MATHS
08 APRIL 2019 [Phase : I]
JEE MAIN PAPER ONLINE

Tangent & Normal

1. The shortest distance between the line y = x and the curve $y^2 = x-2$ is
रेखा y = x तथा वक्र $y^2 = x-2$ के बीच की न्यूनतम दूरी है :

(1) $\frac{11}{4\sqrt{2}}$ (2) $\frac{7}{8}$ (3) 2 (4) $\frac{7}{4\sqrt{2}}$

A. 4

sol. The shortest distance between line y = x and parabola = the distance between line y = x and tangent of parabola having slope 1.
Let equation of tangent of parabola having slope is

$$y = m(x-2) + \frac{a}{m}$$

where m = 1 and a = $\frac{1}{4}$ equation of tangent y = $x - \frac{7}{4}$

Distance between the line y = x and the tangent

$$= \left|\frac{\frac{7}{4}-0}{\sqrt{1^2+1^2}}\right| = \frac{7}{4\sqrt{2}}$$

Binomial Theorem

2. The sum of the co efficients of all even degree terms in x in the expansion of

$\left(x+\sqrt{x^3-1}\right)^6 + \left(x-\sqrt{x^3-1}\right)^6$, (x > 1) is equal to

$\left(x+\sqrt{x^3-1}\right)^6 + \left(x-\sqrt{x^3-1}\right)^6$, (x > 1) के प्रसार में x के सभी समघातीय पदों के गुणांकों का योग बराबर है :

(1) 24 (2) 32 (3) 26 (4) 29

A. 1

sol. $\left(x+\sqrt{x^3-1}\right)^6 + \left(x-\sqrt{x^3-1}\right)^6$

$= 2[{}^6C_0x^6 + {}^6C_2x^4(x^3-1) + {}^6C_4x^2(x^3-1) + {}^6C_6(x^3-1)^3]$

$= 2[x^6 + 15x^7 - 15x^4 + 15x^8 - 30x^5 + 15x^2 + x^9 - 3x^6 + 3x^3 - 1]$

sum of coefficient of even power of x = 2 [1–15 +15 + 15 – 3 – 1] = 24

Maxima & Minima

3. If S_1 and S_2 are respectively the sets of local minimum and local maximum points of the function,
$f(x) = 9x^4 + 12x^3 - 36x^2 + 25, x \in R$
यदि फलन $f(x) = 9x^4 + 12x^3 - 36x^2 + 25, x \in R$, के स्थानीय निम्नतम तथा स्थानीय उच्चतम बिन्दुओं के समुच्चय क्रमशः S_1 तथा S_2 हैं, तो :

(1) $S_1=\{-2\};S_2=\{0,1\}$ (2) $S_1=\{-2,1\};S_2=\{0\}$

(3) $S_1=\{-1\};S_2=\{0,2\}$ (4) $S_1=\{-2,0\};S_2=\{1\}$

A. 2

sol. $f(x) = 9x^4 + 12x^3 - 36x^2 + 72$

$f'(x) = 36[x^3 + x^2 - 2x] = 36x\,(x-1)\,(x+2)$

$-$ | $+$ | $-$ | $+$

-2 0 1

Whenever derivative changes sign form negative to positive we get local minima and whenever derivative change sign from positive to negative we get local maxima (while moving left to right on x-axis)

$S_1 = \{-2,1\}$

$S_2 = \{0\}$

MOD

4. If $2y=\left(\cot^{-1}\left(\frac{\sqrt{3}\cos x+\sin x}{\cos x-\sqrt{3}\sin x}\right)\right)^2, x\in\left(0,\frac{\pi}{2}\right)$ then $\frac{dy}{dx}$ is equal to

यदि $2y=\left(\cot^{-1}\left(\frac{\sqrt{3}\cos x+\sin x}{\cos x-\sqrt{3}\sin x}\right)\right)^2, x\in\left(0,\frac{\pi}{2}\right)$ है, तो $\frac{dy}{dx}$ बराबर है :

(1) $2x-\frac{\pi}{3}$ (2) $x-\frac{\pi}{6}$ (3) $\frac{\pi}{3}-x$ (4) $\frac{\pi}{6}-x$

A. 2

sol. $$2y=\left[\cot^{-1}\left(\frac{\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x}{\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x}\right)\right]^2$$

$$\Rightarrow 2y = 2y=\left[\cot^{-1}\left(\frac{\cos\left(\frac{\pi}{6}-x\right)}{\sin\left(\frac{\pi}{6}-x\right)}\right)\right]^2$$

$$\Rightarrow \quad 2y=\left[\cot^{-1}\left(\cot\left(\frac{\pi}{6}-x\right)\right)\right]^2 \quad \because \quad \frac{\pi}{6}-x\in\left(-\frac{\pi}{3},\frac{\pi}{6}\right)$$

$$\Rightarrow 2y=\begin{cases}\left(\frac{7\pi}{6}-x\right)^2 & \text{if } \frac{\pi}{6}-x\in\left(-\frac{\pi}{3},0\right)\\ \left(\frac{\pi}{6}-x\right)^2 & \text{if } \frac{\pi}{6}-x\in\left(0,\frac{\pi}{6}\right)\end{cases}$$

$$\frac{dy}{dx}=\begin{cases} x-\frac{7\pi}{6} & \text{if } x\in\left(\frac{\pi}{6},\frac{\pi}{2}\right) \\ x-\frac{\pi}{6} & \text{if } x\in\left(0,\frac{\pi}{6}\right) \end{cases}$$

Note: only one given option is correct

Straight Line

5. A point on the straight line, $3x + 5y = 15$ which is equidistant from the coordinate axes will lie only in :

(1) 4^{th} quadrant

(2) 1^{st} quadrant

(3) 1^{st}, 2^{nd} and 4^{th} quadrants

(4) 1^{st} and 2^{nd} quadrants

सरल रेखा $3x + 5y = 15$ पर स्थित एक बिन्दु, जो निर्देशांक अक्षों से समदूरस्थ है, केवल स्थित है :

(1) चतुर्थ चतुर्थांश में

(2) प्रथम चतुर्थांश में

(3) प्रथम, द्वितीय और चतुर्थ चतुर्थांश में

(4) प्रथम तथा द्वितीय चतुर्थांश में

A. 4

sol. A point which is equidistant from both the axes lies on either $y = x$ and $y = -x$

As it is given that the point lies on the line $3x + 5y = 15$

SO the required point is

$3x + 5y = 15$

$$\frac{x+y=0}{x=-\frac{15}{2}},\ y=\frac{15}{2}\Rightarrow\left(-\frac{15}{2},\frac{15}{2}\right)\ \{2^{nd}\text{ qudrant}\}$$

or $3x + 5y = 15$

$$\frac{x=y}{x=\frac{15}{8}},\ y=\frac{15}{8}\Rightarrow\left(\frac{15}{8},\frac{15}{8}\right)\left\{1^{st}\text{ quadrant}\right\}$$

Indefinite Integration

6. $\int\frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx$ is equal to

(where c is a constant of integration)

$\int\frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx$ बराबर है :

(जहाँ c एक समाकलन अचर है।)

(1) $2x + \sin x + 2\sin 2x + c$ (2) $x + 2\sin x + 2\sin 2x + c$
(3) $x + 2\sin x + \sin 2x + c$ (4) $2x + \sin x + \sin 2x + c$

A. 3

sol. $\int \frac{\sin\left(\frac{5x}{2}\right)}{\sin\left(\frac{x}{2}\right)} dx$

$= \int \frac{2\cos\frac{x}{2}.\sin\frac{5x}{2}}{2\cos\frac{x}{2}.\sin\frac{x}{2}} dx$

$= \int \frac{\sin 3x + \sin 2x}{\sin x} dx$

Now use $\sin 2x = 2\sin x \cos x$ and $\sin 3x = 3\sin x - 4\sin^3 x$

$= \int (1 + 2\cos x + 2\cos 2x)\, dx$

$= x + 2\sin x + \sin 2x + c$

Ellipse

7. If the tangents on the ellips $4x^2 + y^2 = 8$ at the points (1,2) and (a,b) are perependicular to each other, then a^2 is equal to

यदि दीर्घवृत्त $4x^2 + y^2 = 8$ के बिन्दुओं (1, 2) तथा (a, b) पर खींची गई स्पर्श रेखाएँ परस्पर लम्बवत् हैं, तो a^2 बरबार है :

(1) $\frac{64}{17}$ (2) $\frac{2}{17}$ (3) $\frac{4}{17}$ (4) $\frac{128}{17}$

A. 2

sol. Equation of tangent at A(1, 2);

$4x + 2y = 8 \quad \Rightarrow \quad 2x + y = 4$

So tangent at B(a, b) can be assumed as

$x - 2y = c \quad \Rightarrow \quad y = \frac{1}{2}\times - \frac{c}{2}$

Condition for tangency ;

$-\frac{c}{2} = \pm\sqrt{2\left(\frac{1}{2}\right)^2 + 8} = \pm\sqrt{\frac{17}{2}}$

$\Rightarrow \quad c = \pm\sqrt{34}$

Equation of tangent; $x - 2y = \pm 34$ ...(i)

Equation of tangent at P(a, b); $4ax + by = 8$ ...(ii)

Comparing both the equations ;

$\frac{4a}{1} = \frac{b}{-2} = \frac{8}{\pm\sqrt{34}}$

$\Rightarrow \quad a = \pm\frac{2}{\sqrt{34}} \quad \Rightarrow \quad a^2 = \frac{2}{17}$

Circle

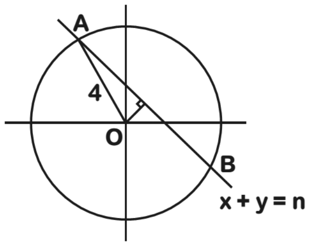
8. The sum of the squares of the lengths of the chords intercepted on the circle, $x^2 + y^2 = 16$, by the lines, $x + y = n$, $n \in N$, where N is the set of all natural numbers, is

वृत्त $x^2 + y^2 = 16$ पर रेखाओं $x + y = n$, $n \in N$ जहाँ N सभी प्राकृत संख्याओं का समुच्चय है, द्वारा काटी गई जीवाओं की लम्बाइयों के वर्गों का योग है :

(1) 105 (2) 160 (3) 320 (4) 210

A. 4

sol.



Let the chord $x + y = n$ cuts the circle $x^2 + y^2 = 16$ at A and B

length of perpendicular from O on $AB = \left|\frac{0+0-n}{\sqrt{1^2+1^2}}\right| = \frac{n}{\sqrt{2}}$

Length of chord $AB = 2\sqrt{4^2 - \left(\frac{n}{\sqrt{2}}\right)^2}$

$$= 2\sqrt{16 - \frac{n^2}{2}}$$

Here possible values of n are 1, 2, 3, 4, 5. Sum of square of length of chords $= \sum_{n=1}^{5} 4\left(16 - \frac{n^2}{2}\right)$

$$= 64 \times 5 - 2.\frac{5 \times 6 \times 11}{6} = 210$$

Function

9. If $f(x) = \log_e\left(\frac{1-x}{1+x}\right)$ $|x| < 1$ then $f\left(\frac{2x}{1+x^2}\right)$ is equal to

यदि $f(x) = \log_e\left(\frac{1-x}{1+x}\right)$ $|x| < 1$ है, तो $f\left(\frac{2x}{1+x^2}\right)$ बराबर है –

(1*) $2f(x)$ (2) $2f(x^2)$ (3) $-2f(x)$ (4) $(f(x))^2$

A. 1

sol. $\because \quad f(x) = ln\left(\frac{1-x}{1+x}\right)$

$$f\left(\frac{2x}{1+x^2}\right) = ln\left(\frac{1-\frac{2x}{1+x^2}}{1+\frac{2x}{1+2x^2}}\right)$$

$$= ln\left(\frac{1+x^2-2x}{1+x^2+2x}\right)$$

$$= ln\left(\frac{1-x}{1+x}\right)^2$$

$$= 2ln\left(\frac{1-x}{1+x}\right)$$

$$= 2f(x)$$

Differential Equation

10. Let $y = y(x)$ be the solution of the differential equation $(x^2+1)^2\frac{dy}{dx} + 2x\left(x^2+1\right)y = 1$ such that $y(0) = 0$. If $\sqrt{a}\,y(1) = \frac{\pi}{32}$ then the value of 'a' is

माना $y = y(x)$, अवकल समीकरण $(x^2+1)^2\frac{dy}{dx} + 2x\left(x^2+1\right)y = 1$ का हल है, जबकि $y(0) = 0$ है। यदि $\sqrt{a}\,y(1) = \frac{\pi}{32}$ है, तो 'a' का मान है :

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) 1 (4) $\frac{1}{16}$

A. 4

sol. $(1+x^2)\frac{2dy}{dx} + 2x(1+x^2)y = 1$

$\Rightarrow \quad \frac{dy}{dx} + \left(\frac{2x}{1+x^2}\right)y = \frac{1}{(1+x^2)^2}$

It is a linear differential equation

$\text{I.F.} = e^{\int\frac{2x}{1+x^2}dx} = e^{\ln(1+x^2)} = 1+x^2$

$\Rightarrow \quad y.(1+x^2) = \int\frac{dx}{1+x^2} + c$

$\Rightarrow \quad y\,(1+x^2) = \tan^{-1}x + c$

If $x = 0$ then $y = 0$

So, $0 = 0 + c$

$\Rightarrow \quad c = 0$

$\Rightarrow \quad y\,(1+x^2) = \tan^{-1}x$

put $x = 1$

$2y = \frac{\pi}{4}$

$\Rightarrow \quad 2\left(\frac{\pi}{32\sqrt{a}}\right) = \frac{\pi}{4}$

$\Rightarrow \quad \sqrt{a} = \frac{1}{4}$

$\Rightarrow \quad a = \frac{1}{16}$

ITF

11. If $\alpha = \cos^{-1}\left(\frac{3}{5}\right), \beta = \tan^{-1}\left(\frac{1}{3}\right)$ where $0 < \alpha, \beta < \frac{\pi}{2}$ then $\alpha - \beta$ is equal to

यदि $\alpha = \cos^{-1}\left(\frac{3}{5}\right), \beta = \tan^{-1}\left(\frac{1}{3}\right)$ हैं, जहाँ $0 < \alpha, \beta < \frac{\pi}{2}$, तो $\alpha - \beta$ बराबर है :

(1) $\tan^{-1}\left(\frac{9}{14}\right)$ (2) $\cos^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (3) $\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (4) $\tan^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

A. 3

sol. $\because \quad \cos\alpha = \frac{3}{5}$

$\Rightarrow \quad \tan\alpha = \frac{4}{3}$

$\Rightarrow \quad \text{and } \tan\beta = \frac{1}{3}$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha.\tan\beta}$$

$$= \frac{\frac{4}{3} - \frac{1}{3}}{1 + \frac{4}{9}} = \frac{1}{\frac{13}{9}}$$

$$= \frac{9}{13}$$

$$\alpha - \beta = \tan^{-1}\left(\frac{9}{13}\right) = \sin^{-1}\left(\frac{9}{5\sqrt{10}}\right) = \cos^{-1}\left(\frac{13}{5\sqrt{10}}\right)$$

Monotonocity

12. Let $f : [0,2] \rightarrow R$ be a twice differetiable function such that $f''(x) > 0$ for all $x \in (0,2)$. If $\phi(x) = f(x) + f(2-x)$ then ϕ is

(1) Decreasing on (0,2) (2) Increasing (0,2)

(3) Decreasing on (0,1) and increasing on (1,2) (4) Increasing on (0,1) and decreasing on (1,2)

$f : [0,2] \rightarrow R$ में दो बार अवकलनीय फलन इस प्रकार है कि सभी $x \in (0,2)$ के लिए $f''(x) > 0$ है। यदि $\phi(x) = f(x) + f(2-x)$ है, तो ϕ

(1) अन्तराल (0,2) में ह्रासमान है।

(2) अन्तराल (0,2) में वर्धमान है।

(3) अन्तराल (0,1) में ह्रासमान तथा अन्तराल (1,2) में वर्धमान है।

(4) अन्तराल (0,1) में वर्धमान तथा अन्तराल (1,2) में ह्रासमान है।

A. 3

sol. $\phi(x) = f(x) + f(2-x)$

differentiating w.r.t. x

$\phi'(x) = f'(x) - f'(2-x)$

For $\phi(x)$ to be increasing $\phi'(x) > 0$

$\Rightarrow f'(x) > f'(2-x)$

($\because$ $f''(x) > 0$ then $f'(x)$ is an increasing function)

$\Rightarrow x > 2 - x$

$\Rightarrow x > 1$

So $\phi(x)$ is increasing in (1, 2) and decreasing in

(0, 1)

Binomial Theorem

13. The sum of the series $2.{}^{20}C_0 + 5.{}^{20}C_1 + 8.{}^{20}C_2 + 11.{}^{20}C_3 + ... + 62.{}^{20}C_{20}$ is equal to

श्रेणी $2.{}^{20}C_0 + 5.{}^{20}C_1 + 8.{}^{20}C_2 + 11.{}^{20}C_3 + ... + 62.{}^{20}C_{20}$ का योग बरबार है :

(1) 2^{23} (2) 2^{25} (3) 2^{24} (4) 2^{26}

A. 2

sol. $2.{}^{20}C_0 + 5.{}^{20}C_1 + 8.{}^{20}C_2 + ... 62.{}^{20}C_{20}$

$= \sum_{r=0}^{20}(3r+2)\,{}^{20}C_r = 3\sum_{r=0}^{20} r.\,{}^{20}C_r + 2\sum_{r=0}^{20} {}^{20}C_r$

$= 60 \times 2^{19} + 2 \times 2^{20}$

$= 2^{21}[15 + 1] = 2^{25}$

P & C

14. All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is :

सभी अंकों 1, 1, 2, 2, 2, 2, 3, 4, 4 को एक साथ लेकर सभी संभव संख्यायें बनाई गई हैं। इस प्रकार की संख्याओं, जिनमें विषम अंक सम स्थानों पर हैं, की संख्या है :

(1) 180 (2) 175 (3) 162 (4) 160

A. 1

sol. There are total 9 digits; out of which only 3 digits are odd.

— — — — — — — — —
  ↑   ↑   ↑   ↑

Number of ways to arrange odd digits first

$= {}^4C_3.\frac{\lfloor 3}{\lfloor 2}$

Total number of 9 digit numbers $= \left({}^4C_3.\frac{\lfloor 3}{\lfloor 2}\right).\frac{\lfloor 6}{\lfloor 2 \lfloor 4}$

$= 180$

Determinant

15. The greatest value of $c \in R$ for which the system of linear equations

$x - cy - cz = 0$

$cx - y + cz = 0$
$cx + cy - z = 0$
has a non-trivial solution, is :
$c \in R$ का अधिकतम मान, जिसके लिए रैखिक समीकरण निकाय
$x - cy - cz = 0$
$cx - y + cz = 0$
$cx + cy - z = 0$
का एक अतुच्छ हल है, है :

(1) –1 (2) 0 (3) 2 (4) $\frac{1}{2}$

A. 4

sol. If the system of equations has non-trivial solutions, then

$$\begin{vmatrix} 1 & -c & -c \\ c & -1 & c \\ c & c & -1 \end{vmatrix} = 0$$

$\Rightarrow \quad (1 - c^2) + c(-c - c^2) - c(c^2 + c) = 0$
$\Rightarrow \quad (1 + c)(1 - c) - 2c^2(1 + c) = 0$
$\Rightarrow \quad (1 + c)(1 - c - 2c^2) = 0$
$\Rightarrow \quad (1 + c)^2(1 - 2c) = 0$
$\Rightarrow \quad c = -1 \text{ or } \frac{1}{2}$

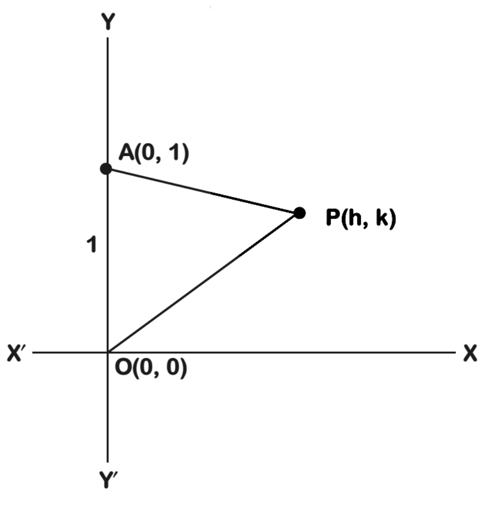
Straight Line

16. Let O(0,0) and A(0,1) be two fixed points. Then the locus of a point such that the perimeter of ΔAOP is 4, is
माना O(0,0) तथा A(0,1) दो निश्चित बिन्दु हैं, तो ऐसे बिन्दु P जिनके लिए ΔAOP का परिमाप 4 हो, का बिन्दुपथ है :

(1) $8x^2 - 9y^2 + 9y = 18$ (2) $9x^2 + 8y^2 - 8y = 16$
(3) $9x^2 - 8y^2 + 8y = 161$ (4) $8x^2 + 9y^2 - 9y = 18$

A. 2

sol. Let point P(h, k)
$\because \quad OA = 1$
So, $OP + AP = 3$
$\sqrt{h^2 + k^2} + \sqrt{h^2 + (k-1)^2} = 3$



$\Rightarrow \quad h^2 + (k-1)^2 = 9 + h^2 + k^2 - 6\sqrt{h^2+k^2}$

$\Rightarrow \quad 6\sqrt{h^2+k^2} = 2k + 8$

$\Rightarrow \quad 9(h^2 + k^2) = k^2 + 16 + 8k$

$\Rightarrow \quad 9h^2 + 8k^2 - 8k - 16 = 0$

Locus of point P will be,

$9x^2 + 8y^2 - 8y - 16 = 0$

Probability

17. Let A and B be two non-null events such that $A \subset B$. Then, which of the following statements is always correct?

माना A तथा B दो ऐसी अरिक्त (non-null) घटनायें हैं कि $A \subset B$ है, तो निम्न में से कौनसा कथन हमेशा सही है?

(1) $P(A|B) = P(B) - P(A)$ (2) $P(A|B) \le P(A)$

(3) $P(A|B) \ge P(A)$ (4) $P(A|B) = 1$

A. 3

sol. $\because \; A \subset B$; so $A \cap B = A$

Now, $P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)}$

$\Rightarrow \quad P\left(\frac{A}{B}\right) = \frac{P(A)}{P(B)}$

$\because \; P(B) \le 1$

So, $P\left(\frac{A}{B}\right) \ge P(A)$

Complex Number

18. If α and β be the roots of the equation $x^2 - 2x + 2 = 0$ then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is

यदि समीकरण $x^2 - 2x + 2 = 0$ के मूल α तथा β हैं, तो n का न्यूनतम मान, जिसके लिए $\left(\frac{\alpha}{\beta}\right)^n = 1$ है, है :

(1) 4 (2) 5 (3) 3 (4) 2

A. 1

sol. $x^2 - 2x + 2 = 0$

roots of this equation are $\frac{2 \pm -4}{2} = 1 \pm i$

Then $\frac{\alpha}{\beta} = \frac{1+i}{1-i} = \frac{(1+i)^2}{1-i^2} = i$

or $\frac{\alpha}{\beta} = \frac{1-i}{1+i} = \frac{(1-i)^2}{1-i^2} = i$

So, $\frac{\alpha}{\beta} = \pm i$

Now, $\left(\frac{\alpha}{\beta}\right)^n = 1 \quad \Rightarrow \quad (\pm i)^n = 1$

$\Rightarrow$ n must be a multiple of 4 minimum value of n = 4

Area Under Curve

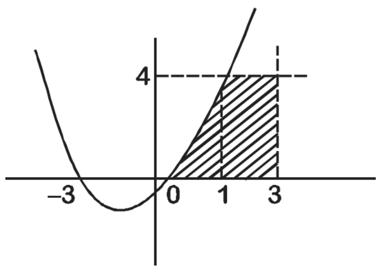
19. The area (in sq units) of the region $A = \{(x,y) \in R \times R \mid 0 \le x \le 3, 0 \le y \le 4, y \le x^2 + 3x\}$ is

क्षेत्र $A = \{(x,y) \in R \times R \mid 0 \le x \le 3, 0 \le y \le 4, y \le x^2 + 3x\}$ का क्षेत्रफल (वर्ग इकाइयों में) है :

(1) $\frac{26}{3}$ (2) $\frac{59}{6}$ (3) 8 (4) $\frac{53}{6}$

A. 2

sol. $y \le x^2 + 3x$ represents region below the parabola.



Area of the required region

$= \int_0^1 (x^2 + 3x)dx + \int_1^3 4.dx$

$= \frac{1}{3} + \frac{3}{2} + 8$

$= \frac{59}{6}$

Definite Integration

20. If $f(x) = \frac{2 - x\cos x}{2 + x\cos x}$ and $g(x) = \log_e x, (x > 0)$ then the value of the integral $\int_{-\pi/4}^{\pi/4} g(f(x))dx$ is

यदि $f(x) = \frac{2 - x\cos x}{2 + x\cos x}$ तथा $g(x) = \log_e x, (x > 0)$ हैं, तो समाकल $\int_{-\pi/4}^{\pi/4} g(f(x))dx$ का मान है :

(1) $\log_e 1$ (2) $\log_e 3$ (3) $\log_e 2$ (4) $\log_e e$

A. 1

sol. $g(f(x)) = \ln\left(\frac{2 - x\cos x}{2 + x\cos x}\right)$

Let $I = \int_{-\pi/4}^{\pi/4} ln\left(\frac{2 - x\cos x}{2 + x\cos x}\right)dx$ .....(i)

$\left(\text{Using property } \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right)$

$I = \int_{-\pi/4}^{\pi/4} ln\left(\frac{2 + x\cos x}{2 - x\cos x}\right)dx$ .....(ii)

Adding (i) and (ii)

$\Rightarrow \quad 2I = \int_{-\pi/2}^{\pi/2} ln(1)dx = 0$

$\Rightarrow \quad I = 0 = ln\ 1$

3 D

21. The equation of a plane containing the line of intersection of the planes $2x - y - 4 = 0$ and $y + 2z - 4 = 0$ and passing through the point (1, 1, 0) is :

समतलों $2x - y - 4 = 0$ तथा $y + 2z - 4 = 0$ की प्रतिच्छेदन रेखा को अंतर्विष्ट करने वाले तथा बिन्दु (1, 1, 0) से होकर जाने वाले समतल का समीकरण है :

(1) $2x - z = 2$ (2) $x - 3y - 2z = -2$
(3) $x - y - z = 0$ (4) $x + 3y + z = 4$

A. 3

sol. Let the equation of required plane be;

$(2x - y - 4) + \lambda(y + 2z - 4) = 0$

$\because$ This plane passes through (1, 1, 0) then

$(2 - 1 - 4) + \lambda(1 + 0 - 4) = 0 \Rightarrow \lambda = -1$

Equation of required plane will be

$(2x - y - 4) - (y + 2z - 4) = 0$

$\Rightarrow \quad 2x - 2y - 2z = 0$

$\Rightarrow \quad x - y - z = 0$

Trig Ratio

22. If $\cos(\alpha+\beta) = \frac{3}{5}$, $\sin(\alpha-\beta) = \frac{5}{13}$ and $0 < \alpha,\beta < \frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to

यदि $\cos(\alpha+\beta) = \frac{3}{5}$, $\sin(\alpha-\beta) = \frac{5}{13}$ तथा $0 < \alpha,\beta < \frac{\pi}{4}$ हैं, तो $\tan(2\alpha)$ बराबर है :

(1) $\frac{21}{16}$ (2) $\frac{63}{52}$ (3) $\frac{33}{52}$ (4) $\frac{63}{16}$

A. 4

sol. $\because \quad \alpha + \beta$ and $\alpha - \beta$ both are acute angles.

$\cos(\alpha + \beta) = \frac{3}{5}$

$\Rightarrow \quad \tan(\alpha + \beta) = \frac{4}{3}$

And $\sin(\alpha-\beta)=\frac{5}{13}$

$\Rightarrow \quad \tan(\alpha-\beta)=\frac{5}{12}$

Now, $\tan 2\alpha = \tan((\alpha+\beta)+(\alpha-\beta))$

$$=\frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta).\tan(\alpha-\beta)}$$

$$=\frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}.\frac{5}{12}}=\frac{63}{16}$$

Matrices

23. Let $A=\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}$, $(\alpha\in R)$ such that $A^{32}=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$ then a value of α is

माना $A=\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}$, $(\alpha\in R)$ इस प्रकार है कि $A^{32}=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$, तो α का एक मान है :

(1) $\frac{\pi}{32}$ (2) $\frac{\pi}{64}$ (3) 0 (4) $\frac{\pi}{16}$

A. 2

sol. $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

$$A^2=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}=\begin{bmatrix}\cos 2\alpha & -\sin 2\alpha\\ \sin 2\alpha & \cos 2\alpha\end{bmatrix}$$

Then $A^4=A^2.A^2=\begin{bmatrix}\cos 4\alpha & -\sin 4\alpha\\ \sin 4\alpha & \cos 4\alpha\end{bmatrix}$

Similarly $A^8=A^4.A^4=\begin{bmatrix}\cos 8\alpha & -\sin 8\alpha\\ \sin 8\alpha & \cos 8\alpha\end{bmatrix}$

and so on $A^{32}=\begin{bmatrix}\cos 32\alpha & -\sin 32\alpha\\ \sin 32\alpha & \cos 32\alpha\end{bmatrix}=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$

So $\sin 32\alpha = 1$ and $\cos 32\alpha = 0$

$\Rightarrow \quad 32\alpha = 2n\pi+\frac{\pi}{2} \quad \Rightarrow \quad \alpha=\frac{n\pi}{16}+\frac{\pi}{64}$ where $n\in Z$

put $n=0$, $\alpha=\frac{\pi}{64}$

Mathematical Reasoning

24. The contrapositive of the statements " if your are boran in india then you are a citizen of india" is

(1) If you are born in india then you are not a citizen of india

(2) if your are not boran in india then you are not citizen of india

(3) If your are a citinzen of india then you are born in india

(4) If you are not a citizen of india then you are not born in india

कथन " यदि आपका भारत में जन्म हुआ है तो आप भारत के नागरिक है।" का प्रतिधनात्मक होगा –

(1) यदि आपका भारत में जन्म हुआ है तो आप भारत के नागरिक नहीं है।

(2) यदि आपका भारत में जन्म नहीं हुआ है तो आप भारत के नागरिक नहीं है।

(3) यदि आप भारत के नागरिक है तो आपका जन्म भारत में हुआ है।

(4) यदि आपका भारत में नागरिक नहीं है, तो आपका जन्म भारत में नहीं हुआ है।

A. 4

sol. S: "If you are born in India, then you are a citizen of India."

Contrapositive of $p \to q$ is $\sim q \to \sim p$

So contrapositive of statement S will be :

"If you are not a citizen of India, then you are not born in India."

Quadratic Equation

25. The sum of the solutions of the equation $|\sqrt{x}-2|+\sqrt{x}\left(\sqrt{x}-4\right)+2=0, (x>0)$ is equal to

समीकरण $|\sqrt{x}-2|+\sqrt{x}\left(\sqrt{x}-4\right)+2=0, (x>0)$ के हलों का योग बराबर है :

(1) 4 (2) 10 (3) 9 (4) 12

A. 2

sol. Let $\sqrt{x}=t$

$|t-2| + t(t-4) + 2 = 0$

$\Rightarrow \quad |t-2| + t^2 - 4t + 4 - 2 = 0$

$\Rightarrow \quad |t-2| + (t-2)^2 - 2 = 0$

Let $|t-2| = z$ (Clearly $z \geq 0$)

$\Rightarrow \quad z + z^2 - 2 = 0$

$\Rightarrow \quad z = 1$ or -2 (rejected)

$\Rightarrow \quad |t-2| = 1 \quad \Rightarrow \quad t = 1, 3$

If $\sqrt{x}=1 \quad \Rightarrow \quad x = 1$

If $\sqrt{x}=3 \quad \Rightarrow \quad x = 9$

Sum of solutions = 10

3 D

26. The magnitude of the projection of the vector $2\hat{i}+3\hat{j}+\hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$ is

सदिश $2\hat{i}+3\hat{j}+\hat{k}$ के सदिशों $\hat{i}+\hat{j}+\hat{k}$ तथा $\hat{i}+2\hat{j}+3\hat{k}$ को अन्तर्विष्ट करने वाले समतल के लंबवर्तीय सदिश पर प्रक्षेप का परिमाण है :

(1) $\sqrt{\frac{3}{2}}$ (2) $3\sqrt{6}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\sqrt{6}$

A. 1

sol. Let $\bar{a} = \hat{i} + \hat{j} + \hat{k}$ and $\bar{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ vector perpendicular to $\bar{a}$ and $\bar{b}$ is $\bar{a} \times \bar{b}$

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \hat{i} - 2\hat{j} + \hat{k}$$

Projection of vector $\bar{c} = 2\hat{i} - 3\hat{j} + \hat{k}$ on $\bar{a} \times \bar{b}$ is

$$= \left| \frac{\bar{c}.(\bar{a} \times \bar{b})}{|\bar{a} \times \bar{b}|} \right| = \left| \frac{2-6+1}{\sqrt{6}} \right|$$

$$= \frac{3}{\sqrt{6}} = \sqrt{\frac{3}{2}}$$

Statistics

27. The mean and variance of seven observations are 8 and 16, respectively. If 5 of the observations are 2, 4, 10, 12, 14, then the product of the remaining two observations is:

सात प्रेक्षणों के माध्य तथा प्रसरण क्रमशः 8 तथा 16 हैं। यदि इनमें से 5 प्रेक्षण 2, 4, 10, 12, 14 है, तो शेष दो प्रेक्षणों का गुणनफल है :

(1) 45 (2) 40 (3) 48 (4) 49

A. 3

sol. Let the remaining numbers are x and y

$$\text{Mean } (\bar{x}) = \frac{\sum x_i}{N} = \frac{2+4+10+12+14+x+y}{7} = 8$$

$$\Rightarrow x + y = 14 \quad .....(i)$$

$$\text{Variance } (\sigma^2) = \frac{\sum x_i^2}{N} - (\bar{x})^2 = 16$$

$$\Rightarrow \frac{2^2+4^2+10^2+12^2+14^2+x^2+y^2}{7} - (8)^2 = 16$$

$$\Rightarrow x^2 + y^2 = 100 \quad ......(ii)$$

From (i) and (ii) (x, y) = (6, 8) or (8, 6) xy = 48

Limit

28. $\lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}}$ equals

$\lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}}$ बरबार है –

(1) $\sqrt{2}$ (2) $2\sqrt{2}$ (3) 4 (4) $4\sqrt{2}$

A. 4

sol. $\lim_{x\to 0}\frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}} = \lim_{x\to 0}\frac{\sin^2 x}{\sqrt{2}-\sqrt{2\cos^2\frac{x}{2}}}\left[\frac{0}{0}\right]$

$= \lim_{x\to 0}\frac{\sin^2 x}{\sqrt{2}\left[1-\cos\frac{x}{2}\right]}$

$= \lim_{x\to 0}\frac{\sin^2 x}{2\sqrt{2}\sin^2\frac{x}{4}}$

$= \lim_{x\to 0}\frac{\left(\frac{\sin x}{x}\right)^2 .16}{2\sqrt{2}\left(\frac{\sin\frac{x}{4}}{\frac{x}{4}}\right)^2}$

$= \frac{16}{2\sqrt{2}} = 4\sqrt{2}$

Sequence & Progression

29. The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. (91, n) > 1 is :
ऐसी सभी प्राकृत संख्याओं 'n', जो इस प्रकार हैं कि $100 < n < 200$ तथा H.C.F. (91, n) > 1, का योग है :
(1) 3303 (2) 3121 (3) 3203 (4) 3221

A. 2

sol. $\because$ 91 = 13 × 7

So the required numbers are either divisible by 7 or 13

Sum of such numbers = Sum of no. divisible by 7 + sum of the no. divisible by 13 – Sum of the numbers divisible by 91

= (105 + 112 + …… + 196) + (104 + 117 + ……+ 195) – 182

= 2107 + 1196 – 182

= 3121

3 D

30. The length of the perpendicular from the point (2,–1,4) on the straight line $\frac{x+3}{10}=\frac{y-2}{-7}=\frac{z}{1}$ is

(1) Greather than 3 but less than 4 (2) Greather than 2 but less than 3
(3) Greather than 4 (4) Less than 2

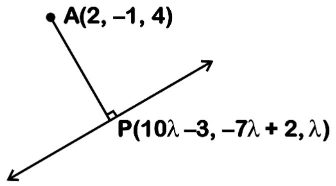
बिन्दु (2,–1,4) से सरल रेखा $\frac{x+3}{10}=\frac{y-2}{-7}=\frac{z}{1}$ पर खींचे गए लम्ब की लम्बाई :

(1) 3 से अधिक परन्तु 4 से कम है। (2) 2 से अधिक परन्तु 3 से कम है।

(3) 4 से अधिक है।   (4) 2 से कम है।

A. 1

sol.



Let P be the foot of perpendicular from point A(2, –1, 4) on the given line. So P can be assumed as P(10λ – 3, –7λ + 2, λ)

DR's of AP ∝ to 10λ – 5, –7λ + 3, λ – 4

∵ AP and given line are perpendicular, so

$10(10\lambda - 5) - 7(-7\lambda + 3) + 1(\lambda - 4) = 0$

$\Rightarrow \quad \lambda = \frac{1}{2}$

$AP = \sqrt{(10\lambda - 5)^2 + (-7\lambda + 3)^2 + (\lambda - 4)^2}$

$= \sqrt{0 + \frac{1}{4} + \frac{49}{4}}$

$= \sqrt{12.5}\,; \sqrt{12.5} \in (3, 4)$

1	41652912772	Differential Equation
2	41652912785	Mathematical Reasoning
3	41652912758	Quadratic Equation
4	41652912773	Straight Line
5	41652912779	3 D
6	41652912768	Maxima & Minima
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28	41652912764	Binomial Theorem
29	41652912767	Monotonocity
30	41652912776	Tangent & Normal