MATHS
08 APRIL 2019 [Phase : II]
JEE MAIN PAPER ONLINE

3 D

1. The vector equation of plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$ is :

समतलों $x + y + z = 1$ तथा $2x + 3y + 4z = 5$ की प्रतिच्छेदन रेखा से होकर जाने वाले तथा समतल $x - y + z = 0$ के लम्बवत् समतल का सदिश समीकरण है :

(1) $\vec{r}.(\hat{i}-\hat{k})+2=0$ (2) $\vec{r}.(\hat{i}-\hat{k})-2=0$ (3) $\vec{r}\times(\hat{i}-\hat{k})+2=0$ (4) $\vec{r}\times(\hat{i}+\hat{k})+2=0$

A. 1

sol. Equation of the plane passing through the line of intersection of $x + y + z = 1$ and $2x + 3y + 4z = 5$ is

$(2x + 3y + 4z - 5) + \lambda (x + y + z - 1) = 0$

$(2 + \lambda) x + (3 + \lambda) y + (4 + \lambda) z + (-5 - \lambda) = 0$ …(i)

(i) is perpendicular to $x - y + z = 0$

$\Rightarrow$ $(2 + \lambda)(1) + (3 + \lambda)(-1) + (4 + \lambda)(1) = 0$

$2 + \lambda - 3 - \lambda + 4 + \lambda = 0$

$\lambda = -3$

$\Rightarrow$ Equation of required plane is

$-x + z - 2 = 0$

$\Rightarrow$ $x - z + 2 = 0$

$\Rightarrow$ $\vec{r}.(\hat{r}-\hat{k})+2=0$

Mathematical Reasoning

2. Which one of the following statements is not a tautology ?

निम्न कथनों में से कौन–सा एक, एक पुनरूक्ति (tautology) नहीं है?

(1) $(p \wedge q) \rightarrow (\sim p) \vee q$ (2) $(p \wedge q) \rightarrow p$

(3) $(p \vee q) \rightarrow (p \vee (\sim q))$ (4) $p \rightarrow (p \vee q)$

A. 3

sol. By help of truth table :

p	q	~q	pv~q	~p	p∧~q	pvq	p→pvq	p∧q	(p∧q)→p	~pvq
T	T	F	T	F	F	T	T	T	T	T
T	F	T	T	F	T	T	T	F	T	F
F	T	F	F	T	F	T	T	F	T	T
F	F	T	T	T	F	F	T	F	T	T

$(p \wedge q) \rightarrow (\sim p) vq$	$(pvq) \rightarrow (pv(\sim q))$
T	T
T	T
T	F
T	T

P & C

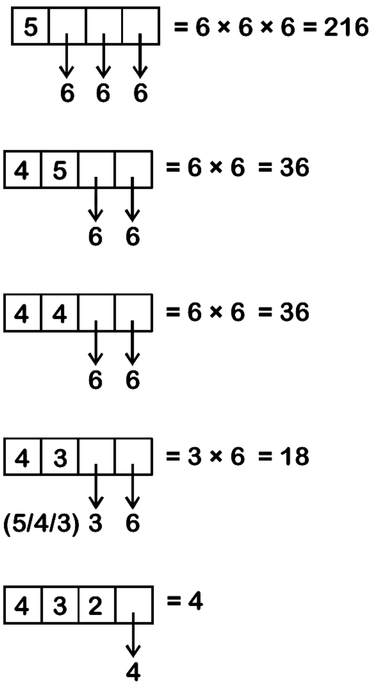
3. The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0, 1, 2, 3, 4, 5 (repetition of digits is allowed) is :

अंकों 0, 1, 2, 3, 4, 5 को प्रयोग करके (जहाँ अंकों को दोहराया जा सकता है) बनाई जा सकने वाली चार अंकों की संख्याओं, जो 4321 से अधिक (strictly greater) हों, की संख्या है :

(1) 360 (2) 306 (3) 288 (4) 310

A. 4

sol. 0, 1, 2, 3, 4, 5



$\Rightarrow$ Required numbers = 216 + 36 + 36 + 18 + 4 = 310

Ellipse

4. In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $\left(0, 5\sqrt{3}\right)$, then the length of its latus rectum is :

यदि एक दीर्घवृत्त जिसका केन्द्र मूलबिन्दु पर है, के दीर्घ अक्ष तथा लघु अक्ष की लम्बाईयों का अन्तर 10 है तथा एक नाभिकेन्द्र $\left(0, 5\sqrt{3}\right)$ पर है, तो इसके नाभिलम्ब की लम्बाई है :

(1) 5 (2) 6 (3) 8 (4) 10

A. 1

sol. $\because$ Focus is $\left(0, 5\sqrt{3}\right) \Rightarrow |b| > |a|$

Let b > a > 0

foci $(0, \pm be)$

$a^2 = b^2 - b^2e^2 \Rightarrow b^2e^2 = b^2 - a^2$

$be = \sqrt{b^2 - a^2}$

$\Rightarrow b^2 - a^2 = 75$ .....(i)

$2b - 2a = 10$

$\Rightarrow \quad b - a = 5 \quad \text{....(ii)}$

From (i) and (ii)

$b + a = 15 \quad \text{...(iii)}$

$\Rightarrow \quad b = 10, a = 5$

Length of L.R. $= \frac{2a^2}{b} = \frac{50}{10} = 5$

Statistics

5. A student scores the following marks in five tests : 45, 54, 41, 57, 43. His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is :

एक विद्यार्थी पाँच परीक्षाओं में निम्न अंक प्राप्त करता है : 45, 54, 41, 57, 43। उसके द्वारा छटी परीक्षा में प्राप्त अंक ज्ञात नहीं हैं। यदि छः परीक्षाओं में प्राप्त अंकों का माध्य 48 है तो छः परीक्षाओं में प्राप्त अंकों का मानक विचलन है :

(1) $\frac{100}{\sqrt{3}}$ (2) $\frac{10}{\sqrt{3}}$ (3) $\frac{100}{3}$ (4) $\frac{10}{3}$

A. 2

sol.
$$\bar{x} = \frac{41+45+54+57+43+x}{6} = 48$$

$x + 240 = 288$

$x = 48$

$$\sigma^2 = \frac{1}{6}\begin{bmatrix} (48-41)^2 + (48-45)^2 + (48-54)^2 \\ +(48-57)^2 + (48-43)^2 + (48-48)^2 \end{bmatrix}$$

$$= \frac{1}{6}(49+9+36+81+25)$$

$$\frac{200}{6} = \frac{100}{3}$$

$$\sigma = \frac{10}{\sqrt{3}}$$

Circle

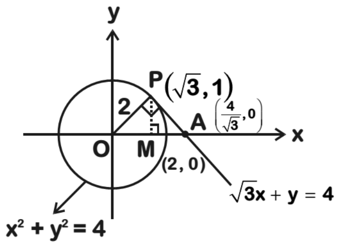
6. The tangent and the normal lines at the point $\left(\sqrt{3},1\right)$ to the circle $x^2 + y^2 = 4$ and the x-axis form a triangle. The area of this triangle (in square units) is :

वृत्त $x^2 + y^2 = 4$ के बिन्दु $\left(\sqrt{3},1\right)$ पर खींची गई स्पर्श रेखा और अभिलम्ब तथा x-अक्ष एक त्रिभुज बनाते है। इस त्रिभुज का (वर्ग इकाइयों में) क्षेत्रफल है :

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{4}{\sqrt{3}}$ (3) $\frac{1}{3}$ (4) $\frac{1}{\sqrt{3}}$

A. 1

sol. Equation of tangent to circle at point $\left(\sqrt{3},1\right)$ is $\sqrt{3}x + y = 4$



$\therefore$ Coordinate of $A = \left(\frac{4}{\sqrt{3}}, 0\right)$

$$\text{Area} = \frac{1}{2} \times OA \times PM$$

$$= \frac{1}{2} \times \frac{4}{\sqrt{3}} \times 1 = \frac{2}{\sqrt{3}} \text{ Square units}$$

Binomial Theorem

7. If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1+\log_{10} x}}} + x^{\frac{1}{12}}\right)^6$ is equal to 200, and $x > 1$, then the value of x is:

यदि $\left(\sqrt{\frac{1}{x^{1+\log_{10} x}}} + x^{\frac{1}{12}}\right)^6$ के द्विपद प्रसार का चौथा पद 200 है तथा $x > 1$ है, तो x का मान है :

(1) 10 (2) 10^3 (3) 100 (4) 10^4

A. 1

sol. $$T_4 = {}^6C_3 \left(\sqrt{x^{\left(\frac{1}{1+\log_{10} x}\right)}}\right)^3 \left(x^{\frac{1}{12}}\right)^3 = 200$$

$$\Rightarrow \frac{3}{20x^{2(1+\log_{10} x)}} . x^{\frac{1}{4}} = 200$$

$$x^{\frac{1}{4}+\frac{3}{2(1+\log_{10} x)}} = 10$$

Taking $\log_{10}$ on both sides and put $\log_{10} x = t$

$$\left(\frac{1}{4} + \frac{3}{2(1+t)}\right) t = 1$$

$$\left(\frac{(1+t)+6}{4(1+t)}\right) \times t = 1 \quad \Rightarrow \quad t^2 + 7t = 4 + 4t$$

$$t^2 + 3t - 4 = 0 \quad \Rightarrow \quad t^2 + 4t - t - 4 = 0$$

$$\Rightarrow \quad t(t+4) - 1(t+4) = 0$$

$$\Rightarrow \quad t = 1 \text{ or } t = -4$$

$$\log_{10} x = 1$$

$\Rightarrow$ $x = 10$ or if $\log_{10}x = -4$

$\Rightarrow$ $x = 10^{-4}$

Note: There seems a printing error in this question in the original question paper.

Parabola

8. The tangent to the parabola $y^2 = 4x$ at the point where it intersects the circle $x^2 + y^2 = 5$ in the first quadrant, passes through the point :

परवलय $y^2 = 4x$ के उस बिन्दु, जहाँ यह वृत्त $x^2 + y^2 = 5$ को प्रथम चतुर्थांश में काटता है, पर खींची गई स्पर्श रेखा जिस बिन्दु से होकर जाती है, वह है :

(1) $\left(\frac{3}{4},\frac{7}{4}\right)$ (2) $\left(-\frac{1}{3},\frac{4}{3}\right)$ (3) $\left(\frac{1}{4},\frac{3}{4}\right)$ (4) $\left(-\frac{1}{4},\frac{1}{2}\right)$

A. 1

sol. Intersection point of

$x^2 + y^2 = 5$, $y^2 = 4x$

$\Rightarrow$ $x^2 + 4x - 5 = 0$

$\Rightarrow$ $x^2 + 5x - x - 5 = 0$

$\Rightarrow$ $x(x + 5) - 1(x + 5) = 0$

$\Rightarrow$ $x = 1, -5$

Intersection point in 1st quadrant be (1, 2) equation of tangent to $y^2 = 4x$ at (1, 2) is

$y \times 2 = 2(x + 1)$

$\Rightarrow$ $y = x + 1$

$\Rightarrow$ $x - y + 1 = 0$ ...(i)

$\left(\frac{3}{4},\frac{7}{4}\right)$ lies on (i)

Sequence & Progression

9. If three distinct numbers a, b, c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct ?

(1) d, e, f are in A.P. (2) $\frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in G.P.

(3) $\frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in A.P. (4) d, e, f are in G.P.

यदि तीन भिन्न संख्याएं a, b, c गुणोत्तर श्रेढ़ी में हैं तथा समीकरण $ax^2 + 2bx + c = 0$ और $dx^2 + 2ex + f = 0$ का एक उभयनिष्ठ मूल है, तो निम्न में से कौनसा एक कथन सत्य है?

(1) d, e, f समान्तर श्रेढ़ी में हैं। (2) $\frac{d}{a},\frac{e}{b},\frac{f}{c}$ गुणोत्तर श्रेढ़ी में है।

(3) $\frac{d}{a},\frac{e}{b},\frac{f}{c}$ समान्तर श्रेढ़ी में हैं। (4) d, e, f गुणोत्तर श्रेढ़ी में है।

A. 3

sol. Since a, b, c are in G.P.

$\Rightarrow$ $b^2 = ac$

Given, $ax^2 + 2bx + c = 0$

$\Rightarrow \quad ax^2 + 2\sqrt{ac}\,x + c = 0$

$(\sqrt{a}\,x + \sqrt{c})^2 = 0$

$\Rightarrow \quad x = -\sqrt{\frac{c}{a}}$

$\because \quad ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have common root

$\Rightarrow \quad x = -\sqrt{\frac{c}{a}}$ must satisfy $dx^2 + 2ex + f = 0$

$\Rightarrow \quad d.\frac{c}{a} + 2e\left(-\sqrt{\frac{c}{a}}\right) + f = 0$

$\frac{d}{a} - \frac{2e}{\sqrt{ac}} + \frac{f}{c} = 0$

$\Rightarrow \quad \frac{d}{a} - \frac{2e}{b} + \frac{f}{c} = 0$

$\Rightarrow \quad \frac{2e}{b} = \frac{d}{a} + \frac{f}{c}$

$\Rightarrow \quad \frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

Complex Number

10. If $z = \frac{\sqrt{3}}{2} + \frac{i}{2}\left(i = \sqrt{-1}\right)$, then $(1 + iz + z^5 + iz^8)^9$ is equal to

यदि $z = \frac{\sqrt{3}}{2} + \frac{i}{2}\left(i = \sqrt{-1}\right)$ है, तो $(1 + iz + z^5 + iz^8)^9$ बराबर है :

(1) 0 (2) $(-1 + 2i)^9$ (3) -1 (4) 1

A. 3

sol. $z = \frac{\sqrt{3}}{2} + \frac{i}{2} = -i\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -i\omega$

where ω is not real cube root of unity

$\Rightarrow \quad (1 + iz + z^5 + iz^8)^9 = (1 + \omega - i\omega^2 + i\omega^2)^9$

$= (1 + \omega)^9$

$= (-\omega^2)^9$

$= -\omega^{18}$

$= -1$

Quadratic Equation

11. The number of integral values of m for which the equation $(1 + m^2)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$ has no real root is :

m के उन पूर्णांक मानों की संख्या, जिनके लिए समीकरण, $(1 + m^2)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$ के कोई भी वास्तविक मूल नहीं हैं, है :

(1) Infinitely many अनन्त (2) 3 (3) 2 (4) 1

A. 1

sol. $(1 + m^2)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$

equation has no real solution

$\Rightarrow \quad D < 0$

$4(1 + 3m)^2 < 4(1 + m^2)(1 + 8m)$

$1 + 9m^2 + 6m < 1 + 8m + m^2 + 8m^3$

$8m^3 - 8m^2 + 2m > 0$

$2m(4m^2 - 4m + 1) > 0$

$2m(2m - 1)^2 > 0$

$m > 0,\ m \neq \frac{1}{2}$

$\Rightarrow$ number of integral values of m are infinitely many.

Matrices

12. Let the numbers 2, b, c be in an A.P. and $A\begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$. If det(A) $\in$ [2, 16], then c lies in the interval :

माना संख्याएं 2, b, c एक समान्तर श्रेढ़ी में हैं तथा $A\begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$. यदि det(A) $\in$ [2, 16], तो c निम्न में से किस अन्तराल में है :

(1) [2, 3] (2) $(2 + 2^{3/4}, 4)$ (3) $[3, 2, + 2^{3/4}]$ (4) [4, 6]

A. 4

sol. $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{vmatrix} \qquad c_2 \to c_2 - c_1,\ c_3 \to c_3 - c_1$

$|A| = \begin{vmatrix} 1 & 0 & 0 \\ 2 & b-2 & c-2 \\ 4 & (b-2)(b+2) & (c-2)(c+2) \end{vmatrix} = (b-2)(c-2)(c-b)$

2, b, c are in A.P. $\Rightarrow$ $(b - 2) = (c - b) = d,\ c - 2 = 2d$

$\Rightarrow \quad |A| = d.2d.\ d = 2d^3$

$\because \quad |A| \in [2,16] \Rightarrow 1 \le d^3 \le 8 \Rightarrow 1 \le d \le 2$

$4 \le 2d + 2 \le 6 \quad \Rightarrow \quad 4 \le c \le 6$

Probability

13. The minimum number of times one has to toss a fair coin so that the probability of observing at least one head is at least 90% is :

एक अनभिनत सिक्के को कम से कम कितनी बार उछाला जाए ताकि कम से कम एक चित्त आने की प्रायिकता, कम से कम 90% हो?

(1) 5 (2) 2 (3) 4 (4) 3

A. 3

sol. $p = P(H) = \frac{1}{2}, q = 1 - p = \frac{1}{2}$

$P(x \geq 1) \geq \frac{9}{19}$

$1 - P(x = 1) \geq \frac{9}{10}$

$1 - {}^nC_0\left(\frac{1}{2}\right)^n \geq \frac{9}{10}$

$\frac{1}{2^n} \leq 1 - \frac{9}{10} \Rightarrow \frac{1}{2^n} \leq \frac{1}{10}$

$2^n \geq 10 \quad \Rightarrow \quad n \geq 4$

$\Rightarrow \quad n_{min} = 4$

Vector

14. Let $\overline{a} = 3\hat{i} + 2\hat{j} + x\hat{k}$ and $\overline{b} = \hat{i} - \hat{j} + \hat{k}$, for some real x. Then $|\overline{a} \times \overline{b}| = r$ is possible if :

माना किसी वास्तविक संख्या x के लिए $\overline{a} = 3\hat{i} + 2\hat{j} + x\hat{k}$ तथा $\overline{b} = \hat{i} - \hat{j} + \hat{k}$ है, तो $|\overline{a} \times \overline{b}| = r$ तभी सम्भव है, जब :

(1) $3\sqrt{\frac{3}{2}} < r < 5\sqrt{\frac{3}{2}}$ (2) $\sqrt{\frac{3}{2}} < r \leq 3\sqrt{\frac{3}{2}}$ (3) $0 < r \leq \sqrt{\frac{3}{2}}$ (4) $r \geq 5\sqrt{\frac{3}{2}}$

A. 4

sol. $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & x \\ 1 & -1 & 1 \end{vmatrix} = (2+x)\hat{i} + (x-3)\hat{j} - 5\hat{k}$

$|\vec{a} \times \vec{b}| = r = \sqrt{(2+x)^2 + (x-3)^2 + (-5)^2}$

$\Rightarrow r = \sqrt{4 + x^2 + 4x + x^2 + 9 - 6x + 25}$

$= \sqrt{2x^2 - 2x + 38} = \sqrt{2\left(x^2 - x + \frac{1}{4}\right) + 38 - \frac{1}{2}}$

$= \sqrt{2\left(x - \frac{1}{2}\right)^2 + \frac{75}{2}}$

$\Rightarrow r \geq \sqrt{\frac{75}{2}} \Rightarrow r \geq 5\sqrt{\frac{3}{2}}$

Sequence & Progression

15. The sum $\sum_{k=1}^{20} k \frac{1}{2^k}$ is equal to :

योग $\sum_{k=1}^{20} k\frac{1}{2^k}$ बराबर है :

(1) $2-\frac{3}{2^{17}}$ (2) $1-\frac{11}{2^{20}}$ (3) $2-\frac{21}{2^{20}}$ (4) $2-\frac{11}{2^{19}}$

A. 4

sol. $S=\sum_{k=1}^{20} k.\frac{1}{2^k}$

$$S=\frac{1}{2}+2.\frac{1}{2^2}+3.\frac{1}{2^3}+......+20.\frac{1}{2^{20}}$$

$$\frac{1}{2}S=\frac{1}{2^2}+2.\frac{1}{2^3}+.....+19\frac{1}{2^{20}}+20\frac{1}{2^{21}}$$

On subtracting

$$\frac{S}{2}=\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+.....+\frac{1}{2^{20}}\right)-20\frac{1}{2^{21}}$$

$$=\frac{\frac{1}{2}\left(1-\frac{1}{2^{20}}\right)}{1-\frac{1}{2}}-20.\frac{1}{2^{21}}=1-\frac{1}{2^{20}}-10.\frac{1}{2^{20}}$$

$$\frac{S}{2}=1-11.\frac{1}{2^{20}} \Rightarrow S=2-11.\frac{1}{2^{19}}=2-\frac{11}{2^{19}}$$

Height & Distance

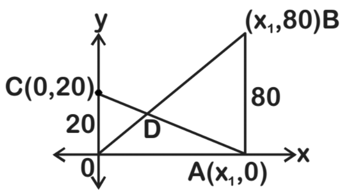
16. Two vertical poles of heights, 20 m and 80 m stand apart on a horizontal plane. The height (in meters) of the point of intersection of the lines joining the top of each pole to the foot of the other, from this horizontal plane is:

20 m तथा 80 m ऊँचाई वाले दो खम्भे, एक क्षैतिज समतल पर सीधे खड़े हैं। प्रत्येक खंभे के शिखर को दूसरे खंभे के पाद से मिलाने वाली रेखाओं के प्रतिच्छेदन बिन्दु की इस समतल से ऊँचाई (मीटर में) है :

(1) 16 (2) 18 (3) 15 (4) 12

A. 1

sol.



equation of line OB and AC are respectively

$$y=\frac{80}{x_1}x \quad .....(i)$$

$$\frac{x}{x_1}+\frac{y}{20}=1 \quad .....(ii)$$

For intersection point, from equation (i) and (ii)

$$\frac{y}{80}+\frac{y}{20}=1$$

$\Rightarrow$ $y + 4y = 80$

$\Rightarrow$ $y = 16$ m

$\Rightarrow$ Height of intersection point is 16 m

Continuity & Diff

17. Let $f : [-1, 3] \rightarrow R$ be defined as

$$f(x)=\begin{cases}|x|+[x], & -1\le x<1\\ x+|x|, & 1\le x<2\\ x+[x], & 2\le x\le 3,\end{cases}$$

where [t] denotes the greatest integer less than or equal to t. Then, f is discontinuous at :

(1) Only one point (2) Only two points (3) Only three points (4) Four or more points

माना $f : [-1, 3] \rightarrow R$ इस प्रकार परिभाषित है कि

$$f(x)=\begin{cases}|x|+[x], & -1\le x<1\\ x+|x|, & 1\le x<2\\ x+[x], & 2\le x\le 3,\end{cases}$$

जहाँ [t], t या उससे कम अधिकतम पूर्णांक को दर्शाता है, तो f असंतत है :

(1) केवल एक बिन्दु पर (2) केवल दो बिन्दुओं पर (3) केवल तीन बिन्दुओं पर (4) चार अथवा उससे अधिक बिन्दुओं पर

A. 3

sol. $$f(x)=\begin{cases}|x|+[x], & -1\le x<1\\ x+|x|, & 1\le x<2\\ x+[x], & 2\le x\le 3\end{cases}$$

$$=\begin{cases}-x-1, & -1\le x<0\\ x, & 0\le x<1\\ 2x, & 1\le x<2\\ x+2, & 2\le x<3\\ 6, & x=3\end{cases}$$

$\Rightarrow$ $f(-1) = 0, f(-1^{+}) = 0$

$f(0^{-}) = -1, f(0) = 0, f(0^{+}) = 0$

$f(1^{-}) = 1, f(1) = 2, f(1^{+}) = 2$

$f(2^{-}) = 4, f(2) = 4, f(2+) = 4$

$f(3^{-}) = 5, f(3) = 6$

f(x) is discontinuous at $x = \{0, 1, 3\}$

Differential Equation

18. Given that the slope of the tangent to a curve $y = y(x)$ at any point (x, y) is $\frac{2y}{x^2}$. If the curve passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, then its equation is :

दिया है कि वक्र $y = y(x)$ के किसी बिन्दु (x, y) पर खींची गई स्पर्श रेखा की ढ़ाल (slope) $\frac{2y}{x^2}$ है।

यदि यह वक्र, वृत्त $x^2 + y^2 - 2x - 2y = 0$ के केन्द्र से होकर जाता है, तो इसका समीकरण है :

(1) $x \log_e|y| = -2(x-1)$ (2) $x \log_e|y| = x - 1$ (3) $x \log_e|y| = 2(x-1)$ (4) $x^2 \log_e|y| = -2(x-1)$

A. 3

sol. $\frac{dy}{dx} = \frac{2y}{x^2}$ $\qquad$ $\frac{dy}{y} = 2\frac{dx}{x^2}$

$ln\,|y| = -\frac{2}{x} + C$ ....(i)

(i) passes through (1, 1)

$\Rightarrow$ $C = 2$

$ln\,|y| = -\frac{2}{x} + 2$

$x\,ln\,|y| = -2 + 2x$

$x\,ln\,|y| = -2(1-x) = 2(x-1)$

Straight Line

19. Suppose that the points (h, k), (1, 2) and (–3, 4) lie on the line L_1. If a line L_2 passing through the points (h, k) and (4, 3) is perpendicular to L_1, then k/h equals :

माना बिन्दु (h, k), (1, 2) तथा (–3, 4) एक रेखा L_1 पर स्थित हैं। यदि बिन्दुओं (h, k) तथा (4, 3) से होकर जाने वाली रेखा L_2, रेखा L_1 के लम्बवत् है, तो k/h बराबर है :

(1) 3 (2) $-\frac{1}{7}$ (3) 0 (4) $\frac{1}{3}$

A. 4

sol. (h, k), (1, 2) and (–3, 4) and collinear

$$\begin{vmatrix} h & k & 1 \\ 1 & 2 & 1 \\ -3 & 4 & 1 \end{vmatrix} = 0 \qquad -2h - 4k + 10 = 0$$

$\Rightarrow$ $h + 2k = 5$ .....(i)

$m_{L_1} = \frac{4-2}{-3-1} = \frac{2}{-4} = -\frac{1}{2}$ $\quad m_{L_2} = 2$

$\Rightarrow$ $m_{L_2} = \frac{k-3}{h-4} = 2 \Rightarrow k - 3 = 2h - 8$

$2h - k = 5$ .....(ii)

from (i) and (ii)

$h = 3, k = 1$ $\qquad \frac{k}{h} = \frac{1}{3}$

Area Under Curve

20. Let $S(\alpha) = \{(x, y) : y^2 \le x, 0 \le x \le \alpha\}$ and $A(\alpha)$ is area of the region $S(\alpha)$.

If for a λ, $0 < \lambda < 4$, $A(\lambda) : A(4) = 2 : 5$, then λ equals :

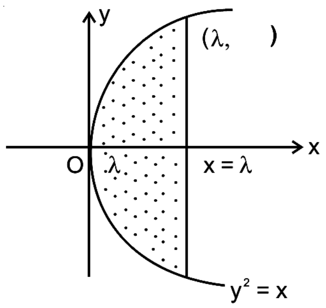
माना $S(\alpha) = \{(x, y) : y^2 \le x, 0 \le x \le \alpha\}$ तथा $A(\alpha)$, क्षेत्र $S(\alpha)$ का क्षेत्रफल है। यदि किसी λ, $0 < \lambda < 4$ के लिए

$A(\lambda) : A(4) = 2 : 5$ है, तो λ बराबर है :

(1) $2\left(\frac{2}{5}\right)^{1/3}$ (2) $2\left(\frac{4}{25}\right)^{1/3}$ (3) $4\left(\frac{2}{5}\right)^{1/3}$ (4) $4\left(\frac{4}{25}\right)^{1/3}$

A. 4

sol.



$$A(\lambda) = 2 \times \frac{2}{3}\lambda \times \sqrt{\lambda} = \frac{4}{3}\lambda$$

$$\Rightarrow \frac{A(\lambda)}{A(4)} = \frac{2}{5} \qquad \frac{\lambda}{8} = \frac{2}{5}$$

$$\lambda = \frac{16}{5}^{2/3} = 4 . \frac{4}{25}^{1/3}$$

Vector

21. If a point R(4, y, z) lies on the line segment joining the points P(2, –3, 4) and Q(8, 0, 10), then the distance of R from the origin is :

यदि एक बिन्दु R(4, y, z), बिन्दुओं P(2, –3, 4) तथा Q(8, 0, 10) को मिलाने वाले रेखाखण्ड पर स्थित है, तो R की मूलबिन्दु से दूरी है :

(1) $\sqrt{53}$ (2) $2\sqrt{21}$ (3) 6 (4) $2\sqrt{14}$

A. 4

sol. P, Q, R are collinear

$\Rightarrow$ PR = λPQ

$2\hat{i} + (y+3)\hat{j} + (z-4)\hat{k} = \lambda[6\hat{i} + 3\hat{j} + 6\hat{k}]$

$\Rightarrow$ $6\lambda = 2$, $y + 3 = 3\lambda$, $z - 4 = 6\lambda$

$\Rightarrow$ $\lambda = \frac{1}{3}, y = -2, z = 6$

$\Rightarrow$ point R (4, –2, 6)

$\Rightarrow$ $OR = \sqrt{(4)^2 + (-2)^2 + (6)^2} = \sqrt{16 + 4 + 36}$

$= \sqrt{56} = 2\sqrt{14}$

Solution of Triangle

22. If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is :

यदि एक त्रिभुज की भुजाओं की लम्बाईयाँ समान्तर श्रेढ़ी में है तथा इसका सबसे बड़ा कोण सबसे छोटे कोण का दुगुना है, तो त्रिभुज की भुजाओं की लम्बाईयों का एक अनुपात है :

(1) 4 : 5 : 6 (2) 3 : 4 : 5 (3) 5 : 9 : 13 (4) 5 : 6 : 7

A. 1

sol. Let a > b > c

$\because$ A = 2C

$\Rightarrow$ A + B + C = π

$\Rightarrow$ B = π – 3C …(i)

$\because$ a + c = 2b

$\Rightarrow$ sinA + sinC = 2sinB …(ii)

$\Rightarrow$ sinA = sin(2C), sinB = sin3C

$\Rightarrow$ From (ii), sin2C + sinC = 2sin3C

$\Rightarrow$ (2cosC + 1)sinC = 2sinC (3 – 4 $\sin^2$C)

2cosC + 1 = 6 – 8(1 – cos2C)

$\Rightarrow$ $8\cos^2 C - 2\cos C - 3 = 0$

$\Rightarrow$ $\cos C = \frac{3}{4}$ or $\cos C = -\frac{1}{2}$

$\because$ C is acute angle

$\Rightarrow$ $\cos C = \frac{3}{4}, \sin A = 2\sin C \cos C = 2\times\frac{\sqrt{7}}{4}\times\frac{3}{4}$

$\Rightarrow$ $\sin C = \frac{\sqrt{7}}{4}, \sin B = \frac{3\sqrt{7}}{4} - \frac{4\sqrt{7}}{4}\times\frac{7}{16} = \frac{5\sqrt{7}}{16}$

$\Rightarrow$ sin A : sin B : sin C : : a : b : c is 6 : 5 : 4

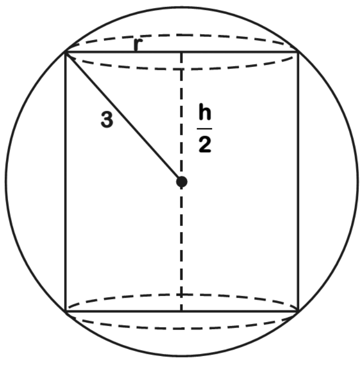
Maxima & Minima

23. The height of a right circular cylinder of maximum volume inscribed in a sphere of radius 3 is :
एक गोले जिसकी त्रिज्या 3 है, के अन्तर्गत बने अधिकतम आयतन के लंबवृत्तीय बेलन की ऊँचाई है :

(1) $\frac{2}{3}\sqrt{3}$ (2) $2\sqrt{3}$ (3) $\sqrt{3}$ (4) $\sqrt{6}$

A. 2

Sol. Let radius of base and height of cylinder be r and h respectively



$\therefore \ r^2 + \frac{h^2}{4} = 9$ .....(i)

$\because$ Volume of cylinder

$V = \pi r^2 h$

$$V = \pi h\left(9 - \frac{h^2}{4}\right)$$

$$V = 9\pi h - \frac{\pi}{4}h^3$$

$$\therefore \quad \frac{dV}{dh} = 9\pi - \frac{3}{4}\pi h^2$$

For maxima / minima

$$\frac{dV}{dh} = 0$$

$\Rightarrow \ h = \sqrt{12}$

and $\frac{d^2V}{dh^2} = -\frac{3}{2}\pi h$

$$\therefore \quad \left(\frac{d^2V}{dh^2}\right)_{h=\sqrt{12}} < 0$$

$\Rightarrow$ Volume is maximum when $h = 2\sqrt{3}$

Determinant

24. If the system of linear equations

$x - 2y + kz = 1$

$2x + y + z = 2$

$3x - y - kz = 3$

has a solution (x, y, z), $z \neq 0$, then (x, y) lies on the straight line whose equation is :

यदि रैखिक समीकरण निकाय

$x - 2y + kz = 1$

$2x + y + z = 2$

$3x - y - kz = 3$

का एक हल (x, y, z), $z \neq 0$ है, तो (x, y) जिस रेखा पर स्थित है, उसका समीकरण है :

(1) $3x - 4y - 4 = 0$ (2) $3x - 4y - 1 = 0$ (3) $4x - 3y - 1 = 0$ (4) $4x - 3y - 4 = 0$

A. 4

sol. $x - 2y + kz = 1,\ 2x + y + z = 2,\ 3x - y - kz = 3$

$$\Delta = \begin{vmatrix} 1 & -2 & k \\ 2 & 1 & 1 \\ 3 & -1 & -k \end{vmatrix} = 1(-k+1) + 2(-2k-3) + k(-2-k)$$

$= -k + 1 - 4k - 6 - 5k = -10k - 5 = -5(2k + 1)$

$$\Delta_1 = \begin{vmatrix} 1 & -2 & k \\ 2 & 1 & 1 \\ 3 & -1 & -k \end{vmatrix} = -5(2k+1)$$

$$\Delta_2 = \begin{vmatrix} 1 & 1 & k \\ 2 & 2 & 1 \\ 3 & 3 & -k \end{vmatrix} = 0, \Delta_3 = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & 2 \\ 3 & -1 & 3 \end{vmatrix} = 0$$

$\because\ z \neq 0$

$\Rightarrow \quad \Delta = 0$

$\Rightarrow \quad k = -\frac{1}{2}$

$\therefore$ System of equation has infinite many solution.

Let $z = \lambda \neq 0$ then $x = \frac{10-3\lambda}{10}$ and $y = -\frac{2\lambda}{5}$

$\therefore$ (x, y) must lie on line $4x - 3y - 4 = 0$

MOD

25. If f(1) = 1, f'(1) = 3, then the derivative of $f(f(f(x))) + (f(x))^2$ at x = 1 is :

यदि f(1) = 1, f'(1) = 3, तो $f(f(f(x))) + (f(x))^2$ का x = 1 पर अवकलज होगा –

(1) 33 (2) 12 (3) 9 (4) 15

A. 1

sol. Let $g(x) = f(f(f(x))) + (f(x))^2$

On differentiating both sides w.r.t. x we get

$g'(x) = f'(f(f(x)))\,f'(f(x))\,f'(x) + 2f(x)\,f'(x)$

$g'(1) = f'(f(f(1)))\,f'(f(1))\,f'(1) + 2f(1)\,f'(1)$

$= f'(f(1))\,f'(1)\,f'(1) + 2f(1)\,f'(1)$

$= 3 \times 3 \times 3 + 2 \times 1 \times 3 = 27 + 6 = 33$

Definite Integration

26. Let $f(x) = \int_0^x g(t)\,dt$, where g is a non-zero even function. If f(x + 5) = g(x), then $\int_0^x f(t)\,dt$, equals :

माना $f(x) = \int_0^x g(t)\,dt$ है, जहाँ g एक शून्येत्तर सम फलन है। यदि f(x + 5) = g(x) है, तो $\int_0^x f(t)\,dt$ बराबर है :

(1) $\int_{x+5}^{x} g(t)\,dt$ (2) $2\int_{5}^{x+5} g(t)\,dt$ (3) $\int_{5}^{x+5} g(t)\,dt$ (4) $5\int_{x+5}^{5} g(t)\,dt$

A. 1

sol. $f(x) = \int_0^x g(t)dt,$ ... (i)

$g(-x) = g(x),$ ....(ii)

$f(x+5) = g(x)$ ....(iii)

From (i)

$f'(x) = g(x)$

Let $I=\int_0^x f(t)dt$,

Put $t=\lambda-5$

$\Rightarrow \quad I=\int_5^{x+5} f(\lambda-5)\,d\lambda$

$\because \ f(x+5)=g(x)$

$\Rightarrow \ f(-x+5)=g(-x)=g(x)$ ....(iv)

$I=\int_5^{x+5} f(\lambda-5)d\lambda,$

$I=\int_5^{x+5} -f(5-\lambda)d\lambda$

($\because$ f(0) = 0, g(x) is even $\Rightarrow$ f(x) is odd)

$I=-\int_5^{x+5} g(\lambda)d\lambda=\int_{x+5}^{5} g(t)dt$ (from iv))

Function

27. Let $f(x)=a^x\ (a>0)$ be written as $f(x)=f_1(x)+f_2(x)$, where $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x+y)+f_1(x-y)$ equals :
माना $f(x)=a^x\ (a>0)$ को $f(x)=f_1(x)+f_2(x)$ के रूप में लिखा गया है जबकि $f_1(x)$ एक सम फलन है और $f_2(x)$ एक विषम फलन है, तो $f_1(x+y)+f_1(x-y)$ बराबर है :
(1) $2f_1(x)f_1(y)$ (2) $2f_1(x+y)f_1(x-y)$ (3) $2f_1(x+y)f_2(x-y)$ (4) $2f_1(x)f_2(y)$

A. 1

sol. $f(x)=a^x=\left(\frac{a^x+a^{-x}}{2}\right)+\left(\frac{a^x-a^{-x}}{2}\right)$

where $f_1(x)=\frac{a^x+a^{-x}}{2}$ is even function

$\Rightarrow \quad f_1(x+y)+f_1(x-y)=\left(\frac{a^{x+y}+a^{-x-y}}{2}\right)+\left(\frac{a^{x-y}+a^{-x+y}}{2}\right)$

$=\frac{1}{2}\left[a^x(a^y+a^{-y})+a^{-x}(a^y+a^{-y})\right]$

$=\frac{(a^x+a^{-x})(a^y+a^{-y})}{2}$

$=2f_1(x).f_1(y)$

Indefinite Integration

28. If $\int \frac{dx}{x^3(1+x^6)^{2/3}}=xf(x)(1+x^6)^{\frac{1}{3}}+C$ where C is a constant of integration, then the function f(x) is equal to:

यदि $\int \frac{dx}{x^3(1+x^6)^{2/3}} = xf(x)(1+x^6)^{\frac{1}{3}} + C$ जहाँ C एक समाकलन अचर है, तो फलन f(x) बराबर है :

(1) $-\frac{1}{2x^3}$ (2) $\frac{3}{x^2}$ (3) $-\frac{1}{6x^3}$ (4) $-\frac{1}{2x^2}$

A. 1

sol. $I = \int \frac{dx}{x^3(1+x^6)^{2/3}} = \int \frac{dx}{x^7(1+x^{-6})^{2/3}}$

Put $1 + x^{-6} = t^3$

$\Rightarrow -6x^{-7}dx = 3t^2\,dt$

$\Rightarrow \frac{dx}{x^7} = -\frac{1}{2}t^2dt$

$\Rightarrow I = \int -\frac{1}{2}\frac{t^2dt}{t^2}$

$= -\frac{1}{2}t + C$

$= -\frac{1}{2}(1+x^{-6})^{\frac{1}{3}} + C$

$= -\frac{1}{2}\frac{(1+x^{-6})^{\frac{1}{3}}}{x^2} + C$

$= -\frac{1}{2x^3}x(1+x^{-6})^{\frac{1}{3}} + C$

$\Rightarrow f(x) = -\frac{1}{2x^3}$

Hyperbola

29. If the eccentricity of the standard hyperbola passing through the point (4, 6) is 2, then the equation of the tangent to the hyperbola at (4, 6) is :

यदि बिन्दु (4, 6) से होकर जाने वाले मानक अतिपरवलय की उत्केन्द्रता 2 है, तो (4, 6) पर अतिपरवलय पर खींची गई स्पर्श रेखा का समीकरण है :

(1) $2x - 3y + 10 = 0$ (2) $x - 2y + 8 = 0$ (3) $3x - 2y = 0$ (4) $2x - y - 2 = 0$

A. 4

sol. Let equation of hyperbola be

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ....(i)

$\Rightarrow b^2 = a^2(e^2 - 1)$

$\because e = 2 \Rightarrow \boxed{b^2 = 3a^2}$ ...(ii)

(i) passes through (4, 6)

$\Rightarrow \frac{16}{a^2} - \frac{36}{b^2} = 1$ ...(iii)

From (ii) and (iii)

$a^2 = 4, b^2 = 12$

$\Rightarrow$ Equation of hyperbola is $\frac{x^2}{4} - \frac{y^2}{12} = 1$

Equation of tangent to the hyperbola at (4, 6) is $\frac{4x}{4} - \frac{6y}{12} = 1$

$\Rightarrow \quad x - \frac{y}{2} = 1$

$\Rightarrow \quad 2x - y = 2$

MOD

30. Let $f : R \to R$ be a differentiable function satisfying $f'(3) + f'(2) = 0$

Then $\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ is equal to :

माना $f : R \to R$ एक अवकलनीय फलन है जो कि $f'(3) + f'(2) = 0$ को संतुष्ट करता है, तो $\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ बराबर है :

(1) e (2) 1 (3) e^2 (4) e^{-1}

A. 2

sol. $I = \lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$

form : 1^∞

$\Rightarrow \quad I = e^{I_1}$, where

$$I_1 = \lim_{x\to 0}\left(\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)} - 1\right)\frac{1}{x}\right)$$

$$= \lim_{x\to 0}\left(\left(\frac{f(3+x)-f(3)-f(2-x)+f(2)}{1+f(2-x)-f(2)}\right)\frac{1}{x}\right)$$

form : $\frac{0}{0}$

Using L.H. Rule

$$I_1 = \lim_{x\to 0}\left(\frac{f'(3+x)+f'(2-x)}{1}\right) . \lim_{x\to 0}\left(\frac{1}{1+f(2-x)-f(2)}\right)$$

$= f'(3) + f'(2) = 0$

$\Rightarrow \quad I = e^{I_1} = 1$