**MATHS**

**08 Jan. 2020 [EVENING]**

**JEE MAIN PAPER ONLINE**

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## *Differential Calculus*

### *Monotonocity*

**1.** Let S be the set of all functions $f:[0,1]\rightarrow R$, which are continuous on [0, 1] and differentiable on (0,1). Then for every f in S, there exists a $c\in(0,1)$, depending on f, such that :

माना सभी फलनों $f:[0,1]\rightarrow R$, जो कि [0, 1] पर संतत है तथा (0,1) पर अवकलनीय हैं, का समुच्चय S है। तो S में प्रत्येक f के लिए f पर निर्भर एक $c\in(0,1)$ का अस्तित्व इस प्रकार है कि :

(1) $\dfrac{f(1)-f(c)}{1-c}=f'(c)$

(2) $|f(c)-f(1)|<|f'(c)|$

(3*) $|f(c)-f(1)|<(1-c)\,|f'(c)|$

(4) $|f(c)+f(1)|<(1+c)\,|f'(c)|$

Question ID : 4050361752

Option 1 ID : 4050366336

Option 2 ID : 4050366333

Option 3 ID : 4050366334

Option 4 ID : 4050366335

**Sol.** All four options are incorrect if

f(x) is a constant function.

## *Algebra*

### *Probability*

**2.** Let A and B be two events such that the probability that exactly one of them occurs is $\frac{2}{5}$ and the probability that A or B occurs is $\frac{1}{2}$, then the probability of both of them occur together is :

माना A तथा B दो घटनायें इस प्रकार हैं कि दोनों में से मात्र एक के होने की प्रायिकता $\frac{2}{5}$ है तथा A या B के होने की प्रायिकता $\frac{1}{2}$ है, तो दोनों के एक साथ होने की प्रायिकता है :

(1*) 0.10 (2) 0.01 (3) 0.02 (4) 0.20

Question ID : 4050361762

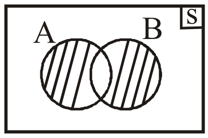
Option 1 ID : 4050366375

Option 2 ID : 4050366373

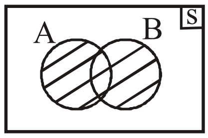
Option 3 ID : 4050366374

Option 4 ID : 4050366376

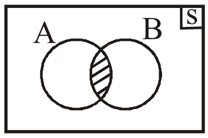
**Sol.** P(Exactly one) $= \frac{2}{5}$




P(A or B) $= \frac{1}{2}$




$P(A \cap B) =$




$P(A \cap B) = \frac{1}{2} - \frac{2}{5} = \frac{1}{10} = 0.10$

***Vectors***

***3D Geometry***

**3.** The mirror image of the point (1, 2, 3) in a plane is $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$. Which of the following points lies on this plane ?

बिन्दु (1, 2, 3) का एक समतल में प्रतिबिम्ब $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$ है। निम्न में से कौनसा बिन्दु इस समतल पर स्थित है ?

(1) (1, 1, 1) (2*) (1, –1, 1) (3) (–1, –1, 1) (4) (–1, –1, –1)

Question ID : 4050361759

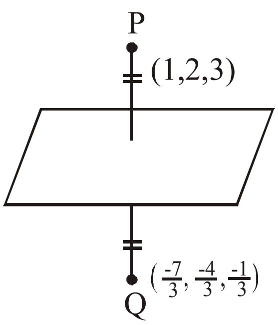
Option 1 ID : 4050366362

Option 2 ID : 4050366363

Option 3 ID : 4050366364

Option 4 ID : 4050366361

**Sol.**




$\overrightarrow{PQ} = \frac{-10}{3}i - \frac{-10}{3}j - \frac{-10}{3}k$

$\overrightarrow{PQ} = \frac{-10}{3}(i + j + k)$

DR's of normal to the plane = 1, 1, 1

Mid point of PQ = $\left(\frac{-2}{3}, \frac{1}{3}, \frac{4}{3}\right)$

Equation of plane = $1\cdot\left(x+\frac{2}{3}\right)+1\cdot\left(y-\frac{1}{3}\right)+1\cdot\left(z-\frac{1}{3}\right)=0$

$\Rightarrow x+y+z=1$

Now check options.

### *Integral Calculus*

### *Area Under Curve*

**4.** The area (in sq. units) of the region $\{(x, y) \in R^2 : x^2 \le y \le 3-2x\}$, is :

क्षेत्र $\{(x, y) \in R^2 : x^2 \le y \le 3-2x\}$ का क्षेत्रफल (वर्ग इकाईयों में) है :

(1) $\frac{31}{3}$ (2) $\frac{34}{3}$ (3) $\frac{29}{3}$ (4*) $\frac{32}{3}$

Question ID : 4050361755

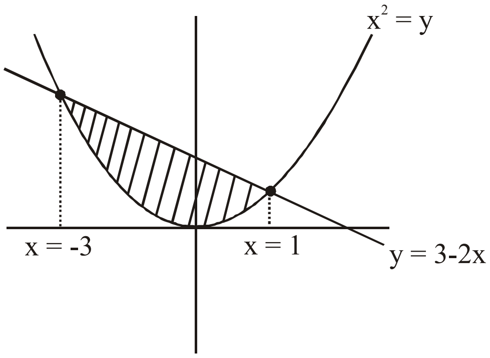
Option 1 ID : 4050366347

Option 2 ID : 4050366348

Option 3 ID : 4050366346

Option 4 ID : 4050366345

**Sol.** $S = \{(x, y) \in R^2 : x^2 \le y \le 3-2x\}$




To find points of intersection $\Rightarrow$

$x^2 = 3-2x$

$x^2 + 2x - 3 = 0$

$x = -3, 1$

Required Area = $\int_{-3}^{1}\left((3-2x)-x^2\right)dx$

$= 3x - x^2 - \frac{x^3}{3}\Big|_{-3}^{1} \qquad = \frac{32}{3}$

### *Algebra*

### *Binomial theorem*

**5.** If $\alpha$ and $\beta$ be the coefficients of $x^4$ and $x^2$ respectively in the expansion of $\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6$, then :

यदि $\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6$ के प्रसार में $x^4$ तथा $x^2$ के गुणांक क्रमशः $\alpha$ तथा $\beta$ हैं, तो :

(1) $\alpha+\beta=-30$ (2) $\alpha+\beta=60$ (3) $\alpha-\beta=60$ (4*) $\alpha-\beta=-132$

Question ID : 4050361749

Option 1 ID : 4050366323

Option 2 ID : 4050366321

Option 3 ID : 4050366324

Option 4 ID : 4050366322

**Sol.** $\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6 = 2\left[{}^6C_0\,x^6+{}^6C_2\,x^4(x^2-1)+{}^6C_4\,x^2(x^2-1)+{}^6C_6(x^2-1)^3\right]$

$= 2\left[32x^6-48x^4+18x^2-1\right]$

Coefficient of $x^2(\alpha)=-96$

Coefficient of $x^2(\beta)=36$ $\alpha-\beta=-132$

***Integral Calculus***

***Differential Equation***

**6.** The differential equation of the family of curves, $x^2=4b(y+b)$, $b\in R$, is :

वक्रों $x^2=4b(y+b)$, $b\in R$ के कुल का अवकल समीकरण है :

(1*) $x(y')^2=x+2yy'$ (2) $xy''=y'$ (3) $x(y')^2=x-2yy'$ (4) $x(y')^2=2yy'-x$

Question ID : 4050361756

Option 1 ID : 4050366351

Option 2 ID : 4050366349

Option 3 ID : 4050366350

Option 4 ID : 4050366352

**Sol.** $x^2=4b(y+b)$, $b\in R$

Dwrt x

$2x=4by'$

$b=\dfrac{x}{2y'}$

$DE \Rightarrow x^2=4\left(\dfrac{x}{2y'}\right)\left(y+\dfrac{x}{2y'}\right)$

$x^2=\dfrac{x(2yy'+x)}{(y')^2}$ $\Rightarrow xy'=x+2yy'$

***Algebra***

***Matrices***

**7.** If $A=\begin{pmatrix}2 & 2\\ 9 & 4\end{pmatrix}$ and $I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$, then $10A^{-1}$ is equal to :

यदि $A = \begin{pmatrix} 2 & 2 \\ 9 & 4 \end{pmatrix}$ तथा $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ हैं, तो $10A^{-1}$ बराबर है :

(1*) $A - 6I$ (2) $6I - A$ (3) $4I - A$ (4) $A - 4I$

Question ID : 4050361747

Option 1 ID : 4050366316

Option 2 ID : 4050366313

Option 3 ID : 4050366314

Option 4 ID : 4050366315

**Sol.** $A = \begin{pmatrix} 2 & 2 \\ 9 & 4 \end{pmatrix}$ ; $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

tr(A) = 6, |A| = – 10

Characteristic equation of A

$A^2 - 6A - 10I = 0$

$A^{-1}A^2 - 6A^{-1}A - 10^{-1}I = 0$

$A - 6I - 10A^{-1} = 0$

$10A^{-1} = A - 6I$

***Differential Calculus***

***Function***

**8.** Let $f : (1, 3) \rightarrow R$ be a function defined by $f(x) = \frac{x[x]}{1+x^2}$, where [x] denotes the greatest integer $\le x$. Then the range of f is :

माना $f : (1, 3) \rightarrow R$ एक फलन है, जो $f(x) = \frac{x[x]}{1+x^2}$, द्वारा परिभाषित है जहां [x] महत्तम पूर्णांक $\le x$ को दर्शाता है। तो f का परिसर है :

(1) $\left(\frac{3}{5}, \frac{4}{5}\right)$ (2*) $\left(\frac{2}{5}, \frac{3}{5}\right] \cup \left(\frac{3}{4}, \frac{4}{5}\right)$ (3) $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$ (4) $\left(\frac{2}{5}, \frac{4}{5}\right]$

Question ID : 4050361744

Option 1 ID : 4050366301

Option 2 ID : 4050366304

Option 3 ID : 4050366303

Option 4 ID : 4050366302

**Sol.** $f(x) = \frac{x[x]}{1+x^2}$ ; $x \in (1, 3)$

$$f(x) = \begin{cases} \frac{x}{1+x^2} & x \in (1,2) \\ \frac{2x}{1+x^2} & X \in [2,3) \end{cases}$$

$$f'(x) = [x]\left(\frac{(1+x^2)\cdot 1 - x \cdot 2x}{(1+x^2)^2}\right)$$

$f'(x) = [x]\dfrac{(1-x)(1+x)}{(1+x^2)^2}$

For $x \in (1, 3)$

$f'(x) < 0 \Rightarrow f(x) \downarrow$

Range of $f(x)$

$= \left(\dfrac{2}{5}, \dfrac{1}{2}\right) \cup \left(\dfrac{3}{5}, \dfrac{4}{5}\right]$

$f(1^+) = \dfrac{1}{2}^-$

$f(2^-) = \dfrac{2}{5}^+$

$f(2) = \dfrac{4}{5}$

$f(3^-) = \dfrac{3}{5}^+$

***Integral Calculus***

***Definite Integration***

**9.** $\lim\limits_{x\to 0} \dfrac{\int_0^x t\sin(10t)dt}{x}$ is equal to :

$\lim\limits_{x\to 0} \dfrac{\int_0^x t\sin(10t)dt}{x}$ बराबर है :

(1) $-\dfrac{1}{5}$ (2) $-\dfrac{1}{10}$ (3) $\dfrac{1}{10}$ (4*) 0

Question ID : 4050361751

Option 1 ID : 4050366329

Option 2 ID : 4050366331

Option 3 ID : 4050366330

Option 4 ID : 4050366332

**Sol.** $\lim\limits_{x\to 0} \dfrac{\int\limits_0^x t\sin(10t)\,dt}{x}\left(\dfrac{0}{0}\right)$

Using L' Hospital Rule

$\lim\limits_{x\to 0} x\sin(10x) = 0$

***Algebra***

***Complex Number***

**10.** Let $\alpha = \dfrac{-1+i\sqrt{3}}{2}$. If $a = (1+\alpha)\sum\limits_{k=0}^{100}\alpha^{2k}$ and $b = \sum\limits_{k=0}^{100}\alpha^{3k}$, then a and b are the roots of the quadratic equation :

माना $\alpha = \dfrac{-1+i\sqrt{3}}{2}$ है। यदि $a = (1+\alpha)\sum\limits_{k=0}^{100}\alpha^{2k}$ तथा $b = \sum\limits_{k=0}^{100}\alpha^{3k}$, तो a तथा b निम्न में से किस द्विघात समीकरण के मूल हैं ?

(1) $x^2 + 102x + 101 = 0$ (2) $x^2 - 101x + 100 = 0$ (3*) $x^2 - 102x + 101 = 0$ (4) $x^2 + 101x + 100 = 0$

Question ID : 4050361745

Option 1 ID : 4050366306

Option 2 ID : 4050366307

Option 3 ID : 4050366308

Option 4 ID : 4050366305

**Sol.** $\alpha = \dfrac{-1+i\sqrt{3}}{2} = \omega$

$a = (1 + \omega)(1 + \omega^2 + \omega^4 + .........+ \omega^{200})$

$a = (1 + \omega)\ \dfrac{(1-(\omega^2)^{101})}{(1-\omega^2)}$

$a = \dfrac{1-\omega^{202}}{1-\omega} = \dfrac{1-\omega}{1-\omega} = 1$

$b = 1 + \omega^3 + \omega^6 + ............+ \omega^{300} = 101$

$S = a + b = 102$

$P = a \cdot b = 101$

Equation. $x^2 - 102x + 101 = 0$

***Vectors***

***Vectors***

**11.** Let $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ be two vectors. If $\vec{c}$ is a vector such that $\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$ and $\vec{c} \cdot \vec{a} = 0$, then $\vec{c} \cdot \vec{b}$ is equal to :

माना दो सदिश $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ तथा $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ हैं। यदि एक सदिश $\vec{c}$ इस प्रकार है कि $\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$ तथा $\vec{c} \cdot \vec{a} = 0$ हैं, तो $\vec{c} \cdot \vec{b}$ बराबर है :

(1) $-\dfrac{3}{2}$ (2) $-1$ (3*) $-\dfrac{1}{2}$ (4) $\dfrac{1}{2}$

Question ID : 4050361760

Option 1 ID : 4050366368

Option 2 ID : 4050366367

Option 3 ID : 4050366366

Option 4 ID : 4050366365

**Sol.** $\vec{a} = i - 2j + k$

$\vec{b} = i - j + k$

$\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$ and $\vec{c} \cdot \vec{a} = 0$

m L ı

$\vec{b} \parallel \vec{c} - \vec{a}$

$\vec{c} - \vec{a} = \lambda\vec{b}$

$\vec{c} = \vec{a} + \lambda\vec{b}$

$\vec{c} \cdot \vec{a} = \vec{a} \cdot \vec{a} + \lambda\vec{a} \cdot \vec{b} = \vec{0}$

$6 + \lambda.4 = 0$

$\lambda = -\dfrac{3}{2}$

$\vec{c} = \vec{a} - \dfrac{3}{2}\vec{b}$

$$\vec{c}\cdot\vec{b}=\vec{a}\cdot\vec{b}-\frac{3}{2}\,\vec{b}\cdot\vec{b} = 4-\frac{3}{2}\times 3=-\frac{1}{2}$$

***Algebra***

***Sequence & progression***

**12.** If the $10^{th}$ term of an A.P. is $\frac{1}{20}$ and its $20^{th}$ term is $\frac{1}{10}$, then the sum of its first 200 terms is :

यदि एक समान्तर श्रेढ़ी का $10^{वां}$ पद $\frac{1}{20}$ है तथा इसका $20^{वां}$ पद $\frac{1}{10}$ है, तो इसके प्रथम 200 पदों का योग है :

(1) $50\frac{1}{4}$ (2) 100 (3) 50 (4*) $100\frac{1}{2}$

Question ID : 4050361750

Option 1 ID : 4050366326

Option 2 ID : 4050366327

Option 3 ID : 4050366325

Option 4 ID : 4050366328

**Sol.**
$$\left.\begin{aligned}T_{10}&=a+9d=\frac{1}{20}\\ T_{20}&=a+19d=\frac{1}{10}\end{aligned}\right\}$$

$$a=\frac{1}{200} \qquad d=\frac{1}{200}$$

$$S_{200}=\frac{200}{2}\left[2\times\frac{1}{200}+199\times\frac{1}{200}\right]$$

$$S_{200}=100\frac{1}{2}$$

***Integral Calculus***

***Definite Integration***

**13.** If $I=\int_1^2\frac{dx}{\sqrt{2x^3-9x^2+12x+4}}$, then :

यदि $I=\int_1^2\frac{dx}{\sqrt{2x^3-9x^2+12x+4}}$ है, तब :

(1) $\frac{1}{16}<I^2<\frac{1}{9}$ (2) $\frac{1}{8}<I^2<\frac{1}{4}$ (3*) $\frac{1}{9}<I^2<\frac{1}{8}$ (4) $\frac{1}{6}<I^2<\frac{1}{2}$

Question ID : 4050361754

Option 1 ID : 4050366342

Option 2 ID : 4050366343

Option 3 ID : 4050366341

Option 4 ID : 4050366344

**Sol.** $I = \int_1^2 \frac{dx}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$

Let $f(x) = 2x^3 - 19x^2 + 12x + 4$

$f'(x) = 6x^2 - 18x + 12$

$f'(x) = 6(x-1)(x+2)$

$f'(x) < 0 \text{ in } x \in (1, 2)$

$f(x) \downarrow$

$\frac{1}{\sqrt{f(x)}} \uparrow$

$(2-1) \times \frac{1}{\sqrt{f(1)}} < I < (2-1)\frac{1}{\sqrt{f(2)}}$

$\frac{1}{3} < I < \frac{1}{2\sqrt{2}}$

$\frac{1}{9} < I^2 < \frac{1}{8}$

## Coordinate Geometry

### Circle

**14.** It a line, $y = mx + c$ is a tangent to the circle, $(x-3)^2 + y^2 = 1$ and it is perpendicular to a line $L_1$, where $L_1$ is the tangent to the circle, $x^2 + y^2 = 1$ at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ ; then :

यदि एक रेखा $y = mx + c$, वृत्त $(x-3)^2 + y^2 = 1$ की एक स्पर्श रेखा है तथा यह एक रेखा $L_1$ पर लम्ब है, जहां $L_1$ वृत्त $x^2 + y^2 = 1$ के बिन्दु $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ पर स्पर्श रेखा है, तो :

(1) $c^2 + 7c + 6 = 0$ (2) $c^2 - 7c + 6 = 0$ (3*) $c^2 + 6c + 7 = 0$ (4) $c^2 - 6c + 7 = 0$

Question ID : 4050361757

Option 1 ID : 4050366354

Option 2 ID : 4050366353

Option 3 ID : 4050366355

Option 4 ID : 4050366356

**Sol.** Equation $L_1 \Rightarrow x \cdot \frac{1}{\sqrt{2}} + y \cdot \frac{1}{\sqrt{2}} = 1$

$x + y = \sqrt{2}$

$m_{L_1} = -1$

$m = 1$

$C : (x-3)^2 + y^2 = 1$

$L : y = x + c \qquad \Rightarrow x - y + c = 0$

Condition of tangency for circle $= \left|\frac{3-0+c}{\sqrt{2}}\right| = 1$

$(c + 3)^2 = 2$

$c^2 + 6c + 7 = 0$

***Differential Calculus***

***Tangent and normal***

**15.** The length of the perpendicular from the origin, on the normal to the curve, $x^2 + 2xy - 3y^2 = 0$ at the point (2, 2) is :

वक्र $x^2 + 2xy - 3y^2 = 0$ के बिन्दु (2, 2) पर खींचे गचे अभिलम्ब पर मूल बिन्दु से डाले गये लम्ब की लम्बाई है :

(1) 2 (2) $4\sqrt{2}$ (3*) $2\sqrt{2}$ (4) $\sqrt{2}$

Question ID : 4050361753

Option 1 ID : 4050366338

Option 2 ID : 4050366340

Option 3 ID : 4050366339

Option 4 ID : 4050366337

**Sol.** $x^2 + 2xy - 3y^2 = 0$

$(x - y)(x + 3y) = 0$

$x - y = 0$ & $x + 3y = 0$

(2, 2) satisfies $x - y = 0$

Equation of normal $x + y = \lambda$

at (2, 2)

$x + y = 4$

Length of perpendicular from

origin $= \left|\frac{0+0-4}{\sqrt{2}}\right| = 2\sqrt{2}$

***JEE Main Only topics***

***Mathematical Reasoning***

**16.** Which of the following statements is a tautology ?

निम्न में से कौनसा कथन एक पुनरुक्ति है ?

(1) $p \vee (\sim q) \rightarrow p \wedge q$ (2) $\sim(p \vee \sim q) \rightarrow p \wedge q$ (3) $\sim(p \wedge \sim q) \rightarrow p \vee q$ (4*) $\sim(p \vee \sim q) \rightarrow p \vee q$

Question ID : 4050361763

Option 1 ID : 4050366377

Option 2 ID : 4050366378

Option 3 ID : 4050366379

Option 4 ID : 4050366380

**Sol.**

| p | q | ~ q | pv ~ q | ~ (pv ~ q) | $p \vee q$ | ~ $(pv \vee q) \to p \vee q$ |
|---|---|---|---|---|---|---|
| T | T | F | T | F | T | T |
| T | F | T | T | F | T | T |
| F | T | F | F | T | T | T |
| F | F | T | T | F | F | T |

## *Algebra*
### *Quadratic Equation*

**17.** Let S be the set of all real roots of the equation, $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$. Then S :

(1) contains exactly two elements
(2) is an empty set
(3) contains at least four elements
(4*) is a singleton

माना समीकरण $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$ के सभी वास्तविक मूलों का समुच्चय S है। तो S :

(1) में मात्र दो अवयव हैं।
(2) एक रिक्त समुच्चय है।
(3) में कम से कम चार अवयव हैं।
(4) एक ही अवयव वाला समुच्चय है।

Question ID : 4050361746

Option 1 ID : 4050366311

Option 2 ID : 4050366309

Option 3 ID : 4050366312

Option 4 ID : 4050366310

**Sol.** $3^x (3^x - 1) + 2 = |3^x - 1| + 1\ |3^x - 2|$

Let 3x = t > 0

$t(t - 1) + 2 = |t - 1| + |t - 2|$

$t^2 - t + 2 = |t - 1| + |y - 2|$

Case-1 $t \in [2, \infty)$

$t^2 - t + 2 = t - 1 + t - 2$

$t^2 - 3t + 5 = 0$

$D < 0$

Case-2 $t \in [1, 2)$

$t^2 - t + 2 = t - 1 + 2 - t$

$t^2 - t + 2 = 1 - t + 2 - t$

$t^2 + t - 1 = 0$

$D < 0$

Case-3 $t \in (0, 1)$

$t^2 - t + 2 = t - 1 + 2 - t$

$t^2 - t + 2 = 1 - t + 2 - t$

$t^2 + t - 1 = 0$

$t = \frac{-1 \pm \sqrt{1+4}}{2}$ $(t > 0)$

$t = \frac{-1 + \sqrt{5}}{2}$ E(0, 1)

## *JEE Main Only topics*

## *Statistics*

**18.** The mean and variance of 20 observations are found to be 10 and 4, respectively. On rechecking, it was found that an observation 9 was incorrect and the correct observation was 11. Then the correct variance is :

20 प्रेक्षणों के माध्य तथा प्रसरण क्रमशः 10 तथा 4 पाये गये। पुनः जांच करने पर पाया गया कि एक प्रेक्षण 9 गलत था तथा सही प्रेक्षण 11 था। तो सही प्रसरण है :

(1) 3.98 (2) 4.01 (3*) 3.99 (4) 4.02

Question ID : 4050361761

Option 1 ID : 4050366369

Option 2 ID : 4050366371

Option 3 ID : 4050366370

Option 4 ID : 4050366372

**Sol.** $\overline{x} = 10 \Rightarrow \sum x_i = 20 \times 10 = 200$

$\sigma^2 = 4 \Rightarrow \frac{\sum x_i^2}{20} - (\overline{x})^2 = 4$

$\sum x_i^2 = 2080$

Actual $\sum x_i = 200 - 9 + 11 = 202$

$\sum x_i^2 = 2080 - 9^2 + 11^2 = 2120$

$\sigma^2 = \frac{2120}{20} - \left(\frac{202}{20}\right)^2$

$\sigma^2 = 3.99$

## *Algebra*

## *Determinant*

**19.** The system of linear equations

रेखिक समीकरण निकाय

$\lambda x + 2y + 2z = 5$

$2\lambda x + 3y + 5z = 8$

$4x + \lambda y + 6z = 10$ has :

(1) a unique solution when $\lambda = -8$

(2*) no solution when $\lambda = 2$

(3) no solution when $\lambda = 8$

(4) infinitely many solutions when $\lambda = 2$

(1) का मात्र एक हल है जब $\lambda = -8$ (2) का कोई हल नहीं है जब $\lambda = 2$

(3) का कोई हल नहीं है जब $\lambda = 8$ (4) के अनन्त हल हैं जब $\lambda = 2$

Question ID : 4050361748

Option 1 ID : 4050366317

Option 2 ID : 4050366320

Option 3 ID : 4050366318

Option 4 ID : 4050366319

**Sol.** $D = \begin{vmatrix} \lambda & 2 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{vmatrix} = (\lambda + 8)(2 - \lambda)$

For unique solution $\lambda \neq 2, -8$
for $\lambda = 2$

$D_x = \begin{vmatrix} 5 & 2 & 2 \\ 8 & 3 & 5 \\ 10 & 2 & 6 \end{vmatrix}$

$D_x = 16 \neq 0$ (No solution)

***Coordinate Geometry***

***Hyperbola***

**20.** If a hyperbola passes through the point P(10, 16) and it has vertices at $(\pm 6, 0)$, then the equation of the normal to it at P is :

यदि एक अतिपरवलय बिन्दु P(10, 16) से होकर जाता है तथा इसके शीर्ष $(\pm 6, 0)$ पर हैं, तो P पर इसके अभिलम्ब का समीकरण है :

(1) $x + 3y = 58$ (2) $3x + 4y = 94$ (3) $x + 2y = 42$ (4*) $2x + 5y = 100$

Question ID : 4050361758

Option 1 ID : 4050366358

Option 2 ID : 4050366357

Option 3 ID : 4050366359

Option 4 ID : 4050366360

**Sol.** $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $a = 6$

$\frac{x^2}{36} - \frac{y^2}{b^2} = 1$ ; p(10, 16) $b^2 = 144$

$\frac{x^2}{36} - \frac{y^2}{144} = 1$

Equation of normal at (10, 16) $\Rightarrow \frac{36.x}{10} + \frac{144.y}{16} = 36 + 144$

$\Rightarrow 2x + 5y = 100$

**INTEGER TYPE QUESTIONS**

***Coordinate Geometry***

***Parabola***

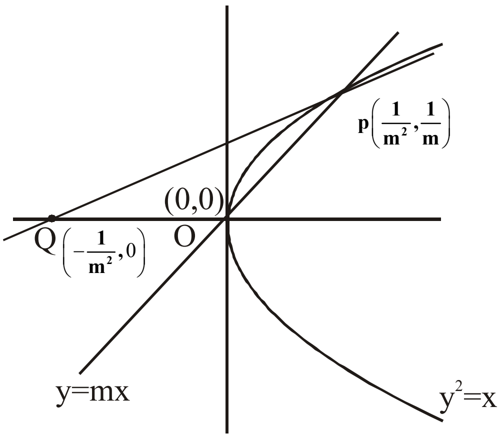
**21.** Let a line $y = mx\ (m > 0)$ intersect the parabola, $y^2 = x$ at a point P, other than the origin. Let the tangent to it at P meet the x-axis at the point Q. If area $(\Delta OPQ) = 4$ sq. units, then m is equal to______.

माना एक रेखा $y = mx\ (m > 0)$, परवलय $y^2 = x$ को मूल बिन्दु के अतिरिक्त एक बिन्दु P पर काटती है। माना P पर इसकी स्पर्श रेखा x- अक्ष को बिन्दु Q पर मिलती है। यदि $\Delta OPQ$ का क्षेत्रफल 4 वर्ग इकाई है, तो m बराबर है_______ ।

Ans. 0.5

Question ID : 4050361767

**Sol.**




Equation of tangent at P

T = 0

$$y \cdot \frac{1}{m} = \left(\frac{x + \frac{1}{m^2}}{2}\right)$$

$$\frac{2y}{m} = x + \frac{1}{m^2}$$

$$Q\left(-\frac{1}{m^2}, 0\right)$$

Area $\Delta OPQ = 4$

$$\frac{1}{2} \times \frac{1}{m^2} \times \frac{1}{m} = 4$$

$$m = \frac{1}{2} = 0.5$$

***Algebra***

***Sequence & progression***

**22.** The sum, $\sum_{n=1}^{7}\frac{n(n+1)(2n+1)}{4}$ is equal to________.

योगफल $\sum_{n=1}^{7}\frac{n(n+1)(2n+1)}{4}$ बराबर है________ ।

Ans. 504

Question ID : 4050361765

**Sol.** $\sum_{n=1}^{7}\frac{n(n+1)(2n+1)}{4}$

$= \frac{6}{4}\sum_{n=1}^{7}\frac{n(n+1)(2n+1)}{6}$

$= \frac{3}{2}\sum_{n=1}^{7}(\sum n^2)$

$= \frac{3}{2}\left(1^2+(1^2+2^2)+......+(1^2+2^2+....7^2)\right)$

$= \frac{3}{2}\left(7\times 1^2+6\times 2^2+5\times 3^2+4\times 4^2+3\times 5^2+2\times 6^2+1\times 7^2\right)$

$= 504$

***Trigonometry***

***Trigonometric Ratio and Identities***

**23.** If $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}}=\frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}}=\frac{1}{\sqrt{10}}$, $\alpha,\beta\in\left(0,\frac{\pi}{2}\right)$, then $\tan(\alpha+2\beta)$ is equal to_______.

यदि $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}}=\frac{1}{7}$ तथा $\sqrt{\frac{1-\cos 2\beta}{2}}=\frac{1}{\sqrt{10}}$, $\alpha,\beta\in\left(0,\frac{\pi}{2}\right)$ हैं, तो $(\alpha+2\beta)$ बराबर है_______ ।

Ans. 1

Question ID : 4050361768

**Sol.** $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}}=\frac{1}{7}$ $\qquad \alpha,\beta\in\left(0,\frac{\pi}{2}\right)$

$\frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha}=\frac{1}{7}$ $\qquad \Rightarrow \tan\alpha=\frac{1}{7}$

$\sqrt{\frac{1-\cos 2\beta}{2}}=\frac{1}{\sqrt{10}}$

$\sin\beta=\frac{1}{\sqrt{10}}$

$\tan\beta=\frac{1}{3}$

$\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan 2\beta}{1-\tan\alpha\cdot\tan 2\beta}$

$\tan 2\beta = \dfrac{2.\frac{1}{3}}{1-\frac{1}{9}} = \dfrac{3}{4}$

$\tan(\alpha+2\beta) = \dfrac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\times\frac{3}{4}}$

$\tan(\alpha+2\beta) = \dfrac{25}{25} = 1$

***Differential Calculus***

***Maxima & Minima***

**24.** Let f(x) be a polynomial of degree 3 such that f(–1) = 10, f(1) = – 6, f(x) has a critical point at x = – 1 and f '(x) has a critical point at x = 1. Then f(x) has a local minima at x = ________.

माना घात 3 का एक बहुपद f(x) इस प्रकार है कि f(–1) = 10, f(1) = – 6, f(x) का एक क्रांतिक बिन्दु x = – 1 है तथा f '(x) का एक क्रांतिक बिन्दु x = 1 है। तो f(x) का एक स्थानीय निम्ननिष्ठ है x = ________ ।

Ans. 3

Question ID : 4050361766

**Sol.** $f''(x) = \lambda(x-1)$

$f'(x) = \lambda\left(\dfrac{x^2}{2} - x\right) + c_1$

$f'(-1) = \lambda\left(\dfrac{1}{2}+1\right) + c_1 = 0$

$c_1 = \dfrac{-3}{2}\lambda$

$f'(x) = \lambda\left(\dfrac{x^2}{2} - x - \dfrac{3}{2}\right)$

$f(x) = \lambda\left(\dfrac{x^2}{6} - \dfrac{x^2}{2} - \dfrac{3}{2}x\right) + c_2$

$f(1) = \lambda\left(\dfrac{1}{6} - \dfrac{1}{2} - \dfrac{3}{2}\right) + c_2 = -6$

$f(-1) = \lambda\left(\dfrac{-1}{6} - \dfrac{1}{2} + \dfrac{3}{2}\right) + c_2 = 10$

$f(1) - f(-1) = -16$

$\lambda\left(\frac{1}{3}-3\right)=-16 \qquad \Rightarrow \lambda = 6$

Hence

$f'(x)=\lambda\left(\frac{x^2-2x-3}{2}\right)$

$f'(x)=3(x-3)(x+1)$

+   –   +
–1   3

f(x) has local mini at n = 3

***Algebra***

***P &C***

**25.** The number of 4 letter words (with or without meaning) that can be formed from the eleven letters of the word 'EXAMINATION' is_______.

शब्द 'EXAMINATION' के ग्यारह अक्षरों से बन सकने वाले 4 अक्षरों के शब्दों (अर्थ वाले तथा अथविहीन) की संख्या है _______ ।

Ans. 2454

Question ID : 4050361764

**Sol.** EXAMINATION

E – 1

X – 1

A – 2

M – 1

I – 2

N – 2

T – 1

O – 1

| Ways to select | Selection | Permutation | Words |
|---|---|---|---|
| 2A + 2A | $^3C_2$ | $\frac{4!}{2!2!}$ | 18 |
| 2A + 2D | $^3C_1 \cdot {}^7C_2$ | $\frac{4!}{2!}$ | 756 |
| 4D | $^8C_4$ | 4! | 1680 |

Total = 2454