MATHS
09 APRIL 2019 [Phase : I]
JEE MAIN PAPER ONLINE

Quadratic Equation

1. Let p, q $\in$ R. If $2-\sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then :

माना p, q $\in$ R, यदि $2-\sqrt{3}$ द्विघाती समीकरण $x^2 + px + q = 0$ का एक मूल है, तो :

(1) $q^2 - 4p - 16 = 0$ (2) $p^2 - 4q + 12 = 0$ (3) $p^2 - 4q - 12 = 0$ (4) $q^2 + 4p + 14 = 0$

Ans. **(3)**

sol. p, q are rational numbers

$\therefore$ $2+\sqrt{3}$ in the other root

Now, p = – 4, q = 1

$\Rightarrow$ $p^2 - 4q - 12 = 16 - 4 - 12$

$= 0$

Note :- p, q should be given as rational numbers instead of real numbers

3D

2. If the line, $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{4}$ meets the plane, $x + 2y + 3z = 15$ at a point P, then the distance of P from the origin is :

यदि रेखा $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{4}$, समतल $x + 2y + 3z = 15$ को बिन्दु P पर मिलती है, तो P की मूल बिन्दु से दूरी है :

(1) $2\sqrt{5}$ (2) $\frac{9}{2}$ (3) $\frac{7}{2}$ (4) $\frac{\sqrt{5}}{2}$

Ans. **(2)**

sol. Let point on line is p(2r + 1, 3r – 1, 4r + 2)

It lies on the plane $x + 2y + 3z = 15$

$\therefore$ $2r + 1 + 6r - 2 + 12r + 6 = 15$

$\Rightarrow$ $r = \frac{1}{2}$

$\therefore$ $P = \left(2, \frac{1}{2}, 4\right)$

$\therefore$ $OP = \sqrt{4 + \frac{1}{4} + 16} = \sqrt{\frac{81}{4}} = \frac{9}{2}$

Circle

3. If a tangent to the circle $x^2 + y^2 = 1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is :

यदि वृत्त $x^2 + y^2 = 1$ की एक स्पर्शरेखा निर्देशांक अक्षों को भिन्न बिन्दुओं P तथा Q पर प्रतिच्छेद करती है, तो PQ के मध्यबिन्दु का बिन्दुपथ (locus) है :

(1) $x^2 + y^2 - 16x^2y^2 = 0$ (2) $x^2 + y^2 - 2x^2y^2 = 0$

(3) $x^2 + y^2 - 4x^2y^2 = 0$ (4) $x^2 + y^2 - 2xy = 0$

Ans. **(3)**

sol. Let any tangent to circle $x^2 + y^2 = 1$ is

$x \cos\theta + y \sin\theta = 1$

$\therefore \quad P\left(\frac{1}{\cos\theta}, 0\right); Q\left(0, \frac{1}{\sin\theta}\right)$

$\therefore$ Mid-point of PQ let $M\left(\frac{1}{2\cos\theta}, \frac{1}{2\sin\theta}\right) = (h, k)$

$\Rightarrow \quad \cos\theta = \frac{1}{2h}; \quad \sin\theta = \frac{1}{2k}$

$\therefore$ On squaring and adding

$\frac{1}{h^2} + \frac{1}{k^2} = 4 \quad \Rightarrow \quad x^2 + y^2 = 4x^2y^2$

Maxima & Minima

4. If f(x) is a non-zero polynomial of degree four, having local extreme points at $x = -1, 0, 1$; then the set $S = \{x \in R : f(x) = f(0)\}$ contains exactly :

(1) Four irrational numbers (2) Four rational numbers

(3) Two irrational and one rational number (4) Two irrational and two rational numbers

यदि f(x), घात चार का एक शून्येत्तर बहुपद है, जिसके स्थानीय चरम बिन्दु $x = -1, 0, 1$ पर हैं, तो समुच्चय $S = \{x \in R : f(x) = f(0)\}$ रखता है –

(1) चार अपरिमेय संख्याऐं (2) चार परिमेय संख्याऐं

(3) दो अपरिमेय तथा एक परिमेय संख्या (4) दो अपरिमेय तथा दो परिमेय संख्याऐं

Ans. **(3)**

sol. $f'(x) = A(x + 1)\, x\, (x - 1)$

$= A(x^3 - x)$

$\Rightarrow \quad f(x) = A\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + C$

Now $f(0) = C$

$\therefore \quad f(x) = f(0) \quad \Rightarrow \quad A\left(\frac{x^4}{4} - \frac{x^2}{2}\right) = 0$

$$\Rightarrow \quad \frac{x^2}{2}\left(\frac{x^2}{2}-1\right)=0$$

$$\Rightarrow \quad x = 0, -\sqrt{2}, \sqrt{2}$$

$$\therefore \quad S = \{0, -\sqrt{2}, \sqrt{2}\}$$

Statistics

5. If the standard deviation of the numbers –1, 0, 1, k is $\sqrt{5}$ where $k > 0$, then k is equal to :

यदि संख्याओं –1, 0, 1, k का मानक विचलन $\sqrt{5}$ है, जहाँ $k > 0$ है, तो k बराबर है :

(1) $\sqrt{6}$ (2) $2\sqrt{6}$ (3) $2\sqrt{\frac{10}{3}}$ (4) $4\sqrt{\frac{5}{3}}$

Ans. **(2)**

sol. Mean of given observation $= \frac{k}{4}$

$\therefore \quad \sigma^2 = 5(\text{given}) \quad ...(i)$

$$\text{Also } \sigma^2 = \frac{\left(\frac{k}{4}+1\right)^2+\left(\frac{k}{4}\right)^2+\left(\frac{k}{4}-1\right)^2+\left(\frac{3k}{4}\right)^2}{4} \quad ....(ii)$$

$\therefore$ from (i) and (ii)

$$\frac{\frac{12k^2}{16}+2}{4}=5 \quad \Rightarrow \quad \frac{12k^2}{16}=18$$

$$\Rightarrow \quad k^2 = 24 \quad \Rightarrow \quad k = 2\sqrt{6}$$

Binomial Theorem

6. If the fourth term in the Binomial expansion of $\left(\frac{2}{x}+x^{\log_8 x}\right)^6$ $(x > 0)$ is 20×8^7, then a value of x is :

यदि $\left(\frac{2}{x}+x^{\log_8 x}\right)^6$ $(x > 0)$ के द्विपद प्रसार का चौथा पद 20×8^7 है, तो x का एक मान है :

(1) 8^3 (2) 8 (3) 8^{-2} (4) 8^2

Ans. **(4)**

sol. $$T_4 = 20\times 8^7 = {}^6C_3\left(\frac{2}{x}\right)^3 \times (x^{\log 8x})^3$$

$$\Rightarrow \quad 8\times 20\times\left(\frac{x^{\log 8x}}{x}\right)^3 = 20\times 8^7$$

$\Rightarrow \quad \left(\dfrac{x^{\log 8x}}{x}\right)^3 = (8^2)^3$

$\Rightarrow \quad \dfrac{x^{\log 8x}}{x} = 64$ Take $\log_8$ both side

$\Rightarrow \quad (\log_8 x)^2 - (\log_8 x) = 2$

$\Rightarrow \quad \log_8 x = -1 \quad$ or $\quad \log_8 x = 2$

$\Rightarrow \quad x = \dfrac{1}{8} \quad$ or $\quad x = 8^2$

Indefinite Integration

7. The integral $\int \sec^{2/3}x \operatorname{cosec}^{4/3}x dx$ is equal to :

(Here C is a constant of integration)

समाकल $\int \sec^{2/3}x \operatorname{cosec}^{4/3}x dx$ बराबर है :

(यहाँ C एक समाकलन अचर है)

(1) $-3\cot^{-1/3}x + C$ (2) $-3\tan^{-1/3}x + C$ (3) $-\dfrac{3}{4}\tan^{-4/3}x + C$ (4) $3\tan^{-1/3}x + C$

Ans. **(2)**

sol. $I = \int \sec^{\frac{2}{3}} x . \operatorname{cosec}^{\frac{4}{3}} dx$

$I = \int \dfrac{\sec^2 x dx}{\tan^{\frac{4}{3}} x}$ put $\tan x = t$

$\Rightarrow \sec^2 x \, dx = dt$

$\Rightarrow \quad I = \dfrac{t^{\frac{-1}{3}}}{\left(\frac{-1}{3}\right)} + C \quad \Rightarrow \quad I = -3(\tan x)^{\frac{-1}{3}} + C$

Parabola

8. If one end of a focal chord of the parabola, $y^2 = 16x$ is at (1, 4), then the length of this focal chord is :

यदि परवलय $y^2 = 16x$ की एक नाभिजीवा का एक छोर (1, 4) पर है, तो इस नाभिजीवा की लम्बाई है :

(1) 24 (2) 20 (3) 22 (4) 25

Ans. **(4)**

sol. $\because \quad y^2 = 16x \quad \Rightarrow \quad a = 4$

One end of focal chord (1, 4) $\quad \therefore \quad 2\,at = 4$

$\Rightarrow \quad t = \dfrac{1}{2}$

Length of focal chord $= = a\left(t+\frac{1}{t}\right)^2$

$$= 4\times\left(2+\frac{1}{2}\right)^2$$

$$= 25$$

Continuity & Diff

9. If the function f defined on $\left(\frac{\pi}{6},\frac{\pi}{3}\right)$ by $f(x)=\begin{cases}\frac{\sqrt{2}\cos x-1}{\cot x-1}, & x\neq\frac{\pi}{4}\\ k, & x=\frac{\pi}{4}\end{cases}$ is continuous, then k is equal to :

यदि फलन f, $\left(\frac{\pi}{6},\frac{\pi}{3}\right)$ पर इस प्रकार परिभाषित है कि $f(x)=\begin{cases}\frac{\sqrt{2}\cos x-1}{\cot x-1}, & x\neq\frac{\pi}{4}\\ k, & x=\frac{\pi}{4}\end{cases}$ संतत है, तो k बराबर है :

(1) 1 (2) $\frac{1}{2}$ (3) $\frac{1}{\sqrt{2}}$ (4) 2

Ans. (2)

sol. $\lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}\cos x-1}{\cot x-1}=k$ $\quad\therefore$ By L hospital rule.

$\Rightarrow \lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}\sin x}{\operatorname{cosec}^2 x}=k \quad\Rightarrow\quad k=\frac{1}{2}$

Complex Number

10. All the points in the set

$S=\left\{\frac{\alpha+i}{\alpha-i}:\alpha\in R\right\}\left(i=\sqrt{-1}\right)$

lie on a :

(1) Straight line whose slope is 1
(2) Circle whose radius is $\sqrt{2}$
(3) Circle whose radius is 1
(4) Straight line whose slope is –1

समुच्चय $S=\left\{\frac{\alpha+i}{\alpha-i}:\alpha\in R\right\}\left(i=\sqrt{-1}\right)$ के सभी बिन्दु जिस पर स्थित हैं, वह है :

(1) एक सरल रेखा जिसका ढाल (slope) 1 है।
(2) एक वृत्त जिसकी त्रिज्या $\sqrt{2}$ है।
(3) एक वृत्त जिसकी त्रिज्या 1 है।
(4) एक सरल रेखा जिसकी ढ़ाल –1 है।

Ans. **(3)**

sol. $\because \quad S = \dfrac{\alpha + i}{\alpha - i}$

$\Rightarrow \quad x + iy = \dfrac{(\alpha + i)^2}{\alpha^2 + 1}$ (by rationalisation)

$\Rightarrow \quad x + iy = \dfrac{(\alpha^2 - 1)}{\alpha^2 + 1} + \dfrac{i(2\alpha)}{\alpha^2 + 1}$ (On comparing both sides)

$\Rightarrow \quad x = \dfrac{\alpha^2 - 1}{\alpha^2 + 1}$ ...(i) $\qquad y = \dfrac{2\alpha}{\alpha^2 + 1}$ ....(ii)

By squaring and adding

$\Rightarrow \quad x^2 + y^2 = 1$

Vector

11. Let $\vec{\alpha} = 3\hat{i} + \hat{j}$ and $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$. If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to :

माना $\vec{\alpha} = 3\hat{i} + \hat{j}$ तथा $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$ हैं। यदि $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$ है, जहाँ $\vec{\beta}_1$ सदिश $\vec{\alpha}$ के समान्तर है तथा $\vec{\beta}_2$ सदिश $\vec{\alpha}$ के लम्बवत् है, तो $\vec{\beta}_1 \times \vec{\beta}_2$ बराबर है :

(1) $\frac{1}{2}\left(3\hat{i} - 9\hat{j} + 5\hat{k}\right)$ (2) $\frac{1}{2}\left(-3\hat{i} + 9\hat{j} + 5\hat{k}\right)$ (3) $-3\hat{i} + 9\hat{j} + 5\hat{k}$ (4) $3\hat{i} - 9\hat{j} - 5\hat{k}$

Ans. **(2)**

sol. $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$ ....(i)

$\therefore \quad \vec{\beta}_2 . \vec{\alpha} = 0$

and Let $\vec{\beta}_1 = \lambda \vec{\alpha}$

$\vec{\alpha} . \vec{\beta} = \vec{\alpha} . \vec{\beta}_1 - \vec{\alpha} . \vec{\beta}_2$

$\Rightarrow \quad 5 = \lambda \alpha^2$

$\Rightarrow \quad 5 = \lambda \times 10$

$\Rightarrow \quad \lambda = \dfrac{1}{2}$

$\therefore \quad \boxed{\vec{\beta}_1 = \dfrac{\vec{\alpha}}{2}}$

Cross product with $\vec{\beta}_1$ in equation (i)

$\Rightarrow \quad \vec{\beta} \times \vec{\beta}_1 = -\vec{\beta}_2 \times \vec{\beta}_1$

$\Rightarrow \quad \boxed{\vec{\beta} \times \vec{\beta}_1 = \vec{\beta}_1 \times \vec{\beta}_2} = \dfrac{\left(\vec{\beta} \times \vec{\alpha}\right)}{2}$

$\Rightarrow \quad \vec{\beta}_1 \times \vec{\beta}_2 = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 3 & 1 & 0 \end{vmatrix}$

$= \frac{1}{2}\left[-3\hat{i} - \hat{j}(-9) + \hat{k}(5)\right]$

$= \frac{1}{2}\left[-3\hat{i} - 9\hat{j} + 5\hat{k}\right]$

Function

12. Let $\sum_{k=1}^{10} f(a+k) = 16(2^{10}-1)$, where the function f satisfies $f(x+y) = f(x)\,f(y)$ for all natural numbers x, y and $f(1) = 2$. Then the natural number 'a' is :

माना $\sum_{k=1}^{10} f(a+k) = 16(2^{10}-1)$ है, जहाँ सभी प्राकृत संख्याओं x, y के लिए, फलन f, $f(x+y) = f(x)\,f(y)$ को संतुष्ट करता है तथा $f(1) = 2$ है, तो प्राकृत संख्या 'a' बराबर है :

(1) 2   (2) 3   (3) 16   (4) 4

Ans. **(2)**

sol. $\because \quad f(x+y) = f(x)\,.\,f(y)$

$\therefore \quad$ Let $\boxed{f(x) = b^x}$

$\because \quad f(1) = 2$

$\therefore \quad b = 2$

$\Rightarrow \quad \boxed{f(x) = 2^x}$

Now, $\sum_{k=1}^{10} 2^{a+k} = 16(2^{10}-1)$

$\Rightarrow \quad 2^a \sum_{k=1}^{10} 2^k = 16(2^{10}-1)$

$\Rightarrow \quad 2^a \times \frac{\left((2^{10})-1\right)\times 2}{(2-1)} = 16\times(2^{10}-1)$

$\Rightarrow \quad 2^a = 8$

$\Rightarrow \quad \boxed{a = 3}$

Matrices

13. If $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\cdot\ldots\cdot\begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$, then the inverse of $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ is :

यदि $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\cdot....\cdot\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$ है, तो $\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$ का व्युत्क्रम (inverse) है :

(1) $\begin{bmatrix}1 & 0\\13 & 1\end{bmatrix}$ (2) $\begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$ (3) $\begin{bmatrix}1 & -12\\0 & 1\end{bmatrix}$ (4) $\begin{bmatrix}1 & 0\\12 & 1\end{bmatrix}$

Ans. (2)

sol. $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 4\\0 & 1\end{bmatrix}.....\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1 & 1+2+3+....+(n-1)\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$

$\Rightarrow \frac{(n-1)n}{2}=78$

$\Rightarrow n=13$

Now, inverse of $\begin{bmatrix}1 & 13\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$

Trig Equation

14. Let $S=\{\theta\in[-2\pi,2\pi];\ 2\cos^2\theta+3\sin\theta=0\}$. Then the sum of the elements of S is :

माना $S=\{\theta\in[-2\pi,2\pi];\ 2\cos^2\theta+3\sin\theta=0\}$ है, तो S के अवयवों का योगफल है :

(1) π (2) 2π (3) $\frac{13\pi}{6}$ (4) $\frac{5\pi}{3}$

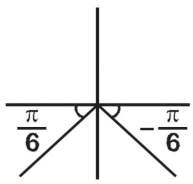
Ans. (2)

sol. $\because\ 2\cos^2\theta+3\sin\theta=0$

$\Rightarrow 2\sin^2\theta-3\sin\theta-2=0$

$\Rightarrow (2\sin\theta+1)(\sin\theta-2)=0$

$\Rightarrow \sin\theta=-\frac{1}{2};\ \boxed{\sin\theta=2}\rightarrow$ Not Possible



$\therefore$ Sum of all solutions in $[-2\pi, 2\pi]$ is

$=\left(\pi+\frac{\pi}{6}\right)+\left(2\pi-\frac{\pi}{6}\right)+\left(-\frac{\pi}{6}\right)+\left(-\pi+\frac{\pi}{6}\right)=2\pi$

Continuity & Diff

15. Let $f(x) = 15 - |x - 10|; x \in R$. Then the set of all values of x, at which the function, $g(x) = f(f(x))$ is not differentiable, is :

माना $f(x) = 15 - |x - 10|; x \in R$ है, तो x के उन सभी मानों का समुच्चय, जिन पर फलन $g(x) = f(f(x))$ अवकलनीय नहीं है, है :

(1) (10, 15) (2) {5, 10, 15, 20} (3) {10} (4) {5, 10, 15}

Ans. **(4)**

sol. Given $f(x) = 15 - |(10 - x)|$

$\Rightarrow$ $f(f(x)) = 15 - ||10 - x| - 5|$

$\therefore$ Non-differentiable at points where

$10 - x = 0$ and $|10 - x| = 5$

$\Rightarrow$ $x = 10$ and $x - 10 = \pm 5$

$\Rightarrow$ $x = 10$ and $x = 15, 5$

Area Under Curve

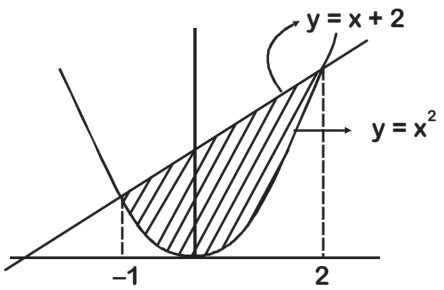
16. The area (in sq. units) of the region $A = \{(x, y) : x^2 \le y \le x + 2\}$ is :

क्षेत्र $A = \{(x, y) : x^2 \le y \le x + 2\}$ का क्षेत्रफल (वर्ग इकाइयों में) है :

(1) $\frac{31}{6}$ (2) $\frac{10}{3}$ (3) $\frac{9}{2}$ (4) $\frac{13}{6}$

Ans. **(3)**

sol.



$\therefore$ Required area $= \int_{-1}^{2}\left((x+2) - x^2\right)dx$

$= \left(\frac{x^2}{2} + 2x - \frac{x^3}{3}\right)_{-1}^{2}$

$= \left(2 + 4 - \frac{8}{3}\right) - \left(+\frac{1}{2} - 2 + \frac{1}{3}\right)$

$= 8 - 3 - \frac{1}{2}$

$$= 5 - \frac{1}{2} = \frac{9}{2}$$

Hyperbola

17. In the line $y = mx + 7\sqrt{3}$ is normal to the hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$, then a value of m is :

यदि रेखा $y = mx + 7\sqrt{3}$, अतिपरवलय $\frac{x^2}{24} - \frac{y^2}{18} = 1$ का अभिलंब है, तो m का एक मान है :

(1) $\frac{2}{\sqrt{5}}$ (2) $\frac{3}{\sqrt{5}}$ (3) $\frac{\sqrt{15}}{2}$ (4) $\frac{\sqrt{5}}{2}$

Ans. **(1)**

sol. $mx - y + 7\sqrt{3} = 0$ is normal to hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$

then $\frac{24}{m^2} - \frac{18}{(-1)^2} = \frac{(24+18)^2}{(7\sqrt{3})^2}$

$\Rightarrow \quad \frac{24}{m^2} - 18 = \frac{42 \times 42}{7 \times 7 \times 3}$

$\Rightarrow \quad m = \frac{2}{\sqrt{5}}$

If $lx + my + n = 0$ is a normal to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$,

then $\frac{a^2}{l^2} - \frac{b^2}{m^2} = \frac{\left(a^2 + b^2\right)^2}{n^2}$

Complex Number

18. Let α and β be the roots of the equation $x^2 + x + 1 = 0$. Then for $y \neq 0$ in R, $\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$ is equal to:

माना α तथा β, समीकरण $x^2 + x + 1 = 0$ के मूल हैं, तो R में $y \neq 0$ के लिए $\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$ बराबर है :

(1) $y(y^2 - 1)$ (2) $y^3 - 1$ (3) $y(y^2 - 3)$ (4) y^3

Ans. **(4)**

sol. Let $\alpha = \omega$ and $\beta = \omega^2$ are roots of $x^2 + x + 1 = 0$

$$\therefore \quad \begin{vmatrix} y+\omega & 1 & \omega^2 \\ 1 & y+\omega^2 & \omega \\ \omega^2 & \omega & 1+y \end{vmatrix} \text{ operate } c_1 \to c_1 + c_2 + c_3$$

$$y = \begin{vmatrix} 1 & 1 & \omega^2 \\ 1 & y+\omega^2 & \omega \\ 1 & \omega & 1+y \end{vmatrix} \left(\text{By } \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix} \right)$$

$$= y\begin{vmatrix} 1 & 1 & \omega^2 \\ 0 & y+\omega^2-1 & \omega-\omega^2 \\ 0 & \omega-1 & 1+y-\omega^2 \end{vmatrix}$$

$= y\{(y + \omega^2 - 1)(1 + y - \omega^2) - \omega(\omega - 1)(1 - \omega)\}$

$= y(y^2 - (\omega^2 - 1)^2) + y\omega(\omega - 1)^2$

$= y^3 + y(\omega - 1)^2 (\omega - (\omega + 1)^2) = y^3$

Trig Ratio

19. The value of $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ is :

$\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ का मान है :

(1) $\frac{3}{4}$ (2) $\frac{3}{2}(1+\cos 20°)$ (3) $\frac{3}{2}$ (4) $\frac{3}{4}+\cos 20°$

Ans. **(1)**

sol. $\left(\frac{1+\cos 20^o}{2}\right)+\left(\frac{1+\cos 100^o}{2}\right)-\frac{1}{2}(2\cos 10^o \cos 50^o)$

$=1+\frac{1}{2}(\cos 20^o+\cos 100^o)-\frac{1}{2}[\cos 60^o+\cos 40^o]$

$=\left(1-\frac{1}{4}\right)+\frac{1}{2}[\cos 20^o+\cos 100^o-\cos 40^o]$

$=\frac{3}{4}+\frac{1}{2}[2\cos 60^o\times\cos 40^o-\cos 40^o]$

$=\frac{3}{4}$

P & C

20. A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then :

8 पुरूषों तथा 5 महिलाओं में से 11 सदस्यों की एक कमेटी बनाई जाती है। यदि कम से कम 6 पुरूषों वाली कमेटी बनाने के m तरीके

हैं तथा कम से कम 3 महिलाओं वाली कमेटी बनाने के n तरीके हैं, तो :

(1) $m = n = 68$ (2) $m + n = 68$ (3) $m = n = 78$ (4) $n = m - 8$

Ans. **(3)**

sol. Here $m = {}^8C_6 \cdot {}^5C_5 + {}^8C_7 \cdot {}^5C_4 + {}^8C_8 \cdot {}^5C_3 = 78$

$n = {}^5C_3 \cdot {}^8C_8 + {}^5C_4 \cdot {}^8C_7 + {}^5C_5 \cdot {}^8C_6 = 78$

So $m = n = 78$

Straight Line

21. Slope of a line passing through P(2, 3) and intersecting the line, $x + y = 7$ at a distance of 4 units from P, is :

P(2, 3) से होकर जाने वाली एक रेखा, जो रेखा $x + y = 7$ को P से 4 इकाई की दूरी पर प्रतिच्छेदित करती है, की ढाल (slope) है :

(1) $\frac{\sqrt{7}-1}{\sqrt{7}+1}$ (2) $\frac{1-\sqrt{7}}{1+\sqrt{7}}$ (3) $\frac{\sqrt{5}-1}{\sqrt{5}+1}$ (4) $\frac{1-\sqrt{5}}{1+\sqrt{5}}$

Ans. **(2)**

sol. Point at 4 units from P(2, 3) will be A($4\cos\theta + 2, 4\sin\theta + 3$) will satisfy $x + y = 7$

$\Rightarrow \quad \cos\theta + \sin\theta = \frac{1}{2}$ on squaring

$\Rightarrow \quad \boxed{\sin 2\theta = \frac{-3}{4}} \quad \Rightarrow \quad \frac{2\tan\theta}{1+\tan^2\theta} = -\frac{3}{4}$

$\Rightarrow \quad 3\tan^2\theta + 8\tan\theta + 3 = 0$

$\Rightarrow \quad \tan\theta = \frac{-8 \pm 2\sqrt{7}}{6}$ (ignoring -ve sign THINK !)

$\Rightarrow \quad \tan\theta = \frac{-8 \pm 2\sqrt{7}}{6} = \frac{1-\sqrt{7}}{1+\sqrt{7}}$

Definite Integration

22. The value of $\int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx$ is :

$\int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx$ का मान है –

(1) $\frac{\pi-2}{4}$ (2) $\frac{\pi-2}{8}$ (3) $\frac{\pi-1}{4}$ (4) $\frac{\pi-1}{2}$

Ans. **(3)**

sol. $I=\int_0^{\frac{\pi}{2}}\frac{\sin^3 x\,dx}{\sin x+\cos x}$

$\Rightarrow \quad I=\int_0^{\frac{\pi}{2}}\frac{\cos^3 x\,dx}{\sin x+\cos x}$

$\Rightarrow \quad 2I=\int_0^{\frac{\pi}{2}}\left(1-\frac{1}{2}\sin(2x)\right)dx$

$\Rightarrow \quad I=\frac{1}{2}\left[x+\frac{1}{4}\cos 2x\right]_0^{\frac{\pi}{2}}$

$I=\frac{1}{2}\left(\frac{\pi-1}{2}\right)=\frac{\pi-1}{4}$

Tangent & Normal

23. Let S be the set of all values of x for which the tangent to the curve $y=f(x)=x^3-x^2-2x$ at (x, y) is parallel to the line segment joining the points (1, f(1)) and (–1, f(–1)), then S is equal to :

माना S, x के उन सभी मानों का समुच्चय है, जिन पर वक्र $y=f(x)=x^3-x^2-2x$ के बिन्दु (x, y) पर खींची गई स्पर्श रेखा बिन्दुओं (1, f(1)) तथा (–1, f(–1)) को मिलाने वाले रेखाखण्ड के समान्तर है, तो S बराबर है :

(1) $\left\{\frac{1}{3},-1\right\}$ (2) $\left\{-\frac{1}{3},1\right\}$ (3) $\left\{-\frac{1}{3},-1\right\}$ (4) $\left\{\frac{1}{3},1\right\}$

Ans. **(2)**

sol. $y=f(x)=x^3-x^2-2x$

$\frac{dy}{dx}=3x^2-2x-2$

$f(1)=-1-2=-2, \quad f(-1)=-1-1+2=0$

According to question, $3x^2-2x-2$

$=\frac{f(1)-f(-1)}{1-(-1)}$

$\Rightarrow \quad 3x^2-2x-2=\frac{-2-0}{2}$

$\Rightarrow \quad 3x^2-2x-1=0$

$\Rightarrow \quad x=\frac{2+4}{6}=1,\frac{-1}{3}$

So, $S=\left\{\frac{-1}{3},1\right\}$

3D

24. A plane passing through the points (0, –1, 0) and (0, 0, 1) and making an angle $\frac{\pi}{4}$ with the plane $y-z+5=0$, also passes through the point :

बिन्दुओं (0, –1, 0) तथा (0, 0, 1) से होकर जाने वाला एक समतल, जो समतल y – z + 5 = 0 के साथ $\frac{\pi}{4}$ का कोण बनाता है, निम्न में से किस बिन्दु से होकर जाता है?

(1) $\left(\sqrt{2},1,4\right)$ (2) $\left(\sqrt{2},-1,4\right)$ (3) $\left(-\sqrt{2},-1,-4\right)$ (4) $\left(-\sqrt{2},1,-4\right)$

Ans. **(1)**

sol. Let the required plane be $\frac{x}{a}+\frac{y}{-1}+\frac{z}{1}=1$ given plane is $y-z+5=0$

$\therefore \quad \cos\frac{\pi}{4}=\frac{-1-1}{\sqrt{\frac{1}{a^2}+1+1}\sqrt{2}}$

$\Rightarrow \quad a^2=\frac{1}{2}$

$\Rightarrow \quad \boxed{\frac{1}{a}=\pm\sqrt{2}}$

$\Rightarrow \quad \boxed{\pm\sqrt{2}x-y+z=1}$

$\therefore \quad (\sqrt{2},1,4)$ satisfies $-\sqrt{2}\,x-y+z=1$

Differential Equation

25. The solution of the differential equation $x\frac{dy}{dx}+2y=x^2(x\neq 0)$ with $y(1)=1$, is :

अवकल समीकरण $x\frac{dy}{dx}+2y=x^2(x\neq 0)$ का हल जिसके लिए y(1) = 1 है, है :

(1) $y=\frac{4}{5}x^3+\frac{1}{5x^2}$ (2) $y=\frac{x^3}{5}+\frac{1}{5x^2}$ (3) $y=\frac{x^2}{4}+\frac{3}{4x^2}$ (4) $y=\frac{3}{4}x^2+\frac{1}{4x^2}$

Ans. **(3)**

sol. $\frac{dy}{dx}+\frac{2}{x}y=x$ $\qquad y(1)=1$ (given)

I.F $=e^{\int\frac{2}{x}dx}=x^2$

$\Rightarrow \quad yx^2=\frac{x^4}{4}+c$

$\Rightarrow$ at $x=1$; $y=1$

$\Rightarrow \quad c = \frac{3}{4}$

$\therefore \quad y = \frac{x^2}{4} + \frac{3}{4x^2}$

Sequence & Progression

26. Let the sum of the first n terms of a nonconstant A.P., a_1 a_2, a_3, ... be $50n + \frac{n(n-7)}{2}A$, where A is a constant. If d is the commondifference of this A.P., then the ordered pair (d, a_{50}) is equal to :

माना भिन्न पदों वाली समान्तर श्रेढ़ी (non-constant A.P.) , a_1 a_2, a_3, ... के प्रथम n पदों का योगफल $50n + \frac{n(n-7)}{2}A$ है, जहाँ A एक अचर है। यदि इस समान्तर श्रेढ़ी का सार्वअन्तर d है, तो क्रमित युग्म (d, a_{50}) बराबर है :

(1) (50, 50 + 46A) (2) (A, 50 + 45A) (3) (A, 50 + 46A) (4) (50, 50 + 45A)

Ans. **(3)**

sol. $\because \quad S_n = \left(50 - \frac{7A}{2}\right)n + n^2 \times \frac{A}{2}$

$\therefore \quad$ Common difference $= \frac{A}{2} \times 2 = \boxed{A}$

$a_{50} = a_1 + 49 \times d$
$= (50 - 3A) + 49A$
$= 50 + 46A$

So, $(d, a_{50}) = (A, 50 + 46A)$

Mathematical Reasoning

27. For any two statements p and q, the negation of the expression $p \vee (\sim p \wedge q)$ is :

किन्हीं दो कथनों p तथा q के लिए, व्यंजक $p \vee (\sim p \wedge q)$ का निषेधन (negation) है :

(1) $\sim p \wedge \sim q$ (2) $\sim p \vee \sim q$ (3) $p \wedge q$ (4) $p \leftrightarrow q$

Ans. **(1)**

sol. $\sim(p \vee (\sim p \wedge q)) = \sim(\sim p \wedge q) \wedge \sim p$

$= (\sim q \vee p) \wedge \sim p$

$= \sim p \wedge (p \vee \sim q)$

$= (\sim q \wedge \sim p) \wedge (p \wedge \sim p)$

$= (\sim p \wedge \sim q)$

Tangent & Normal

28. If the tangent to the curve, $y = x^3 + ax - b$ at the point (1, –5) is perpendicular to the line, $-x + y + 4 = 0$, then which one of the following points lies on curve?

यदि वक्र $y = x^3 + ax - b$ के बिन्दु $(1, -5)$ पर खींची गई स्पर्शरेखा, रेखा $-x + y + 4 = 0$ पर लम्बवत् है, तो निम्न में से कौनसा एक बिन्दु, वक्र पर स्थित है?

(1) $(-2, 1)$ (2) $(2, -2)$ (3) $(2, -1)$ (4) $(-2, 2)$

Ans. **(2)**

sol. $f(x) = x^3 + ax - b \Rightarrow f'(x) = 3x^2 + a$

$f(1) = -5$ and $f'(1) = 3 + a$

$\Rightarrow 1 + a - b = -5$

$\Rightarrow a - b = -6$ ...(1)

Also slope of tangent $P(1, -5) = -1 = f'(1)$

$\therefore 3 + a = -1$

$\Rightarrow a = -4$

$\Rightarrow b = 2$

$\therefore$ Equation of the curve is $f(x) = x^3 - 4x - 2$

$\therefore$ $(2, -2)$ lies on the curve

Function

29. If the function $f : R - \{1, -1\} \to A$ defined by $f(x) = \frac{x^2}{1-x^2}$, is surjective, then A is equal to :

यदि फलन $f : R - \{1, -1\} \to A$, $f(x) = \frac{x^2}{1-x^2}$ द्वारा परिभाषित है तथा आच्छादी (surjective) है, तो A बराबर है :

(1) $[0, \infty)$ (2) $R - \{-1\}$ (3) $R - (-1, 0)$ (4) $R - [-1, 0)$

Ans. **(4)**

sol. $f(x) = \frac{x^2}{1-x^2}$

$\Rightarrow f(-x) = \frac{x^2}{1-x^2} = f(x)$

$\Rightarrow f'(x) = \frac{2x}{(1-x^2)^2}$

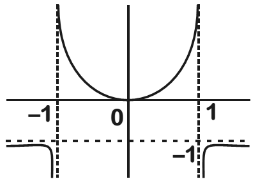
$\therefore$ f(x) increasing in $x \in (0, \infty)$

Also $f(0) = 0$

$\lim_{x \to \pm\infty} f(x) = -1$ and f(x) is even function

$\therefore$ Set $A \to R - [-1, 0)$

$\therefore$ Graph of function



Probability

30. Four persons can hit a target correctly with probabilities $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{8}$ respectively. If all hit at the target independently, then the probability that the target would be hit, is :

चार व्यक्तियों के एक लक्ष्य पर ठीक प्रकार से प्रहार करने की प्रायिकताएँ क्रमशः $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ तथा $\frac{1}{8}$ हैं। यदि सभी इस लक्ष्य पर स्वतंत्र रूप से प्रहार करते हैं, तो लक्ष्य पर आघात होने की प्रायिकता है :

(1) $\frac{7}{32}$ (2) $\frac{25}{192}$ (3) $\frac{1}{192}$ (4) $\frac{25}{32}$

Ans. **(4)**

sol. P(at least one) = – P(none)

$$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{7}{8}$$

$$= 1 - \frac{7}{32} = \boxed{\frac{25}{32}}$$