1}\left(\frac{2}{3x}\right)+\cos^{-1}\left(\frac{3}{4x}\right)=\frac{\pi}{2}\left(x>\frac{3}{4}\right)$ है, तो x बराबर है :

(1) $\frac{\sqrt{146}}{12}$ (2) $\frac{\sqrt{145}}{12}$ (3) $\frac{\sqrt{145}}{10}$ (4) $\frac{\sqrt{145}}{11}$

A. 2

Sol. $\cos(\alpha+\beta)=\cos\cos^{-1}\left(\frac{3}{4x}\right)\times\cos\left(\cos^{-1}\frac{2}{3x}\right)-\sin\left(\cos^{-1}\left(\frac{3}{4x}\right)\right).\sin\left(\cos^{-1}\left(\frac{2}{3x}\right)\right)=0$

$$\Rightarrow \frac{3}{4x}\times\frac{2}{3x}-\sqrt{1-\frac{9}{16x^2}}.\sqrt{1-\frac{4}{9x^2}}=0$$

$\Rightarrow (9x^2-4)(16x^2-9)=36$

$\Rightarrow 144x^4-145x^2+36=36$

$$\Rightarrow x^2=\frac{145}{144}$$

Question ID : 41652910134

Option 1 ID : 41652939995

Option 2 ID : 41652939994

Option 3 ID : 41652939997

Option 4 ID : 41652939996

9. $\lim_{y\to 0} \frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$

(1) Exists and equals $\frac{1}{2\sqrt{2}(\sqrt{2}+1)}$ (2) Exists and equals $\frac{1}{4\sqrt{2}}$

(3) Exists and equals $\frac{1}{2\sqrt{2}}$ (4) Does not exist

(1) अस्तित्व है तथा $\frac{1}{2\sqrt{2}(\sqrt{2}+1)}$ के बराबर है। (2) अस्तित्व है तथा $\frac{1}{4\sqrt{2}}$ के बराबर है।

(3) अस्तित्व है तथा $\frac{1}{2\sqrt{2}}$ के बराबर है। (4) अस्तित्व नहीं है।

A. 2

Sol. $\lim_{y\to 0} \frac{\left(\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}\right)\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}$

$= \lim_{y\to 0} \frac{\left(\sqrt{1+y^4}-1\right)\left(\sqrt{1+y^4}+1\right)}{y^4\times\left(2\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$

$= \lim_{y\to 0} \frac{y^4}{y^4.\left(2\sqrt{2}\right).2} = \frac{1}{4\sqrt{2}}$

Question ID : 41652910115

Option 1 ID : 41652939921

Option 2 ID : 41652939920

Option 3 ID : 41652939919

Option 4 ID : 41652939918

10. The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2, 3) to it and the y - axis is:

परवलय $y = x^2 - 1$, इस परवलय पर स्थित एक बिंदु (2, 3) पर खींची गई स्पर्श रेखा तथा y-अक्ष से घिरे क्षेत्र का क्षेत्रफल (वर्ग इकाइयों में) है :

(1) $\frac{14}{3}$ (2) $\frac{56}{3}$ (3) $\frac{32}{3}$ (4) $\frac{8}{3}$

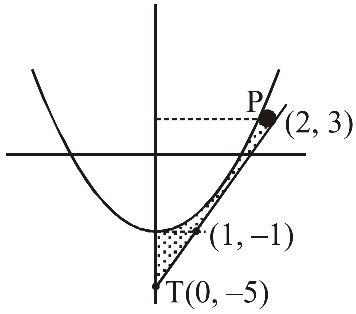
A. 4

Sol. $PT \Rightarrow \left(\frac{y+3}{2}\right) = 2x - 1$

$4x - y - 5 = 0$

$A = \int_{-5}^{3}\left(\frac{y+5}{4}\right)dy - \int_{-1}^{3}\left(\sqrt{y+1}\right)dy$

$\{y+1 \geq 0\}$



$= \frac{1}{4}\left(\frac{y^2}{2} + 5y\right)_5^3 - \frac{2}{3}\left[(y+1)^{\frac{3}{2}}\right]_1^3$

$= \frac{1}{4}\left(\frac{9}{2} + 15 - \frac{25}{2} + 25\right) - \frac{16}{3}$

$= \frac{1}{4}(17 + 15) - \frac{16}{3}$

$= 8 - \frac{16}{3} = \frac{8}{3}$

Question ID : 41652910121

Option 1 ID : 41652939943

Option 2 ID : 41652939944

Option 3 ID : 41652939945

Option 4 ID : 41652939942

11. Let f : R → R be a function defined as:

$$f(x) = \begin{cases} 5, \text{ if} & x \leq 1 \\ a + bx, \textit{ if} & 1 < x < 3 \\ b + 5x, \textit{ if} & 3 \leq x < 5 \\ 30, \textit{ if} & x \geq 5 \end{cases}$$

Then, f is :

(1) Continuous if a = – 5 and b = 10
(2) Continuous if a = 5 and b = 5
(3) Continuous if a = 0 and b = 5
(4) Not continuous for any values of a and b

माना फलन f : R → R be a function defined as:

$$f(x) = \begin{cases} 5, \text{ if} & x \leq 1 \\ a + bx, \textit{ if} & 1 < x < 3 \\ b + 5x, \textit{ if} & 3 \leq x < 5 \\ 30, \textit{ if} & x \geq 5 \end{cases}$$

द्वारा परिभाषित है, तो f :

(1) संतत है यदि a = – 5 तथा b = 10
(2) संतत है यदि a = 5 तथा b = 5

(3) संतत है यदि a = 0 तथा b = 5 (4) a तथा b के किसी भी मान के लिए संतत नहीं है।

A. 4

Sol. $\Rightarrow a + b = 5$ ...(1)

$\Rightarrow b + 25 = 30$ ...(2)

$\Rightarrow a + 3b = b + 15$

$\Rightarrow a + 2b = 15$ ...(3)

for continous (1), (2) and (3) should be satisfy for same value of a, b

So no solution & not continous.

Question ID : 41652910116

Option 1 ID : 41652939922

Option 2 ID : 41652939923

Option 3 ID : 41652939924

Option 4 ID : 41652939925

12. The system of linear equations :

$x + y + z = 2$

$2x + 3y + 2z = 5$

$2x + 3y + (a^2 - 1)z = a + 1$

(1) Is inconsistent when $|a| = \sqrt{3}$ (2) Has a unique solution for $|a| = \sqrt{3}$

(3) Has infinitely many solution for a = 4 (4) is inconsistent when a = 4

रैखिक समीकरण निकाय

$x + y + z = 2$

$2x + 3y + 2z = 5$

$2x + 3y + (a^2 - 1)z = a + 1$

(1) असंगत है जब $|a| = \sqrt{3}$ (2) का $|a| = \sqrt{3}$ के लिए अद्वितीय हल है।

(3) के a = 4 के लिए अनन्त हल हैं। (4) असंगत है जब a = 4

A. 1

Sol. $$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & (a^2-1) \end{vmatrix} = 0$$

$\Rightarrow |a| = \sqrt{3}$

$\Delta_x \neq 0$ for $a = \sqrt{3}$ so system of equations are inconsistent

Question ID : 41652910110

Option 1 ID : 41652939898

Option 2 ID : 41652939901

Option 3 ID : 41652939899

Option 4 ID : 41652939900

13. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then $P(X = 1) + P(X = 2)$ equals:

52 पत्तों की एक अच्छी प्रकार से फेंटी गई ताश की गड्डी में से, एक के बाद एक, दो पत्ते प्रतिस्थापना सहित निकाले गए। मान X, दोनों बार में प्राप्त इक्कों की संख्या को दर्शाने वाला यादृच्छिक चर है, तो $P(X = 1) + P(X = 2)$ बराबर है :

(1) 24/169 (2) 25/169 (3) 52/169 (4) 49/169

A. 2

Sol. $(Ace)(\bar{A}ce)+(\bar{A}ce)(Ace)+(Ace).(Ace)$

$$=\left(\frac{4}{52}\times\frac{48}{52}\right)\times 2+\frac{4}{52}\times\frac{4}{52}=\frac{25}{169}$$

Method II

Binomial Probability Distribution $n = 2, p = 4/52$

Required Probability $= P(X = 1) + P(X = 2)$ where $P(X = r) = {}^{n}C_{r}p^{r}(1-p)^{n-r}$

Question ID : 41652910132

Option 1 ID : 41652939986

Option 2 ID : 41652939987

Option 3 ID : 41652939989

Option 4 ID : 41652939988

14. If a, b and c be three distinct real numbes in G.P. and $a + b + c = xb$, then x cannot be:

यदि तीन भिन्न वास्तविक संख्यायें a, b तथा c एक गुणोत्तर श्रेढ़ी में हैं तथा $a + b + c = xb$, तो x निम्न में से कौन–सा नहीं हो सकता?

(1) 4 (2) – 3 (3) 2 (4) – 2

A. 3

Sol. $(a + c) = b(x - 1)$

$\Rightarrow \alpha + \alpha r^2 = \alpha r(x - 1)$

$\Rightarrow r^2 + (1 - x)r + 1 = 0$

$(1 - x)^2 - 4 \geq 0$

$\Rightarrow x^2 - 2x - 3 \geq 0$

$r \neq 1$

So, $x \neq 2$

Question ID : 41652910114

Option 1 ID : 41652939917

Option 2 ID : 41652939915

Option 3 ID : 41652939916

Option 4 ID : 41652939914

15. For $x \in \mathbf{R} - \{0, 1\}$, let $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, J (x) satisfies $(f_2 oJof_1)(x) = f_3(x)$ then J (x) is equal to :

$x \in \mathbf{R} - \{0, 1\}$ के लिए, माना $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ तथा $f_3(x) = \frac{1}{1-x}$ तीन फलन दिये गये है। यदि एक फलन J (x) सम्बन्ध $(f_2 oJof_1)(x) = f_3(x)$ को सन्तुष्ट करता है तो J (x) का मान होगा –

(1) $f_3(x)$ (2) $f_1(x)$ (3) $f_2(x)$ (4) $\frac{1}{x}f_3(x)$

A. 1

Sol. $f_2(J(f_1(x))) = f_3(x)$

$\Rightarrow 1 - J(f_1(x)) = \frac{1}{1-x}$

$\Rightarrow 1 - J\left(\frac{1}{x}\right) = \frac{1}{1-x}$

$1 - J(x) = \frac{x}{x-1}$

$\Rightarrow J(x) = \frac{-1}{x-1} = \frac{1}{1-x} = f_3(x)$

Question ID : 41652910106

Option 1 ID : 41652939884

Option 2 ID : 41652939882

Option 3 ID : 41652939883

Option 4 ID : 41652939885

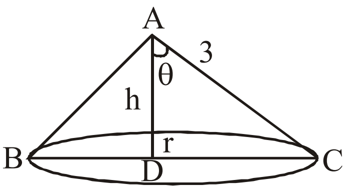
16. The maximum volume (in cu.m) of the right circular cone having slant height 3 m is

3 मी. तिर्यक (slant) ऊँचाई वाले लंबवृत्तीय शंकु का अधिकतम आयतन (घन मी. में) है :

(1) $2\sqrt{3}\pi$ (2) 6π (3) $3\sqrt{3}\pi$ (4) $\frac{4}{3}\pi$

A. 1

Sol.



$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi.(3)^3 \sin^2\theta\cos\theta$

$\frac{dv}{d\theta} = \frac{27\pi}{3}\left(2\sin\theta\cos^2\theta - \sin^3\theta\right) = 0$

$\cos\theta = \frac{1}{\sqrt{3}}$

$V = \frac{1}{3}\times\pi\times 27\times\frac{2}{3}\times\frac{1}{\sqrt{3}}$

$V = 2\sqrt{3}\pi$

Question ID : 41652910118

Option 1 ID : 41652939932

Option 2 ID : 41652939930

Option 3 ID : 41652939931

Option 4 ID : 41652939933

17. Consider the set of all lines $px + qy + r = 0$ such that $3p + 2q + 4r = 0$. Which one of the following statements is true?

(1) The lines are concurrent at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$

(2) Each line passes through the origin

(3) The lines are not concurrent

(4) The lines are all parallel

ऐसी सभी रेखाओं $px + qy + r = 0$ के समुच्चय पर विचार कीजिए जिनके लिए $3p + 2q + 4r = 0$ है, तो निम्न में से कोन–सा एक कथन सत्य है?

(1) रेखायें बिन्दु $\left(\frac{3}{4}, \frac{1}{2}\right)$ पर संगामी है।

(2) प्रत्येक रेखा मूल बिंदु से हो कर जाती है।

(3) रेखाएँ संगामी नहीं हैं।

(4) सभी रेखाएँ समांतर हैं।

A. 1

Sol. $$\left.\begin{array}{l} px + qy + r = 0 \\ \frac{3}{4}p + \frac{2}{4}q + r = 0 \end{array}\right\} x = \frac{3}{4}, y = \frac{1}{2}$$

Question ID : 41652910123

Option 1 ID : 41652939951

Option 2 ID : 41652939953

Option 3 ID : 41652939952

Option 4 ID : 41652939950

18. The equation of the line passing through $(-4, 3, 1)$ parallel to the plane $x + 2y - z - 5 = 0$ and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is:

बिंदु $(-4, 3, 1)$ से हो कर जाने वाली रेखा, जो समतल $x + 2y - z - 5 = 0$ के समांतर है तथा रेखा $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ को काटती है, का समीकरण है :

(1) $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

(2) $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

(3) $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

(4) $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

A. 2

Sol. $$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1} = \lambda \Rightarrow x = -3\lambda - 1, y = 2\lambda + 3, z = -\lambda + 2$$

$(\underset{\ell}{-3\lambda-1+4}).1+(\underset{m}{2\lambda+3-3}).2+(\underset{n}{-\lambda+2-1}).(-1)=0$

$\Rightarrow -3\lambda + 3 + 4\lambda + \lambda - 1 = 0$

$\Rightarrow 2\lambda + 2 = 0 \Rightarrow \lambda = -1$

$\frac{x+4}{6}=\frac{y-3}{-2}=\frac{z-1}{2}$

Question ID : 41652910129

Option 1 ID : 41652939974

Option 2 ID : 41652939976

Option 3 ID : 41652939977

Option 4 ID : 41652939975

19. Equation of a common tangent to the circle, $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$, is :

वृत्त $x^2 + y^2 - 6x = 0$ तथा परवलय $y^2 = 4x$, की एक उभयनिष्ठ स्पर्श रेखा का समीकरण है :

(1) $\sqrt{3}y = x+3$ (2) $\sqrt{3}y = 3x+1$ (3) $2\sqrt{3}y = 12x+1$ (4) $2\sqrt{3}y = -x-12$

A. 1

Sol. $(x-3)^2 + y^2 = 9$

$y^2 = 4x$

Tangent $\rightarrow y = mx + \frac{1}{m}$

$\frac{3m+\frac{1}{m}}{\sqrt{1+m^2}} = 3 \Rightarrow 9m^2 + 6 + \frac{1}{m^2} = 9 + 9m^2$

$\Rightarrow m = \pm\frac{1}{\sqrt{3}}$ for $m = \frac{1}{\sqrt{3}}$

$T \Rightarrow \sqrt{3}y = \sqrt{3}x + 3$

Question ID : 41652910124

Option 1 ID : 41652939957

Option 2 ID : 41652939956

Option 3 ID : 41652939955

Option 4 ID : 41652939954

20. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to:

माना α तथा β समीकरण $x^2 + 2x + 2 = 0$ के दो मूल हैं, तो $\alpha^{15} + \beta^{15}$ बराबर है :

(1) 256   (2) – 256   (3) – 512   (4) 512

A. 2

Sol. $\alpha = -1+i,\ \beta = -1-i,$

$\alpha = \sqrt{2}e^{\frac{i3\pi}{4}},\ \beta = \sqrt{2}e^{\frac{i5\pi}{4}}$

$$\alpha^{15}+\beta^{15}=\left(\sqrt{2}\right)^{15}\left(e^{\frac{i45\pi}{4}}+e^{\frac{i75\pi}{4}}\right)=\left(\sqrt{2}\right)^{15}\left(\frac{-1}{2}-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)$$

$$=\left(\sqrt{2}\right)^{15}\left(-\sqrt{2}\right)$$

$$\alpha^{15}+\beta^{15}=-256$$

Question ID : 41652910108
Option 1 ID : 41652939890
Option 2 ID : 41652939891
Option 3 ID : 41652939893
Option 4 ID : 41652939892

21. If the Boolean expression
$(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to
$p \wedge q$, where $\oplus, \odot \in \{\wedge, \vee\}$, then the ordered pair $(\oplus, \odot)$ is:
यदि बूलीय व्यंजक $(p \oplus q) \wedge (\sim p \odot q)$
$p \wedge q$ के तुल्य है, जहाँ $\oplus$, $\odot \in \{\wedge, \vee\}$ है, तो क्रमित युग्म $(\oplus, \odot)$ है :
(1) $(\vee, \vee)$ (2) $(\wedge, \wedge)$ (3) $(\vee, \wedge)$ (4) $(\wedge, \vee)$

A. 4

Sol. (i) $(p \wedge q) \wedge (\sim p \wedge q)$

$\Rightarrow p \wedge (q \wedge (\sim p \wedge q)) = p \wedge q$ (i) is correct

(ii) $(p \wedge q) \wedge (\sim p \wedge q) = (p \wedge \sim p) \wedge q = \phi$

(iii) $(p \vee q) \wedge (\sim p \vee q) = q$

(iv) $(p \vee q) \wedge (\sim p \vee q) = q \vee (p \wedge \sim p) = q \vee \phi = q$

Question ID : 41652910135
Option 1 ID : 41652940000
Option 2 ID : 41652939998
Option 3 ID : 41652940001
Option 4 ID : 41652939999

22. If $A=\begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}$, then the matrix, A^{-50} when $\theta = \frac{\pi}{12}$, is equal to:

यदि $A=\begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}$, तो आव्यूह A^{-50} जब $\theta = \frac{\pi}{12}$, बराबर है :

(1) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (2) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (3) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ (4) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

A. 1

Sol. $A^2 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

$A^{50} = \begin{bmatrix} \cos 50\theta & -\sin 50\theta \\ \sin 50\theta & \cos 50\theta \end{bmatrix}, A^{-50} = \begin{bmatrix} \cos 50\theta & \sin 50\theta \\ \sin 50\theta & \cos 50\theta \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

Question ID : 41652910109

Option 1 ID : 41652939894

Option 2 ID : 41652939895

Option 3 ID : 41652939896

Option 4 ID : 41652939897

23. For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ the exprssion $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals :

किसी $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ के लिए व्यंजक $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ बराबर है :

(1) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$ (2) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

(3) $13 - 4\cos^2\theta + 6\cos^4\theta$ (4) $13 - 4\cos^6\theta$

A. 4

Sol. $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$3(1-2\sin\theta\cos\theta)^2 + 6(1 + 2\sin\theta\cos\theta) + 4\sin^6\theta$

$\Rightarrow 3(1 + 4\sin^2\theta\cos^2\theta - 4\sin\theta\cos\theta) + 6 + 12\sin\theta\cos\theta + 4\sin^6\theta$

$\Rightarrow 3 + 12\sin^2\theta\cos^2\theta + 6 + 4\sin^6\theta$

$\Rightarrow 9 + 12\cos^2\theta(1 - \cos^2\theta) + 4(1 - \cos^2\theta)^3$

$\Rightarrow 9 + 12\cos^2\theta - 12\cos^4\theta + 4(1 - \cos^6\theta + 3\cos^4\theta - 3\cos^2\theta)$

$\Rightarrow 13 - 4\cos^6\theta$

Question ID : 41652910133

Option 1 ID : 41652939990

Option 2 ID : 41652939992

Option 3 ID : 41652939991

Option 4 ID : 41652939993

24. 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. The variance (in cm^2) of the height of these six student is:

एक कक्षा के 5 विद्यार्थियों की ऊँचाइयों का माध्य 150 cm तथा प्रसरण 18 cm^2 है। 156 cm ऊँचाई वाला एक नए विद्यार्थी उनसे आ मिला। इन छः विद्यार्थियों की ऊँचाइयों का प्रसरण (वर्ग से.मी. में) है :

(1) 18 (2) 16 (3) 20 (4) 22

A. 3

Sol. $x_1, x_2, x_3, x_4, x_5 \leftarrow$ students

$$\bar{x} = \frac{\sum x_i}{5} = 150 \Rightarrow \sum x_i = 750$$

$$\sigma^2 = \frac{\sum x_i^2}{5} - (\bar{x})^2 = 18 \Rightarrow \frac{\sum x_i^2}{5} = 18 + (150)^2$$

$$\Rightarrow \sum x_i^2 = (18 + 22500)5 = 112590$$

$$\bar{x}_{new} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + 156}{6} = \frac{750 + 156}{6} = 151$$

$$(\sigma^2)_{new} = \frac{\sum x_i^2}{6} - (\bar{x})^2 = \frac{112590 + (156)^2}{6} - (151)^2$$

$$(\sigma^2)_{new} = 20$$

Question ID : 41652910131

Option 1 ID : 41652939984

Option 2 ID : 41652939983

Option 3 ID : 41652939982

Option 4 ID : 41652939985

25. If θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then $|\tan\theta|$ is equal to :

यदि वक्रों $y = 10 - x^2$ तथा $y = 2 + x^2$ के बीच एक प्रतिच्छेद बिन्दु पर न्यून कोण θ है, तो $|\tan\theta|$ बराबर है :

(1) $\frac{8}{17}$ (2) $\frac{7}{17}$ (3) $\frac{4}{9}$ (4) $\frac{8}{15}$

A. 4

Sol. $\left.\begin{matrix} C_1 \rightarrow y = 10 - x^2 \\ C_2 \rightarrow y = 2 + x^2 \end{matrix}\right\} \Rightarrow$ POI is ($\pm$2, 6)

$$\text{for } C_1 \rightarrow \frac{dy}{dx} = -2(2) = -4$$

$$\text{for } C_2 \rightarrow \frac{dy}{dx} = 2 \times 2 = 4$$

$$\tan\theta = \left|\frac{M_1 - M_2}{1 + M_1M_2}\right| = \frac{8}{15}$$

Question ID : 41652910117
Option 1 ID : 41652939926
Option 2 ID : 41652939929
Option 3 ID : 41652939928
Option 4 ID : 41652939927

26. If y = y(x) is the solution of the differential equation, $x\frac{dy}{dx} + 2y = x^2$ satisfying y(1) = 1, then $y\left(\frac{1}{2}\right)$ is equal to :

यदि y = y(x), अवकल समीकरण $x\frac{dy}{dx} + 2y = x^2$ का हल है जो y(1) = 1 को संतुष्ट करता है, तो $y\left(\frac{1}{2}\right)$ बराबर है :

(1) $\frac{49}{16}$ (2) $\frac{7}{64}$ (3) $\frac{1}{4}$ (4) $\frac{13}{16}$

A. 1

Sol. $x\frac{dy}{dx} + 2y = x^2$

$\frac{dy}{dx} + \frac{2y}{x} = x \Rightarrow \text{I.F.} = e^{\int \frac{2}{x}dx} = e^{2\ln x} = x^2$

$y.x^2 = \int x^2.x dx = \frac{x^4}{4} + C$

$\Rightarrow 1 = \frac{1}{4} + C \Rightarrow C = \frac{3}{4}$

$\Rightarrow y.x^2 = \frac{x^4}{4} + \frac{3}{4}$

$\Rightarrow \frac{y}{4} = \frac{1}{16 \times 4} + \frac{3}{4}$

$\Rightarrow y = \frac{49}{16}$

Question ID : 41652910122
Option 1 ID : 41652939949
Option 2 ID : 41652939947
Option 3 ID : 41652939946
Option 4 ID : 41652939948

27. Let $0 < \theta < \frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$ is greater than 2, then the length of its latus rectum lies in the inteval :

माना $0 < \theta < \frac{\pi}{2}$ है। यदि अतिपरवलय $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$ की उत्केंद्रता 2 से अधिक है, तो इसके नाभिलंब की लंबाई जिस अन्तराल में है, वह है :

(1) $(3, \infty)$ (2) (2, 3] (3) (1, 3/2] (4) (3/2, 2]

A. 1

Sol. $\frac{x^2}{\cos^2\theta}-\frac{y^2}{\sin^2\theta}=1 \Rightarrow e=\sec\theta>2 \Rightarrow \theta\in\left(\frac{\pi}{3},\frac{\pi}{2}\right)$

$$LL'=\frac{2\sin^2\theta}{\cos\theta}=2\tan\theta\sin\theta$$

$$LL'_{min}=2\times\sqrt{3}\times\frac{\sqrt{3}}{2}$$

$$(LL')_{min}=3$$

$$(LL')_{min}\to\infty$$

Question ID : 41652910127

Option 1 ID : 41652939969D

Option 2 ID : 41652939968

Option 3 ID : 41652939966

Option 4 ID : 41652939967

28. Let $\vec{a}=\hat{i}-\hat{j},\vec{b}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}$ be a vector such that $\vec{a}\times\vec{c}+\vec{b}=\vec{0}$ and $\vec{a}\cdot\vec{c}=4$, then $\left|\vec{c}\right|^2$ is equal to :

माना $\vec{a}=\hat{i}-\hat{j},\vec{b}=\hat{i}+\hat{j}+\hat{k}$ तथा $\vec{c}$ ऐसे संदिश हैं कि $\vec{a}\times\vec{c}+\vec{b}=\vec{0}$ तथा $\vec{a}\cdot\vec{c}=4$ है, तो $\left|\vec{c}\right|^2$ बराबर है :

(1) 9 (2) 8 (3) $\frac{19}{2}$ (4) $\frac{17}{2}$

A. 3

Sol. $\vec{C}=p\hat{i}+q\hat{j}+r\hat{k}$ $\vec{a}.\vec{c}=4$

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 0\\ p & q & r\end{vmatrix}+\left(\hat{i}+\hat{j}+\hat{k}\right)=0$$

$$\Rightarrow(-r+1)\hat{i}+(-r+1)\hat{j}+(p+q+1)\hat{k}=0$$

$$\Rightarrow r=1$$

$$\Rightarrow p+q=-1$$

$$p-q=4$$

$$\Rightarrow p=\frac{3}{2},q=\frac{-5}{2}$$

$$\left|\vec{C}\right|^2=\left(p^2+q^2+r^2\right)=\frac{19}{2}$$

Question ID : 41652910130

Option 1 ID : 41652939980

Option 2 ID : 41652939981

Option 3 ID : 41652939978

Option 4 ID : 41652939979

29. The value of $\int_0^{\pi} |\cos x|^3 \, dx$ is :

$\int_0^{\pi} |\cos x|^3 \, dx$ का मान है :

(1) $-\frac{4}{3}$ (2) $\frac{2}{3}$ (3) $\frac{4}{3}$ (4) 0

A. 3

Sol. $\int_0^{\pi} |\cos^3 x| dx = 2\int_0^{\pi/2} \cos^3 xdx = 2\left(\frac{2}{3}.1\right) = \frac{4}{3}$

walli's formulae

Question ID : 41652910120

Option 1 ID : 41652939941

Option 2 ID : 41652939939

Option 3 ID : 41652939940

Option 4 ID : 41652939938

30. Let $a_1, a_2, ..., a_{30}$ be an A.P., $S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{(2i-1)}$. If $a_5 = 27$ and $S - 2T = 75$, then a_{10} is equal to :

माना $a_1, a_2, ..., a_{30}$ एक समान्तर श्रेढ़ी है, $S = \sum_{i=1}^{30} a_i$ तथा $T = \sum_{i=1}^{15} a_{(2i-1)}$. यदि $a_5 = 27$ तथा $S - 2T = 75$, तो a_{10} बराबर है :

(1) 42 (2) 52 (3) 47 (4) 57

A. 2

Sol. $S - 2T = 75$

$\Rightarrow 15(2a+29d) - 2\frac{15}{2}(2a+14\times 2d) = 75$

$\Rightarrow d = 5$

$a_5 = a + 4d = 27 \Rightarrow a = 7$

$a_{10} = a + 9d = 7 + 45 = 52$

Question ID : 41652910113

Option 1 ID : 41652939910

Option 2 ID : 41652939913

Option 3 ID : 41652939911

Option 4 ID : 41652939912