MATHS

09 Jan. 2020 [EVENING]

JEE MAIN PAPER ONLINE

RED COLOUR IS ANSWER IN JEE-MAIN

Algebra

Binomial theorem

1. In the expansion of $\left(\frac{x}{\cos\theta}+\frac{1}{x\sin\theta}\right)^{16}$, if l_1 is the least value of the term independent of x when $\frac{\pi}{8}\le\theta\le\frac{\pi}{4}$ and l_2 is the least value of the term independent of x when $\frac{\pi}{16}\le\theta\le\frac{\pi}{8}$, then the ratio $l_2 : l_1$ is equal to :

$\left(\frac{x}{\cos\theta}+\frac{1}{x\sin\theta}\right)^{16}$ के प्रसार में, यदि x से स्वतंत्र पद का निम्नतम मान l_1 है जब $\frac{\pi}{8}\le\theta\le\frac{\pi}{4}$ तथा x से स्वतंत्र पद का निम्नतम मान l_2 है जब $\frac{\pi}{16}\le\theta\le\frac{\pi}{8}$, तो अनुपात $l_2 : l_1$ बराबर है :

(1) 1 : 16 (2) 16 : 1 (3) 1 : 8 (4) 8 : 1

A. 2

Question ID : 4050362199

Option 1 ID : 4050367851

Option 2 ID : 4050367853

Option 3 ID : 4050367852

Option 4 ID : 4050367854

Sol. $T_{r+1} = {}^{16}C_r\left(\frac{x}{\cos\theta}\right)^{16-r}\left(\frac{1}{x\sin\theta}\right)^{r}$

$= {}^{16}C_r\ x^{16-2r}\times(\cos\theta)^{r-16}\,(\sin\theta)^{-r}$

$= 16-2r=0 \qquad \Rightarrow r=8$

$= {}^{16}C_8\ (\cos\theta)^{-8}\,(\sin\theta)^{-8}$

Term independent of x $= {}^{16}C_8\times\frac{1}{(\sin\theta\ \cos\theta)^8}$

$= {}^{16}C_8\times\frac{2^8}{(\sin 2\theta)^8}$

$l_1 : \frac{\pi}{4}\le 2\theta\le\frac{\pi}{2} \qquad l_2 : \frac{\pi}{8}\le 2\theta\le\frac{\pi}{4}$

$l_1 : {}^{16}C_8\times 2^8 \qquad l_2 : {}^{16}C_8\times 2^{12}$

$l_2 : l_1 \Rightarrow 2^4 : 1 \Rightarrow 16 : 1$

Coordinate Geometry

Ellipse

2. The length of the minor axis (along y–axis) of an ellipse in the standard form is $\frac{4}{\sqrt{3}}$. If this ellipse touches the line, $x + 6y = 8$; then its eccentricity is :

मानक रूप में एक दीर्घवृत्त के लघु अक्ष (y–अक्ष के अनुदिश) की लम्बाई $\frac{4}{\sqrt{3}}$ है। यदि यह दीर्घवृत्त, रेखा $x + 6y = 8$ को स्पर्श करता है, तो इसकी उत्केन्द्रता है:

(1) $\sqrt{\frac{5}{6}}$ (2) $\frac{1}{3}\sqrt{\frac{11}{3}}$ (3) $\frac{1}{2}\sqrt{\frac{5}{3}}$ (4) $\frac{1}{2}\sqrt{\frac{11}{3}}$

A. 4

Question ID : 4050362208

Option 1 ID : 4050367889

Option 2 ID : 4050367887

Option 3 ID : 4050367890

Option 4 ID : 4050367888

Sol. Equation of tangent $\Rightarrow y = mx \pm\sqrt{a^2m^2 + b^2}$

Compare with $6y = -x + 8$

$$y = -\frac{x}{6} + \frac{4}{3}$$

$$\frac{1}{1} = \frac{m}{-\frac{1}{6}} = \pm\frac{\sqrt{a^2m^2 + b^2}}{\frac{4}{3}}$$

$$1 = -6m = \pm\frac{3}{4}\sqrt{a^2m^2 + b^2}$$

$$m = -\frac{1}{6}$$

$$\frac{\pm 4}{3} = \sqrt{a^2m^2 + b^2}$$

$$b = \frac{2}{\sqrt{3}}$$

$$\Rightarrow a^2 = 16$$

$$e^2 = 1 - \frac{b^2}{a^2}$$

Algebra

Quadratic Equation

3. Let $a, b \in \mathbf{R}, a \neq 0$ be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to :

माना $a, b \in \mathbf{R}, a \neq 0$ इस प्रकार हैं कि समीकरण $ax^2 - 2bx + 5 = 0$ का α पुनरावृत्त मूल है, जो समीकरण $x^2 - 2bx - 10 = 0$ का भी एक मूल है। यदि β इस समीकरण का दूसरा मूल है, तो $\alpha^2 + \beta^2$ बराबर है :

(1) 28 (2) 25 (3) 26 (4) 24

A. 2

Question ID : 4050362195

Option 1 ID : 4050367838

Option 2 ID : 4050367836

Option 3 ID : 4050367837

Option 4 ID : 4050367835

Sol. $ax^2 - 2bx + 5 = 0$ (roots α, α)

$\alpha = \frac{b}{a}$ $\alpha^2 = \frac{5}{a}$

$b^2 = 5a$

$x^2 - 2bx - 10 = 0$ (roots α, β)

$\alpha + \beta = 2b$ $\alpha\beta = -10$

$$\frac{b^2}{a^2} - \frac{2b^2}{a} - 10 = 0$$

$b^2 - 2ab^2 - 10a^2 = 0$

$a = \frac{1}{4}$

$b^2 = \frac{5}{4}$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 4b^2 + 20 = 4 \times \frac{5}{4} + 20 = 25$$

Algebra

Determinant

4. Let a – 2b + c = 1.

If $f(x)=\begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$, then :

माना Let a – 2b + c = 1है। यदि

If $f(x)=\begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$ है, तो :

(1) f(50) = 1 (2) f(50) = – 501 (3) f(–50) = –1 (4) f(–50) = 501

A. 1

Question ID : 4050362197

Option 1 ID : 4050367845

Option 2 ID : 4050367843

Option 3 ID : 4050367846

Option 4 ID : 4050367844

Sol. a – 2b + c = 1

$$f(x)=\begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$$

$R_1 \rightarrow R_1 + R_3 - 2R_2$

$$f(x)=\begin{vmatrix} 2x+a+c-2x-2b & 2x+6-2x-6 & 2x+4-2x-4 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$$

$$f(x)=\begin{vmatrix} 1 & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$$

$= 1((x+3)^2 - (x+4)(x+2)$

$= x^2 + 6x + 9 - x^2 - 6x - 8 = 1$

Coordinate Geometry

Parabola

5. If one end of a focal chord AB of the parabola $y^2 = 8x$ is at $A\left(\frac{1}{2}, -2\right)$, then the equation of the tangent to it at B is :

यदि परवलय $y^2 = 8x$ की एक नाभि जीवा AB का एक छोर $A\left(\frac{1}{2}, -2\right)$ पर है, तो B पर इसकी स्पर्श–रेखा का समीकरण है:

(1) $2x + y - 24 = 0$ (2) $x - 2y + 8 = 0$ (3) $x + 2y + 8 = 0$ (4) $2x - y - 24 = 0$

A. 2

Question ID : 4050362209

Option 1 ID : 4050367891

Option 2 ID : 4050367893

Option 3 ID : 4050367892

Option 4 ID : 4050367894

Sol. $y^2 = 8x$ $a = 2$

$A\left(\frac{1}{2}, -2\right) = (2t^2, 4t)$

$t = -\frac{1}{2}$

$t_1 t_2 = -1$ $\Rightarrow -\frac{1}{2} \times t2 = -1$ $\Rightarrow t_2 = 2$

R(8, 8)

Tangent $\Rightarrow t = 0$ $\Rightarrow y \times 8 = 8\left(\frac{x+8}{2}\right)$ $\Rightarrow 2y = x + 8$

$x - 2y + 8 = 0$

Differential Calculus

Methods of Differentiation

6. Let f and g be differentiable functions on **R** such that fog is the identity function. If for some, $a, b \in \mathbf{R}$, $g'(a) = 5$ and $g(a) = b$, then $f'(b)$ is equal to :

माना R पर अवकलनीय फलन f फलन g इस प्रकार हैं कि fog तत्समक फलन है। यदि किसी $a, b \in \mathbf{R}$ के लिए $g'(a) = 5$ तथा $g(a) = b$ हैं, तो $f'(b)$ बराबर है:

(1) $\frac{1}{5}$ (2) $\frac{2}{5}$ (3) 5 (4) 1

A. 1

Question ID : 4050362203

Option 1 ID : 4050367867

Option 2 ID : 4050367868

Option 3 ID : 4050367870

Option 4 ID : 4050367869

Sol. $f(g(x)) = x$

$f'(g(x))\,g'(x) = 1$

Put $x = a$

$f'(g(a))\,g'(a) = 1$

$f'(b)\,5 = 1$

$f'(b) = \frac{1}{5}$

Differential Calculus

Continuity & Differentiability

7. Let [t] denote the greatest integer $\le$ t and $\lim_{x\to 0} x\left[\frac{4}{x}\right] = A$. Then the function, $f(x) = [x^2]\sin(\pi x)$ is discontinuous, when x is equal to :

माना [t] महत्तम पूर्णांक $\le$ t को दर्शाता है तथा $\lim_{x\to 0} x\left[\frac{4}{x}\right] = A$ है। तो फलन $f(x) = [x^2]\sin(\pi x)$ असंतत है, जब x बराबर है:

(1) $\sqrt{A+5}$ (2) $\sqrt{A+21}$ (3) $\sqrt{A+1}$ (4) $\sqrt{A}$

A. 3

Question ID : 4050362201

Option 1 ID : 4050367861

Option 2 ID : 4050367862

Option 3 ID : 4050367860

Option 4 ID : 4050367859

Sol. $\lim_{x\to 0} x\left[\frac{4}{x}\right] = A \quad \Rightarrow \lim_{x\to 0} x\left(\frac{4}{x} - \left\{\frac{4}{x}\right\}\right)$

$= \lim_{x\to 0} 4 - x\left\{\frac{4}{x}\right\} = A \Rightarrow A = 4$

at $\sqrt{5}$; L.H.L $= 4\sin\sqrt{5}\,\pi$

R.H.L. $= 5\sin\sqrt{5}\,\pi$

Algebra

Sequence & progression

8. Let a_n be the n^{th} term of a G.P. of positive terms. If $\sum_{n=1}^{100} a_{2n+1} = 200$ and $\sum_{n=1}^{100} a_{2n} = 100$, then $\sum_{n=1}^{200} a_n$ is equal to:

माना धनात्मक पदों की एक गुणोत्तर श्रेढ़ी का n वां पद a_n है। यदि $\sum_{n=1}^{100} a_{2n+1} = 200$ तथा $\sum_{n=1}^{100} a_{2n} = 100$, तो $\sum_{n=1}^{200} a_n$ बराबर है:

(1) 300 (2) 150 (3) 225 (4) 175

A. 2

Question ID : 4050362200

Option 1 ID : 4050367855

Option 2 ID : 4050367856

Option 3 ID : 4050367858

Option 4 ID : 4050367857

Sol. $\sum_{n=1}^{100} a_{2n+1} = 200$ $\sum_{n=1}^{100} a_{2n} = 100$

$a_3 + a_5 + \ldots\ldots\ldots + a_{201} = 200$ ....(1)

$a_2 + a_4 + \ldots\ldots\ldots + a_{200} = 100$ ....(2)

$a_1r^2 + a_1r^4 + \ldots\ldots\ldots\ldots\ldots\ldots + a_1r^{200} = 200$

$a_1(r^2 + r^4 + \ldots\ldots\ldots\ldots\ldots\ldots + r^{200} = 200$

$$a_1\left(r^2\left(\frac{r^{200}-1}{r^2-1}\right)\right) = 200 \quad \ldots\ldots\ldots\ldots(1)$$

$a_1r + a_1r^3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + a_1r^{199} = 100$

$$a_1\left(r\left(\frac{r^{200}-1}{r^2-1}\right)\right) = 100 \quad \ldots\ldots\ldots\ldots(2)$$

(1)/(2)

r = 2

$$a_1\left(2\frac{\left(2^{200}-1\right)}{3}\right) = 100$$

$$a_1\left(2^{200}-1\right) = 150$$

$a_1 + \ldots\ldots + a_{200}$

$$= a_1\frac{\left(2^{200}-1\right)}{1} = 150$$

Integral Calculus

Area Under Curve

9. Given : $f(x)=\begin{cases} x & , 0 \le x < \frac{1}{2} \\ \frac{1}{2} & , x=\frac{1}{2} \\ 1-x & , \frac{1}{2} < x \le 1 \end{cases}$ and $g(x)=\left(x-\frac{1}{2}\right)^2$, $x \in \mathbf{R}$. Then the area (in sq. units) of the region bounded by the curves, $y=f(x)$ and $y=g(x)$ between the lines, $2x=1$ and $2x=\sqrt{3}$, is :

दिया है $f(x)=\begin{cases} x & , 0 \le x < \frac{1}{2} \\ \frac{1}{2} & , x=\frac{1}{2} \\ 1-x & , \frac{1}{2} < x \le 1 \end{cases}$ तथा $g(x)=\left(x-\frac{1}{2}\right)^2$, $x \in \mathbf{R}$.तो रेखाओं $2x = 1$ तथा $2x=\sqrt{3}$ के बीच , वक्राकें $y=f(x)$ तथा $y=g(x)$ द्वारा प्रतिबद्ध क्षेत्र का क्षेत्रफल (वर्ग इकाइयों में) है :

(1) $\frac{\sqrt{3}}{4}-\frac{1}{3}$ (2) $\frac{1}{3}+\frac{\sqrt{3}}{4}$ (3) $\frac{1}{2}+\frac{\sqrt{3}}{4}$ (4) $\frac{1}{2}-\frac{\sqrt{3}}{4}$

A. 1

Question ID : 4050362206

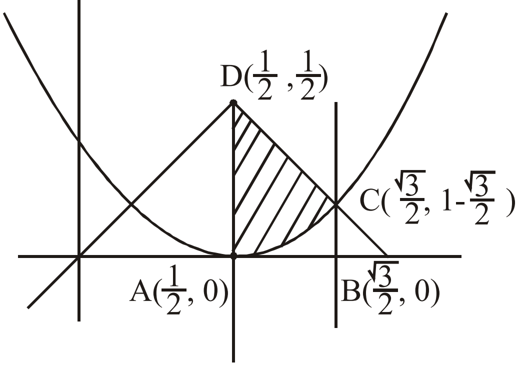
Option 1 ID : 4050367882

Option 2 ID : 4050367880

Option 3 ID : 4050367881

Option 4 ID : 4050367879

Sol.



Required area = Area of trapezium ABCD $-\int_{1/2}^{\sqrt{3}/2}\left(\frac{x-1}{2}\right)^2 dx$

Area of trapezium m = $\frac{1}{2}$ (Sum of parallel sides) (h)

$$=\frac{1}{2}\left(\frac{1}{2}+1-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)$$

$$=\frac{1}{2}\left(\frac{3}{2}-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)$$

$$=\frac{1}{2}\left[\frac{3\sqrt{3}}{4}-\frac{3}{4}-\frac{3}{4}+\frac{\sqrt{3}}{4}\right)=\frac{1}{2}\left(\sqrt{3}-\frac{3}{2}\right)$$

$$=\frac{\sqrt{3}}{2}-\frac{3}{4}$$

$\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}\left(x-\frac{1}{2}\right)^2 dx \qquad =\frac{1}{3}\left(x-\frac{1}{2}\right)^2\Bigg|_{1/2}^{\sqrt{3}/2}$

$$=\frac{1}{3}\left(\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)^3\right)=\frac{1}{3}\times\frac{1}{8}\left(\sqrt{3}-1\right)^3$$

$$=\frac{1}{24}\left(3\sqrt{3}-1+3\times\sqrt{3}-9\right)$$

$$=\frac{1}{24}\left(6\sqrt{3}-10\right)=\frac{1}{12}\left(3\sqrt{3}-5\right)$$

Final Ans. $\quad =\frac{\sqrt{3}}{2}-\frac{3}{4}-\frac{1}{12}\left(3\sqrt{3}-5\right)$

$$=\frac{\sqrt{3}}{2}-\frac{3}{4}-\frac{\sqrt{3}}{4}+\frac{5}{12}$$

$$=\frac{\sqrt{3}}{4}+\frac{5}{12}-\frac{3}{4}=\frac{\sqrt{3}}{4}+\frac{5-9}{12}$$

$$=\frac{\sqrt{3}}{4}-\frac{4}{12}=\frac{\sqrt{3}}{4}-\frac{1}{3}$$

Algebra

Probability

10. A random variable X has the following probability distribution :

X	:	1	2	3	4	5
P(X)	:	K^2	2K	K	2K	$5K^2$

Then $P(X > 2)$ is equal to :

एक यादृच्छिक चर X का प्रायिकता बंटन निम्न है :

X	:	1	2	3	4	5
P(X)	:	K^2	2K	K	2K	$5K^2$

तो $P(X > 2)$ बराबर है:

(1) $\frac{7}{12}$ (2) $\frac{1}{6}$ (3) $\frac{23}{36}$ (4) $\frac{1}{36}$

A. 3

Question ID : 4050362210

Option 1 ID : 4050367896

Option 2 ID : 4050367895

Option 3 ID : 4050367897

Option 4 ID : 4050367898

Sol. $\sum p_i = 1 \Rightarrow 6k^2 + 5k = 1$

$\Rightarrow k = -1 \text{ or } k = \frac{1}{6}$

$p(x > 2) = k + 2 + 5k^2 = \frac{23}{36}$

Integral Calculus

Indefinite Integration

11. If $\int \frac{d\theta}{\cos^2\theta(\tan 2\theta + \sec 2\theta)} =$

$\lambda \tan\theta + 2\log_e|f(\theta)| + C$ where C is a constant of integration, then the ordered pair $(\lambda, f(\theta))$ is equal to :

यदि $\int \frac{d\theta}{\cos^2\theta(\tan 2\theta + \sec 2\theta)} =$

$\lambda \tan\theta + 2\log_e|f(\theta)| + C$ है, जहाँ C एक समाकलन अचर है, तो क्रमित युग्म $(\lambda, f(\theta))$ बराबर है :

(1) $(1, 1 - \tan\theta)$ (2) $(1, 1 + \tan\theta)$ (3) $(-1, 1 + \tan\theta)$ (4) $(-1, 1 - \tan\theta)$

A. 3

Question ID : 4050362205

Option 1 ID : 4050367876

Option 2 ID : 4050367878

Option 3 ID : 4050367875

Option 4 ID : 4050367877

Sol. $\int \frac{d\theta}{\cos^2\theta(\tan 2\theta + \sec 2\theta)} = \lambda \tan\theta + 2\ln|f(\theta)| + c$

$$\int \frac{d\theta}{\cos^2\theta\left(\frac{1+\sin 2\theta}{\cos 2\theta}\right)} = \int \frac{\cos 2\theta\, d\theta}{\cos^2\theta(1+\sin 2\theta)}$$

$$= \int \frac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{\cos^2\theta(\cos\theta + \sin\theta)^2} d\theta$$

$$= \int \sec^2\theta\left(\frac{1-\tan\theta}{1+\tan\theta}\right) d\theta$$

$\tan\theta = t$ $\Rightarrow$ $\sec^2\theta d\theta = dt$

$$\int \left(\frac{1-t}{1+t}\right) dt$$

$1 + t = u$

$dt = du$

$$\int \frac{1-(u-1)}{u} du = \int \frac{2-u}{u} du$$

$$2\int \frac{du}{u} - \int du = 2\ln|u| - u + C$$

$= 2\ln|1 + t| - (1 + t) + C$

$= 2\ln|1 + \tan\theta| - (1 + \tan\theta) + C$

$= 2\ln|1 + \tan\theta| - \tan\theta + C$

Algebra

Complex Number

12. If z be a complex number satisfying $|\text{Re}(z)| + |\text{Im}(z)| = 4$, then $|z|$ cannot be :

यदि z एक ऐसी सम्मिश्र संख्या है जो $|\text{Re}(z)| + |\text{Im}(z)| = 4$ को सन्तुष्ट करती है, तो $|z|$ नहीं हो सकता :

(1) $\sqrt{10}$ (2) $\sqrt{8}$ (3) $\sqrt{\frac{17}{2}}$ (4) $\sqrt{7}$

A. 4

Question ID : 4050362196

Option 1 ID : 4050367841

Option 2 ID : 4050367840

Option 3 ID : 4050367842

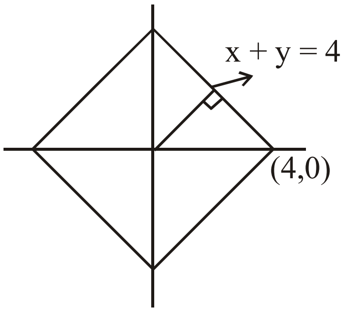
Option 4 ID : 4050367839

Sol. $z = x + iy$ $|z| = \sqrt{x^2 + y^2}$

$\text{Re}(z) = x$ $\text{Im}(z) = y$

$|x| + |y| = 4$

Methods of Differentiation **Differential Calculus**



Differential Calculus
Methods of Differentiation

13. If $x = 2\sin\theta - \sin 2\theta$ and $y = 2\cos\theta - \cos 2\theta,\ \theta \in [0, 2\pi]$, then $\frac{d^2y}{dx^2}$ at $\theta = \pi$ is :

यदि $x = 2\sin\theta - \sin 2\theta$ तथा $y = 2\cos\theta - \cos 2\theta,\ \theta \in [0, 2\pi]$ हैं , तो $\theta = \pi$ पर $\frac{d^2y}{dx^2}$ का मान है :

(1) $-\frac{3}{8}$ (2) $\frac{3}{4}$ (3) $\frac{3}{2}$ (4) $-\frac{3}{4}$

A. Bonus

Question ID : 4050362202

Option 1 ID : 4050367863

Option 2 ID : 4050367865

Option 3 ID : 4050367866

Option 4 ID : 4050367864

Sol. $x = 2\sin\theta - \sin 2\theta$ $y = 2\cos\theta - \cos 2\theta$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{d\theta}}{\frac{dx}{d\theta}}\left(\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}\right)$$

$$\frac{dy}{d\theta} = -2\sin\theta + 2\sin 2\theta$$

$$\frac{dx}{d\theta} = 2\cos\theta - 2\cos 2\theta$$

$$\frac{dy}{dx} = \frac{2\sin 2\theta - 2\sin\theta}{2\cos\theta - 2\cos 2\theta} = \frac{\sin 2\theta - \sin\theta}{\cos\theta - \cos 2\theta}$$

$$\frac{dy}{dx} = \frac{2\cos\left(\frac{3\theta}{2}\right)\sin\frac{\theta}{2}}{2\sin\left(\frac{3\theta}{2}\right)\sin\frac{\theta}{2}} = \cot\left(\frac{3\theta}{2}\right)$$

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\cot\left(\frac{3\theta}{2}\right)=\frac{\frac{d}{d\theta}\cot\left(\frac{3\theta}{2}\right)}{\frac{dx}{d\theta}}=\frac{d}{d\theta}\left(\cot\frac{3\theta}{2}\right)\times\frac{d\theta}{dx}$$

$$=-\cos ec^2\left(\frac{3\theta}{2}\right)\times\frac{3}{2}\times\frac{1}{2\cos\theta-2\cos 2\theta}$$

$$=\frac{3}{4}\left(\frac{\cos ec^2\left(\frac{3\theta}{2}\right)}{\cos 2\theta-\cos\theta}\right)$$

at $\theta=\pi$

$$=\frac{3}{4}\left(\frac{1}{1-(-1)}\right)=\frac{3}{4}\times\frac{1}{2}=\frac{3}{8}$$

JEE Main Only topics

Mathematical Reasoning

14. If $p\rightarrow(p\wedge\sim q)$ is false, then the truth values of p and q are respectively :

यदि $p\rightarrow(p\wedge\sim q)$ असतय है, तो p तथा q के क्रमशः सत्यमान हैं :

(1) F, F (2) F, T (3) T, T (4) T, F

A. 3

Question ID : 4050362213

Option 1 ID : 4050367907

Option 2 ID : 4050367909

Option 3 ID : 4050367910

Option 4 ID : 4050367908

Sol.

p	q	$\vee$	$\wedge$	$p\Rightarrow q$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	T
F	F	F	F	T

$\Rightarrow$ p must be true and $(p\wedge\sim q)$ and this must be false

Integral Calculus

Differential Equation

15. If $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$; $y(1)=1$, then a value of x satisfying $y(x)=e$ is :

यदि $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$; $y(1)=1$ है, तो $y(x)=e$ को सन्तुष्ट करने वाला x का एक मान है :

(1) $\frac{1}{2}\sqrt{3}\,e$ (2) $\frac{e}{\sqrt{2}}$ (3) $\sqrt{2}\,e$ (4) $\sqrt{3}\,e$

A. 4

Question ID : 4050362207

Option 1 ID : 4050367885

Option 2 ID : 4050367886

Option 3 ID : 4050367883

Option 4 ID : 4050367884

Sol. $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$ $\quad v=\frac{y}{x}$

$y=vx \quad \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$v+x\frac{dv}{dx}=\frac{vx^2}{x^2+v^2x^2}$

$v+x\frac{dv}{dx}=\frac{v}{1+v^2}$

$\frac{x\,dv}{dx}=\frac{v}{1+v^2}-v \quad \Rightarrow \frac{x\,dv}{dx}=\frac{v-v-v^3}{1+v^2}$

$\frac{x\,dv}{dx}=\frac{-v^3}{1+v^2}$

$\int\frac{1+v^2}{v^3}dv = -\int\frac{dx}{x}$

$\int v^{-3}dv+\int\frac{dv}{v}=-\int\frac{dx}{x}$

$\frac{-1}{2v^2}+\ln|v|=-\ln|x|+C$

$\frac{-x^2}{2y^2}+\ln\left|\frac{y}{x}\right|=-\ln|x|+C$

$\frac{-1}{2}+\ln 1=-\ln 1+C$

$C=\frac{-1}{2}$

$-\frac{x^2}{2y^2}+\ln\left|\frac{y}{x}\right|=-\ln|x|-\frac{1}{2}$

$$-\frac{x^2}{2e^2}+\ln\left|\frac{e}{x}\right|=-\ln|x|-\frac{1}{2}$$

$$-\frac{x^2}{2e^2}+1-\ln|x|=-\ln|x|-\frac{1}{2}$$

$$\frac{3}{2}=\frac{x^2}{2e^2}$$

$3e^2 = x^2$ $\quad\Rightarrow x = \pm\sqrt{3}\,e$

Integral Calculus

Definite Integration

16. Let a function $f:[0, 5] \rightarrow \mathbf{R}$ be continuous, $f(1) = 3$ and F be defined as :

$F(x)=\int_1^x t^2 g(t)\,dt$, where $g(t)=\int_1^t f(u)\,du$.

Then for the function F, the point x = 1 is :

(1) not a critical point
(2) a point of local maxima
(3) a point of inflection
(4) a point of local minima

माना एक फलन f : $[0, 5] \rightarrow \mathbf{R}$ संतत है, $f(1) = 3$ है तथा F, $F(x)=\int_1^x t^2 g(t)\,dt$ द्वारा परिभाषित है, जहाँ

$g(t)=\int_1^t f(u)\,du$ है तो फलन F के लिए, बिन्दु x = 1एक :

(1) क्रांतिक बिन्दु नहीं है।
(2) स्थानीय उच्चिष्ठ बिन्दु है।
(3) नति परिवर्तन बिन्दु है।
(4) स्थानीय निम्ननिष्ठ बिन्दु है।

A. 4

Question ID : 4050362204

Option 1 ID : 4050367871

Option 2 ID : 4050367872

Option 3 ID : 4050367874

Option 4 ID : 4050367873

Sol. $f:[0,5]\rightarrow R \quad f(1) = 3$

$$F(x)=\int_1^x t^2 g(t)\,dt$$

$$F'(x)=x^2 g(x)$$

$$F'(1)=g(1)$$

$$g(t)=\int_1^t f(u)\,du \qquad \{g(1)=0\}$$

$F'(1) = g(1) = 0$

$F''(x) = x^2 g'(x) + 2xg(x)$

$g'(x) = f(x)$

$F''(x) = x^2 f(x) + 2xg(x)$

$F''(1) = f(1) + 2g(1) \quad = 3$

From (1) and (2), F(x) has local minimum at x = 1

Algebra

Determinant

17. The following system of linear equations

$7x + 6y - 2z = 0$

$3x + 4y + 2z = 0$

$x - 2y - 6z = 0$, has

(1) only the trivial solution

(2) infinitely many solutions, (x, y, z) satisfying $x = 2z$

(3) infinitely many solutions, (x, y, z) satisfying $y = 2z$

(4) no solution

रैखिक समीकरणों के निम्न निकाय

$7x + 6y - 2z = 0$

$3x + 4y + 2z = 0$

$x - 2y - 6z = 0$

(1) का केवल तुच्छ हल हैं।

(2) x = 2z को सन्तुष्ट करने वाले अनन्त हल (x, y, z) हैं।

(3) y = 2z को सन्तुष्ट करने वाले अनन्त हल (x, y, z) हैं।

(4) का होई हल नहीं है।

A. 2

Question ID : 4050362198

Option 1 ID : 4050367848

Option 2 ID : 4050367850

Option 3 ID : 4050367849

Option 4 ID : 4050367847

Sol. $7x + 6y - 2z = 0$

$3x + 4y + 2z = 0$

$x - 2y - 6z = 0$

$$D = \begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix} = 7(-24 + 4) - 6(-18 - 2) - 2(-6 - 4)$$

$= 7(-20) - 6(-20) - 2(-10)$
$= -140 + 120 + 20 = 0$

$$\begin{array}{l} 7x + 6y = 2t \\ 3x + 4y = -2t \\ \hline 21x + 18y = 6t \\ 21x + 28y = -14t \\ \hline -10y = 20t \end{array} \quad \Rightarrow y = -2t$$

$7x + 6(-2t) = 2t$
$7x = 14t \Rightarrow x = 2t$

$x = 2z$

Probability

18. If 10 different balls are to be placed in 4 distinct boxes at random, then the probability that two of these boxes contain exactly 2 and 3 balls is :

यदि 10 भिन्न गेंदें, 4 भिन्न बक्सों में यादृच्छया रखी जानी हैं, तो इनमे से दो बक्सों में मात्र 2 तथा 3 गेंदों के होने की प्रायिकता है :

(1) $\frac{965}{2^{10}}$ (2) $\frac{945}{2^{11}}$ (3) $\frac{945}{2^{10}}$ (4) $\frac{965}{2^{11}}$

A. Bonus

Question ID : 4050362211

Option 1 ID : 4050367902

Option 2 ID : 4050367901

Option 3 ID : 4050367899

Option 4 ID : 4050367900

Sol. The following distrubutions are possible

$P_1(3, 2, 0, 5), P_2(3, 2, 1, 4), P_3(3, 2, 3, 2)$

The respective probabilities are

$$P_1 = \frac{\frac{10!}{3!2!0!5!} \times 4!}{4^{10}}$$

$$P_2 = \frac{\frac{10!}{3!2!1!4!} \times 4!}{4^{10}}$$

$$P_3 = \frac{\frac{10!}{3!2!3!2!2!2!} \times 4!}{4^{10}}$$

Desired probability $= P_1 + P_2 + P_3$

$= \frac{945 \times 17}{2^{15}}$

JEE Main Only topics

Set & Relations

19. If $A=\{x\in\mathbf{R}:|x|<2\}$ and $B=\{x\in\mathbf{R}:|x-2|\geq 3\}$; then :

यदि $A=\{x\in\mathbf{R}:|x|<2\}$ तथा $B=\{x\in\mathbf{R}:|x-2|\geq 3\}$; तो :

(1) $A\cup B=\mathbf{R}-(2,5)$
(2) $A\cap B=(-2,-1)$
(3) $B-A=\mathbf{R}-(-2,5)$
(4) $A-B=[-1,2)$

A. 3

Question ID : 4050362194

Option 1 ID : 4050367834

Option 2 ID : 4050367833

Option 3 ID : 4050367832

Option 4 ID : 4050367831

Sol. $A=\{x\in\mathbf{R}:|x|<2\}$ $B=\{x\in\mathbf{R}:|x-2|\geq 3\}$

$A=\{-2<x<2\}$

For B
$x<2$
$2-x\geq 3$
$x\leq -1$

$x\geq 2$
$x-2\geq 3$
$x\geq 5$

$B=\{x\in(-\infty,-1]\,U[5,+\infty,)\}$

Algebra

Sequence & progression

20. If $x=\sum_{n=0}^{\infty}(-1)^n\tan^{2n}\theta$ and $y=\sum_{n=0}^{\infty}\cos^{2n}\theta$, $0<\theta<\frac{\pi}{4}$, then :

यदि $0<\theta<\frac{\pi}{4}$ के लिए $x=\sum_{n=0}^{\infty}(-1)^n\tan^{2n}\theta$ तथा $y=\sum_{n=0}^{\infty}\cos^{2n}\theta$ हैं ,तो :

(1) $y(1-x)=1$ (2) $y(1+x)=1$ (3) $x(1+y)=1$ (4) $x(1-y)=1$

A. 1

Question ID : 4050362212

Option 1 ID : 4050367904

Option 2 ID : 4050367903

Option 3 ID : 4050367906

Option 4 ID : 4050367905

Sol. $x=1-\tan^2\theta+\tan^4\theta+.....$

$$x=\frac{1}{1-(-\tan^2\theta)}=\frac{1}{1+\tan^2\theta}=\frac{1}{\sec^2\theta}=\cos^2\theta$$

$y = 1 + \cos^2\theta + \cos^4\theta + ...$

$$y=\frac{1}{1-\cos^2\theta}=\frac{1}{\sin^2\theta}=\operatorname{cosec}^2\theta$$

$y(1 - x) = 1$

$\operatorname{cosec}^2\theta\,(1 - \cos^2\theta) = 1$

Vectors

Vectors

21. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|=\sqrt{3}$, $|\vec{b}|=5$, $\vec{b}\cdot\vec{c}=10$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$.

If $\vec{a}$ is perpendicular to the vector $\vec{b}\times\vec{c}$, then $\left|\vec{a}\times\left(\vec{b}\times\vec{c}\right)\right|$ is equal to ____________.

माना तीन सदिश $\vec{a}$, $\vec{b}$ तथा $\vec{c}$ इस प्रकार हैं कि $|\vec{a}|=\sqrt{3}$, $|\vec{b}|=5$, $\vec{b}\cdot\vec{c}=10$ तथा $\vec{b}$ और $\vec{c}$ के बीच का कोण $\frac{\pi}{3}$ है। यदि $\vec{a}$, सदिश $\vec{b}\times\vec{c}$ पर लम्बवत है,तो $\left|\vec{a}\times\left(\vec{b}\times\vec{c}\right)\right|$ बराबर है ____________।

A. 30

Question ID : 4050362218

Sol. $\vec{b}\;\vec{c}=10$

$|\vec{b}|\;|\vec{c}|\cos\theta=10$

$5\times|\vec{c}|\times\frac{1}{2}=10 \qquad \Rightarrow |\vec{c}|=4$

$\vec{a}\cdot(\vec{b}\times\vec{c})=0$

$|\vec{a}\times(\vec{b}\times\vec{c})|=|\vec{a}|\;|\vec{b}\times\vec{c}|\sin\frac{\pi}{2}$

$=|\vec{a}|\;|\vec{b}\times\vec{c}|$

$=\sqrt{3}\;||\vec{b}||\vec{c}|\sin\frac{\pi}{3}\hat{n}|$

$=\sqrt{3}\;||\vec{b}||\vec{c}|\times\frac{\sqrt{3}}{2}$

$=\frac{3}{2}\times5\times4=30$

Algebra

Sequence & progression

22. The number of terms common to the two A.P.'s 3, 7, 11, ..., 407 and 2, 9, 16, ..., 709 is ________.

दो समांतर श्रेढ़ियों 3, 7, 11, ..., 407 तथा 2, 9, 16, ..., 709 में उभयनिष्ठ पदों की संख्या है ________ ।

A. 14

Question ID : 4050362215

Sol. For common A.P.

First term = 23

Common difference = $7 \times 4 = 28$

$T_n \leq 407$

$23 + (n-1)\,28 \leq 407$

$28(n-1) \leq 384$

$n - 1 \leq \frac{384}{28}$

$n \leq \frac{384+28}{28}$

$n \leq \frac{412}{28}$

$n \leq 14.71 \qquad \Rightarrow n = 14$

Vectors

3D Geometry

23. If the distance between the plane, $23x - 10y - 2z + 48 = 0$ and the plane containing the lines

$\frac{x+1}{2} = \frac{y-3}{4} = \frac{z+1}{3}$ and $\frac{x+3}{2} = \frac{y+2}{6} = \frac{z-1}{\lambda}$ $(\lambda \in \mathbf{R})$ is equal to $\frac{k}{\sqrt{633}}$, then k is equal to ______.

यदि समतल $23x - 10y - 2z + 48 = 0$ तथा रेखाओं $\frac{x+1}{2} = \frac{y-3}{4} = \frac{z+1}{3}$ और $\frac{x+3}{2} = \frac{y+2}{6} = \frac{z-1}{\lambda}$ $(\lambda \in \mathbf{R})$ को अंतर्विष्ट करने वाले समतल के बीच की दूरी $\frac{k}{\sqrt{633}}$ है, तो k बराबर है ______ ।

A. 3

Question ID : 4050362217

Sol. Lines must be intersecting

$\frac{x+1}{2} = \frac{y-3}{4} = \frac{z+1}{3} = s \qquad$ ....(1)

$\frac{x+3}{2} = \frac{y+2}{6} = \frac{z-1}{\lambda} = t \qquad$ ....(2)

$(x, y, z) = (2s-1, 4s+3, 3s-1) = (2t-3, 6t-2, \lambda t+1)$

$2s - 1 = 2t - 3$

$2t - 2s = 2 \qquad \Rightarrow t - s = 1$

$6t - 4s = 5$

$t = \frac{1}{2} \qquad s = -\frac{1}{2} \qquad \lambda = -7$

$$\text{Distance} = \left|\frac{23x(-3)-10(-2)-2(1)+48}{\sqrt{(23)^2+(10)^2+(2)^2}}\right| = \frac{3}{\sqrt{633}} \qquad k = 3$$

Coordinate Geometry

Circle

24. If the curves, $x^2 - 6x + y^2 + 8 = 0$ and $x^2 - 8y + y^2 + 16 - k = 0$, $(k > 0)$ touch each other at a point, then the largest value of k is ____________.

यदि वक्र $x^2 - 6x + y^2 + 8 = 0$ तथा $x^2 - 8y + y^2 + 16 - k = 0$, $(k > 0)$ एक दूसरे को एक बिन्दु पर स्पर्श करते हैं तो k का अधिकतम मान है ____________ ।

A. 36

Question ID : 4050362216

Sol. Two circles touch each other

if $C_1C_2 = |r_1 \pm r_2|$

$x^2 - 6x + y^2 + 8 = 0 \quad C_1 : (3, 0) \qquad r_1 = \sqrt{9-8} = 1$

$x^2 + y^2 - 8y + 16 - k = 0 \qquad C_2 : (0, 4)$

$r_2 : \sqrt{16-(16-k)} = \sqrt{k}$

$C_1C_2 = 5$

$5 = |1 + \sqrt{k}|$ or $5 = |1 - \sqrt{k}|$

$\pm 5 = 1 + \sqrt{k} \qquad \pm 5 = 1 - \sqrt{k}$

$\sqrt{k} = \pm 5 - 1 \qquad \sqrt{k} = 1 \pm 5$

$\sqrt{k} = 5 - 1$ or $-5 - 1 \qquad \sqrt{k} = 1 - 5$ or $1 + 5$

$\sqrt{k} = 4 \qquad \sqrt{k} = 6$

$k = 16 \qquad k = 36$

Algebra

Binomial theorem

25. If $C_r \equiv {}^{25}C_r$ and $C_0 + 5.C_1 + 9.C_2 + ... + (101).C_{25} = 2^{25}.k$, then k is equal to ___________.

यदि $C_r \equiv {}^{25}C_r$ तथा $C_0 + 5.C_1 + 9.C_2 + ... + (101).C_{25} = 2^{25}.k$,तो k बराबर है ___________ ।

A. 51

Question ID : 4050362214

Sol. $C_r = {}^{25}C_r$

$C_0 + 5.C_1 + 9.C_2 + ...+ (101)\ C_{25} = 2^{25}.k$

$$\sum_{r=0}^{25}(4r+1)\ {}^{25}C_r = 4\sum_{r=0}^{25} r\ {}^{25}C_r + \sum_{r=0}^{25} {}^{25}C_r$$

$$4\sum_{r=0}^{25} r \times \frac{25}{r}\ {}^{24}C_{r-1} + 2^{25}$$

$$= 100\sum_{r=1}^{25} {}^{24}C_{r-1} + 2^{25}$$

$= 100 \times 2^{24} + 2^{25} = 50 \times 2^{25} + 2^{25}$

$= 2^{25} \times 51$

$= k = 51$