MATHS

09 Jan. 2020 [Morning]

JEE MAIN PAPER ONLINE

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Coordinate Geometry

Hyperbola

1. If e_1 and e_2 are the eccentricities of the ellipse, $\frac{x^2}{18}+\frac{y^2}{4}=1$ and the hyperbola, $\frac{x^2}{9}-\frac{y^2}{4}=1$ respectively and (e_1, e_2) is a point on the ellipse, $15x^2 + 3y^2 = k$, then k is equal to :

यदि e_1 तथा e_2 क्रमशः दीघवृत्त $\frac{x^2}{18}+\frac{y^2}{4}=1$ तथा अतिपरवलय $\frac{x^2}{9}-\frac{y^2}{4}=1$ की उत्केंद्रताएँ है तथा (e_1, e_2) दीर्घवृत्त $15x^2 + 3y^2 = k$ पर स्थित एक बिन्दु है, तो k का मान है :

(1) 15 (2) 14 (3) 16 (4) 17

Ans. 3

Question ID : 4050361984

Option 1 ID : 4050367128

Option 2 ID : 4050367129

Option 3 ID : 4050367127

Option 4 ID : 4050367126

Sol. $4 = 18(1 - e_1^2)$ $4 = 9(e_2^2 - 1)$

$\Rightarrow \frac{2}{9} = 1 - e_1^2$ $\frac{4}{9}+1 = e_2^2$

$e_1^2 = 1-\frac{2}{9}$

$e_1 = \frac{\sqrt{7}}{3}$ ......(1)

$\therefore\ e_2 = \frac{\sqrt{13}}{3}$ ....(2)

(e_1, e_2) lies on $15x^2 + 3y^2 = k$

$\therefore\ k = 15\times\frac{7}{9}+3\times\frac{13}{3} = \frac{48}{3} = 16$

JEE Main Only topics

Mathematical Reasoning

2. Negation of the statement :

'$\sqrt{5}$ is an integer or 5 is irrational' is :

(1) $\sqrt{5}$ is not an integer and 5 is not irrational (2) $\sqrt{5}$ is irrational or 5 is an integer

(3) $\sqrt{5}$ is not an integer or 5 is not irrational (4) $\sqrt{5}$ is an integer and 5 is irrational

कथन,

'$\sqrt{5}$ एक पूर्णांक है या 5 अपरिमेय है' का निषेधन है :

(1) $\sqrt{5}$ एक पूर्णांक नहीं है और 5 अपरिमेय नहीं है। (2) $\sqrt{5}$ एक अपरिमेय है या 5 एक पूर्णांक है।

(3) $\sqrt{5}$ एक पूर्णांक नहीं है या 5 अपरिमेय नहीं है। (4) $\sqrt{5}$ एक पूर्णांक है और 5 अपरिमेय है।

Ans. 1

Question ID : 4050361988

Option 1 ID : 4050367144

Option 2 ID : 4050367142

Option 3 ID : 4050367143

Option 4 ID : 4050367145

Sol. $\sqrt{5}$ is not an integer and 5 is not irrational.

Algebra

Sequence & progression

3. The product $2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}} \cdot$.... to ∞ is equal to :

गुणनफल $2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}} \cdot$.... ∞ तक बराबर है :

(1) 2 (2) 1 (3*) $2^{\frac{1}{2}}$ (4) $2^{\frac{1}{4}}$

Question ID : 4050361975

Option 1 ID : 4050367092

Option 2 ID : 4050367090

Option 3 ID : 4050367091

Option 4 ID : 4050367093

Sol. $2^{\frac{1}{4}+\frac{2}{16}+\frac{3}{48}+\frac{4}{128}+..........\infty}$

$= 2^{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+..........\infty}$

$= 2^{\frac{\frac{1}{4}}{1-\frac{1}{2}}}$

$= 2^{\frac{1}{2}}$

JEE Main Only topics

Statistics

4. Let the observations $x_i \left(1 \le i \le 10\right)$ satisfy the equations, $\sum_{i=1}^{10}\left(x_i - 5\right) = 10$ and $\sum_{i=1}^{10}\left(x_i - 5\right)^2 = 40$. If μ and λ are the mean and the variance of the observations, $x_1 - 3, x_2 - 3, \ldots.., x_{10} - 3$, then the ordered pair (μ, λ) is equal to :

माना प्रेक्षण $x_i\left(1\le i\le 10\right)$ समीकरणों $\sum_{i=1}^{10}\left(x_i-5\right)=10$ तथा $\sum_{i=1}^{10}\left(x_i-5\right)^2=40$ को संतुष्ट करते हैं। यदि μ तथा λ प्रेक्षणों $x_1-3, x_2-3, \ldots, x_{10}-3$ के क्रमशः माध्य तथा प्रसरण हैं, तो क्रमित युग्म (μ, λ) बराबर है :

(1) (6, 3) (2) (6, 6) (3) (3, 6) (4) (3, 3)

Ans. 4

Question ID : 4050361986

Option 1 ID : 4050367136

Option 2 ID : 4050367134

Option 3 ID : 4050367135

Option 4 ID : 4050367137

Sol. $\sum_{i=1}^{10}(x_i-5)=10$ ....(1)

$\sum_{i=1}^{10}(x_i-5)^2=40$ ....(2)

$\text{mean }(x_i-5)=\frac{\sum(x_i-5)}{10}=\frac{10}{10}=1$

$\mu=\text{mean }(x_i-3)=\text{mean }(x_i-5+2)=1+2=3$

$\lambda=\text{var}(x_i-3)\ =\text{var}(x_i-5)=\frac{\sum(x_i-5)^2}{10}-\frac{\sum(x_i-5)}{10}$

$=\frac{40}{10}-1$

$=4-1=3$

Algebra

Matrices

5. If the matrices $A=\begin{bmatrix}1&1&2\\1&3&4\\1&-1&3\end{bmatrix}$, B = adj A and C = 3A, then $\frac{|\text{adj}\,B|}{|C|}$ is equal to :

यदि आव्यूह $A=\begin{bmatrix}1&1&2\\1&3&4\\1&-1&3\end{bmatrix}$, B = adj A तथा C = 3A, हैं, तो $\frac{|\text{adj}\,B|}{|C|}$ का मान है :

(1) 16 (2) 72 (3) 8 (4) 2

Ans. 3

Question ID : 4050361972

Option 1 ID : 4050367080

Option 2 ID : 4050367081

Option 3 ID : 4050367079

Option 4 ID : 4050367078

Sol. $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, B = adjA., C = 3A

$|adj(B)| = |adj(adjA)| = |A|^{(n-1)^2} = |A|^4$ [n = 3]

$|C| = |3A| = 3^3|A| = 27|A|$

$\therefore \frac{|adj(B)|}{|C|} = \frac{|A|^3}{27}$

$\therefore |A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & 7 & 3 \end{vmatrix} = 1(9 + 4) - 1(3 - 4) + 2(-1 - 3)$

$= 13 + 1 - 8$
$= 14 - 8 = 6$

$= \frac{6 \times 6 \times 6}{27} = 8$

Algebra
Probability

6. In a box, there are 20 cards, out of which 10 are labelled as A and the remaining 10 are labelled as B. Cards are drawn at random, one after the other and with replacement, till a second A–card is obtained. The probability that the second A–card appears before the third B–card is :

एक बक्से में 20 कार्ड हैं जिनमें से 10 पर A अंकित किया गया है तथा शेष 10 पर B अंकित किय गया है। बक्से में से यादृच्छया एक के बाद एक (प्रतिस्थापना सहित) कार्ड तब तक निकाले गए जब तक कि दूसरा A से अंकित कार्ड न आ जाए। दूसरे A से अंकित कार्ड के तीसरे B से अंकित कार्ड से पहले आने की प्रायिकता है :

(1) $\frac{15}{16}$ (2) $\frac{9}{16}$ (3) $\frac{13}{16}$ (4) $\frac{11}{16}$

Ans. 4

Question ID : 4050361985

Option 1 ID : 4050367130

Option 2 ID : 4050367133

Option 3 ID : 4050367131

Option 4 ID : 4050367132

Sol. $P(A) = \frac{10}{20} = \frac{1}{2}$

$P(B) = \frac{10}{20} = \frac{1}{2}$

Required prob. = AA + ABA + BAA + ABBA + BBAA + BABA

$= \frac{1}{2}.\frac{1}{2} + \frac{1}{2}.\frac{1}{2}.\frac{1}{2} + \frac{1}{2}.\frac{1}{2}.\frac{1}{2} + \frac{1}{2}.\frac{1}{2}.\frac{1}{2}.\frac{1}{2} + \frac{1}{2}.\frac{1}{2}.\frac{1}{2}.\frac{1}{2} + \frac{1}{2}.\frac{1}{2}.\frac{1}{2}.\frac{1}{2}$

$\frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{11}{16}$

Coordinate Geometry

Straight Line

7. Let C be the centroid of the triangle with vertices (3, – 1), (1, 3) and (2, 4). Let P be the point of intersection of the lines $x + 3y - 1 = 0$ and $3x - y + 1 = 0$. Then the line passing through the points C and P also passes through the point :

माना शीर्षो (3, – 1), (1, 3) तथा (2, 4) वाले त्रिभुज का केंन्द्रक C है । माना रेखाओं $x + 3y - 1 = 0$ तथा $3x - y + 1 = 0$ का प्रतिच्छेदन बिन्दु P है, तो बिन्दुओं C तथा P से गुजरने वाली रेखा, निम्न मे से किस बिन्दु से भी गुजरती है ?

(1) (9, 7) (2) (7, 6) (3) (–9, –7) (4*) (–9, –6)

Ans. 4

Question ID : 4050361982

Option 1 ID : 4050367120

Option 2 ID : 4050367118

Option 3 ID : 4050367119

Option 4 ID : 4050367121

Sol. $C = \left(\frac{6}{3}, \frac{6}{3}\right) = (2\ ,\ 2)$

Let required line be

$(x + 3y - 1) + \lambda(3x - y + 1) = 0$

it passes through C(2, 2)

$(2 + 6 - 1) + \lambda(6 - 2 + 1) = 0$

$7 + 5\lambda = 0$

$\therefore 1 = -\frac{7}{5}$

$5(x + 3y - 1) - 7(3x - y + 1) = 0$

$-16x + 22y - 12 = 0$

$8x - 11y + 6 = 0$

$= (-9,\ -6)$

Differential Calculus

Continuity & Differentiability

8. If $f(x)=\begin{cases}\dfrac{\sin(a+2)x+\sin x}{x} & ; \; x<0\\ b & ; \; x=0\\ \dfrac{(x+3x^2)^{1/3}-x^{1/3}}{x^{4/3}} & ; \; x>0\end{cases}$

is continuous at x = 0, then a + 2b is equal to :

यदि $f(x)=\begin{cases}\dfrac{\sin(a+2)x+\sin x}{x} & ; \; x<0\\ b & ; \; x=0\\ \dfrac{(x+3x^2)^{1/3}-x^{1/3}}{x^{4/3}} & ; \; x>0\end{cases}$

x = 0 पर संतत है, तो a + 2b का मान है :

(1) –1 (2) 0 (3) –2 (4) 1

Ans. 2

Question ID : 4050361976

Option 1 ID : 4050367094

Option 2 ID : 4050367095

Option 3 ID : 4050367097

Option 4 ID : 4050367096

Sol. $\text{LHL} = \lim\limits_{x\to 0-}\dfrac{\sin(a+2)x+\sin x}{x} = (a+2)+1 = a+3$

$$\text{RHL} = \lim_{x\to 0+}\frac{(x+3x^2)^{\frac{1}{3}}-x^{\frac{1}{3}}}{x^{\frac{4}{3}}}$$

$$= \lim_{x\to 0+}\frac{x^{\frac{1}{3}}\left[(1+3x)^{\frac{1}{3}}-1\right]}{x^{\frac{4}{3}}}$$

$$= \lim_{x\to 0+}\frac{(1+3x)^{\frac{1}{3}}-1}{x}$$

$$= \lim_{x\to 0+}\frac{\frac{1}{3}(1+3x)^{-\frac{2}{3}}.3}{1} = 1$$

$a + 3 = b = 1$
$\therefore\ b = 1,\ a = -2$
$a + 2b = -2 + 2 = 0$

Integral Calculus

Indefinite Integration

9. The integral $\int \frac{dx}{(x+4)^{8/7}(x-3)^{6/7}}$ is equal to :

(where C is a constant of integration)

समाकल $\int \frac{dx}{(x+4)^{8/7}(x-3)^{6/7}}$ बराबर है :

(जहाँ C एक समाकलन अचर है)

(1) $\left(\frac{x-3}{x+4}\right)^{1/7}+C$ (2) $-\left(\frac{x-3}{x+4}\right)^{-1/7}+C$ (3) $\frac{1}{2}\left(\frac{x-3}{x+4}\right)^{3/7}+C$ (4) $-\frac{1}{13}\left(\frac{x-3}{x+4}\right)^{-13/7}+C$

Ans. 1

Question ID : 4050361980

Option 1 ID : 4050367111

Option 2 ID : 4050367112

Option 3 ID : 4050367110

Option 4 ID : 4050367113

Sol. $I = \int \frac{dx}{(x+4)^{\frac{8}{7}}(x-3)^{\frac{6}{7}}}$

$= \int \frac{dx}{(x+4)^2\left(\frac{x-3}{x+4}\right)^{\frac{6}{7}}}$

Put $\frac{x-3}{x+4} = z$

$\left(\frac{x+4-x+3}{(x+4)^2}\right)dx = dz$

$\frac{dx}{(x+4)^2} = \frac{dz}{7}$

$= \int \frac{dz}{7.(z)^{\frac{6}{7}}} = \frac{1}{7}.\frac{z^{-\frac{6}{7}+1}}{-\frac{6}{7}+1}+C$

$= z^{\frac{1}{7}}+C$

$= \left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + C$

Algebra

P & C

10. If the number of five digit numbers with distinct digits and 2 at the 10^{th} place is 336 k, then k is equal to :

यदि विभिन्न अंकों वाली पाँच अंकों की संख्याओं, जिनका दहाई का अंक 2 है, की संख्या 336 k है, तो k का मान बराबर है :

(1) 7 (2) 6 (3) 8 (4) 4

Ans. 3

Question ID : 4050361974

Option 1 ID : 4050367087

Option 2 ID : 4050367088

Option 3 ID : 4050367086

Option 4 ID : 4050367089

Sol. $\underline{8}\ \underline{8}\ \underline{7}\ \underline{1}\ \underline{6}$

$= 8 \times 8 \times 7 \times 1 \times 6$

$= 8 \times 336 = 336K$

$\therefore\ k = 8$

Algebra

Quadratic Equation

11. The number of real roots of the equation, $e^{4x} + e^{3x} - 4e^{2x} + e^{x} + 1 = 0$ is :

समीकरण $e^{4x} + e^{3x} - 4e^{2x} + e^{x} + 1 = 0$ के वास्तविक मूलों की संख्या है :

(1) 3 (2*) 1 (3) 2 (4) 4

Ans. 2

Question ID : 4050361970

Option 1 ID : 4050367072

Option 2 ID : 4050367070

Option 3 ID : 4050367071

Option 4 ID : 4050367073

Sol. $e^{4x} + e^{3x} - 4e^{2x} + e^{x} + 1 = 0$

Let $e^{x} = t\ \ (t > 0)$

$t^4 + t^3 - 4t^2 + t + 1 = 0$

$t^3(t-1) + 2t^2(t-1) - 2t(t-1) - (t-1) = 0$

$(t-1)^2 (t^2 + 3t + 1) = 0$

$\Rightarrow t = 1 \qquad t = \frac{3 - \pm\sqrt{9-4}}{2}$

$t = 1$ $\qquad t = \dfrac{-3 \pm \sqrt{5}}{2}$ (– ve roots) ; (Rejected)

$e^x = 1$

$\therefore x = 0$

Differential Calculus

Maxima & Minima

12. Let f be any function continuous on [a, b] and twice differentiable on (a, b). If for all $x \in (a, b), f'(x) > 0$ and $f''(x) < 0$, then for $c \in (a, b)$, $\dfrac{f(c) - f(a)}{f(b) - f(c)}$ is greater than :

माना f कोई फलन है जोकि [a, b] में संतत तथा (a, b) में दो बार अवकलनीय है। यदि सभी $x \in (a, b), f'(x) > 0$ तथा $f''(x) < 0$, हैं, तो किसी भी $c \in (a, b)$, $\dfrac{f(c) - f(a)}{f(b) - f(c)}$ निम्न में से किससे बड़ा है ?

(1) $\dfrac{c-a}{b-c}$ (2) $\dfrac{b-c}{c-a}$ (3) $\dfrac{b+a}{b-a}$ (4) 1

Ans. 1

Question ID : 4050361978

Option 1 ID : 4050367103

Option 2 ID : 4050367102

Option 3 ID : 4050367105

Option 4 ID : 4050367104

Sol. $f'(x) > 0$ and $f''(x) < 0$ $\qquad \forall\, x \in (a, b)$

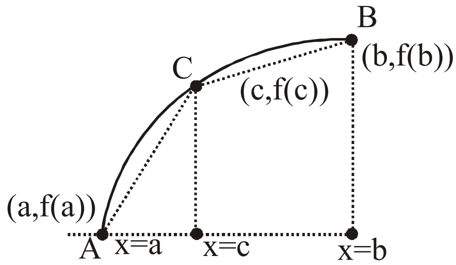
$m_{AC} = \dfrac{f(c) - f(a)}{c - a}$

$m_{CB} = \dfrac{f(b) - f(b)}{b - c}$

$m_{AC} > m_{CB}$

$\therefore \dfrac{f(c) - f(a)}{b - c} > \dfrac{f(b) - f(c)}{b - c}$

$\dfrac{f(c) - f(a)}{f(b) - f(c)} > \dfrac{c - a}{b - c}$



Integral Calculus

Definite Integration

13. The value of $\displaystyle\int_0^{2\pi} \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} dx$ is equal to :

$\displaystyle\int_0^{2\pi} \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} dx$ का मान है :

(1) 4π (2*) π^2 (3) 2π (4) $2\pi^2$

Ans. 2

Question ID : 4050361981

Option 1 ID : 4050367114

Option 2 ID : 4050367117

Option 3 ID : 4050367115

Option 4 ID : 4050367116

Sol. $$I=\int_0^{2\pi}\frac{x\sin^8 x}{\sin^8 x+\cos^8 x}dx \qquad ....(1)$$

Using property

$$\left[\int_a^b f(x)\,dx=\int_a^b f(a+b-x)\,dx\right]$$

$$I=\int_0^{2\pi}\frac{(2\pi-x)(\sin(2\pi-x))^8}{(\sin(2\pi-x)^8+(\cos(2\pi-x))^8}dx \qquad ....(2)$$

(1) + (2)

$$2I=\int_0^{2\pi}\frac{2\pi.\sin^8(x)}{\sin^8 x+\cos^8 x}dx$$

$$I=\pi\int_0^{2\pi}\frac{\sin^8 x}{\sin^8 x+\cos^8 x}dx$$

$$\left[\int_0^{2a} f(x)\,dx=\int_0^{a} f(x)\,dx \quad \because f(x)=f(2a-x)\right]$$

$$I=2\pi\int_0^{\pi}\frac{\sin^8 x}{\sin^8 x+\cos^8 x}dx$$

Again using above property.

$$I=4\pi\int_0^{\pi/2}\frac{\sin^8 x}{\sin^8 x+\cos^8 x}dx \qquad ....(3)$$

$$\text{Let } I_1=\int_0^{\pi/2}\frac{\sin^8 x}{\sin^8 x+\cos^8 x}dx \qquad ....(4)$$

$$I_1=\int_0^{\pi/2}\frac{\sin^8 x}{\cos^8 x+\sin^8 x}dx \qquad ....(5)$$

$$\left[\int_a^b f(x)\,dx=\int_a^b f(a+b-x)\,dx\right]$$

$$2I_1=\int_0^{\pi/2}dx$$

$\therefore\ I = 4\pi \times \frac{\pi}{4} = \pi^2$

Integral Calculus

Definite Integration

14. If for all real triplets (a, b, c), $f(x) = a + bx + cx^2$, then $\int_0^1 f(x)\,dx$ is equal to :

यदि सभी वास्तविक त्रिकों (a, b, c), $f(x) = a + bx + cx^2$ है, तो $\int_0^1 f(x)\,dx$ बराबर है :

(1) $\frac{1}{2}\left\{f(1) + 3f\left(\frac{1}{2}\right)\right\}$

(2) $\frac{1}{6}\left\{f(0) + f(1) + 4f\left(\frac{1}{2}\right)\right\}$

(3) $2\left\{3f(1) + 2f\left(\frac{1}{2}\right)\right\}$

(4) $\frac{1}{3}\left\{f(0) + f\left(\frac{1}{2}\right)\right\}$

Ans. 2

Question ID : 4050361969

Option 1 ID : 4050367067

Option 2 ID : 4050367069

Option 3 ID : 4050367068

Option 4 ID : 4050367066

Sol. $f(x) = a + bx + cx^2$

$$\int_0^1 f(x)\,dx = \left[ax + \frac{bx^2}{2} + \frac{cx^2}{3}\right]_0^1$$

$$= a + \frac{b}{2} + \frac{c}{3}$$

$f(0) = a$

$f(1) = a + b + c$

$$f\left(\tfrac{1}{2}\right) = a + \frac{b}{2} + \frac{c}{4}$$

$$f(0) + f(1) + 4f\left(\tfrac{1}{2}\right) = a + (a + b + c) + (4a + 2b + c)$$

$= 6a + 3b + 2c$

$$\int_0^1 f(x)\,dx = \frac{1}{6}\left\{f(0) + f(1) + 4f\left(\tfrac{1}{2}\right)\right\}$$

Algebra

Determinant

15. If for some α and β in R, the intersection of the following three planes

यदि R में किन्ही α तथा β के लिए निम्न तीन समतलों

$x + 4y - 2z = 1$

$x + 7y - 5z = \beta$

$x + 5y + \alpha z = 5$

is a line in $\mathbf{R}^3$, then $\alpha + \beta$ is equal to :

का प्रतिच्छेदन, $\mathbf{R}^3$ में एक रेखा है, तो $\alpha + \beta$ का मान है :

(1) 2   (2) –10   (3) 0   (4) 10

Ans. 4

Question ID : 4050361973

Option 1 ID : 4050367084

Option 2 ID : 4050367082

Option 3 ID : 4050367083

Option 4 ID : 4050367085

Sol. $x + 4y - 2z = 1$

$x + 7y - 5z = \beta$

$x + 5y + \alpha z = 5$

$\Rightarrow$ System has infinite solutions

$$D = \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0$$

$1(7\alpha + 25) - 4(\alpha + 5) - 2(5 - 7) = 0$

$\Rightarrow 7\alpha + 25 - 4\alpha - 20 + 4 = 0$

$3\alpha + 9 = 0$

$\therefore \alpha = -3$

$$D_x = \begin{vmatrix} 1 & 4 & -2 \\ \beta & 7 & -5 \\ 5 & 5 & -3 \end{vmatrix} = 0$$

$\Rightarrow 1(-21 + 25) - 4(-3\beta + 25) - 2(5\beta - 35) = 0$

$\Rightarrow 4 + 12\beta - 100 - 10\beta + 70 = 0$

$2\beta - 26 = 0$

$\beta = 13$

$$D_y = \begin{vmatrix} 1 & 1 & -2 \\ 1 & \beta & -5 \\ 1 & 5 & -3 \end{vmatrix} = 0$$

$\Rightarrow 1(-3\beta + 25) - 1(-3 + 10) + 1(-5 + 2\beta) = 0$

$-3\beta + 25 - 7 - 5 + 2\beta = 0$

$-\beta + 13 = 0$

$\therefore \beta = 13$

$$D_z = \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & \beta \\ 1 & 5 & 5 \end{vmatrix} = 0$$

$\Rightarrow 1(5-7) - \beta\,(5-4) + 5\,(7-4) = 0$

$-2 - \beta + 15 = 0$

$\beta = 13$

$\alpha + \beta = -3 + 13 = 10$

Differential Calculus

Tangent and normal

16. A spherical iron ball of 10 cm radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate (in cm / min.) at which of the thickness of ice decreases, is :

एक 10 cm त्रिज्या वाली गोलाकार लोहे की गेंद को बर्फ की एक समान मोटाई वाली परत से लेप लिया गया है, जो कि 50 cm^3/min की दर से पिघलती है। जब बर्फ की परत की मोटाई 5 cm है, उस समय बर्फ की मोटाई के घटने की दर (cm / min में), है:

(1) $\frac{1}{54\pi}$ (2) $\frac{1}{36\pi}$ (3) $\frac{5}{6\pi}$ (4*) $\frac{1}{18\pi}$

Ans. 4

Question ID : 4050361977

Option 1 ID : 4050367101

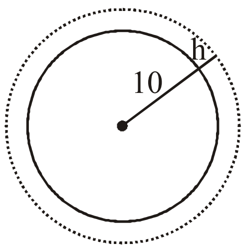
Option 2 ID : 4050367098

Option 3 ID : 4050367099

Option 4 ID : 4050367100

Sol. Let thickness of ice at time t is h cm.

$$V = \frac{4}{3}\pi(10+h)^3 - \frac{4}{3}\pi(10)^3$$



$$\frac{dv}{dt} = \frac{4}{3}\pi.3(10+h)^2.\frac{dh}{dt} - 0$$

$$\Rightarrow \quad -50 = 4\pi.(10+5)^2.\frac{dh}{dt}$$

$$\therefore \frac{dh}{dt} = \frac{50}{4\pi \times 15 \times 15} = \frac{-1}{18\pi}$$

$$\frac{dh}{dt} = \frac{-1}{18\pi}\,\frac{cm}{min}$$

Integral Calculus

Differential Equation

17. If $f'(x) = \tan^{-1}(\sec x + \tan x)$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, and $f(0) = 0$, then $f(1)$ is equal to :

यदि $f'(x) = \tan^{-1}(\sec x + \tan x)$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, तथा $f(0) = 0$ है, तो $f(1)$ का मान है :

(1) $\frac{\pi+2}{4}$ (2) $\frac{\pi+1}{4}$ (3) $\frac{1}{4}$ (4) $\frac{\pi-1}{4}$

Ans. 2

Question ID : 4050361979

Option 1 ID : 4050367107

Option 2 ID : 4050367108

Option 3 ID : 4050367109

Option 4 ID : 4050367106

Sol. $f'(x) = \tan^{-1}(\sec x + \tan x), \frac{-\pi}{2} < x < \frac{\pi}{2}$

$$= \tan^{-1}\left[\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right]$$

$$= \tan^{-1}\left[\frac{1+\sin x}{\cos x}\right]$$

$$= \tan^{-1}\left[\frac{\left(\cos x/2 + \sin x/2\right)^2}{\cos^2 x/2 - \sin^2 x/2}\right]$$

$$= \tan^{-1}\left[\frac{\cos x/2 + \sin x/2}{\cos x/2 - \sin x/2}\right]$$

$$= \tan^{-1}\left[\frac{1+\tan x/2}{1-\tan x/2}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right]$$

$$-\frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{4}$$

$$\Rightarrow 0 < \frac{\pi}{4} + \frac{x}{2} < \frac{\pi}{2}$$

$$f'(x) = \frac{\pi}{4} + \frac{x}{2}$$

$f(x) = \frac{\pi}{4}x + \frac{x^2}{4} + C$

$f(0) = 0 \qquad \therefore\ C = 0$

$f(x) = \frac{\pi}{4}x + \frac{x^2}{4}$

$f(1) = \frac{\pi}{4} + \frac{1}{4} = \frac{\pi+1}{4}$

Trigonometry

Trigonometric Ratio and Identities

18. The value of $\cos^3\left(\frac{\pi}{8}\right).\cos\left(\frac{3\pi}{8}\right) + \sin^3\left(\frac{\pi}{8}\right).\sin\left(\frac{3\pi}{8}\right)$ is :-

$\cos^3\left(\frac{\pi}{8}\right).\cos\left(\frac{3\pi}{8}\right) + \sin^3\left(\frac{\pi}{8}\right).\sin\left(\frac{3\pi}{8}\right)$ का मान है :

(1) $\frac{1}{4}$ (2) $\frac{1}{2}$ (3*) $\frac{1}{2\sqrt{2}}$ (4) $\frac{1}{\sqrt{2}}$

Ans. 3

Question ID : 4050361987

Option 1 ID : 4050367141

Option 2 ID : 4050367139

Option 3 ID : 4050367140

Option 4 ID : 4050367138

Sol. $\cos^3\left(\frac{\pi}{8}\right).\cos\left(\frac{3\pi}{8}\right) + \sin^3\left(\frac{\pi}{8}\right).\sin\left(\frac{3\pi}{8}\right)$

Let $\theta = \frac{\pi}{8}$

$= \cos^3\theta.\cos(3\theta) + \sin^3\theta \sin(3\theta)$

$= \cos^3\theta(4\cos^3\theta - 3\cos\theta) + \sin^3\theta(3\sin\theta - 4\sin^3\theta)$

$= 4(\cos^6\theta - \sin^6\theta) + 3[\sin^4\theta - \cos^4\theta]$

$= 4(\cos^2\theta - \sin^2\theta)(\cos^4\theta + \cos^2\theta . \sin^2\theta + \sin^4\theta) + 3(\sin^2\theta - \cos^2\theta).1$

$= 4\cos2\theta((\cos^2\theta + \sin^2\theta)^2 - \sin^2\theta\cos^2\theta) - 3\cos2\theta$

$= 4\cos2\theta\left(1 - \frac{1}{4}\sin^2 2\theta\right) - \frac{3}{\sqrt{2}}$

$= \frac{4}{\sqrt{2}} \times \frac{7}{8} - \frac{3}{\sqrt{2}}$

$= \frac{7}{2\sqrt{2}} - \frac{6}{2\sqrt{2}} = +\frac{1}{2\sqrt{2}}$

Coordinate Geometry

Circle

19. A circle touches the y–axis at the point (0, 4) and passes through the point (2, 0). Which of the following lines is **not** a tangent to this circle?

एक वृत्त y–अक्ष को बिन्दु (0, 4) पर स्पर्श करता है तथा बिन्दु (2, 0) से होकर जाता है। निम्न में से कौन सी रेखा इस वृत्त की स्पर्श रेखा नहीं है ?

(1) $3x + 4y - 6 = 0$ (2) $4x - 3y + 17 = 0$ (3) $3x - 4y - 24 = 0$ (4) $4x + 3y - 8 = 0$

Ans. 4

Question ID : 4050361983

Option 1 ID : 4050367123

Option 2 ID : 4050367125

Option 3 ID : 4050367122

Option 4 ID : 4050367124

Sol. Let required circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$

$f^2 = c$ ....(1)

(0, 4) lies on it

$16 + 8f + c = 0$ ....(2)

(2, 0) lies on it

$4 + 4g + c = 0$ ....(3)

Using (1) & (2)

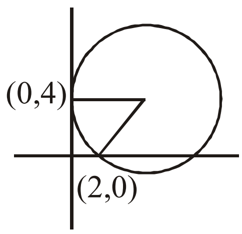
$f^2 + 8f + 16 = 0$

$(f + 4)^2 = 0$ $\therefore$ $f = -4$ C = 16

Using (3) $g = -5$

$\therefore$ $x^2 + y^2 - 10x - 8y + 16 = 0$

Centre = (5, 4) radius = 5



By Condition of tangency

$p = r$

for

$4x + 3y - 8 = 0$

here $(p \neq r)$.

Algebra

Complex Number

20. Let z be a complex number such that $\left|\frac{z-i}{z+2i}\right|=1$ and $|z|=\frac{5}{2}$. Then the value of $|z+3i|$ is :

माना z एक ऐसी सम्मिश्र संख्या है, कि $\left|\frac{z-i}{z+2i}\right|=1$ है तथा $|z|=\frac{5}{2}$ है, तो $|z+3i|$ का मान है :

(1) $2\sqrt{3}$ (2) $\frac{15}{4}$ (3) $\frac{7}{2}$ (4) $\sqrt{10}$

Ans. 3

Question ID : 4050361971

Option 1 ID : 4050367075

Option 2 ID : 4050367074

Option 3 ID : 4050367076

Option 4 ID : 4050367077

Sol. $\left|\frac{z-i}{z+2i}\right|=1$

$\therefore\ |z-i|=|z-(-2i)|$

$\Rightarrow\ \perp^r$ bisector of AB where

A(i) and B(– 2i)

$\therefore$ Equation of $\perp^r$ bisector $y=-\frac{1}{2}$ ....(1)

$|z|=\frac{5}{2}$

Circle with centred at (0, 0) and radius 5/2.

$\therefore\ x^2+y^2=\frac{25}{4}$ ....(2)

Solving (1) & (2)

$x^2+\frac{1}{4}=\frac{25}{4}$

$x^2=6$ $\therefore\ x=\pm\sqrt{6}$

$z=\left(\pm\sqrt{6}+\left(-\frac{1}{2}\right)i\right)$

$\therefore\ |z+3i|=\left|\left(\pm\sqrt{6}+\frac{5}{2}i\right)\right|$

$=\sqrt{6+\frac{25}{4}}=\sqrt{\frac{49}{4}}=\frac{7}{2}$

Trigonometry

Trigonometric Equation

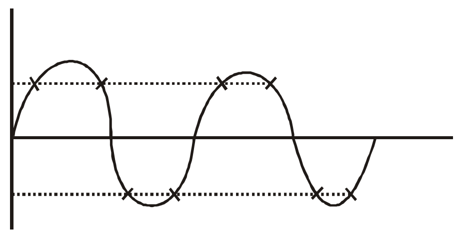
21. The number of distinct solutions of the equation, $\log_{\frac{1}{2}}|\sin x| = 2 - \log_{\frac{1}{2}}|\cos x|$ in the interval $[0, 2\pi]$ is _________.

समीकरण $\log_{\frac{1}{2}}|\sin x| = 2 - \log_{\frac{1}{2}}|\cos x|$ के अंतराल $[0, 2\pi]$ में भिन्न हलों की संख्या है __________ ।

Ans. 8

Question ID : 4050361990

Sol. $\log_{1/2}|\sin x| + \log_{1/2}|\cos x| = 2$



$\Rightarrow \log_{1/2}|\sin x||\cos x| = 2$

$\Rightarrow |\sin x||\cos x| = \frac{1}{4}$

$\Rightarrow |\sin 2x| = \frac{1}{2}$

Number of distinct solutions = 8

Algebra

Binomial theorem

22. The coefficient of x^4 in the expansion of $(1 + x + x^2)^{10}$ is ______.

$(1 + x + x^2)^{10}$ के प्रसार में x^4 का गुणांक है _______ ।

Ans. 615

Question ID : 4050361989

Sol. General term $= \frac{\lfloor 10}{\lfloor \alpha \lfloor \beta \lfloor \gamma} (1)^{\alpha}(x)^{\beta}(x^2)^{\gamma} = \frac{\lfloor 10}{\lfloor \alpha \lfloor \beta \lfloor \gamma} .x^{\beta+2\gamma}$

$\alpha + \beta + \gamma = 10$ ....(1)

$\therefore\ \beta + 2\gamma = 4$ ....(2)

α	β	γ	Coeff of x^4
8	0	2	$\frac{\lfloor 10}{\lfloor 8 \lfloor 2} = 45$
7	2	1	$\frac{\lfloor 10}{\lfloor 7 \lfloor 2 \lfloor 1} = 360$
6	4	0	$\frac{\lfloor 10}{\lfloor 6 \lfloor 4} = 210$

Coeff of x^4 = 615

Vectors

Vectors

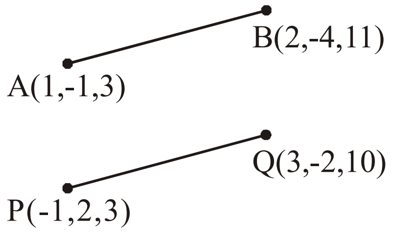
23. The projection of the line segment joining the points (1, –1, 3) and (2, –4, 11) on the line joining the points (–1, 2, 3) and (3, –2, 10) is __________.

बिंदुओं (1, –1, 3) तथा (2, –4, 11) को मिलाने वाले रेखाखण्ड का बिन्दुओं (–1, 2, 3) तथा (3, –2, 10) को मिलाने वाली रेखा पर प्रक्षेप है __________ ।

Ans. 8

Question ID : 4050361993

Sol.



$\vec{a} = \overrightarrow{AB} = \hat{i} - 3\hat{j} + 8\hat{k}$

$\vec{b} = \overrightarrow{PQ} = 4\hat{i} - 4\hat{j} + 7\hat{k}$

Projection of the line segment joining AB on line PQ

$= \vec{a}.\hat{b}$

$$= \left(\hat{i} - 3\hat{j} + 8\hat{k}\right).\frac{\left(4\hat{i} - 4\hat{j} + 7\hat{k}\right)}{\sqrt{16+16+49}}$$

$$= \frac{4+12+56}{9} = \frac{72}{9} = 8$$

Integral Calculus

Differential Equation

24. If for $x \geq 0$, $y = y(x)$ is the solution of the differential equation, $(x + 1)dy = ((x + 1)^2 + y - 3)dx$, $y(2) = 0$, then y(3) is equal to __________.

यदि $x \geq 0$ के लिए $y = y(x)$, अवकल समीकरण $(x + 1)dy = ((x + 1)^2 + y - 3)dx$, $y(2) = 0$, का हल है, तो y(3) का मान है __________ ।

Ans. 3

Question ID : 4050361992

Sol. $x \geq 0$, $y = y(x)$

$(x + 1)dy = ((x + 1)^2 + y - 3)dx$

$$\frac{dy}{dx} = \frac{(x+1)^2 - 3}{x+1} + \frac{y}{x+1}$$

$$\frac{dy}{dx} - \frac{y}{x+1} = \frac{(x+1)^2 - 3}{x+1}$$

$$\text{I.F} = e^{\int -\frac{1}{(x+1)}dx} = e^{-\ln(1+x)} = \frac{1}{1+x}$$

Solution of diff. equation

$$y.\frac{1}{1+x}=\int\frac{(x+1)^2-3}{(x+1)}.\left(\frac{1}{1+x}\right)dx+c$$

$$y\left(\frac{1}{1+x}\right)=\int\left(1-\frac{3}{(1+x)^2}\right)dx+c$$

$y = x(x + 1) + 3 + c(1 + x)$

$y(2) = 0$

$0 = 6 + 3 + c.3$

$\therefore\ c = -3$

$\therefore\ y = x + x^2 + 3 - 3\ - 3x$

$y = x^2 - 2x$

$y(3) = 9 - 6 = 3$

Vectors

Vectors

25. If the vectors, $\vec{p}=(a+1)\hat{i}+a\hat{j}+a\hat{k}$, $\vec{q}=a\hat{i}+(a+1)\hat{j}+a\hat{k}$ and $\vec{r}=a\hat{i}+a\hat{j}+(a+1)\hat{k}$ $(a\in\mathbf{R})$ are coplanar and $3\left(\vec{p}.\vec{q}\right)^2-\lambda\left|\vec{r}\times\vec{q}\right|^2=0$, then the value of λ is __________.

यदि सदिश $\vec{p}=(a+1)\hat{i}+a\hat{j}+a\hat{k}$, $\vec{q}=a\hat{i}+(a+1)\hat{j}+a\hat{k}$ तथा $\vec{r}=a\hat{i}+a\hat{j}+(a+1)\hat{k}$, $(a\in\mathbf{R})$ सहतलीय हैं तथा $3\left(\vec{p}.\vec{q}\right)^2-\lambda\left|\vec{r}\times\vec{q}\right|^2=0$ है, तो λ का मान है __________ ।

Ans. 1

Question ID : 4050361991

Sol. $\therefore$ $[\vec{p}\ \vec{q}\ \vec{r}]$ are coplaner

$[\vec{p}\ \vec{q}\ \vec{r}]=0$

$$\begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix}=0$$

$\Rightarrow a + a+ + a+ 1 = 0$

$\Rightarrow\ a=-\frac{1}{3}$

$\vec{p}.\vec{q}=a(a+1)+a(a+1)+a^2 = 3a^2+2a$

$$\vec{r}\times\vec{q}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & a & a+1 \\ a & a+1 & a \end{vmatrix}$$

$=\ \hat{i}\,(a^2 - a^2 - 2a - 1\ ) - \hat{j}\,(a^2 - a^2 - a) +\ \hat{k}\,(a^2 + a - a^2)$

$\vec{r}\times\vec{q}=-(2a+1)\,\hat{i}+a\hat{j}+a\hat{k}$

$$|\vec{r}\times\vec{q}| = \sqrt{(2a+1)^2 + a^2 + a^2}$$

$$= \sqrt{2a^2 + (2a+1)^2}$$

$$\lambda = \frac{3\left(\vec{p}.\vec{q}\right)^2}{|\vec{r}\times\vec{q}|^2}$$

$$= \frac{3\times(3a^2+2a)^2}{2a^2+(2a+1)^2}$$

$$\therefore \ \lambda = \frac{3\left[3\times\frac{1}{9}-\frac{2}{3}\right]^2}{2\times\frac{1}{9}+\left(-\frac{2}{3}+1\right)^2}$$

$$= \frac{3\left[-\frac{1}{3}\right]^2}{\frac{2}{9}+\frac{1}{9}} = \frac{3\times\left(\frac{1}{3}\right)^2}{\left(\frac{1}{3}\right)}$$

$$= 3\times\frac{1}{3} = 1$$