MATHS

09 APRIL 2019 [Phase : II]

JEE MAIN PAPER ONLINE

Height & Distance

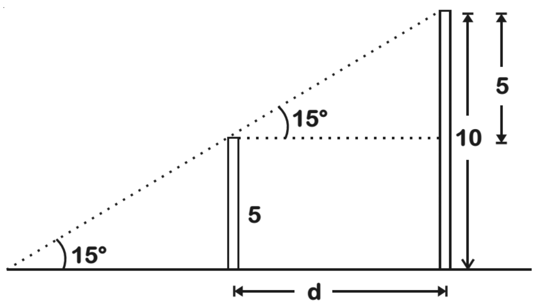
1. Two poles standing on a horizontal ground are of heights 5 m and 10 m respectively. The line joining their tops makes an angle of 15° with the ground. Then the distance (in m) between the poles, is:

क्षैतिज धरातल पर खड़े दो खम्बों की ऊँचाई क्रमशः 5 m तथा 10 m है। उनके शिखरों को मिलाने वाली रेखा धरातल से 15° का कोण बनाती है, तो खम्बों के बीच की दूरी (m में) है :

(1) $5\left(2+\sqrt{3}\right)$ (2) $10\left(\sqrt{3}-1\right)$ (3) $5\left(\sqrt{3}+1\right)$ (4) $\frac{5}{2}\left(2+\sqrt{3}\right)$

A. 1

sol.



$\tan 15^{\circ} = \frac{5}{d} \Rightarrow d = \frac{5}{\tan 15^{\circ}} = \frac{5\left(\sqrt{3}+1\right)}{\sqrt{3}-1}$

$= \frac{5(4+2\sqrt{3})}{2}$

$= 5(+2\sqrt{3})$

Determinant

2. If the system of equations $2x + 3y - z = 0$, $x + ky - 2z = 0$ and $2x - y + z = 0$ has a non-trivial solution (x, y, z), then $\frac{x}{y}+\frac{y}{z}+\frac{z}{x}+k$ is equal to:

यदि समीकरण निकाय $2x + 3y - z = 0$, $x + ky - 2z = 0$ तथा $2x - y + z = 0$ का एक अतुच्छ (non-trivial) हल (x, y, z) है, तो $\frac{x}{y}+\frac{y}{z}+\frac{z}{x}+k$ बराबर है :

(1) $\frac{1}{2}$ (2) -4 (3) $\frac{3}{4}$ (4) $-\frac{1}{4}$

A. 1

sol. $\Delta = 0 \Rightarrow \begin{vmatrix} 2 & 3 & -1 \\ 1 & k & -2 \\ 2 & -1 & 1 \end{vmatrix} = 0 \Rightarrow \boxed{k = \frac{9}{2}}$

$\therefore$ Equations are $2x + 3y - z = 0$ ....(i)

$2x - y + z = 0$ ....(ii)

$2x + 9y - 4z = 0$ ....(iii)

By (i) – (ii) $\boxed{2y = z}$ $\therefore$ $\boxed{z = -4x}$ and $\boxed{2x + y = 0}$

$\therefore \frac{x}{y} + \frac{y}{z} + \frac{z}{x} + k = \frac{-1}{2} + \frac{1}{2} - 4 + \frac{9}{2}$

$= \frac{1}{2}$

Quadratic Equation

3. If m is chosen in the quadratic equation $(m^2 + 1)\, x^2 - 3x + (m^2 + 1)^2 = 0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is:

यदि द्विघातीय समीकरण $(m^2 + 1)\, x^2 - 3x + (m^2 + 1)^2 = 0$ में m इस प्रकार लिया गया है, कि इसके मूलों का योगफल अधिकतम है, तो इसके मूलों के घन का निरपेक्ष अन्तर है :

(1) $8\sqrt{3}$ (2) $10\sqrt{5}$ (3) $4\sqrt{3}$ (4) $8\sqrt{5}$

A. 4

sol. Sum of roots $= \frac{3}{m^2 + 1}$

For maximum $m = 0$

Hence equation becomes $x^2 - 3x + 1 = 0$

$\alpha + \beta = 3,\ \alpha\beta = 1, |\alpha - \beta| = \sqrt{5}$

$|\alpha^3 - \beta^3| = (\alpha - \beta)(\alpha^2 + \beta^2 + \alpha\beta)| = \sqrt{5}(9 - 1) = 8\sqrt{5}$

3 D

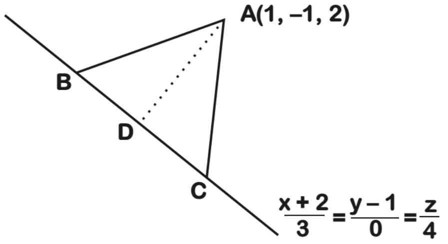
4. The vertices B and C of a ΔABC lie on the line, $\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4}$ such that BC = 5 units. Then the area (in sq. units) of this triangle, given that the point $A(1, -1, 2)$, is:

ΔABC के शीर्ष B तथा C रेखा $\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4}$ पर स्थित हैं तथा BC = 5 इकाई है। यदि दिया है कि बिन्दु $A(1, -1, 2)$ है, तो इस त्रिभुज का क्षेत्रफल (वर्ग इकाइयों में) है:

(1) $5\sqrt{17}$ (2) $\sqrt{34}$ (3) 6 (4) $2\sqrt{34}$

A. 2

sol.



Area of $\Delta ABC = \frac{1}{2}\times BC\times AD$

Given BC = 5 so we need perpendicular distance of A from line BC.

Let a point D on BC = $(3\lambda - 2, 1, 4\lambda)$

$\overrightarrow{AD} = (3\lambda - 3)\hat{i} + 2\hat{j} + (4\lambda - 2)\hat{k}$

Also $\overrightarrow{AD}$ & $\overrightarrow{BC}$ should be perpendicular $\overrightarrow{AD}.\overrightarrow{BC} = 0$

$(3\lambda - 3)3 + 2(0) + (4\lambda - 2)4 = 0$

$9\lambda - 9 + 16\lambda - 8 = 0 \Rightarrow \lambda = \frac{17}{25}$

Hence, $D = \left(\frac{1}{25}, 1, \frac{68}{25}\right)$

$|\overrightarrow{AD}| = \sqrt{\left(\frac{1}{25}-1\right)^2 + (2)^2 + \left(\frac{68}{25}-2\right)^2}$

$= \sqrt{\left(\frac{-24}{25}\right)^2 + 4 + \left(\frac{18}{25}\right)^2}$

$= \sqrt{\frac{(24)^2 + 4(25)^2 + (18)^2}{25^2}}$

$= \sqrt{\frac{576 + 2500 + 324}{25^2}}$

$= \sqrt{\frac{3400}{25^2}}$

$= \frac{\sqrt{34}.10}{25} = \frac{2\sqrt{34}}{5}$

Area of triangle $= \frac{1}{2}\times|\overrightarrow{BC}|\times|\overrightarrow{AD}|$

$= \frac{1}{2}\times 5\times\frac{2\sqrt{34}}{5} = \sqrt{34}$

Differential Equation

5. If $\cos x \frac{dy}{dx} - y\sin x = 6x$, $\left(0 < x < \frac{\pi}{2}\right)$ and $y\left(\frac{\pi}{3}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to :

यदि $\cos x \frac{dy}{dx} - y \sin x = 6x, \left(0 < x < \frac{\pi}{2}\right)$ तथा $y\left(\frac{\pi}{3}\right) = 0$ है, तो $y\left(\frac{\pi}{6}\right)$ बराबर है :

(1) $-\frac{\pi^2}{2}$ (2) $-\frac{\pi^2}{4\sqrt{3}}$ (3) $\frac{\pi^2}{2\sqrt{3}}$ (4) $-\frac{\pi^2}{2\sqrt{3}}$

A. 4

sol. $\cos x dy - (\sin x) y dx = 6x\, dx$

$\Rightarrow \int d(y \cos x) = \int 6x\, dx$

$\Rightarrow y \cos x = 3x^2 + C$

As $y\left(\frac{\pi}{3}\right) = 0 \Rightarrow (0) \times \left(\frac{1}{2}\right) = \frac{3\pi^2}{9} + C \Rightarrow C = \frac{-\pi^2}{3}$

$\Rightarrow y \cos x = 3x^2 - \frac{\pi^2}{3}$

For $y\left(\frac{\pi}{6}\right)$

$y \frac{\sqrt{3}}{2} = \frac{3\pi^2}{36} - \frac{\pi^2}{3}$

$\frac{\sqrt{3}y}{2} = \frac{-3\pi^2}{12} \Rightarrow y = \frac{-\pi^2}{2\sqrt{3}}$

Indefinite Integration

6. If $\int e^{\sec x}(\sec x \tan x f(x) + \sec \times \tan x + \sec^2 x))\, dx = e^{\sec x} f(x) + C$, then a possible choice of f(x) is :

यदि $\int e^{\sec x}(\sec x \tan x f(x) + \sec \times \tan x + \sec^2 x))\, dx = e^{\sec x} f(x) + C$, तो f(x) का एक संभव विकल्प है :

(1) $\sec x - \tan x - \frac{1}{2}$ (2) $\sec x + \tan x + \frac{1}{2}$ (3) $\sec x + x \tan x - \frac{1}{2}$ (4) $x \sec x + \tan x + \frac{1}{2}$

A. 2

sol. $\int e^{\sec x}(\sec x \tan x\, f(x) + (\sec x \tan x + \sec^2 x))dx = e^{\sec x} f(x) + C$

$\because$ We know that

$\int e^{g(x)}((g'(x) f(x)) + f'(x))dx = e^{g(x)} \times f(x) + C$

$\therefore f(x) = \int((\text{sex } x \tan x) + \sec^2 x)\, dx$

$\therefore \boxed{f(x) = \sec x + \tan x + C}$

Binomial Theorem

7. If some three consecutive coefficients in the binomial expansion of $(x + 1)^n$ in powers of x are in the ratio 2:15:70, then the average of these three coefficients is:

यदि $(x + 1)^n$ के x की घातों में द्विपद प्रसार में कोई तीन क्रमागत गुणांक 2:15:70 के अनुपात में हैं, तो इन तीन गुणांकों का औसत

है :

(1) 625 (2) 964 (3) 232 (4) 227

A. 3

sol. Given ${}^{n}C_{r-1} : {}^{n}C_{r} : {}^{n}C_{r+1} = 2 : 15 : 70$

$$\frac{{}^{n}C_{r-1}}{{}^{n}C_{r}} = \frac{2}{15} \ \& \ \frac{{}^{n}C_{r}}{{}^{n}C_{r+1}} = \frac{15}{70}$$

$$\Rightarrow \quad \frac{r}{n-r+1} = \frac{2}{15} \ \& \ \frac{r+1}{n-r} = \frac{3}{14}$$

$\Rightarrow \quad 15r = 2n - 2r + 2 \ \& \ 14r + 14 = 3n - 3r$

$\Rightarrow \quad 17r = 2n + 2 \quad \& \quad 17r = 3n - 14$

i.e., $2n + 2 = 3n - 14 \quad \Rightarrow \quad n = 16 \quad \& \quad r = 2$

$$\text{Mean} = \frac{{}^{16}C_1 + {}^{16}C_2 + {}^{16}C_3}{3} = \frac{16+120+560}{3}$$

$$= \frac{696}{3} = 232$$

Straight Line

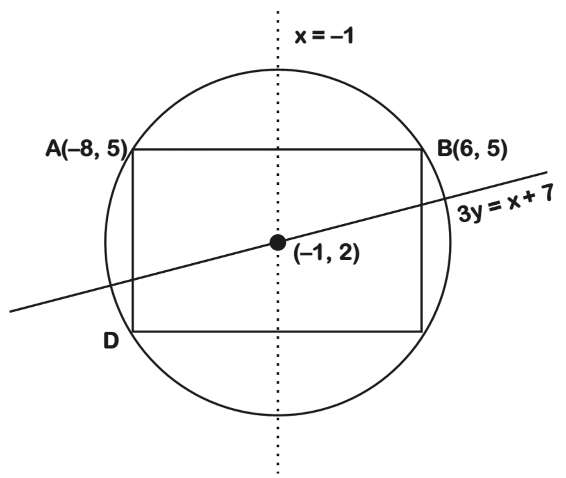
8. A rectangle is inscribed in a circle with a diameter lying along the line $3y = x + 7$. If the two adjacent vertices of the rectangle are $(-8, 5)$ and $(6, 5)$, then the area of the rectangle (in sq. units) is :

एक वृत्त, जिसका एक व्यास रेखा $3y = x + 7$ के अनुदिश है, के अन्तर्गत एक आयत बनाया गया है। यदि आयत के दो संलग्न शीर्ष $(-8, 5)$ तथा $(6, 5)$ हैं, तो आयत का क्षेत्रफल (वर्ग इकाइयों में) है :

(1) 56 (2) 84 (3) 72 (4) 98

A. 2

sol. Given situation



Perpendicular bisector of AB will pass from centre.

$\therefore$ Equation of perpendicular bisector $x = -1$

Hence centre (–1, 2)

Let $D = (\alpha, \beta) \Rightarrow \dfrac{\alpha+6}{2} = -1$ & $\dfrac{\beta+5}{2} = 2$

$\alpha = -8$ & $\beta = -1$ $D \equiv (-8, -1)$

$|AD| = 6$ & $|AB| = 14$

Area = 6 × 14 = 84

Vector

9. If a unit vector $\vec{a}$ makes angles $\dfrac{\pi}{3}$ with $\hat{i}$, $\dfrac{\pi}{4}$ with $\hat{j}$ and $\theta \in (0, \pi)$ with $\hat{k}$, then a value of θ is :

यदि एक मात्रक सदिश $\vec{a}$, $\hat{i}$ से $\dfrac{\pi}{3}$, $\hat{j}$ से $\dfrac{\pi}{4}$ तथा $\hat{k}$ से $\theta \in (0, \pi)$ कोण बनाता है, तो θ का एक मान है :

(1) $\dfrac{5\pi}{12}$ (2) $\dfrac{2\pi}{3}$ (3) $\dfrac{\pi}{4}$ (4) $\dfrac{5\pi}{6}$

A. 2

sol. Let $\cos\alpha, \cos\beta, \cos\gamma$ be direction cosines of $\vec{a}$ Hence, by given data

$\cos\alpha = \cos\dfrac{\pi}{3}, \cos\beta = \cos\dfrac{\pi}{4}$ & $\cos\gamma = \cos\theta$

$\therefore \quad \cos^2\dfrac{\pi}{3} + \cos^2\dfrac{\pi}{4} + \cos^2\theta = 1$

$\cos^2\theta = \dfrac{1}{4} \Rightarrow \cos\theta = \pm\dfrac{1}{2}, \ \theta = \dfrac{\pi}{3}$ or $\dfrac{2\pi}{3}$

Continuity & Diff

10. If the function $f(x) = \begin{cases} a|\pi - x| + 1, & x \le 5 \\ b|x - \pi| + 3, & x > 5 \end{cases}$ is continuous at x = 5, then the value of a – b is:

यदि फलन $f(x) = \begin{cases} a|\pi - x| + 1, & x \le 5 \\ b|x - \pi| + 3, & x > 5 \end{cases}$

x = 5 पर संतत है, तो a – b का मान है :

(1) $\dfrac{2}{\pi-5}$ (2) $\dfrac{-2}{\pi+5}$ (3) $\dfrac{2}{\pi+5}$ (4) $\dfrac{2}{5-\pi}$

A. 4

sol. L.H.L $\lim\limits_{x\to5} b|\pi - 5| + 3 = (5-\pi)b + 3$

$f(5) =$ R.H.L. $\lim\limits_{x\to5} a|5-\pi| + 1 = a(5-\pi) + 1$

For continuity LHL = RHL

$(5-\pi)b + 3 = (5-\pi)a + 1$

$\Rightarrow \quad 2 = (a-b)(5-\pi)$

$\Rightarrow \quad a - b = \dfrac{2}{5-\pi}$

Complex Number

11. Let $z \in C$ be such that $|z| < 1$. If $\omega = \dfrac{5+3z}{5(1-z)}$, then

माना $z \in C$ इस प्रकार है कि $|z| < 1$. यदि $\omega = \dfrac{5+3z}{5(1-z)}$, तो :

(1) $5\,\text{Re}(\omega) > 4$ (2) $5\,\text{Re}(\omega) > 1$ (3) $4\,\text{Im}(\omega) > 5$ (4) $5\,\text{Im}(\omega) < 1$

A. 2

sol. $\omega = \dfrac{5+3z}{5-5z} \quad \Rightarrow \quad 5w - 5wz = 5 + 3z$

$\Rightarrow \quad 5\omega - 5 = z(3 + 5\omega)$

$\Rightarrow \quad z = \dfrac{5(\omega-1)}{3+5\omega}$

Given $|z| < 1$

$\Rightarrow \quad 5|\omega - 1| < |3 + 5\omega|$

$\Rightarrow \quad 25(\omega\bar{\omega} - \omega - \bar{\omega} + 1) < 9 + 25\omega\bar{\omega} + 15\omega + 15\bar{\omega}$

(usin $|z|^2 = z\,\bar{z}$)

$\Rightarrow \quad 16 < 40\omega + 40\bar{\omega}$

$\Rightarrow \quad \omega + \bar{\omega} > \dfrac{2}{5}$

$\Rightarrow \quad 2\,\text{Re}(\omega) > \dfrac{2}{5} \qquad \Rightarrow \quad \text{Re}(\omega) > \dfrac{1}{5}$

3 D

12. Let P be the plane, which contains the line of intersection of the planes, $x + y + z - 6 = 0$ and $2x + 3y + z + 5 = 0$ and it is perpendicular to the xy-plane. Then the distance of the point (0, 0, 256) from P is equal to :

माना P एक समतल है जिसमें समतलों $x + y + z - 6 = 0$ तथा $2x + 3y + z + 5 = 0$ की प्रतिच्छेदन रेखा अंतर्विष्ट है तथा यह xy-तल के लंबवत है, तो बिन्दु (0, 0, 256) की P से दूरी बराबर है :

(1) $63\sqrt{5}$ (2) $\dfrac{17}{\sqrt{5}}$ (3) $205\sqrt{5}$ (4) $\dfrac{11}{\sqrt{5}}$

A. 4

sol. Let the plane be

$P \equiv (2x + 3y + z + 5) + \lambda(x + y + z - 6) = 0$

As the above plane is perpendicular to xy plane

$\Rightarrow \quad ((2+\lambda)\hat{i} + (3+\lambda)\hat{j} + (1+\lambda)\hat{k}).\hat{k} = 0$

$\Rightarrow \quad \lambda = -1$

$P \equiv x + 2y + 11 = 0$

Distance from (0, 0, 256)

$$\left|\frac{0+0+11}{\sqrt{5}}\right| = \frac{11}{\sqrt{5}}$$

Continuity & Diff

13. If $f(x) = [x] - \left[\frac{x}{4}\right], x \in R$, where [x] denotes the greatest integer function, then :

(1) $\lim_{x\to 4+} f(x)$ exists but $\lim_{x\to 4-} f(x)$ does not exist

(2) f is continuous at x = 4

(3) $\lim_{x\to 4-} f(x)$ exists but $\lim_{x\to 4+} f(x)$ does not exist

(4) Both $\lim_{x\to 4-} f(x)$ and $\lim_{x\to 4+} f(x)$ exist but are not equal

यदि $f(x) = [x] - \left[\frac{x}{4}\right], x \in R$ है, जहाँ [x] महत्तम पूर्णांक फलन है, तो :

(1) $\lim_{x\to 4+} f(x)$ का अस्तित्व है परन्तु $\lim_{x\to 4-} f(x)$ का अस्तित्व नहीं है।

(2) x = 4 पर f संतत है।

(3) $\lim_{x\to 4-} f(x)$ का अस्तित्व है परन्तु $\lim_{x\to 4+} f(x)$ का अस्तित्व नहीं है।

(4) $\lim_{x\to 4-} f(x)$ तथा $\lim_{x\to 4+} f(x)$ दोनों का अस्तित्व है परन्तु वह बराबर नहीं हैं।

A. 2

sol. L.H.L $\lim_{x\to 4^-}[x] - \left[\frac{x}{4}\right] = 3 - 0 = 3 \left(x < 4 \Rightarrow [x] = 3 \;\&\; \frac{x}{4} < 1 \Rightarrow \left[\frac{x}{4}\right] = 0\right)$

R.H.L $\lim_{x\to 4^+}[x] - \left[\frac{x}{4}\right] = 4 - 1 = 3 \left(x > 4 \Rightarrow [x] = 4 \;\&\; \frac{x}{4} > 1 \Rightarrow \left[\frac{x}{4}\right] = 1\right)$

$f(4) = [4] - \left[\frac{4}{4}\right] = 4 - 1 = 3$

LHL = f(4) = RHL

Hence f(x) is continuous at x = 4

Circle

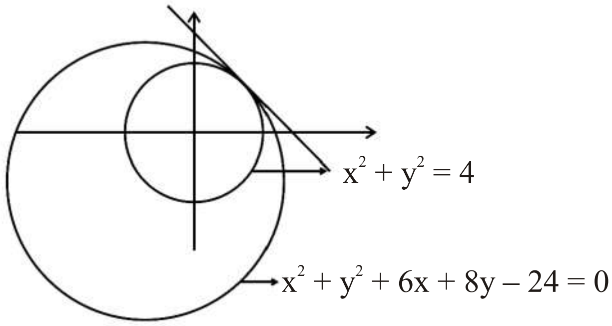
14. The common tangent to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 + 6x + 8y - 24 = 0$ also passes through the point :

वृत्तों $x^2 + y^2 = 4$ तथा $x^2 + y^2 + 6x + 8y - 24 = 0$ की उभयनिष्ठ स्पर्श रेखा निम्न में से किस बिन्दु से होकर जाती है?

(1) (–6, 4) (2) (–4, 6) (3) (4, –2) (4) (6, –2)

A. 4

sol. In given situation $d_{c_1c_2} = |r_1 - r_2|$



Common tangent

$S_1 - S_2 = 0$

$6x + 8y - 20 = 0 \Rightarrow 3x + 4y - 10 = 0$

Hence (6, –2) lies on it

Function

15. The domain of the definition of the function $f(x) = \frac{1}{4-x^2} + \log_{10}(x^3 - x)$ is :

$f(x) = \frac{1}{4-x^2} + \log_{10}(x^3 - x)$ द्वारा परिभाषित फलन का प्रांत है :

(1) $(-1, 0) \cup (1, 2) \cup (3, \infty)$ (2) $(-1, 0) \cup (1, 2) \cup (2, \infty)$

(3) $(1, 2) \cup (2, \infty)$ (4) $(-2, -1) \cup (-1, 0) \cup (2, \infty)$

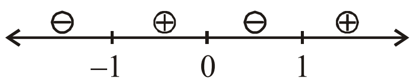
A. 2

sol. For domain denominator $\neq 0$

$4 - x^2 \neq 0 \quad \Rightarrow \quad x \neq \pm 2$ ....(1)

and $x^3 - x > 0$

$\Rightarrow \quad x(x-1)(x+1) > 0$



$x \in (-1, 0) \cup (1, \infty)$ ....(2)

Hence domain is intersection of (1) & (2) i.e.

$x \in (-1, 0) \cup (1, 2) \cup (2, \infty)$

Sequence & Progression

16. The sum of the series $1 + 2 \times 3 + 3 \times 5 + 4 \times 7 + \ldots$ upto 11^{th} term is :

श्रेणी $1 + 2 \times 3 + 3 \times 5 + 4 \times 7 + \ldots$ के 11वें पद तक योगफल है :

(1) 916 (2) 946 (3) 945 (4) 915

A. 2

sol. $1 + 2.3 + 3.5 + 4.7 + \ldots\ldots$

Lets break the sequence as shown

$1+\underbrace{(2.3+3.5+4.7+......)}_{S}$

We find $S_{10} = \sum_{n=1}^{10}(n+1)(2n+1)$

$= \sum_{n=1}^{10}(2n^2+3n+1)$

$= \frac{2n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2} + n(n=10)$

$= \frac{2.10.11.21}{6} + \frac{3.10.11}{2} + 10$

$= 770 + 165 + 10 = 945$

Hence required sum = 1 + 945 = 946

Parabola

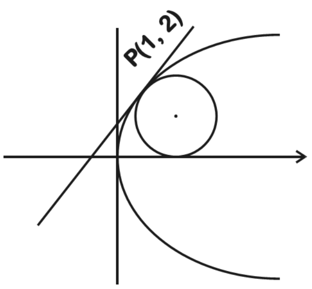
17. The area (in sq. units) of the smaller of the two circles that touch the parabola, $y^2 = 4x$ at the point (1, 2) and the x-axis is :

परवलय $y^2 = 4x$ को बिन्दु (1, 2) पर स्पर्श करने वाले तथा x-अक्ष को स्पर्श करने वाले दो वृत्तों में से छोटे वृत्त का क्षेत्रफल (वर्ग इकाइयों में) है :

(1) $4\pi(3+\sqrt{2})$ (2) $8\pi(2-\sqrt{2})$ (3) $8\pi(3-2\sqrt{2})$ (4) $4\pi(2-\sqrt{2})$

A. 3

sol.



The circle and parabola will have common tangent at P(1, 2)

$\therefore$ Equation of tangent to parabola

$\equiv y\times(2) = 4\frac{(x+1)}{2} \Rightarrow 2y = 2x+2$

Let equation of circle be (using family of circles)

$(x - x_1)^2 + (y - y_1)^2 + \lambda T = 0$

$\Rightarrow \quad c \equiv (x - 1)^2 + (y - 2)^2 + \lambda(x - y + 1) = 0$

Also circle touches x-axis $\Rightarrow$ y-coordinate of centre = radius

$\Rightarrow \quad c \equiv x^2 + y^2 + (\lambda - 2)x + (-\lambda - 4)y + (\lambda + 5) = 0$

$$\frac{\lambda+4}{2}=\sqrt{\left(\frac{\lambda-2}{2}\right)^2+\left(\frac{-\lambda-4}{2}\right)^2-(\lambda+5)}$$

$\Rightarrow \quad \frac{\lambda^2-4\lambda+4}{4}=\lambda+5 \Rightarrow \lambda^2-4\lambda+4=4\lambda+20$

$\Rightarrow \quad \lambda^2-8\lambda-16=0$

$\Rightarrow \quad \lambda=\frac{8\pm\sqrt{64+64}}{2}$

$=4\pm4\sqrt{2}$

$\lambda=4-4\sqrt{2}$ (Other value forms bigger circle)

Hence centre of circle $(2\sqrt{2}-2, 4-2\sqrt{2})$

Radius $=4-2\sqrt{2}$

Area $=\pi(4-2\sqrt{2})^2=8\pi(3-2\sqrt{2})$

Matrices

18. The total number of matrices

$A=\begin{pmatrix}0 & 2y & 1\\ 2x & y & -1\\ 2x & -y & 1\end{pmatrix}$, $(x, y \in R, x \neq y)$ for

Which $A^TA=3I_3$ is :

आव्यूहों $A=\begin{pmatrix}0 & 2y & 1\\ 2x & y & -1\\ 2x & -y & 1\end{pmatrix}$, $(x, y \in R, x \neq y)$

जिनके लिए $A^TA=3I_3$ है, की कुल संख्या है :

(1) 6 (2) 3 (3) 4 (4) 2

A. 3

sol. $A^TA=\begin{bmatrix}0 & 2x & 2x\\ 2y & y & -y\\ 1 & -1 & 1\end{bmatrix}\begin{bmatrix}0 & 2y & 1\\ 2x & y & -1\\ 2x & -y & 1\end{bmatrix}=3I$

$=\begin{bmatrix}8x^2 & 0 & 0\\ 0 & 6y^2 & 0\\ 0 & 0 & 3\end{bmatrix}\begin{bmatrix}3 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 3\end{bmatrix}$

$\Rightarrow \quad 8x^2=3,\ 6y^2=3$

$x = \pm\sqrt{\frac{3}{8}}, y = \pm\sqrt{\frac{1}{2}}$

Total combinations of $(x, y) = 2 \times 2 = 4$

Mathematical Reasoning

19. If $p \Rightarrow (q \vee r)$ is false, then truth values of p, q, r are respectively :

यदि $p \Rightarrow (q \vee r)$ सत्य नहीं है, तो p, q, r के सत्य मान क्रमशः हैं :

(1) T, F, F (2) F, F, F (3) T, T, F (4) F, T, T

A. 1

sol. For $p \Rightarrow (q \vee r)$ to be F

$(q \vee r)$ must be F & p must be T

for $(q \vee r)$ to be F q and r must b F

p, q, r must be T, F, F

Statistics

20. The mean and the median of the following ten numbers in increasing order

10, 22, 26, 29, 34, x, 42, 67, 70, y are 42 and 35 respectively, then $\frac{y}{x}$ is equal to :

वर्धमान क्रम में निम्न दस संख्याओं 10, 22, 26, 29, 34, x, 42, 67, 70, y के माध्य तथा माध्यिका क्रमशः 42 तथा 35 हैं, तो $\frac{y}{x}$ बराबर है :

(1) $\frac{7}{3}$ (2) $\frac{8}{3}$ (3) $\frac{7}{2}$ (4) $\frac{9}{4}$

A. 1

sol. Mean $= \frac{\Sigma x_i}{n} = \frac{x + y + 300}{10} = 42 \Rightarrow x + y = 120$

Median $= \frac{T_5 + T_6}{2} = 35 = \frac{34 + x}{2} \Rightarrow x = 36 \;\&\; y = 84$

Hence $\frac{y}{x} = \frac{84}{36} = \frac{7}{3}$

Sequence & Progression

21. Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are added to the total number of balls used in forming the equilateral triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then the number of balls used to form the equilateral triangle is :

कुछ एक जैसी गेंदें पंक्तियों में इस प्रकार रखी गई हैं कि वह एक समबाहु त्रिभुज बनाती है। पहली पंक्ति में एक गेंद है, दूसरी पंक्ति में दो गेंदें हैं तथा इसी प्रकार अन्य पंक्तियों में गेंदें हैं। समबाहु त्रिभुज बनाने में लगी कुल गेंदों में यदि एक जैसी 99 गेंदें और जोड़ दी जायें तो इन सारी गेंदों को एक ऐसे वर्ग के आकार में रखा जा सकता है जिसकी प्रत्येक भुजा में त्रिभुज की प्रत्येक भुजा से ठीक दो गेंदें कम हैं, तो समबाहु त्रिभुज बनाने में लगी गेंदों की संख्या है :

(1) 157 (2) 225 (3) 262 (4) 190

A. 4

sol. Balls used in equilateral triangle $= \frac{n(n+1)}{2}$

Here side of equiolateral triangle has n-balls No. of balls in each side of square is = $(n-2)$

Given $= \frac{n(n+1)}{2} + 99 = (n-2)^2$

$\Rightarrow \quad n^2 + n + 198 = 2n^2 - 8n + 8$

$\Rightarrow \quad n^2 - 9n - 190 = 0$

$\Rightarrow \quad n^2 - 19n + 10n - 190 = 0$

$\Rightarrow \quad (n-19)(n+10) = 0$

$\Rightarrow \quad n = 19$

Balls used to form triangle

$= \frac{n(n+1)}{2} = \frac{19 \times 20}{2} = 190$

Sets and Relations

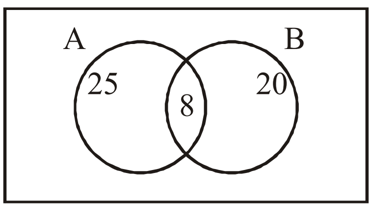
22. Two newspapers A and B are published in a city. It is known that 25% of the city population reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisements is :

एक शहर में दो समाचार पत्र A तथा B प्रकाशित होते हैं। यह ज्ञात है कि शहर की 25% जनसंख्या A पढ़ती है तथा 20% B पढ़ती है जबकि 8% A तथा B दोनों को पढ़ती है। इसके अतिरिक्त, A पढ़ने तथा B न पढ़ने वालों में 30% विज्ञापन देखते हैं और B पढ़ने तथा A न पढ़ने वालों में भी 40% विज्ञापन देखते हैं, जबकि A तथा B दोनों को पढ़ने वालों में से 50% विज्ञापन देखते हैं, तो जनसंख्या में विज्ञापन देखने वालों का प्रतिशत है :

(1) 13.9 (2) 13 (3) 12.8 (4) 13.5

A. 1

sol.



n(A only) = 25 – 8 = 17%

n(B only) = 20 – 8 = 12%

% of people from A only who read advertisement = 17 × 0.3 = 5.1%

% of people from B only who read advertisement = 12 × 0.4 = 4.8%

% of people from A & B both who read advertisement = 8 × 0.5 = 4%

Total % of people who read advertisement = 5.1 + 4.8 + 4 = 13.9%

Tangent & Normal

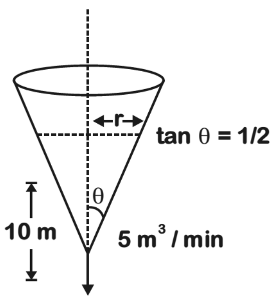
23. A water tank has the shape of an inverted right circular cone, whose semi-vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rate of 5 cubic metre per minute. Then the rate (in m/min.), at which the level of water is rising at the instant when the depth of water in the tank is 10 m; is :

एक पानी की टंकी उल्टे लंब वृत्तीय शंकु के आकार की है, जिसका अर्ध–शीर्ष कोण $\tan^{-1}\left(\frac{1}{2}\right)$ है। इसमें पानी 5 घन मीटर प्रति मिनट की समान दर से डाला जाता है, तो टंकी में पानी की गहराई 10 m होने पर वह दर (मी./मि. में), जिस पर पानी की सतह बढ़ रही है, है :

(1) $\frac{2}{\pi}$ (2) $\frac{1}{15\pi}$ (3) $\frac{1}{10\pi}$ (4) $\frac{1}{5\pi}$

A. 4

sol.



Given $\frac{dv}{dt} = 5m^3 / \text{min}$

$V = \frac{1}{3}\pi r^2 h$ ....(i) (where r is radius and h is height at any time)

Also $\tan\theta = \frac{r}{h} = \frac{1}{2} \Rightarrow h = 2r \Rightarrow \frac{dh}{dt} = \frac{2dr}{dt}$ ....(ii)

Differentiate eq^n. (i), we get

$$\frac{dV}{dt} = \frac{1}{3}\left(\pi 2r\frac{dr}{dt}h + \pi r^2\frac{dh}{dt}\right) = \frac{1}{3}\left(100\pi\frac{1}{2} + 25\pi\right)\frac{dh}{dt}$$

at h = 10, r = 5

$$5 = \frac{75\pi}{3}\frac{dh}{dt}$$

$$\Rightarrow \quad \frac{dh}{dt} = \frac{1}{5\pi} m/min$$

Definite Integration

24. The value of the integral

$$\int_0^1 x\cot^{-1}(1 - x^2 + x^4)\,dx \text{ is}$$

समाकल $\int_0^1 x\cot^{-1}(1 - x^2 + x^4)\,dx$ का मान है :

(1) $\frac{\pi}{4} - \log_e 2$ (2) $\frac{\pi}{2} - \log_e 2$ (3) $\frac{\pi}{2} - \frac{1}{2}\log_e 2$ (4) $\frac{\pi}{4} - \frac{1}{2}\log_e 2$

A. 4

sol. $\int_0^1 x\cot^{-1}\left(1 - x^2 + x^4\right)dx = \int_0^1 x\tan^{-1}\left(\frac{1}{1 + x^4 - x^2}\right)$

$$\Rightarrow \quad \int_0^1 x\tan^{-1}\left(\frac{x^2 - (x^2 - 1)}{1 + x^2(x^2 - 1)}\right)dx$$

$$\Rightarrow \quad \int_0^1 x\tan^{-1}x^2dx - \int_0^1 x\tan^{-1}(x^2 - 1)dx$$

Put $x^2 = t \Rightarrow 2xdx = dt$, (For 1st integration)

put $x^2 - 1 = k \Rightarrow 2xdx = dk$ (For 2nd integration)

$$\Rightarrow \quad \frac{1}{2}\int_0^1 x\tan^{-1}tdt - \frac{1}{2}\int_{-1}^0 1\tan^{-1}kdk$$

$$\Rightarrow \quad \frac{1}{2}\left(\int_0^1 t\tan^{-1}t\Big|_0^1 - \int_0^1 \frac{t}{1+t^2}dt\right) - \frac{1}{2}\left(k\tan^{-1}k\Big|_{-1}^0 - \int_{-1}^0 \frac{k}{1+k^2}dx\right)$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{\pi}{4} - \left(\frac{1}{2}\ln\left(1+t^2\right)\right)\Big|_0^1\right) - \frac{1}{2}\left(0 - \frac{\pi}{4} - \left(\frac{1}{2}\ln\left(1+k^2\right)\right)\Big|_{-1}^0\right)$$

$$\Rightarrow \quad \left(\frac{\pi}{8} - \frac{1}{4}\ln 2\right) - \left(\frac{-\pi}{8} - \frac{1}{4}10 - \ln 2\right)$$

$$\Rightarrow \quad \frac{\pi}{4} - \frac{1}{2}\ln 2$$

Area Under Curve

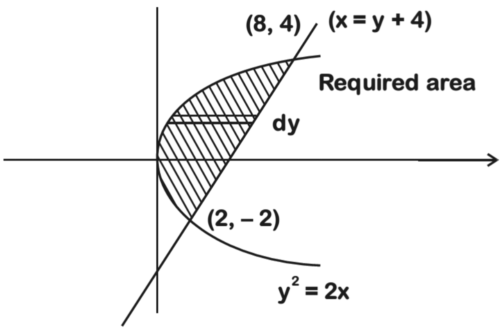
25. The area (in sq. units) of the region $A = \left\{(x, y) : \frac{y^2}{2} \le x \le y + 4\right\}$ is

क्षेत्र $A = \left\{(x, y) : \frac{y^2}{2} \le x \le y + 4\right\}$ का क्षेत्रफल (वर्ग इकाइयों में) है :

(1) 18 (2) 16 (3) $\frac{53}{3}$ (4) 30

A. 1

sol.



Hence area $= \int_{-2}^{4} x\,dy$

$= \int_{-2}^{4} \left(y + 4 - \frac{y^2}{2}\right) dy$

$= \frac{y^2}{2} + 4y - \frac{y^3}{6} \Big|_{-2}^{4} = \left(8 + 16 - \frac{64}{6}\right) - \left(2 - 8 + \frac{8}{6}\right)$

$= \left(24 - \frac{32}{3}\right) - \left(-6 + \frac{4}{3}\right) = \frac{40}{3} + \frac{14}{3} = \frac{54}{3} = 18$

Straight Line

26. If the two lines $x + (a - 1)y = 1$ and $2x + a^2y = 1$ $(a \in R - \{0, 1\})$ are perpendicular, then the distance of their point of intersection from the origin is

यदि दो रेखायें $x + (a - 1)y = 1$ तथा $2x + a^2y = 1$ $(a \in R - \{0, 1\})$ लंबवत् हैं, तो उनके प्रतिच्छेदन बिन्दु की मूल बिन्दु से दूरी है :

(1) $\sqrt{\frac{2}{5}}$ (2) $\frac{\sqrt{2}}{5}$ (3) $\frac{2}{5}$ (4) $\frac{2}{\sqrt{5}}$

A. 1

sol. For perpendicular $m_1 m_2 = -1$

$\left(\frac{-1}{a-1}\right)\left(\frac{-2}{a^2}\right) = -1$

$\Rightarrow \quad 2 = a^2(1 - a)$

$\Rightarrow \quad a^3 - a^2 + 2 = 0$

$\Rightarrow \quad (a+1)(a^2+2a+2)=0$

$\boxed{a=-1}$

Hence lines are $x-2y=1$ and $2x+y=1$

$\therefore \quad$ Intersection point $\left(\frac{3}{5},\frac{-1}{5}\right)$

Distance from origin $=\sqrt{\frac{9}{25}+\frac{1}{25}}=\sqrt{\frac{10}{25}}=\sqrt{\frac{2}{5}}$

Trig Ratio

27. The value of $\sin 10° \sin 30° \sin 50° \sin 70°$ is

$\sin 10° \sin 30° \sin 50° \sin 70°$ का मान है –

(1) $\frac{1}{18}$ (2) $\frac{1}{32}$ (3) $\frac{1}{16}$ (4) $\frac{1}{36}$

A. 3

sol. $\sin(60°+A).\sin(60°-A)\sin A=\frac{1}{4}\sin 3A$

Hence, $\sin 10° \sin 50° \sin 70°$

$=\sin 10° \sin(60°-10°)\sin(60°+10°)=\frac{1}{4}\sin 30°$

Hence, $\sin 10° \sin 30° \sin 50° \sin 70° = \frac{1}{4}\sin^2 30° = \frac{1}{16}$

Ellipse

28. If the tangent to the parabola $y^2=x$ at a point (α,β), $(\beta>0)$ is also a tangent to the ellipse, $x^2+2y^2=1$, then α is equal to

यदि परवलय $y^2 = x$ के एक बिन्दु (α, β), $(\beta > 0)$ पर, स्पर्श रेखा, दीर्घवृत्त $x^2 + 2y^2 = 1$ की भी स्पर्श रेखा है, तो α बराबर है:

(1) $\sqrt{2}-1$ (2) $\sqrt{2}+1$ (3) $2\sqrt{2}+1$ (4) $2\sqrt{2}-1$

A. 2

sol. Let tangent in terms of m to parabola and ellipse

i.e. $y=mx+\frac{1}{4m}$ for parabola at point $\left(\frac{1}{4m^2},\frac{-1}{2m}\right)$ and $y=mx\pm\sqrt{m^2+\frac{1}{2}}$ for ellipse on comparing

$\Rightarrow \quad \frac{1}{4m}=\pm\sqrt{m^2+\frac{1}{2}} \Rightarrow \frac{1}{16m^2}=m^2+\frac{1}{2}$

$\Rightarrow \quad 16m^4+8m^2-1=0$

$m^2=\frac{-8\pm\sqrt{64+64}}{2(16)}=\frac{-8\pm 8\sqrt{2}}{2(16)}=\frac{\sqrt{2}-1}{4}$

$$\alpha = \frac{1}{4m^2} = \frac{1}{4\frac{\sqrt{2}-1}{4}} = \sqrt{2}+1$$

Definite Integration

29. If $f: R \to R$ is a differentiable function and $f(2) = 6$, then $\lim_{x\to 2} \int_6^{f(x)} \frac{2t\ dt}{(x-2)}$ is :

यदि $f: R \to R$ एक अवकलनीय फलन है तथा $f(2) = 6$ है, तो $\lim_{x\to 2} \int_6^{f(x)} \frac{2t\ dt}{(x-2)}$ है :

(1) 0 (2) $2f'(2)$ (3) $12f'(2)$ (4) $24f'(2)$

A. 3

sol. Using L' Hospital rule and Leibnitz theorem

$$\lim_{x\to 2} \frac{\int_6^{f(x)} 2tdt}{(x-2)}$$

$$\lim_{x\to 2} \frac{2f(x)f'(x) - 0}{1}$$

$$2f(2)f'(2) = 12f'(2)$$

Sequence & Progression

30. If the sum and product of the first three terms in an A.P. are 33 and 1155, respectively, then a value of its 11^{th} term is

यदि एक समान्तर श्रेढ़ी के प्रथम तीन पदों का योगफल तथा गुणनफल क्रमशः 33 तथा 1155 है, तो इसके 11वें पद का एक मान है:

(1) –36 (2) 25 (3) –25 (4) –35

A. 3

sol. Let terms be a – d, a, a + d

$\Rightarrow \quad 3a = 33 \Rightarrow 11$

Product of terms

$(a - d)\, a\, (a + d) = 11\,(121 - d^2) = 1155$

$\Rightarrow \quad 121 - d^2 = 105 \Rightarrow d = \pm 4$

if d = 4

$$\left.\begin{matrix} T_1 = 7 \\ T_2 = 11 \\ T_3 = 15 \end{matrix}\right\} \Rightarrow T_{11} = T_1 + 10d = 7 + 10(4) = 47$$

if d = – 4

$$\left.\begin{aligned} T_1 &= 15 \\ T_2 &= 11 \\ T_3 &= 7 \end{aligned}\right\} \Rightarrow T_{11} = T_1 + 10d = 15 + 10(-4) = -25$$