MATHS

07 Jan. 2020 [Morning]

JEE MAIN PAPER ONLINE

RED COLOUR IS ANSWER IN JEE-MAIN

Integral Calculus

Differential Equation

1. Let $x^k + y^k = a^k$, (a, k > 0) and $\frac{dy}{dx} + \left(\frac{y}{x}\right)^{\frac{1}{3}} = 0$, then k is :

यदि $x^k + y^k = a^k$, (a, k > 0) तथा $\frac{dy}{dx} + \left(\frac{y}{x}\right)^{\frac{1}{3}} = 0$,तो k बराबर है :

(1) $\frac{1}{3}$ (2) $\frac{2}{3}$ (3) $\frac{3}{2}$ (4) $\frac{4}{3}$

A. 2

Question ID : 40503658

Option 1 ID : 405036200

Option 2 ID : 405036199

Option 3 ID : 405036202

Option 4 ID : 405036201

Sol. $x_k + y_k = a_k$

Differentiating

$kx^{k-1} + ky^{k-1}\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\left(\frac{x}{y}\right)^{k-1}$

Comparing with $\frac{dy}{dx} = -\left(\frac{x}{y}\right)^{1/3}$

$k - 1 = -\frac{1}{3}$

$\Rightarrow k = \dfrac{2}{3}$

Trigonometry

Trigonometric Ratio and Identities

2. Let α and β be two real roots of the equation $(k+1)\tan^2 x - \sqrt{2}\,.\,\lambda \tan x = (1-k)$, where $k(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha+\beta) = 50$, then a value of λ is :

माना समीकरण $(k+1)\tan^2 x - \sqrt{2}\,.\,\lambda \tan x = (1-k)$, $k(\neq -1)$, $\lambda \in R$) के α तथा β दो वास्तविक मूल हैं। यदि $\tan^2(\alpha+\beta) = 50$ है, तो λ का एक मान है :

(1) $5\sqrt{2}$ (2) $10\sqrt{2}$ (3) 10 (4) 5

A. 3

Question ID : 40503667

Option 1 ID : 405036238

Option 2 ID : 405036237

Option 3 ID : 405036236

Option 4 ID : 405036235

Sol. $\tan\alpha + \tan\beta = \dfrac{\sqrt{2}\lambda}{k+1}$

and $\tan\alpha \tan\beta = \dfrac{k-1}{k+1}$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha + \tan\beta} = \frac{\frac{\sqrt{2}\lambda}{k+1}}{1 - \frac{k-1}{k+1}} = \frac{\lambda}{\sqrt{2}}$$

$\tan^2(\alpha+\beta) = 50 = \dfrac{\lambda^2}{2}$

$\Rightarrow \lambda^2 = 100 \Rightarrow \lambda = \pm 10.$

Integral Calculus

Definite Integration

3. If $f(a+b+1-x) = f(x)$, for all x, where a and b are fixed positive real numbers,

then $\dfrac{1}{a+b}\displaystyle\int_a^b x(f(x) + f(x+1))dx$ is equal to :

यदि सभी x के लिए, $f(a+b+1-x) = f(x)$ है, जबकि a तथा b स्थिर धन वास्तविक संख्याएँ हैं, तो $\dfrac{1}{a+b}\displaystyle\int_a^b x(f(x) + f(x+1))dx$ बराबर है :

(1) $\int_{a-1}^{b-1} f(x+1)dx$ (2) $\int_{a+1}^{b+1} f(x)dx$ (3) $\int_{a-1}^{b-1} f(x)dx$ (4) $\int_{a+1}^{b+1} f(x+1)dx$

A. 3

Question ID : 40503660

Option 1 ID : 405036208

Option 2 ID : 405036209

Option 3 ID : 405036207

Option 4 ID : 405036210

Sol. $I = \frac{1}{a+b}\int_a^b x(f(x)+f(x+1))\,dx$ ....(1)

Using the property $\int_a^b f(x)\,dx = \int_a^b f(a+b+x)\,dx$;

$I = \frac{1}{a+b}\int_a^b (a+b-x)(f(a+b-x)+f(a+b-x+1))\,dx$ ....(2)

But given that $f(a+b+1-x) = f(x) \Rightarrow (a+b-x) = f(1+x)$

So Equation (2) becomes

$I = \frac{1}{a+b}\int_a^b (a+b-x)\,(f(x+1)+f(x))dx$ ....(3)

Adding (1) and (3) $\quad 2I = \int_a^b (f(x+1)+f(x))\,dx$

$\Rightarrow 2I = \int_a^b f(x+1)\,dx + \int_a^b (f(a+b-x)\,dx$

$\Rightarrow 2I = 2\int_a^b f(x+1)\,dx$

$\Rightarrow I = \int_a^b f(a+b-x)\,dx$

Putting $x+1 = t$, $\quad I = \int_{a-1}^{b-1} f(t)\,dt = \int_{a-1}^{b-1} f(x)\,dx$

Algebra
Probability

4. An unbiased coin is tossed 5 times. Suppose that a variable X is assigned the value k when k consecutive heads are obtained for k =3, 4, 5, otherwise X takes the value –1. Then the expected value of X, is :

एक अनभिनत सिक्के को पाँच बार उछाला जाता है। माना, एक चर X को, k = 3, 4, 5 के लिए, मान k दिया जाता है जब सिक्के पर क्रमागत k चित्त आएं तथा अन्य सभी स्थितियों में X का मान –1 है, तो X का अपेक्षित मान है:

(1) $\frac{3}{16}$ (2) $-\frac{1}{8}$ (3) $-\frac{3}{16}$ (4) $\frac{1}{8}$

A. 4

Question ID : 40503668

Option 1 ID : 405036239

Option 2 ID : 405036241

Option 3 ID : 405036240

Option 4 ID : 405036242

Sol. $P\ (x = 5) = \frac{1}{32}$ HHHHH

$P(x = 4) = \frac{2}{32}$ HHHHT or THHHH

$P(x = 3) = \frac{5}{32}$ HHHTT or HHHTH or THHHT or TTHHH or HTHHH

P(None of these) $= 1 - \frac{8}{32} = \frac{3}{4}$

Expectation E(x) $= \sum x_i\, p(x_i)$

$= 5\times\frac{1}{32} + 4\times\frac{2}{32} + 3\times\frac{5}{32} - 1\times\frac{3}{4}$

$= \frac{1}{8}$

Differential Calculus

Monotonocity

5. Let the function , $f: [-7, 0] \to R$ be continuous on $[-7, 0]$ and differentiable on $(-7, 0)$. If $f(-7) = -3$ and $f'(x) \le 2$, for all $x \in (-7,0)$, then for all such functions f, $f(-1) + f(0)$ lies in the interval :

माना फलन : $[-7, 0] \to R$,$[-7, 0]$ पर संतत है तथा $(-7, 0)$ पर अवकलनीय है। यदि $f(-7) = -3$ और सभी $x \in (-7,0)$ के लिए , $f'(x) \le 2$ है, तो ऐसे सभी फलनों f के लिए $f(-1) + f(0)$जिस अंतराल में है, वह है:

(1) $[-3, 11]$ (2) $[-6, 20]$ (3) $[-\infty, 11]$ (4) $[-\infty, 20]$

A. 4

Question ID : 40503659

Option 1 ID : 405036204

Option 2 ID : 405036205

Option 3 ID : 405036203

Option 4 ID : 405036206

Sol. Applying LMVT in interval $[-7, 0]$,

$f'(c_1) = \dfrac{f(0)-f(-7)}{0-(-7)} \le 2$

Þ $f(0) \le 14 - 3 = 11$ .....(1)

Similarly, Applying LMVT in the interval [–7, –1] ;

$f'(c_2) = \dfrac{f(-1)-(-7)}{-1-(-7)} \le 2$

$\Rightarrow f(-1) \le 12 - 3 = 9$ .....(2)

Adding (1) and (2) ; $\Rightarrow f(0) + f(-1) \le 11 + 9 = 20$

JEE Main Only topics

Mathematical Reasoning

6. The logical statement $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to :

तर्कसंगत कथन $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ निम्न कथनों में से तुल्य है ?

(1) p (2) ~ p (3) q (4) ~ q

A. 2

Question ID : 40503670

Option 1 ID : 405036247

Option 2 ID : 405036249

Option 3 ID : 405036248

Option 4 ID : 405036250

Sol. Truth table

p	q	~ p	$p \Rightarrow q$	$q \Rightarrow \sim p$	$(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$
T	T	F	T	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	T	T

This is e quivalent to ~p.

Integral Calculus

Area Under Curve

7. The area of the region, enclosed by the circle $x^2 + y^2 = 2$ which is not common to the region bounded by the parabola $y^2 = x$ an the straight line $y = x$, is :

वृत्त $x^2 + y^2 = 2$ द्वारा परिबद्ध क्षेत्र का वह क्षेत्रफल जो परवलय, $y^2 = x$ तथा सरल रेखा, $y = x$ द्वारा परिबद्ध क्षेत्र में नहीं है, है:

(1) $\frac{1}{6}(12\pi - 1)$ (2) $\frac{1}{6}(24\pi - 1)$ (3) $\frac{1}{3}(12\pi - 1)$ (4) $\frac{1}{3}(24\pi - 1)$

A. 1

Question ID : 40503661

Option 1 ID : 405036212

Option 2 ID : 405036211

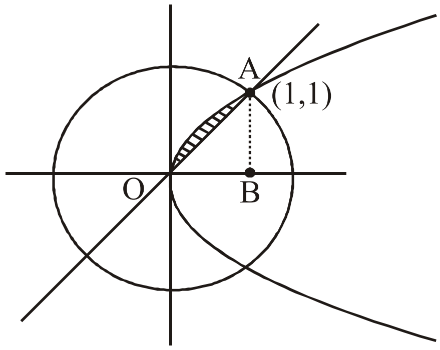
Option 3 ID : 405036214

Option 4 ID : 405036213

Sol. Area of circle

$x^2 + y^2 = 2$ is $\pi r^2 = 2\pi$ units

Area bounded by $y^2 = x$ and $y = x$ is



$$A_2 = \int_0^1 (\sqrt{x} - x)\,dx$$

$$= \left[\frac{2x^{3/2}}{3} - \frac{x^2}{2}\right]_0^1 = \frac{1}{6}$$

So desired area

$$= A_1 - A_2 = 2\pi - \frac{1}{6} = \frac{12\pi - 1}{6} \text{ units}$$

Algebra

Complex Number

8. Let α be a root of the equation $x^2 + x + 1 = 0$ and the matix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}$, the the matrix A^{31} is equal to : 0

यदि समीकरण $x^2 + x + 1 = 0$ का एक मूल a है तथा आव्यूह $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}$ है, तो आव्यूह A^{31} बराबर है:

(1) I_3 (2) A^3 (3) A (4) A^2

A. 2

Question ID : 40503653

Option 1 ID : 405036182

Option 2 ID : 405036181

Option 3 ID : 405036179

Option 4 ID : 405036180

Sol. $A^2 = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}$

$$= \frac{1}{3}\begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

So $A^{31} = A^{28}.A^3 = A^3$

Integral Calculus

Differential Equation

9. If $y = y(x)$ is the solution of the differential equation, $e^y\left(\frac{dy}{dx} - 1\right) = e^x$ such that $y(0) = 0$, then $y(1)$ is equal to:

यदि अवकलन समीकरण, $e^y\left(\frac{dy}{dx} - 1\right) = e^x$, जबकि $y(0) = 0$, का हल $y = (x)$ है, $y(1)$ बराबर है:

(1) $\log_e 2$ (2) $2 + \log_e 2$ (3) $1 + \log_e 2$ (4) $2e$

A. 3

Question ID : 40503662

Option 1 ID : 405036216

Option 2 ID : 405036217

Option 3 ID : 405036215

Option 4 ID : 405036218

Sol. $c^y = t$ $e^y\frac{dy}{dx} = \frac{dt}{dx}$

So $\frac{dt}{dx} - t = e^x$

Integrating factor $= e^{-\int 1dx} = e^{-x}$

So $\int \frac{d}{dx}(e^{-x}.t) = \int 1dx$

$\Rightarrow t.e^{-x} = x + c$ $\Rightarrow t = x\,e^x + c\,e^x$ $\Rightarrow e^y = x\,e^x + c\,e^x$

Since $y(0) = 0$

$\Rightarrow c = 1$

So $e^y = x\,e^x + e^x$

$y(1) = \log_e(e^x(x+1))|_{x=1} = 1 + \log_e 2$

Geometry

Parabola Coordinate

10. If $y = mx + 4$ is a tangent to both the parabolas, $y^2 = 4x$ and $x^2 = 2by$, then b is equal to :

यदि $y = mx + 4$, दोनों परवलयों, $y^2 = 4x$ तथा $x^2 = 2by$ को स्पर्श करती है, तो b बराबर है :

(1) 128 (2) –32 (3) –128 (4) –64

A. 3

Question ID : 40503663

Option 1 ID : 405036220

Option 2 ID : 405036222

Option 3 ID : 405036219

Option 4 ID : 405036221

Sol. Tangent to the parabola $y^2 = 4x$ is

$$y = mx + \frac{1}{m}$$

Comparing with $y = mx + 4$ $\Rightarrow m = \frac{1}{4}$

It is target to another parabola $x^2 = 2by$

$\Rightarrow$ Quadratic $x^2 = 2b\left(\frac{x}{4} + 4\right)$ will have repeated roots

$\Rightarrow 2x^2 - bx - 16b = 0$

$D = 0 \Rightarrow b^2 + 12\ 8b = 0 \Rightarrow b = 0$ or $- 128$

But for the curve to be parabola : $b \neq 0$

$\Rightarrow b = - 128$

Vectors

3D Geometry

11. Let P be a plane passing through the points (2, 1, 0) , (4, 1, 1) and (5, 0, 1) and R be any point (2, 1, 6). Then the image of R in the plane P is :

यदि एक समतल P, तीन बिंदुओं (2, 1, 0) , (4, 1, 1) और (5, 0, 1) से होकर जाता है, तथा कोई और बिन्दु R (2, 1, 6) है, तो समतल P में R का प्रतिबिम्ब है:

(1) (6, 5, –2) (2) (6, 5, 2) (3) (4, 3, 2) (4) (3, 4, –2)

A. 1

Question ID : 40503665

Option 1 ID : 405036227

Option 2 ID : 405036229

Option 3 ID : 405036228

Option 4 ID : 405036230

Sol. Equation of the plane passing through

(2, 1, 0), (0, 1, 1) and (5, 0, 1) is

$$\begin{vmatrix} x-2 & y-1 & z-0 \\ 4-2 & 1-1 & 1-0 \\ 5-2 & 0-1 & 1-0 \end{vmatrix} = 0$$

$\Rightarrow x + y - 2z = 3$

Image of R(2, 1, 6) in the given plane is

$$\frac{x-2}{1} = \frac{y-1}{1} = \frac{z-6}{-2} = \frac{-2(2 \times 1 + 1 \times 1 - 6 \times 2 - 3)}{1^2 + 1^2 + 2^2}$$

$\Rightarrow (x, y, z) = (6, 5, -2)$

Differential Calculus

Methods of Differentiation

12. If $y(a) = \sqrt{2\left(\frac{\tan a + \cot a}{1+\tan^2 a}\right) + \frac{1}{\sin^2 a}}, a \in \left(\frac{3\pi}{4}, \pi\right)$, then $\frac{dy}{da}$ at $a = \frac{5\pi}{6}$ is :

यदि $y(a) = \sqrt{2\left(\frac{\tan a + \cot a}{1+\tan^2 a}\right) + \frac{1}{\sin^2 a}}, a \in \left(\frac{3\pi}{4}, \pi\right)$ है, तो $a = \frac{5\pi}{6}$ पर $\frac{dy}{da}$ का मान है :

(1) $-\frac{1}{4}$ (2) $\frac{4}{3}$ (3) -4 (4) 4

A. 4

Question ID : 40503669

Option 1 ID : 405036244

Option 2 ID : 405036243

Option 3 ID : 405036245

Option 4 ID : 405036246

Sol. $y(a) = \sqrt{\frac{2\left(\frac{\sin a}{\cos a} + \frac{\cos a}{\sin a}\right)}{\sec^2 a} + \text{cosec}^2 a}$, $a \in \left(\frac{3\pi}{4}, \pi\right)$

$= \sqrt{2\cot a + 1 + \cot^2 a}$

$= |1 + \cot a|$

For $a \in \left(\frac{3\pi}{4}, \pi\right)$; $\cot a < -1$ $\Rightarrow 1 + \cot a < 0$

$\Rightarrow y(a) = -1 - \cot a$

$$\left.\frac{dy}{da}\right|_{a=\frac{5\pi}{6}} = \left.\text{cosec}^2 a\right|_{a=\frac{5\pi}{6}} = \text{cosec}^2 \frac{5\pi}{6} = 4$$

Algebra

Binomial theorem

13. The greatest positive integer k, for which $49^k + 1$ is a factor of the sum $49^{125} + 49^{124} + \ldots\ldots 49^2 + 49 + 1$, is :

सबसे बड़ी धन पूर्णांक संख्या k, जिसके लिए $49^k + 1$ योगफल $49^{125} + 49^{124} + ... 49^2 + 49 + 1$ का एक गुणन खंड है, है :

(1) 60 (2) 63 (3) 65 (4) 32

A. 2

Question ID : 40503657

Option 1 ID : 405036196

Option 2 ID : 405036197

Option 3 ID : 405036198

Option 4 ID : 405036195

Sol. $\text{Sum} = \dfrac{49^{126}-1}{49-1} = \dfrac{\left(49^{63}+1\right)\left(49^{63}-1\right)}{49-1}$

So k = 63

Algebra

Sequence & progression

14. Five numbers are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is $-\dfrac{1}{2}$, then the greatest number amongst them is :

पाँच संख्याएँ समान्तर श्रेढ़ी में हैं, जिनका योगफल 25 तथा गुणनफल 2520 है। यदि इन पाँच संख्याओं में से एक $-\dfrac{1}{2}$ है, तो इनमें सबसे बड़ी संख्या है:

(1) 27 (2) $\dfrac{21}{2}$ (3) 7 (4) 16

A. 4

Question ID : 40503656

Option 1 ID : 405036194

Option 2 ID : 405036193

Option 3 ID : 405036191

Option 4 ID : 405036192

Sol. Let the five numbers in A.P. be

$a - 2d, a - d, a, a + d, a + 2d$

$\text{Sum} = 5a = 25 \Rightarrow a = 5$ ....(1)

$\text{Product} = a(a^2 - d^2)(a^2 - 4d^2) = 2520$

$\Rightarrow 5(25 - d^2)(25 - 4d^2) = 2520$

$\Rightarrow 4d^4 - 125d^2 + 121 = 0$

$\Rightarrow d^2 = 1$ or $\frac{121}{4} \Rightarrow d = \pm 1$ or $\pm \frac{11}{2}$.

Since one of the terms is $-\frac{11}{2}$; $d = \frac{11}{2}$

Largest term = a + 2d = $5 + 2 \times \frac{11}{2} = 16$

Algebra

Determinant

15. If the system of linear equation

$2x + 2ay + az = 0$

$2x + 3by + bz = 0$

$2x + 4cy + cz = 0$

where a, b, c, $\in$ R are non-zero and distinct; has a non-zero solution, then:

(1) a + b + c = 0 (2) a, b, c are in A.P. (3) a, b, c are in G.P. (4) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

यदि निम्न रैखिक समीकरण निकाय

$2x + 2ay + az = 0$

$2x + 3by + bz = 0$

$2x + 4cy + cz = 0$

जहाँ a, b तथा c विभिन्न शून्येतर वास्तविक संख्याएँ हैं ; का एक शून्येतर हल है, तो :

(1) a + b + c = 0

(2) a, b, c समान्तर श्रेढ़ी में हैं।

(3) a, b, c गुणोत्तर श्रेढ़ी में हैं।

(4) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ समान्तर श्रेढ़ी में हैं।

A. 4

Question ID : 40503654

Option 1 ID : 405036183

Option 2 ID : 405036186

Option 3 ID : 405036185

Option 4 ID : 405036184

Sol. For the system of equations to have non-zero solutions :

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0 \qquad \Rightarrow 6bc + 4ab + 8ac - 6ab - 8bc - 4ac = 0$$

$\Rightarrow 2ab + 2bc = 4ac$

$\Rightarrow b = \frac{2ac}{a+b} \Rightarrow$ a, b, c are in H.P.

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

Coordinate Geometry

Ellipse

16. If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is :

यदि एक दीर्घवृत्त की नाभियों के बीच की दूरी 6 है तथा इसकी नियताओं के बीच की दूरी 12 है, तो इसकी नाभिलम्ब जीवा की लम्बाई है :

(1) $3\sqrt{2}$ (2) $\frac{2}{\sqrt{2}}$ (3) $2\sqrt{3}$ (4) $\sqrt{3}$

A. 1

Question ID : 40503664

Option 1 ID : 405036225

Option 2 ID : 405036223

Option 3 ID : 405036224

Option 4 ID : 405036226

Sol. Distance between foci = 2ac = 6 ....(1)

Distance between directrices = $\frac{2a}{e} = 12$ ....(2)

From (1) and (2)

$a = 3\sqrt{2}$ and $e = \frac{1}{\sqrt{2}}$

$e^2 = 1 - \frac{b^2}{a^2} \Rightarrow b^2 = 9$

Length of latus rectum = $2\frac{b^2}{a} = 2\times\frac{9}{3\sqrt{2}} = 3\sqrt{2}$

Algebra

Complex Number

17. If $\text{Re}\left(\frac{z-1}{2z+i}\right) = 1$, where z = x + iy, then the point (x, y) lies on a :

(1) straight line whose slope is $-\frac{2}{3}$.

(2) circle whose centre is at $\left(-\frac{1}{2}, -\frac{3}{2}\right)$

(3) Circle whose diameter is $\frac{\sqrt{5}}{2}$

(4) Straight line whose slope is $\frac{3}{2}$

यदि $\text{Re}\left(\frac{z-1}{2z+i}\right)=1$, जहाँ $z = x + iy$, तो बिन्दु (x, y) स्थित है :

(1) एक सरल रेखा पर, जिसका ढ़ाल $-\frac{2}{3}$ है।

(2) एक वृत्त पर, जिसका केन्द्र बिन्दु $\left(-\frac{1}{2},-\frac{3}{2}\right)$ है।

(3) एक वृत्त पर, जिसका व्यास $\frac{\sqrt{5}}{2}$ है।

(4) एक सरल रेखा पर, जिसका ढ़ाल $\frac{3}{2}$

A. 3

Question ID : 40503652

Option 1 ID : 405036175

Option 2 ID : 405036177

Option 3 ID : 405036178

Option 4 ID : 405036176

Sol. $\frac{z-1}{2z+i}=\frac{(x-1)+iy}{2(x+iy)+i}=\frac{x-1+iy}{2x+(2y+1)i}=\frac{((x-1)+iy)(2x-(2y+1)i)}{4x^2+(2y+1)^2}$

$=\frac{2x(x-1)+y(2y-1)+i[-(x-1)(2y+1)+2xy]}{4x^2+4y^2+4y+1}$

$\text{Re}\left(\frac{z-1}{2z+i}\right)=1 \Rightarrow \frac{2x^2-2x+2y^2+y}{4x^2+4y^2+4y+1}=1$

$\Rightarrow 2x^2 + 2y^2 + 2x + 3y + 1 = 0$

$\Rightarrow$ Circle with centre $\left(-\frac{1}{2},-\frac{3}{4}\right)$; radius $=\sqrt{\frac{1}{4}+\frac{9}{16}-\frac{1}{2}}=\frac{\sqrt{5}}{4} \Rightarrow \text{dia}=\frac{\sqrt{5}}{2}$

Vectors

Vectors

18. A vector $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}(\alpha,\beta\in R)$ lies in the plane of the vectors, $\vec{b}=\hat{i}+\hat{j}$ and $\vec{c}=\hat{i}-\hat{j}+4\hat{k}$. If $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$, then :

एक सदिश $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}(\alpha,\beta\in R)$ उस समतल में , जिसमें दोनों सदिश $\vec{b}=\hat{i}+\hat{j}$ तथा $\vec{c}=\hat{i}-\hat{j}+4\hat{k}$ स्थित हैं, स्थित है। यदि $\vec{a}$ सदिशों $\vec{b}$ और $\vec{c}$ के बीच के कोण को समद्विभाजित करता है, तो :

(1) $\vec{a}.\hat{k}+4=0$ (2) $\vec{a}.\hat{k}+2=0$ (3) $\vec{a}.\hat{i}+1=0$ (4) $\vec{a}.\hat{i}+3=0$

A. 2

Question ID : 40503666

Option 1 ID : 405036234

Option 2 ID : 405036232

Option 3 ID : 405036231

Option 4 ID : 405036233

Sol. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are copanar $\Rightarrow \begin{vmatrix} \alpha & 1 & \beta \\ 1 & 1 & 0 \\ 1 & -1 & 4 \end{vmatrix} = 0$ .....(1)

Also $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$

$\Rightarrow$ Let $\vec{a} = \lambda\left(\hat{b}+\hat{c}\right) = \lambda\left(\frac{\hat{i}+\hat{j}}{3\sqrt{2}} + \frac{\hat{i}-\hat{j}+4\hat{k}}{3\sqrt{2}}\right)$

$\Rightarrow \alpha\hat{i}+2\hat{j}+\beta\hat{k} = \lambda\left(\frac{4}{3\sqrt{2}}\hat{i}+\frac{2}{3\sqrt{2}}\hat{j}+\frac{4}{3\sqrt{2}}\hat{k}\right)$

Comparing, $\frac{2\lambda}{3\sqrt{2}} = 2 \Rightarrow \lambda = 3\sqrt{2}$

$\alpha = 4$; $\beta = 4$ $\quad \vec{a} = 4\hat{i}+2\hat{j}+4\hat{k}$

But it does not satisfy any option

So let $\vec{a} = \mu\left(\hat{b}-\hat{c}\right) = \mu\left(\frac{2}{3\sqrt{2}}\hat{i}+\frac{4}{3\sqrt{2}}\hat{j}-\frac{4}{3\sqrt{2}}\hat{k}\right)$

Comparing, $\mu = \frac{3\sqrt{2}}{2} \Rightarrow \vec{a} = \hat{i}+2\hat{j}-2\hat{k}$

$\Rightarrow \vec{a}.\hat{k} = -2 \qquad \Rightarrow \vec{a}.\hat{k}+2 = 0$

Differential Calculus

Function

19. If $g(x) = x^2 + x - 1$ and $(gof)(x) = 4x^2 - 10x + 5$, then $f\left(\frac{5}{4}\right)$ is equal to :

यदि $g(x) = x^2 + x - 1$ तथा $(gof)(x) = 4x^2 - 10x + 5$, तो $f\left(\frac{5}{4}\right)$ बराबर है:

(1) $-\frac{3}{2}$ (2) $-\frac{1}{2}$ (3) $\frac{1}{2}$ (4) $\frac{3}{2}$

A. 2

Question ID : 40503651

Option 1 ID : 405036173

Option 2 ID : 405036171

Option 3 ID : 405036172

Option 4 ID : 405036174

Sol. $g(x) = x^2 + x - 1$ ....(1)

$g(f(x)) = 4x^2 - 10x + 5$ ....(2)

Putting $x = f(5/4)$ in equation (1)

$g(f(5/4)) = (f(5/4))^2 + f(5/4) - 1$ ....(3)

Putting $x = 5/4$ in equation (2)

$g(f(5/4)) = 4 \times (5/4)^2 - 10 \times (5/4) + 5$ ....(4)

From equation (3) and (4)

$(f(5/4))^2 + f(5/4) - 1 = -5/4$

$\Rightarrow f(5/4) = -1/2$

Algebra

P &C

20. Total number of 6-digit numbers in which only and all the five digits 1,3, 5, 7 and 9 appear, is

छः अंकों वाली सभी संख्याओं की कुल संख्या जिनमें केवल तथा सभी पाँच अंक 1,3, 5, 7 और 9 ही हों,

(1) 6! (2) 5^6 (3) $\frac{1}{6}(6!)$ (4) $\frac{5}{2}(6!)$

A. 4

Question ID : 40503655

Option 1 ID : 405036187

Option 2 ID : 405036190

Option 3 ID : 405036188

Option 4 ID : 405036189

Sol. If only and all the five digits out of 1, 3, 5, 7 & 9 appear in a 6 digit numbers, exactly one digit will repeat.

No. of ways of selecting that digit = ${}^5C_1$

No. of permutations of such digits = $\frac{6!}{2}$

So total number of such numbers = ${}^5C_1 \times \frac{6!}{2} = 5 \times \frac{6!}{2}$

Differential Calculus

Limit

21. $\lim\limits_{x \to 2} \frac{3^x + 3^{3-x} - 12}{3^{-x/2} - 3^{1-x}}$ is equal to ______.

$\lim\limits_{x \to 2} \frac{3^x + 3^{3-x} - 12}{3^{-x/2} - 3^{1-x}}$ बराबर है______.

A. 36

Question ID : 40503673

Sol. Let $3^{x/2} = t$; $x \to 2$; $t \to 3$.

$$\text{Limit } L = \lim_{t\to 3}\frac{t^2+\frac{27}{t^2}-12}{\frac{1}{t}-\frac{3}{t^2}} = \lim_{t\to 3}\frac{t^4-12t^2+27}{t-3}$$

$$= \lim_{t\to 3}\frac{t^4-9t^2-3t^2+27}{(t-3)}$$

$$= \lim_{t\to 3}\frac{(t^2-9)(t^2-3)}{(t-3)} = \lim_{t\to 3}(t+3)\,(t^2-3)$$

Differential Calculus

Continuity & Differentiability

22. Let S be the set of points where the function, $f(x) = |2-|x-3||$, $x \in R$, is not differentiable. Then $\sum_{x\in S} f(f(x))$

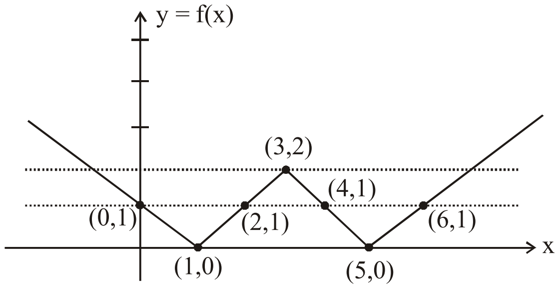
यदि S उन सभी बिन्दुओं का समुच्चय है, जिनके लिए फलन, $f(x) = |2-|x-3||$, $x \in R$ अवकलनीय नहीं है, तो $\sum_{x\in S} f(f(x))$ बराबर है ––––––– ।

A. 3

Question ID : 40503674

Sol. f(x) = |2–|x-31| = ||x-3|-2|

The graph of f(x) is shown below :



From the graph ; the function is not differentiable at x = 1, 3 and 5.

f(1) = 0 ; f(3) = 2 ; f(5) = 0

$\sum$ f(f(x)) = f(f(1)) + f(f(3)) + f(f(5))

= f(0) + f(2) + f(0)

= 1 + 1 + 1 = 3

Coordinate Geometry

Straight Line

23. Let A (1, 0), B(6, 2) and $C\left(\frac{3}{2}, 6\right)$ be the vertices of a triangle ABC. If P is a point inside the tgriangle ABC such that the triangles APC, APB and BPC have equal areas, then the length of the line segment PQ, where Q is the point $\left(-\frac{7}{6}, -\frac{1}{3}\right)$, is_________.

माना A (1, 0), B(6, 2) तथा $C\left(\frac{3}{2}, 6\right)$, एक त्रिभुज ABC के शीर्ष बिन्दु हैं। यदि एक बिन्दु P, ΔABC के अन्दर इस प्रकार है, कि त्रिभुजों APC, APB और BPC के क्षेत्रफल बराबर हैं, तो रेखाखंड PQ, जबकि बिंदु $Q\left(-\frac{7}{6}, -\frac{1}{3}\right)$ है, की लम्बाई है _________ ।

A. 5

Question ID : 40503675

Sol. The point P is centroid of ΔABC ;

$$\text{So } P = \left(\frac{17}{6}, \frac{8}{3}\right) \qquad Q = \left(\frac{-7}{6}, \frac{-1}{3}\right)$$

$$PQ = \sqrt{\left(\frac{17}{6}+\frac{7}{6}\right)^2 + \left(\frac{8}{3}+\frac{1}{3}\right)^2} = 5$$

JEE Main Only topics

Statistics

24. If the variance of the first n natural numbers is 10 and the variance of the first m even natural numbers is 16, then m + n is equal to _____.

यदि प्रथम n प्राकृत संख्याओं का प्रसरण 10 है और प्रथम m सम–प्राकृत संख्याओं का प्रसरण 16 है, तो m + n बराबर है _____ ।

A. 18

Question ID : 40503671

Sol. Variance of first n-natural numbers = 10

$$\Rightarrow \frac{\sum n^2}{n} - \left(\frac{\sum n}{n}\right)^2 = 10 \qquad \Rightarrow \frac{n(n+1)(2n+1)}{6\times n} - \left(\frac{n(n+1)}{2n}\right)^2 = 10$$

$$\Rightarrow \frac{2(2n^2+3n+1) - 3(n2+2n+1)}{12} = 10$$

$$\Rightarrow n^2 - 1 = 120 \qquad \Rightarrow n^2 = 121 \qquad \Rightarrow n = \pm 11$$

But since $n \in N \Rightarrow x = 11$

Similarly variance of first m even natural numbers = 16

$\Rightarrow$ Variance of first m natural numbers = 4

$$\Rightarrow \frac{m^2-1}{12} = 4 \qquad \Rightarrow m^2 = 49 \qquad \Rightarrow m = 7$$

m + n = 7 + 11 = 18

Algebra

Binomial theorem

25. If the sum of the coefficients of all even powers of x in the product $(1 + x + x^2 + ...x^{2n})(1 - x + x^2 - x^3 .... + x^{2n})$ is 61, then n is equal to ________.

यदि गुणनफल $(1 + x + x^2 + ...x^{2n})(1 - x + x^2 - x^3 .... + x^{2n})$ में x के सभी सम–घातों वाले गुणांकों का यांगफल 61 है, तो

n बराबर है ________ ।

A. 30

Question ID : 40503672

Sol. Let $(1 + x + x^2 + \ldots\ldots x^{2n})(1 - x + x^2 - x^3 + \ldots\ldots\ldots x^{2n})$

$= a_0 + a_1 x + a_2x^2 + a_3x^3 + \ldots\ldots a_{4n}x^{4n}$ ....(1)

Put x = 1 in equation (1)

$2n + 1 = a_0 + a_1 + a_2 + a_3 + \ldots\ldots a_{4n}$ ....(2)

Put x = –1 in equation (1)

$2n + 1 = a_0 - a_1 + a_2 - a_3 + \ldots\ldots a_{4n}$ ....(3)

Adding (2) and (3)

$2(a_0 + a_2 + a_4 + \ldots\ldots a_{4n}) = 4n + 2$

$\Rightarrow 2 \times 61 = 2(2n + 1)$ $\Rightarrow 2n + 1 = 61$ $\Rightarrow n = 30$